Title: A New Gravitational Time Dilation Equation

(2nd Revised Edition)

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Abstract:

In a previous paper, I showed that the gravitational time dilation equation, which has been accepted since Einstein published it in 1907, is incorrect. It is incorrect because it is inconsistent with the required outcome at the reunion of the twins in the famous twin 'paradox' of special relativity. In this paper, I describe a new gravitational time dilation (GTD) equation which IS consistent with the required outcome at the reunion of the twins. And my new GTD equation gives the same instantaneous change of the home twin's (her) age, according to the traveling twin (him), when he instantaneously changes his velocity, as is given by the CMIF simultaneity method, but without requiring the assumption that the CMIF method requires. Therefore, my GTD equation asserts that it is the SINGLE correct simultaneity method.

Section 1. Introduction

The Gravitational Time Dilation (GTD) Equation is described in Wikipedia:

https://en.wikipedia.org/wiki/Gravitational_time_dilation

It is also described in Einstein's 1907 paper:

https://einsteinpapers.press.princeton.edu/vol2-trans/319

and in

W. Rindler, "Essential Relativity", revised second edition, page 118.

The GTD equation says, in particular, that for two stationary clocks in a constant and uniform gravitational field of force per unit mass "g", separated by the constant distance "L" in the direction of the field, the clock that is farther from the source of the field will run faster than the other clock, by the "rate ratio" R:

$$R = \exp(g L).$$

The equivalence principle then says that, if there is no gravitational field, that for two clocks that are accelerating with the same acceleration "A", separated by the constant distance "L" in the direction of the acceleration, the leading clock will run faster than the other clock, by the rate ratio

$$R = \exp(A L).$$

The two values “g” and “A” are numerically the same. I call the equivalence principle version of the GTD equation the EPVGTD equation. Note that when “g” or “A” are constant with respect to time, the rate factor is constant with respect to time.

In my previous monograph (“An Inconsistency Between the Gravitational Time Dilation Equation and the Twin Paradox”, https://vixra.org/abs/2109.0076?ref=12745236), I showed that, when applied to an instantaneous velocity increase by the traveling twin in the twin paradox scenario, the EPVGTD equation says that the home twin’s (her) age, during the traveling twin’s (his) instantaneous velocity increase, instantaneously changes by an INFINITE amount. That clearly contradicts the well-known outcome of the twin paradox scenario, that says that her age is FINITE when they are reunited. The outcome of the twin paradox scenario depends only on the well-known time dilation equation for a perpetually-inertial observer (the “PIOTD equation”): she says that anyone moving at speed “v” with respect to her will be ageing at a rate “gamma” times slower than her own rate of ageing, where

gamma = 1 / sqrt { 1 - (v v) } .

His age is finite at their reunion, and her age (gamma times older than his) is therefore also finite, which contradicts what the EPVGTD says happens. In a dispute between the PIOTD equation and the EPVTD equation, the PIOTD equation wins, in my opinion. And if the EPVGTD equation is wrong, then the GTD equation is also wrong, assuming that the equivalence principle is correct. Again, in a dispute between the GTD equation and the equivalence principle, my bet is on the equivalence principle. So I believe that the existing GTD equation is incorrect.

Before going any further, I need to review some of the terminology introduced in my previous paper that is still needed now. Each of the accelerated clocks has a co-located human with it. It is one of those humans whose “perspective” or “point of view” about simultaneity is desired. I call that human the “accelerated observer”, abbreviated as the “AO”. The human co-located with each of the other distributed clocks is called a “helper friend”, abbreviated as an “HF” (and sometimes ending in an integer when I need more than one helper friend, like HF1 and HF2). What I ultimately want to do is arrange for an HF to be momentarily co-located with the (stationary) home twin, at the instant that the AO wants to know her age.

Although the AO and the HF have different ages as the acceleration progresses, they each know, and agree about, what the relationship is between their respective ages. That establishes a “NOW” instant for them, that they both agree about.

Einstein showed how mutually stationary, perpetually-inertial observers and clocks can be (conceptually) distributed in a lattice structure throughout space, so that their ages and clock readings are synchronized (using light signals for the synchronization). That allows the current age of any given distant person (or current reading on any given distant clock), according to a perpetually-inertial observer, to be (eventually) determined. So that establishes a common “NOW” instant for all of those perpetually-inertial observers.

That common “NOW” instant has proven to be very useful in analyzing special relativity scenarios without accelerations. And it can also be helpful in the twin paradox scenario with a traveling twin who changes his velocity instantaneously, provided we ASSUME that the accelerating observer always agrees with the perpetually-inertial observer who happens to be momentarily co-located and mutually stationary with him at that instant. That assumption results in the commonly-used “co-moving inertial frames” (CMIF) simultaneity method.

What is VERY exciting is the fact that the accelerating AO and HF can ALSO agree on a "NOW" instant, WITHOUT any assumption about momentarily co-located and mutually stationary perpetually-inertial observers. It basically extends the idea of Einstein's "lattice of clocks" for a perpetually-inertial observer to an accelerating observer! I've known a not insubstantial number of physicists who believe that simultaneity at a distance is meaningless for an accelerating observer. But the fact that an accelerating observer CAN drag (or push) an array of clocks around with him, which effectively define a "NOW" instant everywhere for him, disproves the notion that simultaneity at a distance has no meaning for him.

Section 2. The Search for a Correct Gravitational Time Dilation Equation

After I had determined, in my previous monograph, that the EPVGTD equation was incorrect, I decided to try (as an alternative) the linearized version of the EPVGTD equation (which I called the LGTD equation), which says that the rate ratio R is

R = (1 + A L).

The rate ratio R just tells us how much faster the HF is ageing, compared to the AO. if "A" is constant with respect to time, the rate ratio R is also constant with respect to time.

The LGTD equation is just the approximation of exp(A L) for small values of (A L). The idea was that maybe that approximation of the exponential might actually be the correct equation for all values of (A L).

Let the duration of the constant acceleration (according to the AO) be denoted by tau. So the change in the AO's age during the acceleration "A" is tau. Then the change in the age of the HF is just the (constant) rate R at which the HF is ageing, multiplied by the duration tau:

Age_change = tau R = tau (1 + A L).

But an acceleration "A", which lasts tau seconds, changes the rapidity theta from zero to

theta = A tau,

so

A = theta / tau.

The Age_change equation then becomes

Age_change = tau (1 + A L) = tau (1 + [(theta L) / tau]) = tau + (theta L)

In the example I chose, L = 7.52 lightseconds, and, when t = tau, theta = 1.317 lightseconds/second (corresponding to a velocity "v" of 0.866 ls/s, which was chosen because the gamma factor has the nice even value of 2.0 for that velocity).

So the Age_change of the HF, between t = 0 and t = tau, is

Age_change = tau + (1.317)(7.52) = tau + 9.904 seconds.

As tau goes to zero, with A going to infinity, so that the product (A tau) is kept constant at 1.317 ls/s, the Age_change of the HF is just 9.904 seconds. So, unlike with the EPVGTD equation, the HF's age doesn't increase by an infinite amount during the instantaneous velocity change of the AO. And because, in the instantaneous velocity change case, the HF and the home twin are co-located during the instantaneous velocity change of the AO, the home twin ALSO instantaneously ages by 9.904 seconds (because it would be absurd for either of them to see, instantaneously, the other age differently from themselves). That is a much better result than the infinite age change which the EPVGTD equation gets, but it is larger than the twin paradox result of 6.51 seconds (as given by the CMIF simultaneity method). So the LGTD equation is better than the EPVGTD equation, but it is still incorrect.

When I got the above result, I noticed that, if I substituted the velocity v = 0.866 ls/s for the rapidity theta = 1.317 ls/s in the above Age_change equation, I got the correct value of 6.51 seconds. So that suggested that I should change the rate equation from

R = (1 + A L)

to what I called the Modified LGTD (the MLGTD equation)

R = (1 + alpha A L),

where

alpha = v / theta = [tanh(theta)] / theta,

and so

R = (1 + [v / theta] A L) = (1 + [tanh(theta) / theta] A L).

The Age_change equation then becomes

Age_change = tau R

= tau (1 + [v / theta] A L)

= tau (1 + [tanh(theta) / theta] A L). But A = theta / tau, so

= tau (1 + [tanh(theta) / theta] {theta / tau} L)

= tau [1 + tanh(theta) { 1 / tau } L]

= tau + L tanh(theta)

= tau + L v.

So, for L = 7.52 lightseconds and v = 0.866 ls/s, we get

Age_change = tau + (7.52) (0.866)

= tau + 6.51.

As tau goes to zero, with A going to infinity, so that the product (A tau) is kept constant at 1.317 ls/s, the Age_change of the HF is 6.51 seconds, which agrees with the twin paradox result (as given by the CMIF simultaneity method).

At first, I thought the above equation for the rate ratio R was the correct replacement for the gravitational time dilation equation. But there is a problem with it: it violates the principle of causality. To see why, note that “R” is a function of the acceleration “A”. We frequently are interested in scenarios where the acceleration instantaneously changes from zero to some constant value “A”, and remains there until the time “tau”, at which time it becomes zero again. Let “t” be the time variable, according to the AO. I.e., let “t” be the age change of the AO, after he starts his acceleration. The acceleration is constant at “A” until “t” reaches “tau”, afterwhich it is zero.

The rate ratio “R” is equal to 1.0 for $t < 0$, and is constant at

$$R = (1 + [\mathrm{v(tau)} / \mathrm{theta(tau)}]\ A\ L)$$

$$= (1 + [\ \mathrm{tanh(\ theta(tau)\ )} / \mathrm{theta(tau)}\ ]\ A\ L)$$

$$= (1 + \mathrm{alpha(tau)}\ A\ L)$$

for $0 <= t <= tau$.

I.e., for the ENTIRE interval $0 <= t <= tau$, when the acceleration is constant at “A”, alpha is the value of v / theta at the END of the acceleration. THAT is what is necessary for the resulting age change to be consistent with the outcome of the twin paradox. But that means that the value of the rate factor R, immediately after the acceleration starts, depends on the velocity when the acceleration ends. That is a violation of the principle of causality, which says that an effect can’t precede its cause. At the beginning of the acceleration, we can’t be certain how long the acceleration will last. So the value of R(t) at the beginning of the acceleration CAN’T depend on the value of “v” at the end of the acceleration.

To make the equation causal, we would need to write it as

$$R(t) = (\ 1 + \mathrm{alpha}(t)\ L\ A\ ),$$

where alpha is a function of the variable “t”, NOT the end value “tau”. Note that R is no longer constant during the acceleration: R varies with t.

But

$$\mathrm{alpha}(t) = v(t) / \mathrm{theta}(t) = [\mathrm{tanh(\ theta\ )}] / \mathrm{theta},$$

so alpha starts out equal to 1.0 at $t = 0$, and then monotonically decreases as “t” increases toward tau. Therefore, using alpha(t) rather that alpha(tau) during the interval [0, tau] causes R(t) to be too large for most of the interval [0, tau], and so the age change of the HF (which is the integral of R) won’t be consistent with the twin paradox outcome.

So how do we fix the R(t) equation so that it is both causal AND consistent with the outcome of the twin paradox? That is the subject of the next section.

Section 3. A Proposed Replacement for the Incorrect GTD Equation

We need to multiply the quantity alpha(t) in the above equation for R(t) by some quantity xi(t) that will make R(t) smaller during the interval 0 <= t <= tau, so that the age change of the HF at the end of the acceleration will be consistent with the outcome of the twin paradox.
I.e., we want

R(t) = (1 + L A xi(t) alpha(t)),

where

alpha(t) = v(t) / theta(t) = [tanh(theta)] / theta,

and where xi(t) is some function that is less than unity in magnitude over all or most of the interval 0 < t < tau. More specifically, we want xi(t) to be such that the age change of the HF at the end the acceleration will be equal to what the age change WOULD HAVE BEEN if we could have used the non-causal equation for R (because we know that that age change agrees with the outcome of the twin paradox). In the non-causal case, the HF's age change (AC) at the end of the acceleration (at t = tau) would be

AC(tau) = tau R

= tau [1 + L A alpha(tau)],

because R is constant for 0 <= t <= tau in the non-causal case.

In the new causal case, R is NOT constant over the interval 0 <= t <= tau, and so the HF's age change (AC) at tau would be

AC(tau) = integral from zero to tau {1 + L A xi(t) alpha(t) dt }.

We REQUIRE that xi(t) is such that those two expressions for AC(tau) be EQUAL:

integral from zero to tau {1 + L A xi(t) alpha(t) dt } = tau [1 + L A alpha(tau)].

This is an integral equation which we need to solve for xi(t), for 0 <= t <= tau. That intimidated me at first, but I finally realized that I could solve it by differentiating both sides of the equation with respect to tau, and thereby getting a new equation that I could easily solve for xi(tau).

First, it helps to simplify the integral equation a bit, before doing the differentiations. The LHS of the integral equation can be written

integral from zero to tau {1 dt} + L A integral from zero to tau {xi(t) alpha(t)} dt

which becomes

tau + L A integral from zero to tau {xi(t) alpha(t) dt}.

And the RHS becomes

tau + LA tau alpha(tau).

So we can subtract tau from both sides, and then divide both sides by (L A), to give

integral from zero to tau {xi(t) alpha(t) dt} = tau alpha(tau).

That latter equation is the one whose two sides we need to differentiate.

The LHS is trivial to differentiate, because integration and differentiation are inverse operations. It is just

xi(tau) alpha(tau) = xi v / theta.

The derivative of the RHS is

RHS = d/d(tau) { tau alpha },

where alpha = v / theta. So
RHS = d/d(tau) { tau alpha }

= d/d(tau) { tau v / theta }

= (v / theta) d/dtau{ tau} + tau d/d(tau){ v / theta }

= (v / theta) + tau d/d(tau) { v / theta }

= (v / theta) + tau [(1/theta) d/d(tau){v} + v d/d(tau)(1/theta)]

= (v / theta) + tau [(1/theta) d/d(tau){v} - v(1/{theta theta}) {d(theta)/d(tau)}]

= (v / theta) + tau [(1/theta) d/d(tau){tanh(theta)} - v(1/{theta theta}) {d(theta)/d(tau)}]

= (v / theta) + tau [(1/theta) d(theta)/d(tau) {sech(theta) sech(theta)} -
v(1/{theta theta}) {d(theta)/d(tau)]}.

Equating the LHS and RHS, we get

xi (v / theta) = (v / theta) + tau [(1/theta) d(theta)/d(tau) {sech(theta) sech(theta)} -
v(1/{theta theta}) {d(theta)/d(tau)]}
so

xi = 1 + tau (theta / v) {(1 / theta) d(theta)/d(tau) {sech(theta) sech(theta)} -
v(1/{theta theta}) {d(theta)/d(tau)]}

= 1 + tau {(1/v) d(theta)/d(tau) {sech(theta) sech(theta)} -
(1/theta) {d(theta)/d(tau)]}

= 1 + tau [(A/v) {sech(theta) sech(theta)} - A/theta]

= 1 + A tau [(1 / v) {sech(theta) sech(theta)} - (1 / theta)]

= 1 + theta [(1 / v) {sech(theta) sech(theta)} - (1 / theta)]

= 1 + (theta / v) {sech(theta) sech(theta)} - 1

= (theta / v) {sech(theta) sech(theta)}

xi = (theta / v) {sech_sqrd(theta)}.

Now that we can calculate xi(t), we can calculate the rate ratio R:

R(t) = (1 + L A xi(t) alpha(t)),

where

alpha(t) = v(t) / theta(t) = [tanh(theta)] / theta,

and

xi = (theta / v) {sech_sqrd(theta)}.

The (theta / v) and (v / theta) factors cancel, and we are left with

R(t) = [1 + L A sech_sqrd(theta)].

But

sech_sqrd(theta) = d{tanh(theta)}/d(theta),

so

R(t) = [1 + L A d{tanh(theta)}/d(theta)].

And once we have R(t), we can calculate the age change AC(tau):

AC(tau) = integral from zero to tau { R(t) dt }.

= integral from zero to tau { [1 + L A d{tanh(theta)}/d(theta)] dt }.

But theta = (A t), so

AC(tau) = integral from zero to tau { [1 + L A d{tanh(A t)}/d(A t)] dt }

= tau + L A * integral from zero to tau { d{tanh(A t)}/d(A t)] dt }

= tau + L * integral from zero to tau { A d{tanh(A t)}/d(A t)] dt }

= tau + L * integral from zero to tau { d{tanh(A t)}/d(A t)] A dt }

= tau + L * integral from zero to (A tau) { d{tanh(A t)}/d(A t)] d(At) }

= tau + L * integral from zero to (A tau) { d{tanh(A t)}

= tau + L * { tanh(A tau) - tanh(0) }

AC(tau) = tau + L * tanh(A tau) .

So we've been able to get an analytical expression for AC(tau)! We don't have to get it from a numerical integration. That's a nice surprise.

As tau goes to infinity, AC goes to tau + L, which goes to infinity. As tau goes to zero, AC goes to zero.

Since we now know that

xi = (theta / v) {sech_sqrd(theta)},

we can analytically determine what the limit of xi(t) is, as "t" goes to zero. "theta" and "v" both go to zero as "t" goes to zero, but the ratio (theta / v) goes to 1.0 as "t" goes to zero. Sech(0) and sech_sqrd(0) equal 1.0 at t = 0. Therefore

limit of xi as t -> 0 is 1.0.

Therefore,

limit of R as t -> 0 is 1 + L A xi(0) {v(0)/theta(0)} = 1 + L A.

As t -> infinity, "v" goes to 1.0 (the speed of light), and theta (the rapidity) goes to infinity. So (theta/v) goes to infinity. The limit of sech(theta) as "t" goes to infinity is zero, and likewise for sech_sqrd. So what is the limit of (theta / v) {sech_sqrd}? The factor (theta / v) goes to infinity linearly, but sech_sqrd goes to zero FASTER than linearly, so their product xi goes to zero as "t" goes to infinity:

limit of xi as t -> infinity is 0.

Therefore,

limit of R as t -> infinity is 1 + L A xi(infinity) {v(infinity)/theta(infinity)} = 1.

So, as t goes to infinity, the rate ratio R goes to 1, which means that time for the AO and the HF (and their clocks) passes at the same rate after an infinite duration of a constant acceleration.

But in the case of gravitation, with no motion, experiments have been done that say that separated clocks click at small but different rates in the gravitational field that our earth has produced for millions of years (see the Wiki reference given earlier). Assuming that the equivalence principle is valid, my results above (with no gravitational field, but rather with acceleration), at first appear to contradict those gravitational measurements. But there is no contradiction: my results apply to the case where both observers (the AO and the HF) always have EXACTLY the same acceleration "A". In contrast, the experimental measurements were made in the gravitational field of the earth, for which the field strength "g" varies with the inverse square of the distance of each clock from the center of the earth. So the two situations (the experimental gravitational measurements, versus my special relativity results) are NOT equivalent, and therefore they do NOT violate the equivalence principle.

Section 4. Some Results Using Numerical Integration

The equation for xi doesn't require integration, but it does require being able to compute the hyperbolic secant function. That is often not given in tables, but sech = 1/cosh. For me, the easiest way to generate values of xi(t) so I could plot them, was to write a C program, because sech( ) and tanh( ) are available in C libraries. Then, R(t) is easy to compute from xi(t), and the age change AC(t) can be obtained either with numerical integration of R(t), or else directly from the equation I derived for AC above. For these calculations, I usually used A = 1.0 ls/s/s (about 40 g's) and L = 7.52 ls. (It is possible, since I'm using dimensionless units with c = 1, to use the same program with units of years instead of seconds, and lightyears instead of lightseconds. A = 1 ly/y/y then corresponds to about 1 g.)

On the next several pages, I show the xi, R, and AC curves, and also the C program and the output files.

I also used the program to confirm that my new gravitational time dilation (GTD) equation agrees with the CMIF simultaneity method about the traveling twin's (his) conclusions about the home twin's (her) ageing during his instantaneous velocity change. The CMIF method says that when their separation is 7.52 ls, and his speed instantaneously changes from zero to 0.866 ls/s (in the direction TOWARD her), her age instantaneously increases by 6.51 seconds. (These same results hold when the time units are in years rather than in seconds … but beware: the accelerations A = 1 ls/s/s and A = 1 ly/y/y are very different. The former is about 40 g's, and the latter is about 1g).

To confirm that my new GTD equation agrees with the CMIF simultaneity method, I first ran my program with the input A = 1.317, L = 7.52, tau (called T in the program) = 1, du = 0.001, and m = 10. That choice produces v = 0.866 at t = tau, as desired. The program says that AC, the age change of the HF during the acceleration, is 7.513 seconds. Then, I re-ran the program, with "A" ten times larger, and tau ten times smaller, so that v is again 0.866 at t = tau. In that case, AC = 6.613. I re-ran the program again, again with "A" ten times larger, and tau ten times smaller, so that v is again 0.866 at t = tau. In that case, AC = 6.518. Finally, I re-ran the program a final time, again with "A" ten times larger, and tau ten times smaller, so that v is again 0.866 at t = tau. In that case, AC = 6.509. Here is a table of those results:

A	L	T	du	m	xi	R	AC
1.317	7.52	1.0	0.001	10	0.380	3.476	7.513
13.17	7.52	0.1	0.001	10	0.380	25.76	6.613
131.7	7.52	0.01	0.001	10	0.380	248.58	6.518
1317.0	7.52	0.001	0.0001	10	0.380	2476.8	6.509

It's clear that in this sequence, AC is converging to 6.51seconds, as required.

In the above, I stated that "The CMIF method says that when their separation is 7.52 ls, and his speed instantaneously changes from zero to 0.866 ls/s (in the direction TOWARD her), her age instantaneously increases by 6.51 seconds. How did I get the number 6.51? The easiest way is to use the "CADO equation", which I have described in my previous paper. I derived that equation several decades ago. It gets the same answer (but much quicker) than that which can be obtained by doing an analysis of a Minkowski diagram with lines of simultaneity (which I describe later in this paper). I'll give the CADO equation analysis here.

The CADO equation (or actually, the simplest version, called the "delta_CADO equation") says that the change in the age of the home twin (her), according to the traveling twin (he), when their separation (according to her) is "L", and when he instantaneously changes his velocity from v1 to v2, is

delta_CADO = (-L) (delta_v),

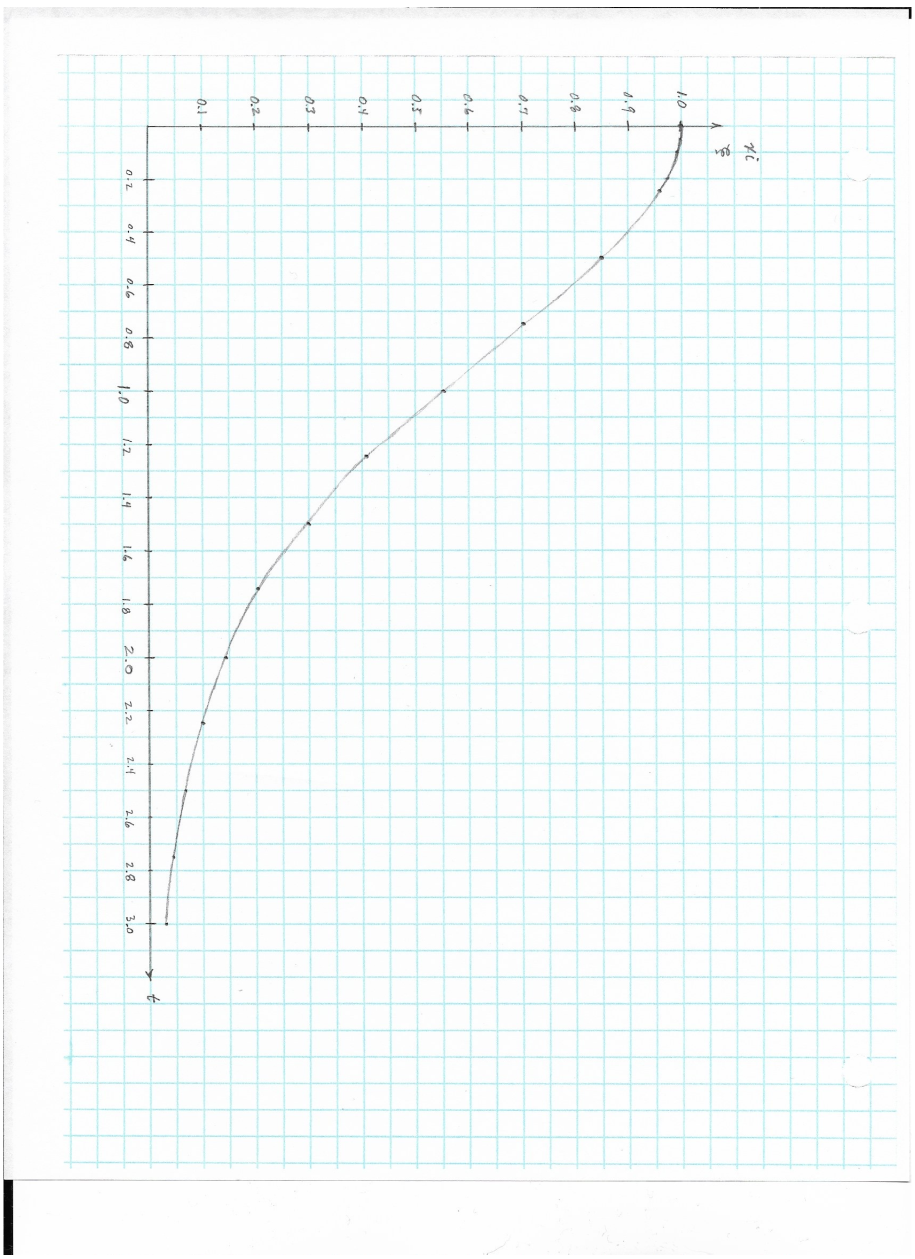
where

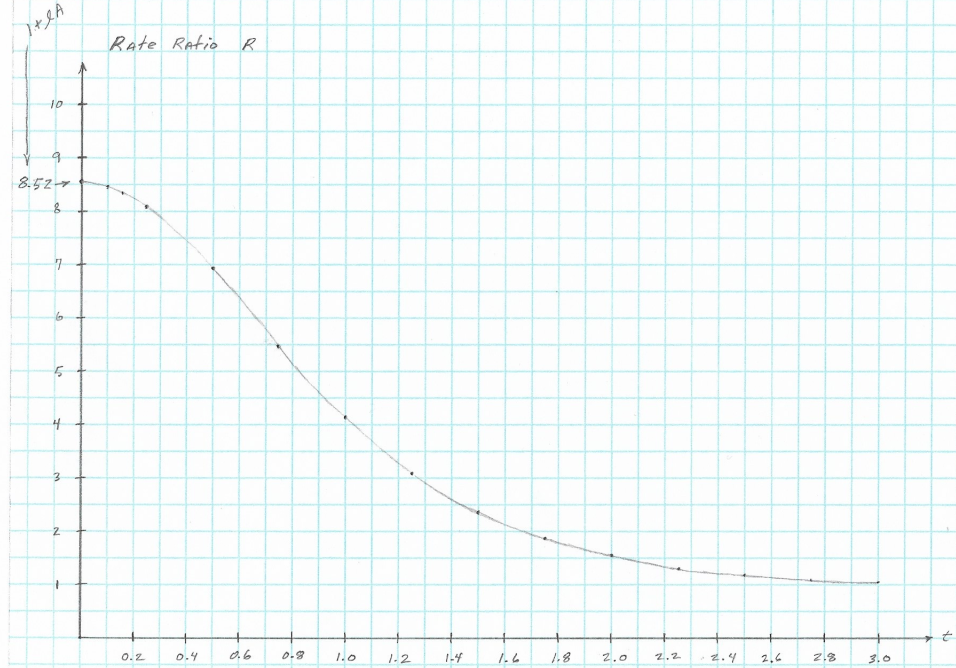
delta_v = v2 - v1 = (-0.866) - (0.0) = -0.866,

and L = 7.52.

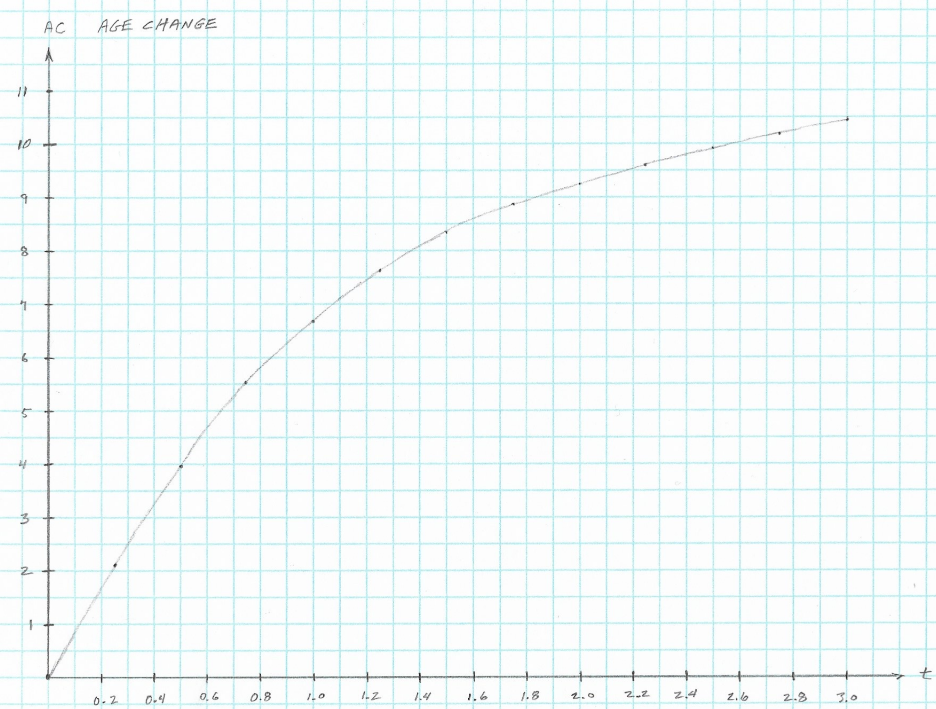
(The negative value for velocity means that he is traveling TOWARD her). So we get

delta_CADO = (-7.52) (-0.866) = 6.51. Very easy!





1×ℓA
Rate Ratio R
10
9
8.52
8
7
6
5
4
3
2
1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
t



AC
AGE CHANGE
11
10
9
8
7
6
5
4
3
2
1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
t

```
/* The final xi program */

#include   <math.h>
main()
      {
   char    ln[80];
   double  theta, v, x;
   double  denom, numer, du;
   double  A, l, T, t, u;
   double  lA, v_ov_theta;
   double  T_sech_sqrd, sech, sech_sqrd;
   int     k, j, m, mskip, kmax;
   double  xi7, r7, r7_prev, r7_avg, ac7, cell_area7, intg7;

      /*
   y = powl(3.14159,2.0);
   printf("\n 3**2 = %lg", y);
   y = powf(3.14159,2.0);
   printf("\n 3**2 = %lg\n\n", y);
      */

    start:
   /* print out every "m" steps. */

          printf("\n\nenter A, l, T, du, m: ");
     gets(ln);
     sscanf(ln, "%lf" "%lf" "%lf" "%lf" "%d", &A, &l, &T, &du, &m);
     printf("\nA = %lf, l = %lf, T = %lf, du = %lf, m = %d\n\n", A, l, T, du
, m);

     if(A == 0)
      {
       printf("\n 'A' must be greater than zero.\n");
       goto start;
      }

     lA = l * A;

     /* do the first starting values manually */

       k = 0;
       u = 0.0;
       theta = 0.0;
       v = 0.0;
       v_ov_theta = 1.0;

       xi7 = 1.0;
       r7 = xi7 * (1.0 + (lA * v_ov_theta));
       r7_prev = r7;
       r7_avg = r7;
       ac7 = 0.0;
       cell_area7 = 0.0;
       intg7 = 0.0;

     kmax = T / du + 0.01;  /* the 0.01 is to correct round-off error! */

     /* printf("\n kmax = %d\n", kmax); */

     mskip = 0;


       printf("\nk = %d  u = %lf  theta = %lf  v = %lf  v/theta = %lf\n",
```

```
     k, u, theta, v, v_ov_theta);
    printf("xi7 = %lf  r7 = %lf  ac7 = %lf\n", xi7, r7, ac7);
    printf("cell_area7 = %lf  intg7 = %lf\n", cell_area7, intg7);


for(k = 1; k <= kmax; k++)
 {
    u = k * du;
    theta = A * u;
    v = tanh(theta);
    sech = 1.0 / cosh(theta);
    sech_sqrd = sech * sech;

    xi7 = (theta / v) * sech_sqrd;
    r7 = 1.0 + xi7 * (lA * v / theta);
    r7_avg = 0.5 * (r7 + r7_prev);
    cell_area7 = r7_avg * du;
    intg7 = intg7 + cell_area7;
    ac7 = intg7;

 if(mskip == 0)
  {

    printf("\nk = %d  u = %lf  theta = %lf  v = %lf  v/theta = %lf\n",
     k, u, theta, v, v_ov_theta);

    printf("xi7 = %lf  r7 = %lf  r7_avg = %lf  sech_sqrd = %lf\n",
     xi7, r7, r7_avg, sech_sqrd);

    printf("ac7 = %lf  cell_area7 = %lf  intg7 = %lf\n",
     ac7, cell_area7, intg7);

  }

    mskip++;
    if(k == 1) mskip++;

    /* reset mskip, to get "mod" function. */

    if(mskip >= m) mskip = 0;

    /* m is the number of loops to skip printing. */

    r7_prev = r7;
 }
   }
```

```
enter A, l, T, du, m:
A = 1.000000, l = 7.520000, T = 2.500000, du = 0.010000, m = 25


k = 0  u = 0.000000  theta = 0.000000  v = 0.000000  v/theta = 1.000000
xi7 = 1.000000  r7 = 8.520000  ac7 = 0.000000
cell_area7 = 0.000000  intg7 = 0.000000

k = 1  u = 0.010000  theta = 0.010000  v = 0.010000  v/theta = 1.000000
xi7 = 0.999933  r7 = 8.519248  r7_avg = 8.519624  sech_sqrd = 0.999900
ac7 = 0.085196  cell_area7 = 0.085196  intg7 = 0.085196

k = 25  u = 0.250000  theta = 0.250000  v = 0.244919  v/theta = 1.000000
xi7 = 0.959517  r7 = 8.068912  r7_avg = 8.085933  sech_sqrd = 0.940015
ac7 = 2.091759  cell_area7 = 0.080859  intg7 = 2.091759

k = 50  u = 0.500000  theta = 0.500000  v = 0.462117  v/theta = 1.000000
xi7 = 0.850918  r7 = 6.914087  r7_avg = 6.941308  sech_sqrd = 0.786448
ac7 = 3.975075  cell_area7 = 0.069413  intg7 = 3.975075

k = 75  u = 0.750000  theta = 0.750000  v = 0.635149  v/theta = 1.000000
xi7 = 0.704464  r7 = 5.486325  r7_avg = 5.514866  sech_sqrd = 0.596586
ac7 = 5.526273  cell_area7 = 0.055149  intg7 = 5.526273

k = 100  u = 1.000000  theta = 1.000000  v = 0.761594  v/theta = 1.000000
xi7 = 0.551441  r7 = 4.158207  r7_avg = 4.182376  sech_sqrd = 0.419974
ac7 = 6.727148  cell_area7 = 0.041824  intg7 = 6.727148

k = 125  u = 1.250000  theta = 1.250000  v = 0.848284  v/theta = 1.000000
xi7 = 0.413209  r7 = 3.108720  r7_avg = 3.126730  sech_sqrd = 0.280415
ac7 = 7.629063  cell_area7 = 0.031267  intg7 = 7.629063

k = 150  u = 1.500000  theta = 1.500000  v = 0.905148  v/theta = 1.000000
xi7 = 0.299465  r7 = 2.358914  r7_avg = 2.371314  sech_sqrd = 0.180707
ac7 = 8.306694  cell_area7 = 0.023713  intg7 = 8.306694

k = 175  u = 1.750000  theta = 1.750000  v = 0.941376  v/theta = 1.000000
xi7 = 0.211575  r7 = 1.855867  r7_avg = 1.863995  sech_sqrd = 0.113812
ac7 = 8.829131  cell_area7 = 0.018640  intg7 = 8.829131

k = 200  u = 2.000000  theta = 2.000000  v = 0.964028  v/theta = 1.000000
xi7 = 0.146574  r7 = 1.531294  r7_avg = 1.536464  sech_sqrd = 0.070651
ac7 = 9.249479  cell_area7 = 0.015365  intg7 = 9.249479

k = 225  u = 2.250000  theta = 2.250000  v = 0.978026  v/theta = 1.000000
xi7 = 0.099993  r7 = 1.326856  r7_avg = 1.330084  sech_sqrd = 0.043465
ac7 = 9.604751  cell_area7 = 0.013301  intg7 = 9.604751

k = 250  u = 2.500000  theta = 2.500000  v = 0.986614  v/theta = 1.000000
xi7 = 0.067383  r7 = 1.199974  r7_avg = 1.201966  sech_sqrd = 0.026592
ac7 = 9.919336  cell_area7 = 0.012020  intg7 = 9.919336
```

```
enter A, l, T, du, m:
A = 1.000000, l = 7.520000, T = 0.500000, du = 0.010000, m = 10


k = 0  u = 0.000000  theta = 0.000000  v = 0.000000  v/theta = 1.000000
xi7 = 1.000000  r7 = 8.520000  ac7 = 0.000000
cell_area7 = 0.000000  intg7 = 0.000000

k = 1  u = 0.010000  theta = 0.010000  v = 0.010000  v/theta = 1.000000
xi7 = 0.999933  r7 = 8.519248  r7_avg = 8.519624  sech_sqrd = 0.999900
ac7 = 0.085196  cell_area7 = 0.085196  intg7 = 0.085196

k = 10  u = 0.100000  theta = 0.100000  v = 0.099668  v/theta = 1.000000
xi7 = 0.993364  r7 = 8.445299  r7_avg = 8.452357  sech_sqrd = 0.990066
ac7 = 0.849491  cell_area7 = 0.084524  intg7 = 0.849491

k = 20  u = 0.200000  theta = 0.200000  v = 0.197375  v/theta = 1.000000
xi7 = 0.973823  r7 = 8.227043  r7_avg = 8.240987  sech_sqrd = 0.961043
ac7 = 1.684239  cell_area7 = 0.082410  intg7 = 1.684239

k = 30  u = 0.300000  theta = 0.300000  v = 0.291313  v/theta = 1.000000
xi7 = 0.942428  r7 = 7.881830  r7_avg = 7.901619  sech_sqrd = 0.915137
ac7 = 2.490637  cell_area7 = 0.079016  intg7 = 2.490637

k = 40  u = 0.400000  theta = 0.400000  v = 0.379949  v/theta = 1.000000
xi7 = 0.900793  r7 = 7.434404  r7_avg = 7.458666  sech_sqrd = 0.855639
ac7 = 3.257175  cell_area7 = 0.074587  intg7 = 3.257175

k = 50  u = 0.500000  theta = 0.500000  v = 0.462117  v/theta = 1.000000
xi7 = 0.850918  r7 = 6.914087  r7_avg = 6.941308  sech_sqrd = 0.786448
ac7 = 3.975075  cell_area7 = 0.069413  intg7 = 3.975075
```

Section 5. How to Determine the Outcome of the Twin Paradox Reunion

In the above sections, I have proposed a new gravitational time dilation (GTD) equation, to replace the existing one. Assuming that the equivalence principle is valid, I've shown in my previous paper that the existing GTD equation isn't correct, because it is inconsistent with the outcome of the twin paradox reunion. But how is the outcome of the twin paradox reunion determined? The outcome at the reunion is easy to determine: the home twin (she) concludes that the traveling twin (he) is, at any instant in her life, ageing gamma times slower than she is, where

gamma = 1 / sqrt { 1 - (v v) }.

If their relative speed is piecewise constant during the trip, she can easily determine how much he ages during each of those piecewise-constant segments during the trip. In the simplest case, if he always has a speed of 0.866, gamma will be equal to 2.0 during the entire trip. So she knows that he will be half her age when they are reunited. And since they are co-located and mutually stationary at and after the reunion, they can't disagree about their respective ages then. Clearly, the twins' ages are both FINITE at the reunion. This contradicts the existing gravitational equation (or, at least, the equivalence principle version of that equation), which says that the home twin's age increases INFINITELY during the instantaneous speed change by the traveling twin.

Section 6. Some Additional Information About the Twin Paradox

The twins DO generally disagree about their current ages DURING the trip, before the reunion. Special Relativity is widely considered to be a completed discipline … a "done deal". It's been more than a hundred years since Einstein presented it to us in 1905. Simultaneity at a distance, according to a perpetually-inertial observer, isn't in dispute. Specifically, the question "How old is that distant person, RIGHT NOW", when asked by a perpetually-inertial observer, is never in dispute (even in the case where the distant person is NOT perpetually inertial).
We can call the perpetually-inertial observer "the home twin", and refer to her as "she". We can refer to "the traveler" who may sometimes accelerate, as "he" or "him". At any instant, he has a velocity relative to her of "v" lightyears/year. The quantity "gamma" depends only on "v", and has the value

gamma = 1 / sqrt { 1 - (v v) },

and according to her, at any instant in her life, he is aging slower than she is, by the factor gamma. For example, if their relative velocity is v = 0.866 ly/y, gamma is equal to 2.0. If he is continually changing his velocity (i.e., continually accelerating), gamma will be changing continually, and so she will conclude that his rate of aging is continually changing. So she will have to integrate that changing rate to compute his current age. But a much easier situation is when he just changes their relative velocity instantaneously, which keeps his rate of aging (compared to hers) constant between his instantaneous velocity changes.

For example, in the standard twin paradox, immediately after they are born, he changes his velocity with respect to her from zero to 0.866 ly/y, and maintains that velocity until he is ready to do his turnaround. At the turnaround, he instantaneously changes his velocity to -0.866 ly/y, and is then heading back toward her. The factor gamma doesn't depend on the direction or the sign of the velocity, so gamma = 2.0 for the entire trip. So she concludes that he is aging half as fast as she is, during the entire trip. Therefore she knows that he will be half as old as she is when he returns at the reunion. If she is 80 years old when they are reunited, he must be 40 years old then. She is just making use of the Time Dilation Equation (TDE) for a perpetually-inertial observer, which is the "gold standard" in special relativity.

Since they are co-located at the reunion, they MUST agree about their respective ages at the reunion. But what is HIS conclusion about how their ages compare during the parts of the trip when they are NOT co-located? I.e., what is HIS answer to the general question, "How old is she (that distant person) right now", at each instant of his life during the trip? He can't just use the time dilation equation (the TDE) during his entire trip (like SHE was able to do), because he is NOT perpetually inertial like she is.

There is disagreement among physicists about the answer to that question. As far as I know, Einstein never addressed that question. Some physicists believe that simultaneity at a distance, according to an accelerating observer, is a meaningless concept, and the question shouldn't even be asked. Some others think that any particular observer is free to choose from among an infinite number of possible answers to that question. I.e., some think simultaneity at a distance should just be regarded as a convention that can be chosen on a whim.

Among those physicists who believe that simultaneity for an accelerating observer IS meaningful, probably the most popular simultaneity method is to specify that the traveling twin (he) should, at each instant of his life, always agree with the answer given, about the home twin's (her) current age, by the perpetually-inertial observer who is momentarily co-located and co-stationary with him at that instant. That method is usually called "the co-moving inertial frames", or "CMIF" method. In the case of the twin paradox scenario, the CMIF method says that on the outbound and inbound legs, he says she ages slower than him by the factor gamma, but that at the turnaround, when he instantaneously changes his velocity in the direction TOWARD her, he says she instantaneously gets OLDER by an amount that turns out to be just large enough so that he will agree with her about their respective ages at the reunion. The CMIF method also says that, if he instantaneously changes his velocity in the direction AWAY FROM her, he says she instantaneously gets YOUNGER. The amount of that instantaneous ageing, either positive or negative, is fairly easy to determine. One can draw a Minkowski diagram, with the two (straight) lines of simultaneity (LOS's) of slope 1/v shown, corresponding to the two different perpetually-inertial observers, one immediately BEFORE and one immediately AFTER the velocity change. Where each of those LOS's cross her worldline gives her age at the turnaround, according to each of the two perpetually-inertial observers. A line of simultaneity (LOS) is just the "RIGHT NOW" line for a perpetually-inertial observer (PIO).

Here is some more detail about how to draw that Minkowski diagram. I prefer to draw her worldline as the horizontal axis, and the distance "X" of objects from her, according to her, on the vertical axis. And so each point "T" on the horizontal axis corresponds to some instant in her life. When both twins are born, they are each located at the origin of the diagram (where the two axes intersect on the left side of the diagram). Immediately after they are born, he instantaneously changes his velocity, with respect to her, from zero to 0.866 ly/y. His distance from her (according to her) then increases linearly according to the equation

$$X = v T = 0.866 T.$$

So, on the outbound leg, HIS worldine is a straight line starting from the origin and sloping upward to the right with a slope of 0.866. That straight line continues until the turnaround point … let's say she says she is 40 years old then. So draw a vertical line that starts at the point T = 40 on the horizontal axis, and extends upward until it intersects his worldline. His distance from her at the turn point, according to her, is (0.866 40) = 34.64, so mark and write that distance on the vertical axis. Using the time dilation equation (TDE) for a perpetually-inertial observer, she knows that he is 20 years old at the turnaround, so label that point on his worldline as 20.

Now, we want to determine the his line of simultaneity (LOS) that passes through his worldline immediately before he reverses his velocity. (That is the LOS of the perpetually-inertial observer (PIO) who is co-located and mutually stationary with him at that instant. That line has a slope of 1/v, or 1/0.866, or 1.155, sloping downward to the left. That LOS forms the hypotenuse of a right triangle, with a vertical side of length 34.64, and with a horizontal base side, extending to the LEFT of the vertical side, whose length we need to determine. The height of the triangle (34.64), divided by the length "L" of the base of the triangle equals the slope 1.155 of the hypotenuse, so we have

34.64 / L = 1.155,

or

L = 34.64 / 1.155 = 30.

So he says her age immediately before he turns around is 40 - L = 40 - 30 = 10 years old.

Note that, in this outbound case, we could have gotten that result immediately from the time dilation equation for a perpetually-inertial observer, because on the outbound leg, he can be considered to be an inertial observer (until he changes his velocity). (If there is any doubt about that, we can say that he and she aren't really twins. Their respective mothers are perpetually inertial, and they just happened to be momentarily co-located when their babies were born. They have always had a relative velocity of 0.866. So in that case he never has accelerated before, and he is certainly entitled to use the time delay equation.) We already have determined that he is 20 years at the turnaround, and according to him, she has been ageing half as fast as he has on the outbound leg, so he says she must be 10 years old when he is 20 years old, immediately before the turnaround. But, nevertheless, it was important to show how he determines her age from his line of simultaneity (LOS).

Next, we need to use the same process to determine how old she is, according to him, immediately AFTER he changes his velocity to -0.866 ly/y. His new line of simultaneity (which is the LOS of the PIO he is NOW co-located with and co-stationary with) forms the hypotenuse of a right triangle, with a vertical side of length 34.64, and with a horizontal base side, extending to the RIGHT of the vertical side, whose length we need to determine. That length is again equal to 30, so now he says that her age immediately after he turns around is 40 + L = 40 + 30 = 70 years old. So he says she instantaneously got 60 years older when he instantaneously changed his velocity from +0.866 to -0.866 (from going AWAY FROM her to going TOWARD her).

Note that he COULD, if he wanted, immediately decide to switch his velocity back to +0.866 from -0.866 (from going TOWARD her to going AWAY FROM her). If he did that, he would conclude that she instantaneously gets 60 years YOUNGER, from 70 years old to 10 years old. Such "back-to-back" equal velocity changes (with no finite time between them) are equivalent to no velocity change at all … the velocity changes cancel each other out. For that reason, if she can instantaneously get older (according to him), it must also be possible that she can instantaneously get younger (according to him). Otherwise, a long series of back-to-back instantaneous velocity changes could make her age (according to him) be arbitrarily large, which would be inconsistent with her certain knowledge of his and her ages at the reunion.

The above back-to-back instantaneous velocity changes are of course not the only scenario where she gets younger, according to him. Whenever there is a finite amount of time between the two opposite velocity changes, the effects don't cancel out. And it is generally true that anytime they are separated and he does a Dirac delta function acceleration (producing an instantaneous velocity change) in the direction TOWARD her, she will instantaneously get YOUNGER.

Besides the Minkowski diagram described above, there is another diagram (that I call the "Age Correspondence Diagram") that is even more important. It (the "ACD") basically graphically shows what the answer is to the question: "For each instant in the life of the accelerating observer (him), what is the current age of the distant person (her)?" For each instant in his life, the ACD plots her corresponding current age, according to him. For example, in the well-known twin paradox scenario where the twins are colocated when they are born, and he immediately instantaneously changes his velocity from zero to 0.866 ly/y (so that his outbound velocity is 0.866 ly/y) and where he does an instantaneous turnaround when he is 20 years old, heading back to her at -0.866 ly/y, the ACD plot starts out at the origin (both twins aged zero), and then rises linearly to the right with a slope of 0.5. That represents the fact that he says she ages gamma times slower than he does on the outbound leg, and gamma equals 2.0. So it says she is 10 years when he is 20 years old, immediately before he changes his velocity to -0.866. Then, at his instantaneous velocity change, the plot goes straight up vertically by 60 years … indicating that she instantaneously gets 60 years older during his velocity change. So at the end of that vertical increase in her age, she is 70 years old, according to him. On the inbound leg, he again says that she is ageing half as fast as he is (because gamma is equal to 2.0). He ages by 20 years on the inbound leg, so he says she ages by 10 years. So at the reunion, he is 40 and she is 80. So the last segment of the ACD plot slopes upward to the right with slope 0.5, and her age increases from 70 to 80 years old.

Some physicists object to that instantaneous ageing in CMIF simultaneity. And the negative ageing in CMIF simultaneity is even MORE abhorrent to a lot of physicists. (Some physicists even DISALLOW negative ageing, while allowing positive ageing, but that is inconsistent with the requirement that back-to-back velocity changes must cancel each other out.) But other physicists have no problem with negative ageing. A prime example of that latter group is Brian Greene, who in his NOVA show "The Fabric of the Cosmos" discusses it very clearly and enthusiastically:

(https://www.pbs.org/wgbh/nova/video/the-fabric-of-the-cosmos-the-illusion-of-time/

(scan forward to the 23:15 point).

(Brian also gives the same example in his book of the same title.)

Besides the CMIF simultaneity method, there are at least three other simultaneity methods that have been proposed. One is the Dolby and Gull "Radar" method (arXiv:gr-qc/0104077), another is the Minguizzi method (arXiv:physics/0411233v1), and a third is my method (http://viXra.org/abs/2109.0076). None of those three methods produce any discontinuities in her age, according to him. So that means there are no vertical rises or vertical drops in the ACD for any of these three simultaneity methods. But both the Dolby and Gull method, and Minguizzi's method, are non-causal: they have an effect on her age (according to him) well BEFORE he decides to change his velocity! In my opinion, that is a disqualifier for a simultaneity method. So the only simultaneity methods that I know of that are causal are the CMIF method and my method. Note: Even though my simultaneity method has no discontinuities in her age, I actually PREFER the CMIF method (because of its simplicity, and because I don't have a problem with instantaneous negative or positive ageing). I previously didn't know which (if either) method is correct.

The ACD for MY simultaneity method, for the scenario I gave above when I described the ACD for the CMIF method, is similar to the CMIF ACD. The plot for the outbound leg is the same for both methods. But in my method, there is no vertical rise in the plot at the turnaround. Instead, in my method, the plot rises linearly from the turnaround point with a steep but finite slope, until it intersects the final section of the diagram where the slope is equal to 0.5 as in the CMIF method. The slope of that steep section can be determined either with a fairly simple equation, or even easier by graphical means. Those details can be found either in my Amazon monograph ("A New Simultaneity Method for Accelerated Observers in Special Relativity", which you can search for on Amazon under my name, Michael Leon Fontenot), or else on viXra at https://vixra.org/abs/2106.0133 .

Section 6b. The CADO Equation

Instead of plotting lines of simultaneity (LOS's), an easier and quicker way to answer the question "How old is that distant person, right now, according to an observer who sometimes accelerates", is to use the "CADO Equation". And even quicker is the "Delta CADO Equation". ("CADO" is just an abbreviation for the "Current Age of a Distant Object".)

For example, instead of using the LOS's of the two perpetually-inertial observers (PIO's) immediately before and immediately after the turnaround as we did above, we can just do this:

First, we need to know their separation "D", according to her, when he instantaneously changes his velocity: D = 34.64 ly.

Then, we need to know what the instantaneous CHANGE in his velocity is:

delta_v = v2 - v1 = (-0.866) - (0.866) = -1.732 ly/y.

Then the instantaneous change in her age, denoted "delta_CADO" is

delta_CADO = (-D) (delta_v) = (-34.64) (-1.732) = 60.0,

so the delta_CADO equation says that she instantaneously gets 60 years older (according to him) when he instantaneously changes his velocity from 0.866 ly/y to -0.866 ly/y. VERY EASY!

The CADO reference frame is the same as the CMIF reference frame. It just uses some new terminology that is designed to reduce errors that are commonly made in working with special relativity. And it also makes use of the very useful (and not well-known) CADO and delta_CADO equations. I first derived the CADO equation in a paper I published more than 20 years ago:

Fontenot, Michael L., "Accelerated Observers in Special Relativity", Physics Essays, December 1999, pp. 629-648.

Section 7. Some Philosophical Thoughts

I've never been able to adopt the "simultaneity at a distance is meaningless" view, mainly for philosophical reasons (which are supposed to be off-limits in physics, but I think everyone is influenced by philosophical thoughts to some extent). I don't believe that my home twin *ceases to exist* whenever we are separated. If she *does* still exist "right now", she must be *doing something specific* right now. And if she is doing something specific right now, she must be a *specific age* right now (because at each instant of a person's life, their brain at that instant is in a state that is uniquely consistent with their actions at that instant). So I believe she *must* have some specific current age. Her current age is *not* just one of a set of equally good "conventions" of simultaneity, as some physicists believe. Therefore there *must* be a single, correct simultaneity method. Before the results in this paper were obtained, I didn't know which of the known simultaneity methods (the CMIF method, and my method) was the correct one, or perhaps some currently unknown method was the correct one. (I preferred the CMIF simultaneity method, but I couldn't contend that it was the correct one). But the results of this paper show that the single correct simultaneity method is the CMIF simultaneity method. And it arrives at that conclusion WITHOUT having to assume that the accelerating observer must accept the opinion of the perpetually-inertial observer with whom he is momentarily mutually-stationary and co-located.

Section 8. A CADO Cartoon

Shortly after I first came up with the CADO equation (several decades ago), and after I started to realize some of its bizarre implications, I created a cartoon (only in my mind) that captures (in only a slightly exaggerated way) the essence of what makes those implications so shocking.

Imagine that a spaceship left Earth many years ago (maybe 20 years ago or so, in ship time), and that the spaceship (at some local date-and-time on the ship) is currently very far away from Earth (maybe 50 lightyears or so, as measured in the Earth frame). The passengers on that ship still remember well their previous lives on Earth, and they still often think about the people they cared about then (and still very much care about). They naturally would wonder if their loved-ones are still alive, and if they are OK. The passengers would probably often try to imagine, if they can figure out their loved-ones' current ages, what they might be currently doing, "right now".

In my imagined cartoon, the ship is having its annual New Year's Eve party. One of the passengers asks the captain, "What is the date right now, back on Earth?" The captain, with his hand on a HUGE throttle, answers, "What date would you LIKE it to be?".

Section 9. Conclusions

I have defined and described in this paper a new gravitational time dilation GTD equation, as a replacement for the existing GTD equation which my previous paper showed to be incorrect. The new equation agrees with the results of the twin paradox as given by the CMIF simultaneity method, but does not require the assumption used to define the CMIF method. Therefore, my GTD equation asserts that it is the SINGLE correct simultaneity method.

My new GTD equation (when converted to a Special Relativity equation via the equivalence principle) allows an accelerating observer to (at least mentally) construct an array of clocks and helper observers throughout space, which defines his "NOW" instant throughout space. That is what Einstein did for perpetually-inertial observers a long time ago, but it has never before been available for accelerating observers. The fact that an accelerating observer can drag (and or push) an array of clocks around with him, effectively defining a "NOW" instant everywhere for him, disproves the belief of some physicists that simultaneity at a distance has no meaning for an accelerating observer.

My GTD equation (actually, the equivalence principle version of it) says that when the SAME constant acceleration is applied to both of the separated clocks for a long enough time, the rate ratio R of the two separated clocks eventually approaches 1.0 … i.e., the two clocks eventually tic at the same rate.

But in the case of gravitation, with no motion, experiments have been done that say that separated clocks click at slightly different rates in the gravitational field that our earth has produced for millions of years (see the Wiki reference given earlier). Assuming that the equivalence principle is valid, my results above (with no gravitational field, but rather with acceleration), at first appear to contradict those gravitational measurements. But there is no contradiction: my results apply to the case where both observers (the AO and the HF) always have EXACTLY the same acceleration "A". In contrast, the experimental measurements were made in the gravitational field of the earth, for which the field strength "g" varies with the inverse square of the distance of each clock from the center of the earth. So the two situations (the experimental gravitational measurements, versus my special relativity results) are NOT equivalent, and therefore they do NOT violate the equivalence principle.