Condensate of Spacetime Quanta

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Abstract

Assuming quanta of spacetime to be spin-2 particles, after developing a statistical theory that has no reference to Boltzmann constant, it is argued that below a certain critical pressure of vacuum, quanta of spacetime form a condensate. The possibility of explanation of Sonoluminescence as a quantum-gravitational effect is also envisaged.

Keywords— spacetime quanta, sonoluminescence, pressure of spacetime, Bose-Einstein condensate, quantum gravity

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1 Introduction

There are sceptics that doubt the existence of spacetime quanta much like Mach and his disciples who opposed atoms and killed Boltzmann in despair. We would be far from a theory of quantum gravity unless we have evidence for the very starting point

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of any such theory. There are already some researchers who pursue this seriously, most notably among them Bose and colleagues [1] who are testing the possibility of quantum superposition of spacetime itself. But from quantum phenomena themselves we know that there are many much more securely-founded quantum phenomena, most significantly *Bose-Einstein condensates*, that need not necessarily invoke the problematic notion of quantum superposition. I have some firm reasons to doubt linearity of quantum mechanics [2] hence prefer not to base serious ontology on the notion of superposition of quantum states. In this paper I consider the possibility of testing quantum gravity via *directly observing condensates of spacetime quanta.* Although there have been some –misguided– attempts [3, 4] that try to –artificially– enforce this idea, they all lack the formal insight of this paper: A statistical theory of quanta of spacetime must

- Not use any notion from anything matter-related, i.e. 'traditional' statistical mechanics.
- Directly involve quantum-gravitational constants such as Planck length $l_P = \sqrt{\hbar G/c^3}$.

In this paper we construct a simple statistical theory of quanta of spacetime that does satisfy these requirements, does not contradict *spin-statistics theorem* if the gist of the theorem is understood, and predicts a condensate of volume states of spacetime.

Any attempt of putting together the notion of *condensate* and 'statistical mechanics' in anyway, immediately faces the anger of academic mind, because for that they 'have a theorem' and fail to recognize that the spin-statistics theorem[5] does not imply at all the exact *content* of the statistics particles obey. In particular the theorem does *not imply* that *there is a parameter* $\beta = 1/k_BT$ using which the statistics *must* be built. As *spin-statistics theorem is indifferent to the value of* β, one can construct statistical theories for almost any physical quantity apart from temperature. For example one can let

$$\tilde{\beta} = \frac{1}{e\varphi},$$

where e is elementary charge and φ electric potential, to define a statistical theory for an ensemble particles in 'electrostatic equilibrium'.

A weak form of this idea was used by Rovelli [6] to calculate the Hawking entropy of a blackhole using the assumption that area is quantized, in which he took microstates to correspond to the different number of ways one can 'make a surface' in spacetime with a certain area A; weak, because he ultimately used β (temperature).

To create a theory that satisfies the above-mentioned requirements we must put aside all the definitions from 'orthodox' statistical mechanics, and guided by formality, define new notions. To that end the first task is to have a clear understanding of what (and what not) spin-statistics theorem says

Theorem (Spin-Statistics)**.** *In three dimensions there are two types of particles*

1. *Bosons, whose multi-particle wavefunction is symmetric under exchange of any two particles. Bosons obey a distribution of the* ***form***

$$\langle n_r \rangle = \frac{1}{e^{\alpha(E_r - \nu)} - 1},$$

in which α and ν are ***not necessarily thermodynamic quantities.***

2. *Fermions, whose multi-particle wavefunction is anti-symmetric under exchange of particles. Fermions obey a distribution of the* ***form***

$$\langle n_r \rangle = \frac{1}{e^{\alpha(E_r - \nu)} + 1},$$

again, in which α *and* ν *are* ***not necessarily thermodynamic quantities.***

2 Statistical mechanics of Spacetime Quanta

Definition (Volume of an ensemble of quanta of spacetime)**.** *Assuming all the spatial volumes in physics are quantized in packets of Planck volume* l_P^3*, let* $N(E)$ *be number of spacetime quanta with a certain fixed energy* E*. Then we 'define' the volume* $\mathcal{V}$ *occupied by these spacetime quanta by*

$$\boxed{\mathcal{V} = l_P^3 \log N(E)} \tag{1}$$

Naturally the next quantity to define is

$$\frac{\partial \mathcal{V}}{\partial E},$$

which has dimension *pressure*$^{-1}$, therefore

Definition (Pressure of an ensemble of spacetime quanta)**.**

$$\boxed{\frac{1}{\mathcal{P}} = \frac{\partial \mathcal{V}}{\partial E}} \tag{2}$$

It is critical to note that this 'pressure' is different from the familiar notion of *vacuum pressure*, because vacuum pressure is a result of residuals of *matter*, while (2) is the result of spacetime *itself.*

2.1 Equipartition theorem

Above definitions allow us to derive the analogue of equipartition theorem for this 'new' statistical theory at once.

Theorem (Equipartition)**.**

$$E = l_P^3 \mathcal{P} \tag{3}$$

Before going further, it would be instrumental for the viability of this new theory if we can prove its use in explaining an already-observed phenomenon.

3 Interlude: Towards explaining Sonoluminescence

Here I –very– briefly point to a possible application of this theory which will be elaborated in another paper.
In short Sonoluminescence is the phenomenon in which *sound is transformed to light.* For a detailed review of the phenomenon see [7] which is a collection of important

literature. Although *sound creates light* summarizes the *quiddity* of the effect, complications of observation and our preconceptions based on the medium (fluid) and conditions in which it occurs, tend to distract us from the essence when attempting a theoretical consideration. As such

Requirement 1 (Light **and Sound**). *Any theoretical model of sonoluminescence must have both light and sound at its core of explanation.*

This seemingly simple requirement dispenses at once with the 'hot spot' models of sonoluminescence (which are almost the consensus) as they consider sonoluminescence to be merely a thermal effect. This requirement will prove to be a powerful razor in excluding remaining theoretical attempts.
To abide by this requirement we must *fully understand both sound and light.* We understand light well enough and we know in low energy regimes it behaves as an electromagnetic wave with energy (density)

$$u = \frac{1}{2}(\epsilon_0|\mathbf{E}|^2 + \frac{1}{\mu_0}|\mathbf{B}|^2) \tag{4}$$

and at high energies with energy packets (quanta) of energy

$$E = \hbar\omega. \tag{5}$$

We have all the reasons to think of sonoluminescence as a high-energy effect. Furthermore since an important aspect of the effect is the spectrum

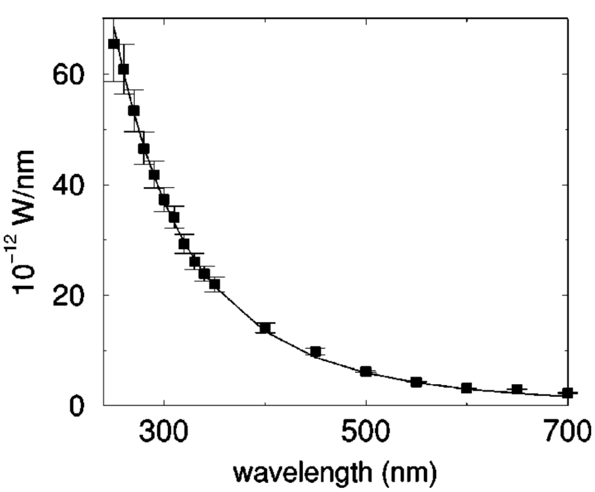

Figure 1: The spectrum of sonoluminescence in water at 22 °C. Taken from [8] with data originally from[9]. The solid curve is a best line fit of a blackbody spectrum of a body at 40 000 K.

and no satisfactory quantitative theory of a spectrum of radiated light is possible without $E = \hbar\omega$ (hence *Bose-Einstein statistics*), we need

Requirement 2 ($E = \hbar\omega$).

Consequently *we must consider a Bose-Einstein–like statistics* of the radiated light from sonoluminescence. To the academic mindset this might result in thinking that we are about to consider a blackbody model of sonoluminescence, but those models do not satisfy **requirement 1**: Blackbody models miss the point as they completely ignore the effect of sound. *Nothing happens without presence of sound.* If blackbody models captured the essence of sonoluminescence the effect could well happen for any blackbody. That is the reason we have used the suffix '-like' for Bose-Einstein statistics as it does not help to solve the problem. Bose-Einstein statistics is a bridge between light *and heat*, not light and sound, while by **requirement 1** *we need a statistical bridge between light and sound.* We have a firm empirical reason to need a *statistical* bridge: a most important failure of the prevailing mathematical model of sonoluminescence, i.e. *Rayleigh-Plesset equation* is that it cannot account for the critical moment

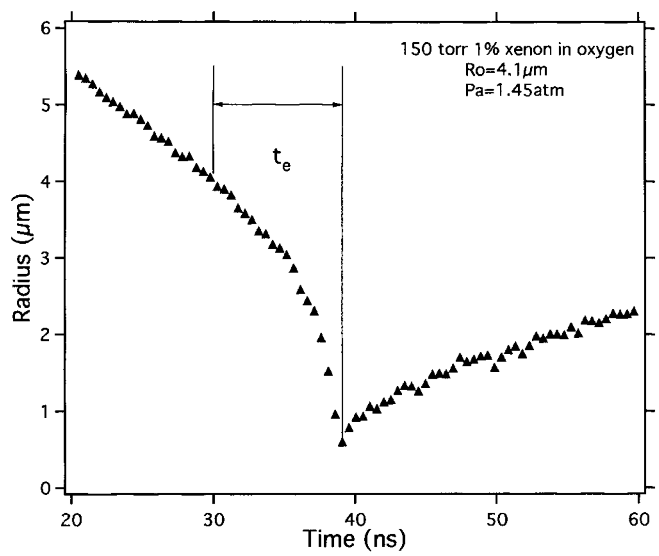

Figure 2: Taken from [10].

The moment closely resembles a *phase transition* familiar from condensed matter physics

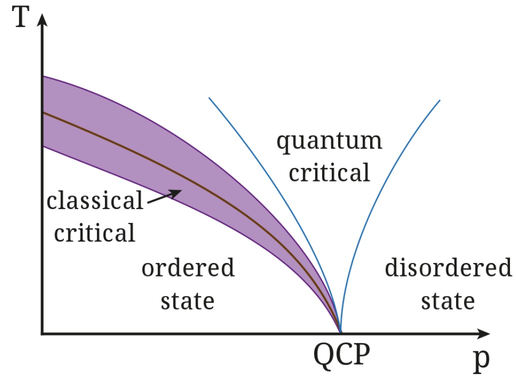

Figure 3: Taken from Wikipedia.

Therefore

Requirement 3 (Phase Transition). *Any satisfactory theory of sonoluminescence must predict a phase transition in volume (radius).*

This requirement is another reason that we need a statistical theory, but we already saw that the current statistical physics would not do. We can however look for formal insights from the Blackbody radiation and its underlying statistical theory. Blackbody radiation is the radiation of light from a heated body: A medium radiates light (outcome = light) when heated (income = heat). In other words it is *light* $\equiv$ *heat*, just like for sonoluminescence, light $\equiv$ sound. The factor that characterizes the statistics of blackbody is

$$\exp\left(\frac{\text{energy of outcome (light)}}{\text{energy of income (heat)}}\right) = \exp(\frac{\hbar\omega}{k_B T}), \tag{6}$$

according to which we expect the statistics of sonoluminescence to be determined by

$$\exp\left(\frac{\text{energy of outcome (light)}}{\text{energy of income (sound)}}\right) = \exp\left(\frac{\hbar\omega}{\text{energy of income (sound)}}\right), \tag{7}$$

but we posses no equation for *energy of income (sound)* which has the same dimension (= energy; *not* energy *density*) as that of $\hbar\omega$. A mundane possibility is to use the familiar energy density of sound

$$u = \frac{p^2}{2\rho_0 v^2},$$

revert to

$$u = \frac{1}{2}(\epsilon_0|\mathbf{E}|^2 + \frac{1}{\mu_0}|\mathbf{B}|^2),$$

hence write

$$\exp\left(\frac{\epsilon_0|\mathbf{E}|^2 + \frac{1}{\mu_0}|\mathbf{B}|^2}{\frac{p^2}{\rho_0 v^2}}\right);$$

although the possibility of this above factor exists it cannot satisfy the second and third requirements.

An energy (not energy density) expression for sound is in fact not altogether alien to the studies of sonoluminescence; the expression

$$E = Vp,$$

where V is volume, is used[10] for energetics of sonoluminescence. Using this expression along the lines of the ideas discussed in [11] we are led to

Definition (Local energy of a particle experiencing sound wave)**.**

$$\boxed{E = l_P^3 p} \tag{8}$$

where l_P^3 is the *Planck volume* (Planck length cubed).

We can now explicitly write down the *bare-bone process of sonoluminescence* as

$$\boxed{l_P^3 p \to \hbar\omega}$$

Appearance of *Planck volume* l_P^3 in (8) is the main reason that *sonoluminescence is probably a quantum-gravitational effect.*
Of course *until shown to be in accord with all the experimental data, this explanation must remain at the level of a hypothesis.*

4 Grand Canonical Ensemble of Spacetime Quanta

Definition (Pressure Capacity of spacetime)**.** *If one adds a certain amount of energy to an ensemble of spacetime quanta, the rise in their pressure is given by*

$$\mathcal{C} = \frac{\partial E}{\partial \mathcal{P}} \tag{9}$$

Definition (Chemical Potential of spacetime)**.** *The energy needed to add a spacetime quantum to an ensemble while keeping $\mathcal{V}$ fixed, is*

$$\nu = -\mathcal{P}\frac{\partial \mathcal{V}}{\partial N} \tag{10}$$

It is now time to ask about the statistics that quanta of spacetime obey. According to *equivalence principle*, in absence of all other interactions, *gravity is the same as the spacetime itself.* As the quanta of gravity are *gravitons* which are *spin-2* particles, by the spin-statistics theorem, **quanta of spacetime obey Bose-Einstein statistics.** Therefore

Definition (Bose-Einstein distribution for spacetime quanta)**.** *Let*

$$\alpha := \frac{1}{l_P^3 \mathcal{P}}, \tag{11}$$

then the grand partition function for a state of spacetime is given by

$$\mathcal{Z}_r = \frac{1}{1 - e^{-\alpha(E_r - \nu)}} \tag{12}$$

Now that we have introduced the grand canonical ensemble for spacetime quanta, it is a matter of calculation to find the number density of particles in ground state of vacuum. Since the calculations are formally the same as those in conventional statistical physics, we only mention the result:
The number of spacetime quanta in the ground state of vacuum as a function of pressure of vacuum is

$$\boxed{\frac{n_0}{n} = 1 - (\frac{\mathcal{P}}{\mathcal{P}_c})^{3/2}} \tag{13}$$

This result might provide a way to test the quantum nature of spacetime as it is stating

Conjecture (Existence of Condensate of Spacetime Quanta in Vacuum)**. There exists a state of spacetime in vacuum below a critical temperature $\mathcal{P}_c$, consisting of a macroscopic number of spacetime quanta**.

It should be noted again that **this 'pressure' is different from *vacuum pressure* which is due to residuals of matter, not spacetime *itself*.**

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