Calculation of the Nuclear Saturation Density

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Abstract

The nuclear saturation density of matter is extracted from a vast amount of charge density distributions found from elastic electron scattering. The established result with error bounds is compared with a calculated value based on 2, π and fundamental constants.

Keywords: Nuclear saturation density, nuclear central density, nuclear charge density distribution, fundamental constants, number constants, universality.

The nuclear saturation density of matter ($\rho_{m,sat}$) is a fundamental property of an infinite symmetric nuclear system without Coulomb interactions, and it is conjectured that the interior of extended nuclei corresponds to this picture. The distribution of matter (ρ_m) in nuclei is difficult to probe precisely, and it is therefore empirically assumed that ρ_m is equal to $(A/Zq_e)\rho$, where A is the mass number or nucleon number and ρ is the distribution of charge in the nucleus. The charge density with the SI unit C/m^3 or q_e/fm^3 can be determined from elastic electron scattering experiments, the results of which for different models are tabulated [1, 2, 3]. The mass of a nucleon within a nucleus is vaguely defined, and the saturation density or central density of matter is therefore often given as a number density with the unit nucleons/fm^3.

The established value of $\rho_{m,sat}$ is about 0.170 nucleons/fm^3, which Bohr and Mottelson [4] explicitly mention based on the work of Hofstadter and co-workers [5]. Since it is difficult to estimate the errors that arise in fitting elastic electron scattering data, little can be found in the literature about the uncertainty of the value of $\rho_{m,sat}$. Many authors state 0.170 nucleons/fm^3 for the density of nucleons at the nuclear core without an error margin and leave it that way. At least there is agreement that the result depends on the density model used and that the best fits are achieved with distributions of charge being almost uniform in the center rather than increasing to infinity as the point charge Coulomb law.

The aim of this work is to determine the mean value with error band of the nuclear saturation density from tabulated experimental data and to compare it with a conjecture made by the author [6]. Whether other theoretical values mentioned in the literature lie within the experimental error limits is not the subject of this work.

To estimate the mean and the mean absolute deviation of $\rho_{m,sat}$, published data on fits of the charge density ρ(r) by means of the Fermi model (2pF+3pF), the Gaussian model (2pG+3pG), or the model-independent Fourier Bessel expansion (FB) were investigated. Evaluations using the one-parameter Fermi model (1pF), that is, where the value of z was fixed in the analysis, were not taken into account. For reasons of consistency, if the calculated root-mean-square radius deviated more than 1% from the radius tabulated in the nuclear data tables, the fit was discarded.

To get consistent results, the Fourier Bessel expansion coefficients a_υ of the experiments from Mazanek [3a, 3b] must be multiplied by Zq_e and divided by the integral from zero to R_{cut} of $j_0(q_\upsilon r)$ over $4\pi r^2 dr$, where $j_0(q_\upsilon r)$ denotes the spherical Bessel function of order zero with q_υ being $\upsilon\pi/R_{cut}$. This integral can be solved analytically and reads $-4(-1)^\upsilon R_{cut}{}^3/\upsilon^2/\pi$.

For all parameterizations, because $\rho(0)$ is the most difficult region to measure experimentally, the maximum density ρ_{max} was also used as a measure of the interior charge density. In addition, for the Fermi model and the Gaussian model, the normalization constant ρ_0 has also been determined as an observable for $\rho_{m,sat}$. The central density depression $w \equiv 1-\rho(0)/\rho_{max}$ is utilized as a parameter in the estimation of the $\rho_{m,sat}$ statistics. The results from the analysis are summarized in Table 1.

In the literature, there is no reliable theory or quantitative explanation available concerning the empirical value of $\rho_{m,sat}$. Accurate computations from first principles with information about the uncertainties are also missing. The author conjectured [6] that the length $2^{-16}\pi^5$ in the unit $\{\lambda_{e,bar}\}_{Codata}$ or ≈1.803 fm, could be the relevant length for the universal value of $\rho_{m,sat}$. Using this hypothesis, a value of **≈ 0.171 nucleons/fm^3** is calculated, which is within the experimental error margins of the Fermi model and the model-independent Fourier Bessel expansion. However, $\rho(0)$ and ρ_{max} of the Gaussian model do not include the conjectured value. Table 1 provides empirical evidence that, as mentioned in [6], the calculated value might be related to $\rho_{m,sat}$.

References

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Table 1: Nuclear saturation densities of matter $\rho_{m,sat}$ extracted from experimental data.

model	**mass region**	ρ_{sat}	**w**	**number of fits**	$\rho_{m,sat} \equiv (A/Zq_e)\rho_{sat}$ **[nucleons/fm^3]**
Fermi (2pF+3pF)	$232 \geq A \geq 12$	ρ_0	norst	162	0.169 (6)
		$\rho(0)$	norst	162	0.168 (6)
		ρ_{max}	norst	162	0.168 (6)
	$40 \geq A=2Z \geq 12$	ρ_0	norst	21	0.171 (7)
		$\rho(0)$	norst	21	0.171 (7)
		ρ_{max}	norst	21	0.171 (7)
	$A=2Z=40$	ρ_0	norst	4	0.173 (3)
		$\rho(0)$	norst	4	0.173 (3)
		ρ_{max}	norst	4	0.173 (3)
Gaussian (2pG+3pG)	$209 \geq A \geq 54$	ρ_0	norst	60	0.171 (8)
		$\rho(0)$	norst	60	0.161 (5) *
		ρ_{max}	norst	60	0.166 (3) *
Fourier-Bessel (FB)	$208 \geq A \geq 12$	$\rho(0)$	norst	93	0.162 (11)
		ρ_{max}	norst	93	0.168 (5)
		$\rho(0)$	< 0.05	59	0.170 (5)
		ρ_{max}	< 0.05	59	0.171 (5)
	$40 \geq A=2Z \geq 12$	$\rho(0)$	< 0.05	5	0.170 (5)
		ρ_{max}	< 0.05	5	0.172 (4)

Note: Summary statistics of mean and mean absolute deviation (in round brackets) of $\rho_{m,sat}$ arranged by charge density models. The equation $A=2Z$ signifies that only symmetrical nuclei with the same number of protons and neutrons were taken into account. The acronym norst means no restrictions on central density depression, and the * character indicates that the calculated value of ≈ 0.171 nucleons/fm^3 is outside the error bound represented by the mean absolute deviation of the data set used.