The Covariant Helmholtzian

The d'alembertian operator on vector doublets may be factored with a pair of four-by-four matrices via simple partial derivatives as elements.

As a generalization of the d'alembertian operator,the Helmholtzian operator on vector doublets may be factored with a pair of four-by-four matrices via simple partial derivatives augmented by adding certain constants as elements thereto.

The Covariant Helmholtzian operator generalizes these, where the elements of the pair of four-by-four matrices are covariant derivatives applying to the vector doublet operated on.

Thus, the d'alembertian operator is a Covariant Helmholtzian operator operated in a flat rectangular Cartesian space; the Helmholtzian operator is a Covariant Helmholtzian operator operated in a space of curvature where all the Christoffel symbols are appropriate constants.

The Helmholtzian operator and its factorization are important, but they are derived-founded on algebra from semigroup binary operation constructed from square matrices as vector basis with integer entries, and variations thereof.

$$\mathbf{u}^{4;0} = \left\{ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \quad \mathbf{u}^{4;1} = \left\{ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\},$$

$$\mathbf{u}^{4;2} = \left\{ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \quad \mathbf{u}^{4;3} = \left\{ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\}$$

under weighted matrix multiplication $(a_{ij})(b_{ij}) = \left(\sum_h \Phi_{hij} a_{ih} b_{hj} \right)$ with bases: $\mathbf{u}^{4;i}$, and weights: $\mathbf{\Phi}_h \equiv \boldsymbol{\varphi}^{4;j}$:

$$\boldsymbol{\varphi}^{4;0} = \left\{ \begin{pmatrix} -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 \end{pmatrix} \right\}, \quad \boldsymbol{\varphi}^{4;1} = \left\{ \begin{pmatrix} -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 \end{pmatrix} \right\},$$

$$\boldsymbol{\varphi}^{4;2} = \left\{ \begin{pmatrix} -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \right\}, \quad \boldsymbol{\varphi}^{4;3} = \left\{ \begin{pmatrix} -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \right\}$$

$\mathbf{u}^{4;0} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;0}$	$\mathbf{u}^{4;1} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;1}$
$\mathbf{u}^{4;0} \circ \mathbf{u}^{4;1} = \mathbf{u}^{4;1}$	$\mathbf{u}^{4;1} \circ \mathbf{u}^{4;1} = -\mathbf{u}^{4;0}$
$\mathbf{u}^{4;0} \circ \mathbf{u}^{4;2} = \mathbf{u}^{4;2}$	$\mathbf{u}^{4;1} \circ \mathbf{u}^{4;2} = \mathbf{u}^{4;3}$
$\mathbf{u}^{4;0} \circ \mathbf{u}^{4;3} = \mathbf{u}^{4;3}$	$\mathbf{u}^{4;1} \circ \mathbf{u}^{4;3} = -\mathbf{u}^{4;2}$
$\mathbf{u}^{4;2} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;2}$	$\mathbf{u}^{4;3} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;3}$
$\mathbf{u}^{4;2} \circ \mathbf{u}^{4;1} = -\mathbf{u}^{4;3}$	$\mathbf{u}^{4;3} \circ \mathbf{u}^{4;1} = \mathbf{u}^{4;2}$
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$\mathbf{u}^{4;2} \circ \mathbf{u}^{4;3} = \mathbf{u}^{4;1}$	$\mathbf{u}^{4;3} \circ \mathbf{u}^{4;3} = -\mathbf{u}^{4;0}$

$\star$	$\mathbf{u}^{4;0}$	$\mathbf{u}^{4;1}$	$\mathbf{u}^{4;2}$	$\mathbf{u}^{4;3}$
$\mathbf{u}^{4;0}$	$-\mathbf{u}^{4;0}$	$\mathbf{u}^{4;1}$	$\mathbf{u}^{4;2}$	$\mathbf{u}^{4;3}$
$\mathbf{u}^{4;1}$	$-\mathbf{u}^{4;1}$	$-\mathbf{u}^{4;0}$	$\mathbf{u}^{4;3}$	$-\mathbf{u}^{4;2}$
$\mathbf{u}^{4;2}$	$-\mathbf{u}^{4;2}$	$-\mathbf{u}^{4;3}$	$-\mathbf{u}^{4;0}$	$\mathbf{u}^{4;1}$
$\mathbf{u}^{4;3}$	$-\mathbf{u}^{4;3}$	$\mathbf{u}^{4;2}$	$-\mathbf{u}^{4;1}$	$-\mathbf{u}^{4;0}$

(1)

$$\begin{array}{llll}
\mathbf{u}^{4;0} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;0}, & \mathbf{u}^{4;1} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;1}, & \mathbf{u}^{4;2} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;2}, & \mathbf{u}^{4;3} \circ \mathbf{u}^{4;0} = -\mathbf{u}^{4;3} \\
\mathbf{u}^{4;0} \circ \mathbf{u}^{4;1} = \mathbf{u}^{4;1}, & \mathbf{u}^{4;1} \circ \mathbf{u}^{4;1} = -\mathbf{u}^{4;0}, & \mathbf{u}^{4;2} \circ \mathbf{u}^{4;1} = -\mathbf{u}^{4;3}, & \mathbf{u}^{4;3} \circ \mathbf{u}^{4;1} = \mathbf{u}^{4;2} \\
\mathbf{u}^{4;0} \circ \mathbf{u}^{4;2} = \mathbf{u}^{4;2}, & \mathbf{u}^{4;1} \circ \mathbf{u}^{4;2} = \mathbf{u}^{4;3}, & \mathbf{u}^{4;2} \circ \mathbf{u}^{4;2} = -\mathbf{u}^{4;0}, & \mathbf{u}^{4;3} \circ \mathbf{u}^{4;2} = -\mathbf{u}^{4;1} \\
\mathbf{u}^{4;0} \circ \mathbf{u}^{4;3} = \mathbf{u}^{4;3}, & \mathbf{u}^{4;1} \circ \mathbf{u}^{4;3} = -\mathbf{u}^{4;2}, & \mathbf{u}^{4;2} \circ \mathbf{u}^{4;3} = \mathbf{u}^{4;1}, & \mathbf{u}^{4;3} \circ \mathbf{u}^{4;3} = -\mathbf{u}^{4;0}
\end{array}$$

So:

$$\begin{aligned}
\mathbf{Z}_1 \circ \mathbf{Z}_2 = {} & \mathbf{u}^{4;1}(Z_1^0 Z_2^1 - Z_1^1 Z_2^0 + Z_1^2 Z_2^3 - Z_1^3 Z_2^2) + \\
& + \mathbf{u}^{4;2}(Z_1^0 Z_2^2 - Z_1^1 Z_2^3 - Z_1^2 Z_2^0 + Z_1^3 Z_2^1) + \\
& + \mathbf{u}^{4;3}(Z_1^0 Z_2^3 + Z_1^1 Z_2^2 - Z_1^2 Z_2^1 - Z_1^3 Z_2^0) + \\
& + \mathbf{u}^{4;0}(-Z_1^0 Z_2^0 - Z_1^1 Z_2^1 - Z_1^2 Z_2^2 - Z_1^3 Z_2^3)
\end{aligned}$$

Recall the Helmholtzian operator factorization.
Consider first, the 2-dimensional version and variation.

$$\mathbf{J} \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2) f^1 \\ (\Box - |m|^2) f^2 \\ (\Box - |m|^2) f^3 \\ (\Box - |m|^2) f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \tag{1}$$

where:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i) \tag{2}$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \ , \ D_i^{\Updownarrow} \equiv \begin{pmatrix} D_i^- & 0 \\ 0 & D_i^+ \end{pmatrix} \ , \ D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \ , \ D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_i^- & 0 \end{pmatrix} \tag{3}$$

For the 1-D space-1-D time situation, consider variations on:

$$\begin{pmatrix} D_{01} & & & D_{11} \\ & D_{01} & D_{11}^{\Leftrightarrow} & \\ & -D_{11}^{\Leftrightarrow} & D_{01} & \\ D_{11}^{\Updownarrow} & & & -D_{01}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_{02}^{\Updownarrow} & & & D_{12} \\ & D_{02}^{\Updownarrow} & -D_{12}^{\Leftrightarrow} & \\ & D_{12}^{\Leftrightarrow} & D_{02}^{\Updownarrow} & \\ D_{12}^{\Updownarrow} & & & -D_{02} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

and:

$$\begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & & & -D_{11} \\ & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & \\ & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & \\ -D_{11}^{\Updownarrow} & & & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

$$\mathbf{J} \equiv \begin{pmatrix} J^1 \\ \\ \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2) f^1 \\ \\ \\ (\Box - |m|^2) f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} D_0 & & & D_1 \\ & D_0 & D_1^{\Leftrightarrow} & \\ & -D_1^{\Leftrightarrow} & D_0 & \\ D_1^{\Updownarrow} & & & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & & & D_1 \\ & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & \\ & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & \\ D_1^{\Updownarrow} & & & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & & & -D_1 \\ & -D_0 & D_1^{\Leftrightarrow} & \\ & -D_1^{\Leftrightarrow} & -D_0 & \\ -D_1^{\Updownarrow} & & & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & & & -D_1 \\ & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & \\ & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & \\ -D_1^{\Updownarrow} & & & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} D_0 D_0^{\Updownarrow} + D_1 D_1^{\Updownarrow} & & & D_0 D_1 - D_1 D_0 \\ & D_0 D_0^{\Updownarrow} + D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} & D_0 D_1^{\Leftrightarrow} - D_1^{\Leftrightarrow} D_0^{\Updownarrow} & \\ & D_1^{\Leftrightarrow} D_0^{\Updownarrow} - D_0 D_1^{\Leftrightarrow} & D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_0 D_0^{\Updownarrow} & \\ D_1^{\Updownarrow} D_0^{\Updownarrow} - D_0^{\Updownarrow} D_1^{\Updownarrow} & & & D_1^{\Updownarrow} D_1 + D_0^{\Updownarrow} D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} \left(D_0 D_0^{\Updownarrow} + D_1 D_1^{\Updownarrow} \right) f^1 + (D_0 D_1 - D_1 D_0) f^0 \\ \\ \\ \left(D_1^{\Updownarrow} D_0^{\Updownarrow} - D_0^{\Updownarrow} D_1^{\Updownarrow} \right) f^1 + \left(D_1^{\Updownarrow} D_1 + D_0^{\Updownarrow} D_0 \right) f^0 \end{pmatrix}$$

$$= \begin{pmatrix} D_0 & & & D_1 \\ & D_0 & D_1^{\Leftrightarrow} & \\ & -D_1^{\Leftrightarrow} & D_0 & \\ D_1^{\Updownarrow} & & & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} f^1 + D_1 f^0 \\ \\ \\ D_1^{\Updownarrow} f^1 - D_0 f^0 \end{pmatrix} = \begin{pmatrix} D_0\left(D_0^{\Updownarrow} f^1 + D_1 f^0\right) + D_1\left(D_1^{\Updownarrow} f^1 - D_0 f^0\right) \\ \\ \\ D_1^{\Updownarrow}\left(D_0^{\Updownarrow} f^1 + D_1 f^0\right) - D_0^{\Updownarrow}\left(D_1^{\Updownarrow} f^1 - D_0 f^0\right) \end{pmatrix}$$

$$= \begin{pmatrix} D_0 D_0^{\Updownarrow} f^1 + D_0 D_1 f^0 + D_1 D_1^{\Updownarrow} f^1 - D_1 D_0 f^0 \\ \\ \\ D_1^{\Updownarrow} D_0^{\Updownarrow} f^1 + D_1^{\Updownarrow} D_1 f^0 - D_0^{\Updownarrow} D_1^{\Updownarrow} f^1 + D_0^{\Updownarrow} D_0 f^0 \end{pmatrix} = \begin{pmatrix} D_0 D_0^{\Updownarrow} f^1 + D_1 D_1^{\Updownarrow} f^1 + D_0 D_1 f^0 - D_1 D_0 f^0 \\ \\ \\ D_1^{\Updownarrow} D_0^{\Updownarrow} f^1 - D_0^{\Updownarrow} D_1^{\Updownarrow} f^1 + D_1^{\Updownarrow} D_1 f^0 + D_0^{\Updownarrow} D_0 f^0 \end{pmatrix}$$

$$= \begin{pmatrix} \left(D_0 D_0^{\Updownarrow} + D_1 D_1^{\Updownarrow}\right) f^1 + (D_0 D_1 - D_1 D_0) f^0 \\ \\ \\ \left(D_1^{\Updownarrow} D_0^{\Updownarrow} - D_0^{\Updownarrow} D_1^{\Updownarrow}\right) f^1 + \left(D_1^{\Updownarrow} D_1 + D_0^{\Updownarrow} D_0\right) f^0 \end{pmatrix}$$

are associative, since the m_j are constants.

Likewise, for the m_j constants :

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} \left(D_0 D_0^{\Updownarrow} + D_3^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_2^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_1 D_1^{\Updownarrow}\right) & \left(D_0 D_3^{\Leftrightarrow} - D_3^{\Leftrightarrow} D_0^{\Updownarrow} - D_2^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_1 D_2^{\Updownarrow}\right) & \left(-D_0 D_2^{\Leftrightarrow} - D_3^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_2^{\Leftrightarrow} D_0^{\Updownarrow} + D_1 D_3^{\Updownarrow}\right) & (D_0 D_1 - D_3^{\Leftrightarrow} D_2 + D \\ \left(D_3^{\Leftrightarrow} D_0^{\Updownarrow} - D_0 D_3^{\Leftrightarrow} - D_1^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_2 D_1^{\Updownarrow}\right) & \left(D_3^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_0 D_0^{\Updownarrow} + D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_2 D_2^{\Updownarrow}\right) & \left(-D_3^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_0 D_1^{\Leftrightarrow} - D_1^{\Leftrightarrow} D_0^{\Updownarrow} + D_2 D_3^{\Updownarrow}\right) & (D_3^{\Leftrightarrow} D_1 + D_0 D_2 - D \\ \left(-D_2^{\Leftrightarrow} D_0^{\Updownarrow} - D_1^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_0 D_2^{\Leftrightarrow} + D_3 D_1^{\Updownarrow}\right) & \left(-D_2^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_1^{\Leftrightarrow} D_0^{\Updownarrow} - D_0 D_1^{\Leftrightarrow} + D_3 D_2^{\Updownarrow}\right) & \left(D_2^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_0 D_0^{\Updownarrow} + D_3 D_3^{\Updownarrow}\right) & (-D_2^{\Leftrightarrow} D_1 + D_1^{\Leftrightarrow} D_2 + \\ \left(D_1^{\Updownarrow} D_0^{\Updownarrow} - D_2^{\Updownarrow} D_3^{\Leftrightarrow} + D_3^{\Updownarrow} D_2^{\Leftrightarrow} - D_0^{\Updownarrow} D_1^{\Updownarrow}\right) & \left(D_1^{\Updownarrow} D_3^{\Leftrightarrow} + D_2^{\Updownarrow} D_0^{\Updownarrow} - D_3^{\Updownarrow} D_1^{\Leftrightarrow} - D_0^{\Updownarrow} D_2^{\Updownarrow}\right) & \left(-D_1^{\Updownarrow} D_2^{\Leftrightarrow} + D_2^{\Updownarrow} D_1^{\Leftrightarrow} + D_3^{\Updownarrow} D_0^{\Updownarrow} - D_0^{\Updownarrow} D_3^{\Updownarrow}\right) & \left(D_1^{\Updownarrow} D_1 + D_2^{\Updownarrow} D_2 + D \end{pmatrix}$$

$$= \begin{pmatrix} \left(D_0 D_0^{\Updownarrow} + D_3^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_2^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_1 D_1^{\Updownarrow}\right) f^1 + \left(D_0 D_3^{\Leftrightarrow} - D_3^{\Leftrightarrow} D_0^{\Updownarrow} - D_2^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_1 D_2^{\Updownarrow}\right) f^2 + \\ \quad + \left(-D_0 D_2^{\Leftrightarrow} - D_3^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_2^{\Leftrightarrow} D_0^{\Updownarrow} + D_1 D_3^{\Updownarrow}\right) f^3 + (D_0 D_1 - D_3^{\Leftrightarrow} D_2 + D_2^{\Leftrightarrow} D_3 - D_1 D_0) f^0 \\ \left(D_3^{\Leftrightarrow} D_0^{\Updownarrow} - D_0 D_3^{\Leftrightarrow} - D_1^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_2 D_1^{\Updownarrow}\right) f^1 + \left(D_3^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_0 D_0^{\Updownarrow} + D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_2 D_2^{\Updownarrow}\right) f^2 + \\ \quad + \left(-D_3^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_0 D_1^{\Leftrightarrow} - D_1^{\Leftrightarrow} D_0^{\Updownarrow} + D_2 D_3^{\Updownarrow}\right) f^3 + (D_3^{\Leftrightarrow} D_1 + D_0 D_2 - D_1^{\Leftrightarrow} D_3 - D_2 D_0) f^0 \\ \left(-D_2^{\Leftrightarrow} D_0^{\Updownarrow} - D_1^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_0 D_2^{\Leftrightarrow} + D_3 D_1^{\Updownarrow}\right) f^1 + \left(-D_2^{\Leftrightarrow} D_3^{\Leftrightarrow} + D_1^{\Leftrightarrow} D_0^{\Updownarrow} - D_0 D_1^{\Leftrightarrow} + D_3 D_2^{\Updownarrow}\right) f^2 + \\ \quad + \left(D_2^{\Leftrightarrow} D_2^{\Leftrightarrow} + D_1^{\Leftrightarrow} D_1^{\Leftrightarrow} + D_0 D_0^{\Updownarrow} + D_3 D_3^{\Updownarrow}\right) f^3 + (-D_2^{\Leftrightarrow} D_1 + D_1^{\Leftrightarrow} D_2 + D_0 D_3 - D_3 D_0) f^0 \\ \left(D_1^{\Updownarrow} D_0^{\Updownarrow} - D_2^{\Updownarrow} D_3^{\Leftrightarrow} + D_3^{\Updownarrow} D_2^{\Leftrightarrow} - D_0^{\Updownarrow} D_1^{\Updownarrow}\right) f^1 + \left(D_1^{\Updownarrow} D_3^{\Leftrightarrow} + D_2^{\Updownarrow} D_0^{\Updownarrow} - D_3^{\Updownarrow} D_1^{\Leftrightarrow} - D_0^{\Updownarrow} D_2^{\Updownarrow}\right) f^2 + \\ \quad + \left(-D_1^{\Updownarrow} D_2^{\Leftrightarrow} + D_2^{\Updownarrow} D_1^{\Leftrightarrow} + D_3^{\Updownarrow} D_0^{\Updownarrow} - D_0^{\Updownarrow} D_3^{\Updownarrow}\right) f^3 + \left(D_1^{\Updownarrow} D_1 + D_2^{\Updownarrow} D_2 + D_3^{\Updownarrow} D_3 + D_0^{\Updownarrow} D_0\right) f^0 \end{pmatrix}$$

are associative.

$(\partial_p \pm m_q)(\partial_j \pm m_k) f^h = (\partial_p \oplus m_q)(\partial_j f^h \pm m_k f^h) = \partial_{pj}^2 f^h \pm m_k \partial_p f^h \oplus m_q \partial_j f^h \oplus \pm m_q m_k f^h$

However, the D terms represent covariant derivatives, so the m_j terms are rather g_{mn}^h functions.
And, the matrix products are NOT associative, so must be applied right to left.

Now, the Helmholtzian operator factorization expresses 2nd order linear PDEs in dimensionns from 2 to 4.
Extennding the Helmholtzian operator factorization beyond constant mass provides a framework for considering solutions by factoring of 2nd order linear PDE probems in dimensionns from 2 to 4 for real-world applications.
Deconstructed from 3-space and time to 2-space and time, then 1-space and time:

For the 1-D space-1-D time situation, consider variations on:

$$\begin{pmatrix} D_{01} & & & D_{11} \\ & D_{01} & D_{11}^{\Leftrightarrow} & \\ & -D_{11}^{\Leftrightarrow} & D_{01} & \\ D_{11}^{\Updownarrow} & & & -D_{01}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_{02}^{\Updownarrow} & & & D_{12} \\ & D_{02}^{\Updownarrow} & -D_{12}^{\Leftrightarrow} & \\ & D_{12}^{\Leftrightarrow} & D_{02}^{\Updownarrow} & \\ D_{12}^{\Updownarrow} & & & -D_{02} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

and:

$$\begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & & & -D_{11} \\ & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & \\ & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & \\ -D_{11}^{\Updownarrow} & & & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix} \tag{1}$$

where:

$$D_{ij}^{+} \equiv (\partial_i + g_{ij}) \quad , \quad D_{ij}^{-} \equiv (\partial_i - g_{ij}) \tag{2}$$

$$D_{ij} \equiv \begin{pmatrix} D_{ij}^{+} & 0 \\ 0 & D_{ij}^{-} \end{pmatrix} \ , \ D_{ij}^{\Updownarrow} \equiv \begin{pmatrix} D_{ij}^{-} & 0 \\ 0 & D_{ij}^{+} \end{pmatrix} \ , \ D_{ij}^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_{ij}^{-} \\ D_{ij}^{+} & 0 \end{pmatrix} \ , \ D_{ij}^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_{ij}^{+} \\ D_{ij}^{-} & 0 \end{pmatrix} \tag{3}$$

for the 1-D space - 1-D time version.

Theorem I.1: For differentiable functions $f^i, f^i_+, f^i_-, g^\lambda_{ijh}; \forall j,h,\lambda \in \{0,1\}$, $\forall i \in \{1,2\}$:

If: $\exists J(x_1,x_0) \ni$

$$J(x_1,x_0) \equiv \begin{pmatrix} J^1 \\ \\ \\ J^0 \end{pmatrix} \equiv \begin{pmatrix} \begin{pmatrix} J^1_+ \\ J^1_- \end{pmatrix} \\ \\ \\ \begin{pmatrix} J^0_+ \\ J^0_- \end{pmatrix} \end{pmatrix}$$

$$\equiv \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & & & -D_{11} \\ & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & \\ & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & \\ -D_{11}^{\Updownarrow} & & & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}$$

$$\equiv \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & & & -D_{11} \\ & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & \\ & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & \\ -D_{11}^{\Updownarrow} & & & D_{01} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} f^1_+ \\ f^1_- \end{pmatrix} \\ \\ \\ \begin{pmatrix} f^0_+ \\ f^0_- \end{pmatrix} \end{pmatrix}$$

where:

$D^+_{ik} \equiv (\partial_i + g^\lambda_{ijk})$, $D^-_{ik} \equiv (\partial_i - g^\lambda_{ijk})$ [where: $D^\pm_{ik} f^m_\sigma = (\partial_i \pm g^\lambda_{ijk}) f^m_\sigma = \partial_i f^m_\sigma \pm g^m_{ijk} f^j_\sigma$]

NOTE: g^m_{nh1} and g^m_{nh2} represent the Christoffel symbols of the second kind due to space-curvature at the first and second operation appication, respectively.

$$D_{ik} \equiv \begin{pmatrix} D^+_{ik} & 0 \\ 0 & D^-_{ik} \end{pmatrix} \ , \ D^{\Updownarrow}_{ik} \equiv \begin{pmatrix} D^-_{ik} & 0 \\ 0 & D^+_{ik} \end{pmatrix} \ ,$$

$$D^{\Leftrightarrow}_{ik} \equiv \begin{pmatrix} 0 & D^-_{ik} \\ D^+_{ik} & 0 \end{pmatrix} \ , \ D^{\Leftrightarrow\Updownarrow}_{ik} \equiv \begin{pmatrix} 0 & D^+_{ik} \\ D^-_{ik} & 0 \end{pmatrix}$$

and:

$$J^i \equiv \begin{pmatrix} J^i_+ \\ J^i_- \end{pmatrix} \ , \ \Phi^i \equiv \begin{pmatrix} \Phi^i_+ \\ \Phi^i_- \end{pmatrix} \ , \ f^i \equiv \begin{pmatrix} f^i_+ \\ f^i_- \end{pmatrix}$$

where:

$$\begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D_{11} f^0 \\ 0 \\ 0 \\ -D^{\Updownarrow}_{11} f^1 + D_{01} f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ 0 \\ 0 \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix} \\ 0 \\ 0 \\ \begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix} \end{pmatrix}$$

then:

$$\begin{pmatrix} J^1 \\ 0 \\ 0 \\ J^0 \end{pmatrix} = \begin{pmatrix} -D_{02}\Phi^1 - D_{12}\Phi^0 \\ 0 \\ -0 \\ -D^{\Updownarrow}_{12}\Phi^1 + D^{\Updownarrow}_{02}\Phi^0 \end{pmatrix} \ \& \ \begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D_{11} f^0 \\ 0 \\ 0 \\ -D^{\Updownarrow}_{11} f^1 + D_{01} f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ 0 \\ 0 \\ \Phi^0 \end{pmatrix}$$

and:

$$
\mathbf{J} = \left(\begin{array}{c}
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_+^1 + \\
+\partial_0([+g_{1h1}^0 - g_{0h1}^1]f_+^h) + \\
+\partial_1([-g_{0h1}^0 - g_{1h1}^1]f_+^h) + \\
+(-g_{1h2}^0 + g_{0h2}^1)\partial_0 f_+^h + \\
+(+g_{0h2}^0 + g_{1h2}^1)\partial_1 f_+^h + \\
([-g_{1k2}^0 - g_{0k2}^1]g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1]g_{1h1}^k)f_+^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_-^1 + \\
+\partial_0([-g_{1h1}^0 + g_{0h1}^1]f_-^h) + \\
+\partial_1([+g_{0h1}^0 + g_{1h1}^1]f_-^h) + \\
+(+g_{1h2}^0 - g_{0h2}^1)\partial_0 f_-^h + \\
+(-g_{0h2}^0 - g_{1h2}^1)\partial_1 f_-^h + \\
([-g_{1k2}^0 - g_{0k2}^1]g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1]g_{1h1}^k)f_-^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_+^0 + \\
+\partial_0([+g_{0h1}^0 + g_{1h1}^1]f_+^h) + \\
+\partial_1([+g_{1h1}^0 - g_{0h1}^1]f_+^h) + \\
+[-g_{0h2}^0 - g_{1h2}^1]\partial_0 f_+^h + \\
+[-g_{1h2}^0 - g_{0h2}^1]\partial_1 f_+^h + \\
+([-g_{0k2}^0 + g_{1k2}^1]g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1]g_{1h1}^k)f_+^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_-^0 + \\
+\partial_0([-g_{0h1}^0 - g_{1h1}^1]f_-^h) + \\
+\partial_1([-g_{1h1}^0 + g_{0h1}^1]f_-^h) + \\
+[+g_{0h2}^0 + g_{1h2}^1]\partial_0 f_-^h + \\
+[+g_{1h2}^0 - g_{0h2}^1]\partial_1 f_-^h + \\
+([-g_{0k2}^0 + g_{1k2}^1]g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1]g_{1h1}^k)f_-^h
\end{array}\right)
\end{array}\right) =
$$

$$
= \left(\begin{array}{c}
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_+^1 + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 + g_{0h2}^1])\partial_0 f_+^h + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{1h1}^0 - g_{0h1}^1)f_+^h + \partial_1(-g_{0h1}^0 - g_{1h1}^1)f_+^h + \\
([-g_{1k2}^0 - g_{0k2}^1]g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1]g_{1h1}^k)f_+^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_-^1 + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_0 f_-^h + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_1 f_-^h + + \\
+\partial_0(-g_{1h1}^0 + g_{0h1}^1)f_-^h + \partial_1(+g_{0h1}^0 + g_{1h1}^1)f_-^h + \\
([-g_{1k2}^0 - g_{0k2}^1]g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1]g_{1h1}^k)f_-^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_+^0 + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_0 f_+^h + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 - g_{0h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{0h1}^0 + g_{1h1}^1)f_+^h + \partial_1(+g_{1h1}^0 - g_{0h1}^1)f_+^h + \\
+([-g_{0k2}^0 + g_{1k2}^1]g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1]g_{1h1}^k)f_+^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_-^0 + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_0 f_-^h + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_1 f_-^h + \\
+\partial_0(-g_{0h1}^0 - g_{1h1}^1)f_-^h + \partial_1(-g_{1h1}^0 + g_{0h1}^1)f_-^h + \\
+([-g_{0k2}^0 + g_{1k2}^1]g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1]g_1^k)f_-^h
\end{array}\right)
\end{array}\right)
$$

and:

$J_+^1 = -\partial_0\Phi_+^1 - \partial_1\Phi_+^0 + (-g_{0j2}^1 - g_{1j2}^0)\Phi_+^j$	$-\partial_0 f_+^1 - \partial_i f_+^0 + (+g_{1j1}^0 - g_{0j1}^1)f_+^j = \Phi_+^1$
$J_-^1 = -\partial_0\Phi_-^1 - \partial_1\Phi_-^0 + (+g_{0j2}^1 + g_{1j2}^0)\Phi_-^j$	$-\partial_0 f_-^1 - \partial_1 f_-^0 + (-g_{0j1}^1 + g_{1j1}^0)f_-^j = \Phi_-^1$
$J_+^0 = -\partial_1\Phi_+^1 + \partial_0\Phi_+^0 + (+g_{1j2}^1 - g_{0j2}^0)\Phi_+^j$	$-\partial_1 f_+^1 + \partial_0 f_+^0 + (+g_{1j1}^1 + g_{0j1}^0)f_+^j = \Phi_+^0$
$J_-^0 = -\partial_1\Phi_-^1 + \partial_0\Phi_+^0 + (-g_{1j2}^1 + g_{0j2}^0)\Phi_-^j$	$-\partial_1 f_-^1 + \partial_0 f_-^0 + (-g_{1j1}^1 - g_{0j1}^0)f_-^j = \Phi_-^0$

or

$\mathbf{J}^1 = -\partial_0\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^0 + (-g_{0j2}^1 - g_{1j2}^0)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\mathbf{\Phi}^j$	$-\partial_0\mathbf{f}^1 - \partial_i\mathbf{f}^0 + (+g_{1j1}^0 - g_{0j1}^1)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\mathbf{f}^j = \mathbf{\Phi}^1$
$\mathbf{J}^0 = -\partial_1\mathbf{\Phi}^1 + \partial_0\mathbf{\Phi}^0 + (+g_{1j2}^1 - g_{0j2}^0)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\mathbf{\Phi}^j$	$-\partial_1\mathbf{f}^1 + \partial_0\mathbf{f}^0 + (+g_{1j1}^1 + g_{0j1}^0)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\mathbf{f}^j = \mathbf{\Phi}^1$

Proof:

$$
\mathbf{J}(x_1,x_0) \equiv \begin{pmatrix} J^1 \\ \\ J^0 \end{pmatrix} \equiv \left(\begin{array}{c} \begin{pmatrix} J_+^1 \\ J_-^1 \end{pmatrix} \\ \\ \begin{pmatrix} J_+^0 \\ J_-^0 \end{pmatrix} \end{array}\right)
$$

$$
= \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & & & -D_{11} \\ & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & \\ & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & \\ -D_{11}^{\Updownarrow} & & & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ \\ \\ f^0 \end{pmatrix}
$$

$$
= \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} f^1 - D_{11} f^0 \\ \\ \\ -D_{11}^{\Updownarrow} f^1 + D_{01} f^0 \end{pmatrix}
$$

$$
\begin{aligned}
&= \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} \Phi^1 \\ 0 \\ 0 \\ \Phi^0 \end{pmatrix} \\
&= \begin{pmatrix} -D_{02}\Phi^1 - D_{12}\Phi^0 \\ 0 \\ 0 \\ -D_{12}^{\Updownarrow}\Phi^1 + D_{02}^{\Updownarrow}\Phi^0 \end{pmatrix} \\
\Rightarrow \begin{pmatrix} J^1 \\ 0 \\ 0 \\ J^0 \end{pmatrix} &= \begin{pmatrix} -D_{02}\Phi^1 - D_{12}\Phi^0 \\ 0 \\ -0 \\ -D_{12}^{\Updownarrow}\Phi^1 + D_{02}^{\Updownarrow}\Phi^0 \end{pmatrix} \;\&\; \begin{pmatrix} -D_{01}^{\Updownarrow} f^1 - D_{11} f^0 \\ 0 \\ 0 \\ -D_{11}^{\Updownarrow} f^1 + D_{01} f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ 0 \\ 0 \\ \Phi^0 \end{pmatrix} \\
&= \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} - \begin{pmatrix} D_{11}^+ & 0 \\ 0 & D_{11}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \\ \\ \\ -\begin{pmatrix} D_{11}^- & 0 \\ 0 & D_{11}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} + \begin{pmatrix} D_{01}^+ & 0 \\ 0 & D_{01}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} -D_{02} & & & -D_{12} \\ & -D_{02} & D_{12}^{\Leftrightarrow} & \\ & -D_{12}^{\Leftrightarrow} & -D_{02} & \\ -D_{12}^{\Updownarrow} & & & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} \\ \\ \\ \begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} -D_{02}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} - D_{12}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \\ \\ \\ -D_{12}^{\Updownarrow}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + D_{02}^{\Updownarrow}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} -\begin{pmatrix} D_{02}^+ & 0 \\ 0 & D_{02}^- \end{pmatrix}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} - \begin{pmatrix} D_{12}^+ & 0 \\ 0 & D_{12}^- \end{pmatrix}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \\ \\ \\ -\begin{pmatrix} D_{12}^- & 0 \\ 0 & D_{12}^+ \end{pmatrix}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + \begin{pmatrix} D_{02}^- & 0 \\ 0 & D_{02}^+ \end{pmatrix}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} \begin{pmatrix} -D_{02}^+ & 0 \\ 0 & -D_{02}^- \end{pmatrix}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + \begin{pmatrix} -D_{12}^+ & 0 \\ 0 & -D_{12}^- \end{pmatrix}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \\ \\ \\ \begin{pmatrix} -D_{12}^- & 0 \\ 0 & -D_{12}^+ \end{pmatrix}\begin{pmatrix} (-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ (-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + \begin{pmatrix} D_{02}^- & 0 \\ 0 & D_{02}^+ \end{pmatrix}\begin{pmatrix} (-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ (-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} \begin{pmatrix} -D_{02}^+(-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ -D_{02}^-(-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + \begin{pmatrix} -D_{12}^+(-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ -D_{12}^-(-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \\ \\ \\ \begin{pmatrix} -D_{12}^-(-D_{01}^- f_+^1 - D_{11}^+ f_+^0) \\ -D_{12}^+(-D_{01}^+ f_-^1 - D_{11}^- f_-^0) \end{pmatrix} + \begin{pmatrix} D_{02}^-(-D_{11}^- f_+^1 + D_{01}^+ f_+^0) \\ D_{02}^+(-D_{11}^+ f_-^1 + D_{01}^- f_-^0) \end{pmatrix} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\begin{array}{c}
\begin{pmatrix} (D^+_{02}D^-_{01}f^1_+ + D^+_{02}D^+_{11}f^0_+) \\ (D^-_{02}D^+_{01}f^1_- + D^-_{02}D^-_{11}f^0_-) \end{pmatrix} + \begin{pmatrix} (D^+_{12}D^-_{11}f^1_+ - D^+_{12}D^+_{01}f^0_+) \\ (D^-_{12}D^+_{11}f^1_- - D^-_{12}D^-_{01}f^0_-) \end{pmatrix} \\
\\
\begin{pmatrix} (D^-_{12}D^-_{01}f^1_+ + D^-_{12}D^+_{11}f^0_+) \\ (D^+_{12}D^+_{01}f^1_- + D^+_{12}D^-_{11}f^0_-) \end{pmatrix} + \begin{pmatrix} (-D^-_{02}D^-_{11}f^1_+ + D^-_{02}D^+_{01}f^0_+) \\ (-D^+_{02}D^+_{11}f^1_- + D^+_{02}D^-_{01}f^0_-) \end{pmatrix}
\end{array} \right) \\
&= \left(\begin{array}{c}
\begin{pmatrix} (D^+_{02}(\partial_0 f^1_+ - g^1_{0h1}f^h_+) + D^+_{02}(\partial_1 f^0_+ + g^0_{1h1}f^h_+)) \\ (D^-_{02}(\partial_0 f^1_- + g^1_{0h1}f^h_-) + D^-_{02}(\partial_1 f^0_- - g^0_{1h1}f^h_-)) \end{pmatrix} + \begin{pmatrix} (D^+_{12}(\partial_1 f^1_+ - g^1_{1h1}f^h_+) - D^+_{12}(\partial_0 f^0_+ + g^0_{0h1}f^h_+)) \\ (D^-_{12}(\partial_1 f^1_- + g^1_{1h1}f^h_-) - D^-_{12}(\partial_0 f^0_- - g^0_{0h1}f^h_-)) \end{pmatrix} \\
\\
\begin{pmatrix} (D^-_{12}(\partial_0 f^1_+ - g^1_{0h1}f^h_+) + D^-_{12}(\partial_1 f^0_+ + g^0_{1h1}f^h_+)) \\ (D^+_{12}(\partial_0 f^1_- + g^1_{0h1}f^h_-) + D^+_{12}(\partial_1 f^0_- - g^0_{1h1}f^h_-)) \end{pmatrix} + \begin{pmatrix} (-D^-_{02}(\partial_1 f^1_+ - g^1_{1h1}f^h_+) + D^-_{02}(\partial_0 f^0_+ + g^0_{0h1}f^h_+)) \\ (-D^+_{02}(\partial_1 f^1_- + g^1_{1h1}f^h_-) + D^+_{02}(\partial_0 f^0_- - g^0_{0h1}f^h_-)) \end{pmatrix}
\end{array} \right) \\
&= \left(\begin{array}{c}
\begin{pmatrix} (D^+_{02}(\partial_0 f^1_+) - D^+_{02}(g^1_{0h1}f^h_+) + D^+_{02}(\partial_1 f^0_+) + D^+_{02}(g^0_{1h1}f^h_+)) \\ (D^-_{02}(\partial_0 f^1_-) + D^-_{02}(g^1_{0h1}f^h_-) + D^-_{02}(\partial_1 f^0_-) - D^-_{02}(g^0_{1h1}f^h_-)) \end{pmatrix} + \begin{pmatrix} (D^+_{12}(\partial_1 f^1_+) - D^+_{12}(g^1_{1h1}f^h_+) - D^+_{12}(\partial_0 f^0_+) - D^+_{12}(g^0_{0h1}f^h_+)) \\ (D^-_{12}(\partial_1 f^1_-) + D^-_{12}(g^1_{1h1}f^h_-) - D^-_{12}(\partial_0 f^0_-) + D^-_{12}(g^0_{0h1}f^h_-)) \end{pmatrix} \\
\\
\begin{pmatrix} (D^-_{12}(\partial_0 f^1_+) - D^-_{12}(g^1_{0h1}f^h_+) + D^-_{12}(\partial_1 f^0_+) + D^-_{12}(g^0_{1h1}f^h_+)) \\ (D^+_{12}(\partial_0 f^1_-) + D^+_{12}(g^1_{0h1}f^h_-) + D^+_{12}(\partial_1 f^0_-) - D^+_{12}(g^0_{1h1}f^h_-)) \end{pmatrix} + \begin{pmatrix} (-D^-_{02}(\partial_1 f^1_+) + D^-_{02}(g^1_{1h1}f^h_+) + D^-_{02}(\partial_0 f^0_+) + D^-_{02}(g^0_{0h1}f^h_+)) \\ (-D^+_{02}(\partial_1 f^1_-) - D^+_{02}(g^1_{1h1}f^h_-) + D^+_{02}(\partial_0 f^0_-) - D^+_{02}(g^0_{0h1}f^h_-)) \end{pmatrix}
\end{array} \right) \\
&= \left(\begin{array}{c}
\begin{pmatrix} (\partial_0(\partial_0 f^1_+) + g^1_{0k2}(\partial_0 f^k_+) - \partial_0(g^1_{0h1}f^h_+) - g^1_{0k2}(g^k_{0h1}f^h_+) + \partial_0(\partial_1 f^0_+) + g^0_{0h2}(\partial_1 f^k_+) + \partial_0(g^0_{1h1}f^h_+) + g^0_{0k2}(g^k_{1h1}f^h_+)) \\ (\partial_0(\partial_0 f^1_-) - g^1_{0k2}(\partial_0 f^k_-) + \partial_0(g^1_{0h1}f^h_-) - g^1_{0k2}(g^k_{0h1}f^h_-) + \partial_0(\partial_1 f^0_-) - g^0_{0k2}(\partial_1 f^k_-) - \partial_0(g^0_{1h1}f^h_-) + g^0_{0k2}(g^k_{1h1}f^h_-)) \end{pmatrix} + \\
+ \begin{pmatrix} (\partial_1(\partial_1 f^1_+) + g^1_{1k2}(\partial_1 f^k_+) - \partial_1(g^1_{1h1}f^h_+) - g^1_{1k2}(g^k_{1h1}f^h_+) - \partial_1(\partial_0 f^0_+) - g^0_{1k2}(\partial_0 f^k_+) - \partial_1(g^0_{0h1}f^h_+) - g^0_{1k2}(g^k_{0h1}f^h_+)) \\ (\partial_1(\partial_1 f^1_-) - g^1_{1k2}(\partial_1 f^k_-) + \partial_1(g^1_{1h1}f^h_-) - g^1_{1k2}(g^k_{1h1}f^h_-) - \partial_1(\partial_0 f^0_-) + g^0_{1k2}(\partial_0 f^k_-) + \partial_1(g^0_{0h1}f^h_-) - g^0_{1k2}(g^k_{0h1}f^h_-)) \end{pmatrix} \\
\\
\begin{pmatrix} (\partial_1(\partial_0 f^1_+) - g^1_{1k2}(\partial_0 f^k_+) - \partial_1(g^1_{0h1}f^h_+) + g^1_{1k2}(g^k_{0h1}f^h_+) + \partial_1(\partial_1 f^0_+) - g^0_{1k2}(\partial_1 f^k_+) + \partial_1(g^0_{1h1}f^h_+) - g^0_{1k2}(g^k_{1h1}f^h_+)) \\ (\partial_1(\partial_0 f^1_-) + g^1_{1k2}(\partial_0 f^k_-) + \partial_1(g^1_{0h1}f^h_-) + g^1_{1k2}(g^k_{0h1}f^h_-) + \partial_1(\partial_1 f^0_-) + g^0_{1k2}(\partial_1 f^k_-) - \partial_1(g^0_{1h1}f^h_-) - g^0_{1k2}(g^k_{1h1}f^h_-)) \end{pmatrix} + \\
+ \begin{pmatrix} (-\partial_0(\partial_1 f^1_+) + g^1_{0k2}(\partial_1 f^k_+) + \partial_0(g^1_{1h1}f^h_+) - g^1_{0k2}(g^k_{1h1}f^h_+) + \partial_0(\partial_0 f^0_+) - g^0_{0k2}(\partial_0 f^k_+) + \partial_0(g^0_{0h1}f^h_+) - g^0_{0k2}(g^k_{0h1}f^h_+)) \\ (-\partial_0(\partial_1 f^1_-) - g^1_{0k2}(\partial_1 f^k_-) - \partial_0(g^1_{1h1}f^h_-) - g^1_{0k2}(g^k_{1h1}f^h_-) + \partial_0(\partial_0 f^0_-) + g^0_{0k2}(\partial_0 f^k_-) - \partial_0(g^0_{0h1}f^h_-) - g^0_{0k2}(g^k_{0h1}f^h_-)) \end{pmatrix}
\end{array} \right) \\
&= \left(\begin{array}{c}
\begin{pmatrix} (\partial^2_0 f^1_+ + g^1_{0k2}\partial_0 f^k_+ - \partial_0(g^1_{0h1})f^h_+ - g^1_{0h1}\partial_0 f^h_+ - g^1_{0k2}g^k_{0h1}f^h_+ + \partial_0\partial_1 f^0_+ + g^0_{0k2}\partial_1 f^k_+ + \partial_0(g^0_{1h1})f^h_+ + g^0_{1h1}\partial_0 f^h_+ + g^0_{0k2}g^k_{1h1}f^h_+) \\ (\partial^2_0 f^1_- - g^1_{0k2}\partial_0 f^k_- + \partial_0(g^1_{0h1})f^h_- + g^1_{0h1}\partial_0 f^h_- - g^1_{0k2}g^k_{0h1}f^h_- + \partial_0\partial_1 f^0_- - g^0_{0k2}\partial_1 f^k_- - \partial_0(g^0_{1h1})f^h_- - g^0_{1h1}\partial_0 f^h_- + g^0_{0k2}g^k_{1h1}f^h_-) \end{pmatrix} + \\
+ \begin{pmatrix} (\partial^2_1 f^1_+ + g^1_{1k2}\partial_1 f^k_+ - \partial_1(g^1_{1h1})f^h_+ - g^1_{1h1}\partial_1 f^h_+ - g^1_{1k2}g^k_{1h1}f^h_+ - \partial_1\partial_0 f^0_+ - g^0_{1k2}\partial_0 f^k_+ - \partial_1(g^0_{0h1})f^h_+ - g^0_{0h1}\partial_1 f^h_+ - g^0_{1k2}g^k_{0h1}f^h_+) \\ (\partial^2_1 f^1_- - g^1_{1k2}\partial_1 f^k_- + \partial_1(g^1_{1h1})f^h_- + g^1_{1h1}\partial_1 f^h_- - g^1_{1k2}g^k_{1h1}f^h_- - \partial_1\partial_0 f^0_- + g^0_{1k2}\partial_0 f^k_- + \partial_1(g^0_{0h1})f^h_- + g^0_{0h1}\partial_1 f^h_- - g^0_{1k2}g^k_{0h1}f^h_-) \end{pmatrix} \\
\\
\begin{pmatrix} (\partial_1\partial_0 f^1_+ - g^1_{1k2}\partial_0 f^k_+ - \partial_1(g^1_{0h1})f^h_+ - g^1_{0h1}\partial_1 f^h_+ + g^1_{1k2}g^k_{0h1}f^h_+ + \partial^2_1 f^0_+ - g^0_{1k2}\partial_1 f^k_+ + \partial_1(g^0_{1h1})f^h_+ + g^0_{1h1}\partial_1 f^h_+ - g^0_{1k2}g^k_{1h1}f^h_+) \\ (\partial_1\partial_0 f^1_- + g^1_{1k2}\partial_0 f^k_- + \partial_1(g^1_{0h1})f^h_- + g^1_{0h1}\partial_1 f^h_- + g^1_{1k2}g^k_{0h1}f^h_- + \partial^2_1 f^0_- + g^0_{1k2}\partial_1 f^k_- - \partial_1(g^0_{1h1})f^h_- - g^0_{1h1}\partial_1 f^h_- - g^0_{1k2}g^k_{1h1}f^h_-) \end{pmatrix} + \\
+ \begin{pmatrix} (-\partial_0\partial_1 f^1_+ + g^1_{0k2}\partial_1 f^k_+ + \partial_0(g^1_{1h1})f^h_+ + g^1_{1h1}\partial_0 f^h_+ - g^1_{0k2}g^k_{1h1}f^h_+ + \partial^2_0 f^0_+ - g^0_{0k2}\partial_0 f^k_+ + \partial_0(g^0_{0h1})f^h_+ + g^0_{0h1}\partial_0 f^h_+ - g^0_{0k2}g^k_{0h1}f^h_+) \\ (-\partial_0\partial_1 f^1_- - g^1_{0k2}\partial_1 f^k_- - \partial_0(g^1_{1h1})f^h_- - g^1_{1h1}\partial_0 f^h_- - g^1_{0k2}g^k_{1h1}f^h_- + \partial^2_0 f^0_- + g^0_{0k2}\partial_0 f^k_- - \partial_0(g^0_{0h1})f^h_- - g^0_{0h1}\partial_0 f^h_- - g^0_{0k2}g^k_{0h1}f^h_-) \end{pmatrix}
\end{array} \right)
\end{aligned}
$$

$$
=
\begin{pmatrix}
\begin{pmatrix}
\partial_0^2 f_+^1 + \partial_1^2 f_+^1 + \\
+\partial_0\partial_1 f_+^0 - \partial_1\partial_0 f_+^0 + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 + g_{0h2}^1])\partial_0 f_+^h + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{1h1}^0 - g_{0h1}^1) f_+^h + \\
+\partial_1(-g_{0h1}^0 - g_{1h1}^1) f_+^h + \\
([-g_{1k2}^0 - g_{0k2}^1] g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1] g_{1h1}^k) f_+^h
\end{pmatrix} \\
\begin{pmatrix}
\partial_0^2 f_-^1 + \partial_1^2 f_-^1 + \\
+\partial_0\partial_1 f_-^0 - \partial_1\partial_0 f_-^0 + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_0 f_-^k + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_1 f_-^h + \\
+\partial_0(-g_{1h1}^0 + g_{0h1}^1) f_-^h + \\
+\partial_1(+g_{0h1}^0 + g_{1h1}^1) f_-^h + \\
([-g_{1k2}^0 - g_{0k2}^1] g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1] g_{1h1}^k) f_-^h
\end{pmatrix} \\
\\
\begin{pmatrix}
+\partial_1^2 f_+^0 + \partial_0^2 f_+^0 + \\
+\partial_1\partial_0 f_+^1 - \partial_0\partial_1 f_+^1 + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_0 f_+^h + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 + g_{0h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{0h1}^0 + g_{1h1}^1) f_+^h + \\
+\partial_1(+g_{1h1}^0 - g_{0h1}^1) f_+^h + \\
+([-g_{02k}^0 + g_{12k}^1] g_{01h}^k + [-g_{12k}^0 - g_{02k}^1] g_{11h}^k) f_+^h
\end{pmatrix} \\
\begin{pmatrix}
+\partial_1^2 f_-^0 + \partial_0^2 f_-^0 + \\
\partial_1\partial_0 f_-^1 - \partial_0\partial_1 f_-^1 + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_0 f_-^h + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_1 f_-^h + \\
+\partial_0(-g_{0h1}^0 - g_{1h1}^1) f_-^h + \\
+\partial_1(-g_{11h1}^0 + g_{0h1}^1) f_-^h + \\
+([-g_{0k2}^0 + g_{1k2}^1] g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1] g_{1h1}^k) f_-^h
\end{pmatrix}
\end{pmatrix}
$$

$$
=
\begin{pmatrix}
\begin{pmatrix}
(\partial_0^2 + \partial_1^2) f_+^1 + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 + g_{0h2}^1])\partial_0 f_+^h + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{1h1}^0 - g_{0h1}^1) f_+^h + \partial_1(-g_{0h1}^0 - g_{1h1}^1) f_+^h + \\
([-g_{1k2}^0 - g_{0k2}^1] g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1] g_{1h1}^k) f_+^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2 + \partial_1^2) f_-^1 + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_0 f_-^h + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_1 f_-^h + + \\
+\partial_0(-g_{1h1}^0 + g_{0h1}^1) f_-^h + \partial_1(+g_{0h1}^0 + g_{1h1}^1) f_-^h + \\
([-g_{1k2}^0 - g_{0k2}^1] g_{0h1}^k + [+g_{0k2}^0 - g_{1k2}^1] g_{1h1}^k) f_-^h
\end{pmatrix} \\
\\
\begin{pmatrix}
(\partial_1^2 + \partial_0^2) f_+^0 + \\
+([+g_{0h1}^0 + g_{1h1}^1] + [-g_{0h2}^0 - g_{1h2}^1])\partial_0 f_+^h + \\
+([+g_{1h1}^0 - g_{0h1}^1] + [-g_{1h2}^0 + g_{0h2}^1])\partial_1 f_+^h + \\
+\partial_0(+g_{0h1}^0 + g_{1h1}^1) f_+^h + \partial_1(+g_{1h1}^0 - g_{0h1}^1) f_+^h + \\
+([-g_{0k2}^0 + g_{1k2}^1] g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1] g_{1h1}^k) f_+^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2 + \partial_0^2) f_-^0 + \\
+([-g_{0h1}^0 - g_{1h1}^1] + [+g_{0h2}^0 + g_{1h2}^1])\partial_0 f_-^h + \\
+([-g_{1h1}^0 + g_{0h1}^1] + [+g_{1h2}^0 - g_{0h2}^1])\partial_1 f_-^h + \\
+\partial_0(-g_{0h1}^0 - g_{1h1}^1) f_-^h + \partial_1(-g_{1h1}^0 + g_{0h1}^1) f_-^h + \\
+([-g_{0k2}^0 + g_{1k2}^1] g_{0h1}^k + [-g_{1k2}^0 - g_{0k2}^1] g_1^k) f_-^h
\end{pmatrix}
\end{pmatrix}
$$

$$
=\left(\begin{array}{l}
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_+^1+\\
+\partial_0([+g_{1h1}^0-g_{0h1}^1]f_+^h)+\\
+\partial_1([-g_{0h1}^0-g_{1h1}^1]f_+^h)+\\
+(-g_{1h2}^0+g_{0h2}^1)\partial_0 f_+^h+\\
+(+g_{0h2}^0+g_{1h2}^1)\partial_1 f_+^h+\\
+([-g_{1k2}^0-g_{0k2}^1]g_{0h1}^k+[+g_{0k2}^0-g_{1k2}^1]g_{1h1}^k)f_+^h
\end{array}\right)\\
\left(\begin{array}{l}
(\partial_0^2+\partial_1^2)f_-^1+\\
+\partial_0([-g_{1h1}^0+g_{0h1}^1]f_-^h)+\\
+\partial_1([+g_{0h1}^0+g_{1h1}^1]f_-^h)+\\
+(+g_{1h2}^0-g_{0h2}^1)\partial_0 f_-^h+\\
+(-g_{0h2}^0-g_{1h2}^1)\partial_1 f_-^h+\\
+([-g_{1k2}^0-g_{0k2}^1]g_{0h1}^k+[+g_{0k2}^0-g_{1k2}^1]g_{1h1}^k)f_-^h
\end{array}\right)\\
\\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_+^0+\\
+\partial_0([+g_{0h1}^0+g_{1h1}^1]f_+^h)+\\
+\partial_1([+g_{1h1}^0-g_{0h1}^1]f_+^h)+\\
+[-g_{0h2}^0-g_{1h2}^1]\partial_0 f_+^h+\\
+[-g_{1h2}^0+g_{0h2}^1]\partial_1 f_+^h+\\
+([-g_{0k2}^0+g_{1k2}^1]g_{0h1}^k+[-g_{1k2}^0-g_{0k2}^1]g_{1h1}^k)f_+^h
\end{array}\right)\\
\left(\begin{array}{l}
(\partial_1^2+\partial_0^2)f_-^0+\\
+\partial_0([-g_{0h1}^0-g_{1h1}^1]f_-^h)+\\
+\partial_1([-g_{1h1}^0+g_{0h1}^1]f_-^h)+\\
+[+g_{0h2}^0+g_{1h2}^1]\partial_0 f_-^h+\\
+[+g_{1h2}^0-g_{0h2}^1]\partial_1 f_-^h+\\
+([-g_{0k2}^0+g_{1k2}^1]g_{0h1}^k+[-g_{1k2}^0-g_{0k2}^1]g_{1h1}^k)f_-^h
\end{array}\right)
\end{array}\right)
$$

And:

$J^1=-D_{02}\Phi^1-D_{12}\Phi^0$	$-D_{01}^{\Updownarrow}f^1-D_{11}f^0=\Phi^1$
$J^0=-D_{12}^{\Updownarrow}\Phi^1+D_{02}^{\Updownarrow}\Phi^0$	$-D_{11}^{\Updownarrow}f^1+D_{01}f^0=\Phi^0$

$\Downarrow$

$\begin{pmatrix}J_+^1\\J_-^1\end{pmatrix}=-\begin{pmatrix}D_{02}^+&0\\0&D_{02}^-\end{pmatrix}\begin{pmatrix}\Phi_+^1\\\Phi_-^1\end{pmatrix}-\begin{pmatrix}D_{12}^+&0\\0&D_{12}^-\end{pmatrix}\begin{pmatrix}\Phi_+^0\\\Phi_-^0\end{pmatrix}$	$-\begin{pmatrix}D_{01}^-&0\\0&D_{01}^+\end{pmatrix}\begin{pmatrix}f_+^1\\f_-^1\end{pmatrix}-\begin{pmatrix}D_{11}^+&0\\0&D_{11}^-\end{pmatrix}\begin{pmatrix}f_+^0\\f_-^0\end{pmatrix}=\begin{pmatrix}\Phi_+^1\\\Phi_-^1\end{pmatrix}$
$\begin{pmatrix}J_+^0\\J_-^0\end{pmatrix}=-\begin{pmatrix}D_{12}^-&0\\0&D_{12}^+\end{pmatrix}\begin{pmatrix}\Phi_+^1\\\Phi_-^1\end{pmatrix}+\begin{pmatrix}D_{02}^-&0\\0&D_{02}^+\end{pmatrix}\begin{pmatrix}\Phi_+^0\\\Phi_-^0\end{pmatrix}$	$-\begin{pmatrix}D_{11}^-&0\\0&D_{11}^+\end{pmatrix}\begin{pmatrix}f_+^1\\f_-^1\end{pmatrix}+\begin{pmatrix}D_{01}^+&0\\0&D_{01}^-\end{pmatrix}\begin{pmatrix}f_+^0\\f_-^0\end{pmatrix}=\begin{pmatrix}\Phi_+^0\\\Phi_-^0\end{pmatrix}$

$\Downarrow$

$J_+^1=-D_{02}^+\Phi_+^1-D_{12}^+\Phi_+^0$	$-D_{01}^-f_+^1-D_{11}^+f_+^0=\Phi_+^1$
$J_-^1=-D_{02}^-\Phi_-^1-D_{12}^-\Phi_-^0$	$-D_{01}^+f_-^1-D_{11}^-f_-^0=\Phi_-^1$
$J_+^0=-D_{12}^-\Phi_+^1+D_{02}^-\Phi_+^0$	$-D_{11}^-f_+^1+D_{01}^+f_+^0=\Phi_+^0$
$J_-^0=-D_{12}^+\Phi_-^1+D_{02}^+\Phi_-^0$	$-D_{11}^+f_-^1+D_{01}^-f_-^0=\Phi_-^0$

$\Downarrow$

$J_+^1=-\partial_0\Phi_+^1-g_{0j2}^1\Phi_+^j-\partial_1\Phi_+^0-g_{1j2}^0\Phi_+^j$	$-\partial_0 f_+^1+g_{0j1}^1f_+^{j1}-\partial_i f_+^0-g_{1j1}^0f_+^j=\Phi_+^1$
$J_-^1=-\partial_0\Phi_-^1+g_{0j2}^1\Phi_-^j-\partial_1\Phi_-^0+g_{1j2}^0\Phi_-^j$	$-\partial_0 f_-^1-g_{0j1}^1f_-^j-\partial_1 f_-^0+g_{1j1}^0f_-^j=\Phi_-^1$
$J_+^0=-\partial_1\Phi_+^1+g_{1j2}^1\Phi_+^j+\partial_0\Phi_+^0-g_{0j2}^0\Phi_+^j$	$-\partial_1 f_+^1+g_{1j1}^1f_+^j+\partial_0 f_+^0+g_{0j1}^0f_+^j=\Phi_+^0$
$J_-^0=-\partial_1\Phi_-^1-g_{1j2}^1\Phi_-^j+\partial_0\Phi_+^0+g_{0j2}^0\Phi_-^j$	$-\partial_1 f_-^1-g_{1j1}^1f_-^j+\partial_0 f_-^0-g_{0j1}^0f_-^j=\Phi_-^0$

$\Downarrow$

$J_+^1=-\partial_0\Phi_+^1-\partial_1\Phi_+^0+(-g_{0j2}^1-g_{1j2}^0)\Phi_+^j$	$-\partial_0 f_+^1-\partial_i f_+^0+(+g_{1j1}^0-g_{0j1}^1)f_+^j=\Phi_+^1$
$J_-^1=-\partial_0\Phi_-^1-\partial_1\Phi_-^0+(+g_{0j2}^1+g_{1j2}^0)\Phi_-^j$	$-\partial_0 f_-^1-\partial_1 f_-^0+(-g_{0j1}^1+g_{1j1}^0)f_-^j=\Phi_-^1$
$J_+^0=-\partial_1\Phi_+^1+\partial_0\Phi_+^0+(+g_{1j2}^1-g_{0j2}^0)\Phi_+^j$	$-\partial_1 f_+^1+\partial_0 f_+^0+(+g_{1j1}^1+g_{0j1}^0)f_+^j=\Phi_+^0$
$J_-^0=-\partial_1\Phi_-^1+\partial_0\Phi^0+(-g_{1j2}^1+g_{0j2}^0)\Phi_-^j$	$-\partial_1 f_-^1+\partial_0 f_-^0+(-g_{1j1}^1-g_{0j1}^0)f_-^j=\Phi_-^0$

$\Downarrow$

$\mathbf{J}^1 = -\partial_0\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^0 + (-g^1_{0j2} - g^0_{1j2})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{\Phi}^j$	$-\partial_0\mathbf{f}^1 - \partial_i\mathbf{f}^0 + (+g^0_{1j1} - g^1_{0j1})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{f}^j = \mathbf{\Phi}^1$
$\mathbf{J}^0 = -\partial_1\mathbf{\Phi}^1 + \partial_0\mathbf{\Phi}^0 + (+g^1_{1j2} - g^0_{0j2})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{\Phi}^j$	$-\partial_1\mathbf{f}^1 + \partial_0\mathbf{f}^0 + (+g^1_{1j1} + g^0_{0j1})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{f}^j = \mathbf{\Phi}^1$

□

Corollary I.2 For differentiable functions $f^i_+, f^i_-, g^i_{jhk}; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$:

Given theorem I.1;

Whenever $g^i_{jhk} = g^j_{ihk} \quad ; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$

$$\Rightarrow J^m_\sigma = (\partial^2_0 + \partial^2_1)f^m_\sigma + (\sigma(-1)^m[+g^0_{01h} + g^1_{11h}] - [+g^0_{02h} + g^1_{12h}])\partial_m f^h_\sigma + \\ + \left(\sigma(-1)^m\partial_m[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} + (-1)^m g^1_{02k}]g^k_{01h} + \left[(-1)^{1-m}g^0_{02k} - g^1_{12k}\right]g^k_{11h}\right)f^h_\sigma$$

$$(m \in \{0,1\}; \sigma \in \{+,-\})$$

Proof:

$$\mathbf{J} = \begin{pmatrix} \begin{pmatrix} (\partial^2_0 + \partial^2_1)f^1_+ + \\ +([-g^0_{01h} - g^1_{11h}] + [+g^0_{02h} + g^1_{12h}])\partial_1 f^h_+ + \\ +(\partial_1[-g^0_{01h} - g^1_{11h}] + [-g^0_{12k} - g^1_{02k}]g^k_{01h} + [+g^0_{02k} - g^1_{12k}]g^k_{11h})f^h_+ \end{pmatrix} \\ \begin{pmatrix} (\partial^2_0 + \partial^2_1)f^1_- + \\ +([+g^0_{01h} + g^1_{11h}] + [-g^0_{02k} - g^1_{12k}])\partial_1 f^h_- + \\ +(\partial_1[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} - g^1_{02k}]g^k_{01h} + [+g^0_{02k} - g^1_{12k}]g^k_{11h})f^h_- \end{pmatrix} \\ \\ \begin{pmatrix} (\partial^2_1 + \partial^2_0)f^0_+ + \\ +([+g^0_{01h} + g^1_{11h}] + [-g^0_{02k} - g^1_{12k}])\partial_0 f^h_+ + \\ +(+\partial_0[+g^0_{01h} + g^1_{11h}] + [-g^0_{02k} + g^1_{12k}]g^k_{01h} + [-g^0_{12k} - g^1_{02k}]g^k_{11h})f^h_+ \end{pmatrix} \\ \begin{pmatrix} (\partial^2_1 + \partial^2_0)f^0_- + \\ +([-g^0_{01h} - g^1_{11h}] + [+g^0_{02k} + g^\lambda_{12k}])\partial_0 f^h_- + \\ +(\partial_0[-g^0_{01h} - g^1_{11h}] + [-g^0_{02k} + g^\lambda_{12k}]g^k_{01h} + [-g^0_{12k} - g^\lambda_{02k}]g^k_{11h})f^h_- \end{pmatrix} \end{pmatrix}$$

m/σ	1	0
+	$(-[+g^0_{01h} + g^1_{11h}] - [+g^0_{02h} + g^1_{12h}])\partial_1 f^h_+$	$(+[+g^0_{01h} + g^1_{11h}] - [+g^0_{02k} + g^1_{12k}])\partial_0 f^h_+$
−	$(+[+g^0_{01h} + g^1_{11h}] - [+g^0_{02k} + g^1_{12k}])\partial_1 f^h_-$	$(-[+g^0_{01h} + g^1_{11h}] - [+g^0_{02k} + g^\lambda_{12k}])\partial_0 f^h_-$

$$\Rightarrow (\sigma(-1)^m[+g^0_{01h} + g^1_{11h}] - [+g^0_{02h} + g^1_{12h}])\partial_m f^h_\sigma$$

m/σ	1	0
+	$(-\partial_1[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} - g^1_{02k}]g^k_{01h} + [+g^0_{02k} - g^1_{12k}]g^k_{11h})f^h_+$	$(+\partial_0[+g^0_{01h} + g^1_{11h}] + [-g^0_{02k} + g^1_{12k}]g^k_{01h} + [-g^0_{12k} - g^\lambda_{02k}]g^k_{11h})f^h_+$
−	$(+\partial_1[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} - g^1_{02k}]g^k_{01h} + [+g^0_{02k} - g^1_{12k}]g^k_{11h})f^h_-$	$(-\partial_0[+g^0_{01h} + g^1_{11h}] + [-g^0_{02k} + g^\lambda_{12k}]g^k_{01h} + [-g^0_{12k} - g^\lambda_{02k}]g^k_{11h})f^h_-$

$$\Rightarrow \left(\sigma(-1)^m\partial_m[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} + (-1)^m g^1_{02k}]g^k_{01h} + \left[(-1)^{1-m}g^0_{02k} - g^1_{12k}\right]g^k_{11h}\right)f^h_\sigma$$

$$\Rightarrow J^m_\sigma = (\partial^2_0 + \partial^2_1)f^m_\sigma + (\sigma(-1)^m[+g^0_{01h} + g^1_{11h}] - [+g^0_{02h} + g^1_{12h}])\partial_m f^h_\sigma + \\ + \left(\sigma(-1)^m\partial_m[+g^0_{01h} + g^1_{11h}] + [-g^0_{12k} + (-1)^m g^1_{02k}]g^k_{01h} + \left[(-1)^{1-m}g^0_{02k} - g^1_{12k}\right]g^k_{11h}\right)f^h_\sigma$$

□

Corollary I.2 For differentiable functions $f^i_+, f^i_-, g^i_{jhk}; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$:

Given theorem I.1;

Whenever $g^i_{jhk} = \delta^i_h g_{jk} \quad ; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$

$$\Rightarrow \mathbf{J} = \begin{pmatrix} \begin{pmatrix} (\partial_0^2+\partial_1^2)f_+^1 + \\ +(+g_{11}-g_{12})\partial_0 f_+^0 + (-g_{01}+g_{02})\partial_0 f_+^1 + \\ +(-g_{01}+g_{02})\partial_1 f_+^0 + (-g_{11}+g_{12})\partial_1 f_+^1 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01})-g_{12}g_{01}+g_{02}g_{11})f_+^0 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-g_{02}g_{01}-g_{12}g_{11})f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_0^2+\partial_1^2)f_-^1 + \\ +(-g_{11}+g_{12})\partial_0 f_-^0 + (+g_{01}-g_{02})\partial_0 f_-^1 + \\ +(+g_{01}-g_{02})\partial_1 f_-^0 + (+g_{11}-g_{12})\partial_1 f_-^1 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01})-g_{12}g_{01}+g_{02}g_{11})f_-^0 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-g_{02}g_{01}-g_{12}g_{11})f_-^1 \end{pmatrix} \\ \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_+^0 + \\ +(+g_{01}-g_{02})\partial_0 f_+^0 + (+g_{11}-g_{12})\partial_0 f_+^1 + \\ +(+g_{11}-g_{12})\partial_1 f_+^0 + (-g_{01}+g_{02})\partial_1 f_+^1 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-g_{02}g_{01}-g_{12}g_{11})f_+^0 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01})+g_{12}g_{01}-g_{02}g_{11})f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_-^0 + \\ +(-g_{01}+g_{02})\partial_0 f_-^0 + (-g_{11}+g_{12})\partial_0 f_-^1 + \\ +(-g_{11}+g_{11})\partial_1 f_-^0 + (+g_{01}-g_{02})\partial_1 f_-^1 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-g_{02}g_{01}-g_{12}g_{11})f_-^0 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01})+g_{12}g_{01}-g_{02}g_{11})f_-^1 \end{pmatrix} \end{pmatrix}$$

Proof:

$$\text{Whenever } g^i_{jhk} = \delta^i_h g_{jk} \quad ;\forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$$

$$\mathbf{J} = \begin{pmatrix} \begin{pmatrix} (\partial_0^2+\partial_1^2)f_+^1 + \\ +\partial_0([+\delta^0_h g_{11}-\delta^1_h g_{01}]f_+^h) + \\ +\partial_1([-\delta^0_h g_{01}-\delta^1_h g_{11}]f_+^h) + \\ +(-\delta^0_h g_{12}+\delta^1_h g_{02})\partial_0 f_+^h + \\ +(+\delta^0_h g_{02}+\delta^1_h g_{12})\partial_1 f_+^h + \\ ([-\delta^0_k g_{12}-\delta^1_k g_{02}]\delta^k_h g_{01}+[+\delta^0_k g_{02}-\delta^1_k g_{12}]\delta^k_h g_{11})f_+^h \end{pmatrix} \\ \begin{pmatrix} (\partial_0^2+\partial_1^2)f_-^1 + \\ +\partial_0([-\delta^0_h g_{11}+\delta^1_h g_{01}]f_-^h) + \\ +\partial_1([+\delta^0_h g_{01}+\delta^1_h g_{11}]f_-^h) + \\ +(+\delta^0_h g_{12}-\delta^1_h g_{02})\partial_0 f_-^h + \\ +(-\delta^0_h g_{02}-\delta^1_h g_{12})\partial_1 f_-^h + \\ ([-\delta^0_k g_{12}-\delta^1_k g_{02}]\delta^k_h g_{01}+[+\delta^0_k g_{02}-\delta^1_k g_{12}]\delta^k_h g_{11})f_-^h \end{pmatrix} \\ \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_+^0 + \\ +\partial_0([+\delta^0_h g_{01}+\delta^1_h g_{11}]f_+^h) + \\ +\partial_1([+\delta^0_h g_{11}-\delta^1_h g_{01}]f_+^h) + \\ +[-\delta^0_h g_{02}-\delta^1_h g_{12}]\partial_0 f_+^h + \\ +[-\delta^0_h g_{12}+\delta^1_h g_{02}]\partial_1 f_+^h + \\ +([-\delta^0_k g_{02}+\delta^1_k g_{12}]\delta^k_h g_{01}+[-\delta^0_k g_{12}-\delta^1_k g_{02}]\delta^k_h g_{11})f_+^h \end{pmatrix} \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_-^0 + \\ +\partial_0([-\delta^0_h g_{01}-\delta^1_h g_{11}]f_-^h) + \\ +\partial_1([-\delta^0_h g_{11}+\delta^1_h g_{01}]f_-^h) + \\ +[+\delta^0_h g_{02}+\delta^1_h g_{12}]\partial_0 f_-^h + \\ +[+\delta^0_h g_{11}-\delta^1_h g_{02}]\partial_1 f_-^h + \\ +([-\delta^0_k g_{02}+\delta^1_k g_{12}]\delta^k_h g_{01}+[-\delta^0_k g_{12}-\delta^1_k g_{02}]\delta^k_h g_{11})f_-^h \end{pmatrix} \end{pmatrix}$$

$$
\begin{aligned}
&= \begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_1^2)f_+^1 + \\
+\partial_0(+g_{11}f_+^0 - g_{01}f_+^1) + \\
+\partial_1(-g_{01}f_+^0 - g_{11}f_+^1) + \\
-g_{12}\partial_0 f_+^0 + g_{02}\partial_0 f_+^1 + \\
+g_{02}\partial_1 f_+^0 + g_{12}\partial_1 f_+^1 + \\
-g_{12}g_{01}f_+^0 - g_{02}g_{01}f_+^1 + g_{02}g_{11}f_+^0 - g_{12}g_{11}f_+^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_1^2)f_-^1 + \\
+\partial_0(-g_{11}f_-^0 + g_{01}f_-^1) + \\
+\partial_1(+g_{01}f_-^0 + g_{11}f_-^1) + \\
+g_{12}\partial_0 f_-^0 - g_{02}\partial_0 f_-^1 + \\
-g_{02}\partial_1 f_-^0 - g_{12}\partial_1 f_-^1 + \\
-g_{12}g_{01}f_-^0 - g_{02}g_{01}f_-^1 + g_{02}g_{11}f_-^0 - g_{12}g_{11}f_-^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_0^2)f_+^0 + \\
+\partial_0(+g_{01}f_+^0 + g_{11}f_+^1) + \\
+\partial_1(+g_{11}f_+^0 - g_{01}f_+^1) + \\
-g_{02}\partial_0 f_+^0 - g_{12}\partial_0 f_+^1 + \\
-g_{12}\partial_1 f_+^0 + g_{02}\partial_1 f_+^1 + \\
-g_{02}g_{01}f_+^0 + g_{12}g_{01}f_+^1 - g_{12}g_{11}f_+^0 - g_{02}g_{11}f_+^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_0^2)f_-^0 + \\
+\partial_0(-g_{01}f_-^0 - g_{11}f_-^1) + \\
+\partial_1(-g_{11}f_-^0 + g_{01}f_-^1) + \\
+g_{02}\partial_0 f_-^0 + g_{12}\partial_0 f_-^1 + \\
+g_{11}\partial_1 f_-^0 - g_{02}\partial_1 f_-^1 + \\
-g_{02}g_{01}f_-^0 + g_{12}g_{01}f_-^1 + -g_{12}g_{11}f_-^0 - g_{02}g_{11}f_-^1
\end{pmatrix}
\end{pmatrix} \\
&= \begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_1^2)f_+^1 + \\
+(+g_{11}-g_{12})\partial_0 f_+^0 + (-g_{01}+g_{02})\partial_0 f_+^1 + \\
+(-g_{01}+g_{02})\partial_1 f_+^0 + (-g_{11}+g_{12})\partial_1 f_+^1 + \\
+(+\partial_0(g_{11}) - \partial_1(g_{01}) - g_{12}g_{01} + g_{02}g_{11})f_+^0 + \\
+(-\partial_0(g_{01}) - \partial_1(g_{11}) - g_{02}g_{01} - g_{12}g_{11})f_+^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_1^2)f_-^1 + \\
+(-g_{11}+g_{12})\partial_0 f_-^0 + (+g_{01}-g_{02})\partial_0 f_-^1 + \\
+(+g_{01}-g_{02})\partial_1 f_-^0 + (+g_{11}-g_{12})\partial_1 f_-^1 + \\
+(-\partial_0(g_{11}) + \partial_1(g_{01}) - g_{12}g_{01} + g_{02}g_{11})f_-^0 + \\
+(+\partial_0(g_{01}) + \partial_1(g_{11}) - g_{02}g_{01} - g_{12}g_{11})f_-^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_0^2)f_+^0 + \\
+(+g_{01}-g_{02})\partial_0 f_+^0 + (+g_{11}-g_{12})\partial_0 f_+^1 + \\
+(+g_{11}-g_{12})\partial_1 f_+^0 + (-g_{01}+g_{02})\partial_1 f_+^1 + \\
+(+\partial_0(g_{01}) + \partial_1(g_{11}) - g_{02}g_{01} - g_{12}g_{11})f_+^0 + \\
+(+\partial_0(g_{11}) - \partial_1(g_{01}) + g_{12}g_{01} - g_{02}g_{11})f_+^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_0^2)f_-^0 + \\
+(-g_{01}+g_{02})\partial_0 f_-^0 + (-g_{11}+g_{12})\partial_0 f_-^1 + \\
+(-g_{11}+g_{11})\partial_1 f_-^0 + (+g_{01}-g_{02})\partial_1 f_-^1 + \\
+(-\partial_0(g_{01}) - \partial_1(g_{11}) - g_{02}g_{01} - g_{12}g_{11})f_-^0 + \\
+(-\partial_0(g_{11}) + \partial_1(g_{01}) + g_{12}g_{01} - g_{02}g_{11})f_-^1
\end{pmatrix}
\end{pmatrix}
\end{aligned}
$$

□

Corollary I.3 For differentiable functions $f_+^i, f_-^i, g_{jhk}^i; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$:
Given theorem I.1;
Whenever $g_{jhk}^i = \delta_h^i g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$

$$\Rightarrow \mathbf{J} = \begin{pmatrix} \begin{pmatrix} (\partial_0^2+\partial_1^2)f_+^1 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^0 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_0^2+\partial_1^2)f_-^1 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^0 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_-^1 \end{pmatrix} \\ \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_+^0 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_+^0 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_-^0 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_-^0 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^1 \end{pmatrix} \end{pmatrix}$$

Proof:

Given theorem I.1;

Whenever $g_{jhk}^i = \delta_h^i g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$

$$\Rightarrow \mathbf{J} = \begin{pmatrix} \begin{pmatrix} (\partial_0^2+\partial_1^2)f_+^1 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^0 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_0^2+\partial_1^2)f_-^1 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^0 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_-^1 \end{pmatrix} \\ \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_+^0 + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_+^0 + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^1 \end{pmatrix} \\ \begin{pmatrix} (\partial_1^2+\partial_0^2)f_-^0 + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-(g_{01}^2+g_{11}^2))f_-^0 + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^1 \end{pmatrix} \end{pmatrix}$$

□

Corollary I.4 For differentiable functions $f_+^i, f_-^i, g_{jhk}^i; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$:

Given theorem I.1;

Whenever $g_{jhk}^i = \delta_h^i g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1\} \quad , \quad \forall k \in \{1,2\}$

$$AND \quad \partial_0(g_{11}) = \partial_1(g_{01})$$

$$\Rightarrow \mathbf{J} = \begin{pmatrix} \left((\partial_0^2+\partial_1^2)f_+^1 - \left(e^{-\int g_{01}\partial x^0}\partial_0^2\left(g_{01}e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1^2\left(g_{11}e^{\int g_{11}\partial x^1}\right)\right)f_+^1\right) \\ \left((\partial_0^2+\partial_1^2)f_-^1 - \left(e^{\int g_{01}\partial x^0}\partial_0^2\left(g_{01}e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1^2\left(g_{11}e^{-\int g_{11}\partial x^1}\right)\right)f_-^1\right) \\ \\ \left((\partial_1^2+\partial_0^2)f_+^0 - \left(e^{\int g_{01}\partial x^0}\partial_0\left(e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1^2\left(e^{-\int g_{11}\partial x^1}\right)\right)f_+^0\right) \\ \left((\partial_1^2+\partial_0^2)f_-^0 - \left(e^{-\int g_{01}\partial x^0}\partial_0^2\left(e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1^2\left(e^{\int g_{11}\partial x^1}\right)\right)f_-^0\right) \end{pmatrix}$$

Proof:

$$\Rightarrow \mathbf{J} = \begin{pmatrix} ((\partial_0^2+\partial_1^2)f_+^1 + (-\partial_0(g_{01})-\partial_1(g_{11})-g_{01}^2-g_{11}^2)f_+^1) \\ ((\partial_0^2+\partial_1^2)f_-^1 + (+\partial_0(g_{01})+\partial_1(g_{11})-g_{01}^2-g_{11}^2)f_-^1) \\ \\ ((\partial_1^2+\partial_0^2)f_+^0 + (+\partial_0(g_{01})+\partial_1(g_{11})-g_{01}^2-g_{11}^2)f_+^0 +) \\ ((\partial_1^2+\partial_0^2)f_-^0 + (-\partial_0(g_{01})-\partial_1(g_{11})-g_{01}^2-g_{11}^2)f_-^0 +) \end{pmatrix}$$

$$
= \begin{pmatrix}
\left((\partial_0^2+\partial_1^2)f_+^1 - \left(e^{-\int g_{01}\partial x^0}\partial_0\left(g_{01}e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1\left(g_{11}e^{\int g_{11}\partial x^1}\right)\right)f_+^1\right) \\
\left((\partial_0^2+\partial_1^2)f_-^1 + \left(e^{\int g_{01}\partial x^0}\partial_0\left(g_{01}e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1\left(g_{11}e^{-\int g_{11}\partial x^1}\right)\right)f_-^1\right) \\
\\
\left((\partial_1^2+\partial_0^2)f_+^0 + \left(e^{\int g_{01}\partial x^0}\partial_0\left(g_{01}e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1\left(g_{11}e^{-\int g_{11}\partial x^1}\right)\right)f_+^0\right) \\
\left((\partial_1^2+\partial_0^2)f_-^0 - \left(e^{-\int g_{01}\partial x^0}\partial_0\left(g_{01}e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1\left(g_{11}e^{\int g_{11}\partial x^1}\right)\right)f_-^0\right)
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\left((\partial_0^2+\partial_1^2)f_+^1 - \left(e^{-\int g_{01}\partial x^0}\partial_0^2\left(g_{01}e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1^2\left(g_{11}e^{\int g_{11}\partial x^1}\right)\right)f_+^1\right) \\
\left((\partial_0^2+\partial_1^2)f_-^1 - \left(e^{\int g_{01}\partial x^0}\partial_0^2\left(g_{01}e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1^2\left(g_{11}e^{-\int g_{11}\partial x^1}\right)\right)f_-^1\right) \\
\\
\left((\partial_1^2+\partial_0^2)f_+^0 - \left(e^{\int g_{01}\partial x^0}\partial_0\left(e^{-\int g_{01}\partial x^0}\right) + e^{\int g_{11}\partial x^1}\partial_1^2\left(e^{-\int g_{11}\partial x^1}\right)\right)f_+^0\right) \\
\left((\partial_1^2+\partial_0^2)f_-^0 - \left(e^{-\int g_{01}\partial x^0}\partial_0^2\left(e^{\int g_{01}\partial x^0}\right) + e^{-\int g_{11}\partial x^1}\partial_1^2\left(e^{\int g_{11}\partial x^1}\right)\right)f_-^0\right)
\end{pmatrix}
$$

□

Corollary I.5 For differentiable functions $f_+^i, f_-^i, g_{jhk}^i; \forall i,j,h \in \{0,1\}\ ,\ \forall k \in \{1,2\}$:

Given theorem I.1;

Whenever $g_{jhk}^i = \delta_h^i g_{jk}$ *AND* $g_{j2} = g_{j1}$ $; \forall i,j,h \in \{0,1\}\ ,\ \forall k \in \{1,2\}$

AND g_{01} *AND* g_{11} are constants

$$
\Rightarrow \mathbf{J} = \begin{pmatrix}
((\partial_0^2+\partial_1^2)f_+^1 - (g_{01}^2+g_{11}^2)f_+^1) \\
((\partial_0^2+\partial_1^2)f_-^1 - (g_{01}^2+g_{11}^2)f_-^1) \\
\\
((\partial_1^2+\partial_0^2)f_+^0 - (g_{01}^2+g_{11}^2)f_+^0) \\
((\partial_1^2+\partial_0^2)f_-^0 - (g_{01}^2+g_{11}^2)f_-^0)
\end{pmatrix} \Rightarrow J_\sigma^m = [(\partial_0^2+\partial_1^2) - (g_{01}^2+g_{11}^2)]f_\sigma^m
$$

$$
; \forall m \in \{0,1\}\ ,\ \forall \sigma \in \{+,-\}
$$

□

NOTE: Even at the two dimensional lavel, it does not take a vivid imagination to envision corollaries I.2 thru I.5 yielding particle oscillation and zero mass particles with non-zero "rest-mass".

The previous theorem leads to a four-dimensional version.
Consider (For the 3-D space-1-D time situation):

$$
\begin{pmatrix}
-D_{02} & D_{32}^{\rightleftarrows} & -D_{22}^{\rightleftarrows} & -D_{12} \\
-D_{32}^{\rightleftarrows} & -D_{02} & D_{12}^{\rightleftarrows} & -D_{22} \\
D_{22}^{\rightleftarrows} & -D_{12}^{\rightleftarrows} & -D_{02} & -D_{32} \\
-D_{12}^{\Updownarrow} & -D_{22}^{\Updownarrow} & -D_{32}^{\Updownarrow} & D_{02}^{\Updownarrow}
\end{pmatrix}
\begin{pmatrix}
-D_{01}^{\Updownarrow} & -D_{31}^{\rightleftarrows} & D_{21}^{\rightleftarrows} & -D_{11} \\
D_{31}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{11}^{\rightleftarrows} & -D_{21} \\
-D_{21}^{\rightleftarrows} & D_{11}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{31} \\
-D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01}
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

for the 3-D space - 1-D time version.

Theorem II.1: For differentiable functions $\Phi, f^l, f_+^l, f_-^l, g_{ij}; \forall i,j \in \mathbb{N}$:

If: $\exists \mathbf{J}(x_3,x_2,x_1,x_0) \ni$

$$
\mathbf{J}(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix}
-D_{02} & D_{32}^{\rightleftarrows} & -D_{22}^{\rightleftarrows} & -D_{12} \\
-D_{32}^{\rightleftarrows} & -D_{02} & D_{12}^{\rightleftarrows} & -D_{22} \\
D_{22}^{\rightleftarrows} & -D_{12}^{\rightleftarrows} & -D_{02} & -D_{32} \\
-D_{12}^{\Updownarrow} & -D_{22}^{\Updownarrow} & -D_{32}^{\Updownarrow} & D_{02}^{\Updownarrow}
\end{pmatrix}
\begin{pmatrix}
-D_{01}^{\Updownarrow} & -D_{31}^{\rightleftarrows} & D_{21}^{\rightleftarrows} & -D_{11} \\
D_{31}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{11}^{\rightleftarrows} & -D_{21} \\
-D_{21}^{\rightleftarrows} & D_{11}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{31} \\
-D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01}
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

and

$$
\exists \Phi(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix}
-D_{01}^{\Updownarrow} & -D_{31}^{\rightleftarrows} & D_{21}^{\rightleftarrows} & -D_{11} \\
D_{31}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{11}^{\rightleftarrows} & -D_{21} \\
-D_{21}^{\rightleftarrows} & D_{11}^{\rightleftarrows} & -D_{01}^{\Updownarrow} & -D_{31} \\
-D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01}
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

where:

$$
D_{ij}^{+} \equiv (\partial_i + g_{ijh}^{\lambda})\ \ ,\ \ D_{ij}^{-} \equiv (\partial_i - g_{ijh}^{\lambda}) \qquad [\text{ where: } D_{ik}^{\pm} f_\sigma^m = (\partial_i \pm g_{ijk}^{\lambda}) f_\sigma^m = \partial_i f_\sigma^m \pm g_{ijk}^m f_\sigma^j \text{ }]
$$

$$D_{ij} \equiv \begin{pmatrix} D^{+}_{ij} & 0 \\ 0 & D^{-}_{ij} \end{pmatrix} \ , \ D^{\Updownarrow}_{ij} \equiv \begin{pmatrix} D^{-}_{ij} & 0 \\ 0 & D^{+}_{ij} \end{pmatrix} \ ,$$

$$D^{\Leftrightarrow}_{ij} \equiv \begin{pmatrix} 0 & D^{-}_{ij} \\ D^{+}_{ij} & 0 \end{pmatrix} \ , \ D^{\Leftrightarrow\Updownarrow}_{ij} \equiv \begin{pmatrix} 0 & D^{+}_{ij} \\ D^{-}_{ij} & 0 \end{pmatrix}$$

and:

$$J^{i} \equiv \begin{pmatrix} J^{i}_{+} \\ J^{i}_{-} \end{pmatrix} \ , \ \Phi^{i} \equiv \begin{pmatrix} \Phi^{i}_{+} \\ \Phi^{i}_{-} \end{pmatrix} \ , \ f^{i} \equiv \begin{pmatrix} f^{i}_{+} \\ f^{i}_{-} \end{pmatrix}$$

then:

$$\begin{pmatrix} -D_{02}\Phi^{1} + D^{\Leftrightarrow}_{32}\Phi^{2} - D^{\Leftrightarrow}_{22}\Phi^{3} - D_{12}\Phi^{0} \\ -D^{\Leftrightarrow}_{32}\Phi^{1} - D_{02}\Phi^{2} + D^{\Leftrightarrow}_{12}\Phi^{3} - D_{22}\Phi^{0} \\ D^{\Leftrightarrow}_{22}\Phi^{1} - D^{\Leftrightarrow}_{12}\Phi^{2} - D_{02}\Phi^{3} - D_{32}\Phi^{0} \\ -D^{\Updownarrow}_{12}\Phi^{1} - D^{\Updownarrow}_{22}\Phi^{2} - D^{\Updownarrow}_{32}\Phi^{3} + D^{\Updownarrow}_{02}\Phi^{0} \end{pmatrix} = \begin{pmatrix} J^{1} \\ J^{2} \\ J^{3} \\ J^{0} \end{pmatrix} \quad \& \quad \begin{pmatrix} -D^{\Updownarrow}_{01}f^{1} - D^{\Leftrightarrow}_{31}f^{2} + D^{\Leftrightarrow}_{21}f^{3} - D_{11}f^{0} \\ D^{\Leftrightarrow}_{31}f^{1} - D^{\Updownarrow}_{01}f^{2} - D^{\Leftrightarrow}_{11}f^{3} - D_{21}f^{0} \\ -D^{\Leftrightarrow}_{21}f^{1} + D^{\Leftrightarrow}_{11}f^{2} - D^{\Updownarrow}_{01}f^{3} - D_{31}f^{0} \\ -D^{\Updownarrow}_{11}f^{1} - D^{\Updownarrow}_{21}f^{2} - D^{\Updownarrow}_{31}f^{3} + D_{01}f^{0} \end{pmatrix} \equiv \begin{pmatrix} \Phi^{1} \\ \Phi^{2} \\ \Phi^{3} \\ \Phi^{0} \end{pmatrix}$$

and:

$J^{1}_{+} = -\partial_0\Phi^{1}_{+} - \partial_1\Phi^{0}_{+} + \partial_3\Phi^{2}_{-} - \partial_2\Phi^{3}_{-} + (-g^{1}_{0j2} - g^{0}_{1j2})\Phi^{j}_{+} + (-g^{2}_{3j2} + g^{3}_{2j2})\Phi^{j}_{-}$	$-\partial_0 f^{1}_{+} - \partial_1 f^{0}_{+} - \partial_3 f^{2}_{-} + \partial_2 f^{3}_{-} + (+g^{1}_{0j1} - g^{0}_{1j1})f^{j}_{+} + (+g^{2}_{3j1} - g^{3}_{2j1})f^{j}_{-} = \Phi^{1}_{+}$
$J^{1}_{-} = -\partial_0\Phi^{1}_{-} - \partial_1\Phi^{0}_{-} + \partial_3\Phi^{2}_{+} - \partial_2\Phi^{3}_{+} + (+g^{2}_{3j2} - g^{3}_{2j2})\Phi^{j}_{+} + (+g^{1}_{0j2} + g^{0}_{1j2})\Phi^{j}_{-}$	$-\partial_0 f^{1}_{-} - \partial_1 f^{0}_{-} - \partial_3 f^{2}_{+} + \partial_2 f^{3}_{+} + (-g^{2}_{3j1} + g^{3}_{2j1})f^{j}_{+} + (-g^{1}_{0j1} + g^{0}_{1j1})f^{j}_{-} = \Phi^{1}_{-}$
$J^{2}_{+} = -\partial_0\Phi^{2}_{+} - \partial_2\Phi^{0}_{+} + \partial_3\Phi^{1}_{-} + \partial_1\Phi^{3}_{-} + (-g^{2}_{0j2} - g^{0}_{2j2})\Phi^{j}_{+} + (-g^{1}_{3j2} - g^{3}_{1j2})\Phi^{j}_{-}$	$-\partial_0 f^{2}_{+} - \partial_2 f^{0}_{+} + \partial_3 f^{1}_{-} - \partial_1 f^{3}_{-} + (-g^{0}_{2j1} + g^{2}_{0j1})f^{j}_{+} + (-g^{1}_{3j1} + g^{3}_{1j1})f^{j}_{-} = \Phi^{2}_{+}$
$J^{2}_{-} = -\partial_0\Phi^{2}_{-} - \partial_2\Phi^{0}_{-} + \partial_3\Phi^{1}_{+} + \partial_1\Phi^{3}_{+} + (+g^{1}_{3j2} + g^{3}_{1j2})\Phi^{j}_{+} + (+g^{2}_{0j2} + g^{0}_{2j2})\Phi^{j}_{-}$	$-\partial_0 f^{2}_{-} - \partial_2 f^{0}_{-} + \partial_3 f^{1}_{+} - \partial_1 f^{3}_{+} + (+g^{1}_{3j1} - g^{3}_{1j1})f^{j}_{+} + (-g^{2}_{0j1} + g^{0}_{2j1})f^{j}_{-} = \Phi^{2}_{-}$
$J^{3}_{+} = -\partial_0\Phi^{3}_{+} - \partial_3\Phi^{0}_{+} + \partial_2\Phi^{1}_{-} - \partial_1\Phi^{2}_{-} + (-g^{3}_{0j2} - g^{0}_{3j2})\Phi^{j}_{+} + (-g^{1}_{2j2} + g^{2}_{1j2})\Phi^{j}_{-}$	$-\partial_0 f^{3}_{+} - \partial_3 f^{0}_{+} - \partial_2 f^{1}_{-} + \partial_1 f^{2}_{-} + (-g^{0}_{3j1} + g^{3}_{0j1})f^{j}_{+} + (+g^{1}_{2j1} - g^{2}_{1j1})f^{j}_{-} = \Phi^{3}_{+}$
$J^{3}_{-} = -\partial_0\Phi^{3}_{-} - \partial_3\Phi^{0}_{-} + \partial_2\Phi^{1}_{+} - \partial_1\Phi^{2}_{+} + (+g^{1}_{2j2} - g^{2}_{1j2})\Phi^{j}_{+} + (+g^{3}_{0j2} + g^{0}_{3j2})\Phi^{j}_{-}$	$-\partial_0 f^{3}_{-} - \partial_3 f^{0}_{-} - \partial_2 f^{1}_{+} + \partial_1 f^{2}_{+} + (-g^{1}_{2j1} + g^{2}_{1j1})f^{j}_{+} + (+g^{0}_{3j1} - g^{3}_{0j1})f^{j}_{-} = \Phi^{3}_{-}$
$J^{0}_{+} = +\partial_0\Phi^{0}_{+} - \partial_1\Phi^{1}_{+} - \partial_2\Phi^{2}_{+} - \partial_3\Phi^{3}_{+} + (+g^{1}_{1j2} + g^{2}_{2j2} + g^{3}_{3j2} - g^{0}_{0j2})\Phi^{j}_{+}$	$+\partial_0 f^{0}_{+} - \partial_1 f^{1}_{+} - \partial_2 f^{2}_{+} - \partial_3 f^{3}_{+} + (+g^{0}_{0j1} + g^{1}_{1j1} + g^{2}_{2j1} + g^{3}_{3j1})f^{j}_{+} = \Phi^{0}_{+}$
$J^{0}_{-} = +\partial_0\Phi^{0}_{-} - \partial_1\Phi^{1}_{-} - \partial_2\Phi^{2}_{-} - \partial_3\Phi^{3}_{-} + (-g^{1}_{1j2} - g^{2}_{2j2} - g^{3}_{3j2} + g^{0}_{0j2})\Phi^{j}_{-}$	$+\partial_0 f^{0}_{-} - \partial_1 f^{1}_{-} - \partial_2 f^{2}_{-} - \partial_3 f^{3}_{-} + (-g^{0}_{0j1} - g^{1}_{1j1} - g^{2}_{2j1} - g^{3}_{3j1})f^{j}_{-} = \Phi^{0}_{-}$

or

$$\mathbf{J}^{1} = \begin{pmatrix} J^{1}_{+} \\ J^{1}_{-} \end{pmatrix} = -\partial_0\mathbf{\Phi}^{1} - \partial_1\mathbf{\Phi}^{0} + \partial_3\mathbf{\Phi}^{2} - \partial_2\mathbf{\Phi}^{3} + \left[(-g^{1}_{0j2} - g^{0}_{1j2}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g^{2}_{3j2} + g^{3}_{2j2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{\Phi}^{j}$$

$$\mathbf{J}^{2} = \begin{pmatrix} J^{2}_{+} \\ J^{2}_{-} \end{pmatrix} = -\partial_0\mathbf{\Phi}^{2} - \partial_2\mathbf{\Phi}^{0} + \partial_3\mathbf{\Phi}^{1} + \partial_1\mathbf{\Phi}^{3} + \left[(-g^{2}_{0j2} - g^{0}_{2j2}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g^{1}_{3j2} - g^{3}_{1j2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{\Phi}^{j}$$

$$\mathbf{J}^{3} = \begin{pmatrix} J^{3}_{+} \\ J^{3}_{-} \end{pmatrix} = -\partial_0\mathbf{\Phi}^{3} - \partial_3\mathbf{\Phi}^{0} + \partial_2\mathbf{\Phi}^{1} - \partial_1\mathbf{\Phi}^{2} + \left[(-g^{3}_{0j2} - g^{0}_{3j2}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g^{1}_{2j2} + g^{2}_{1j2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{\Phi}^{j}$$

$$\mathbf{J}^{0} = \begin{pmatrix} J^{0}_{+} \\ J^{0}_{-} \end{pmatrix} = +\partial_0\mathbf{\Phi}^{0} - \partial_1\mathbf{\Phi}^{1} - \partial_2\mathbf{\Phi}^{2} - \partial_3\mathbf{\Phi}^{3} + (+g^{1}_{1j2} + g^{2}_{2j2} + g^{3}_{3j2} - g^{0}_{0j2}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{\Phi}^{j}$$

$$-\partial_0\mathbf{f}^{1} - \partial_1\mathbf{f}^{0} - \partial_3\mathbf{f}^{2} + \partial_2\mathbf{f}^{3} + \left[(+g^{1}_{0j1} - g^{0}_{1j1}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (+g^{2}_{3j1} - g^{3}_{2j1}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{f}^{j} = \begin{pmatrix} \Phi^{1}_{+} \\ \Phi^{1}_{-} \end{pmatrix} = \mathbf{\Phi}^{1}$$

$$-\partial_0\mathbf{f}^{2} - \partial_2\mathbf{f}^{0} + \partial_3\mathbf{f}^{1} - \partial_1\mathbf{f}^{3} + \left[(-g^{0}_{2j1} + g^{2}_{0j1}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g^{1}_{3j1} + g^{3}_{1j1}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{f}^{j} = \begin{pmatrix} \Phi^{2}_{+} \\ \Phi^{2}_{-} \end{pmatrix} = \mathbf{\Phi}^{2}$$

$$-\partial_0\mathbf{f}^{3} - \partial_3\mathbf{f}^{0} - \partial_2\mathbf{f}^{1}_{-} + \partial_1\mathbf{f}^{2}_{-} + \left[(-g^{0}_{3j1} + g^{3}_{0j1}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (+g^{1}_{2j1} - g^{2}_{1j1}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{f}^{j} = \begin{pmatrix} \Phi^{3}_{+} \\ \Phi^{3}_{-} \end{pmatrix} = \mathbf{\Phi}^{3}$$

$$-\partial_1\mathbf{f}^{1} - \partial_2\mathbf{f}^{2} - \partial_3\mathbf{f}^{3} + \partial_0\mathbf{f}^{0} + \left[(+g^{0}_{0j1} + g^{1}_{1j1} + g^{2}_{2j1} + g^{3}_{3j1}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \mathbf{f}^{j} = \begin{pmatrix} \Phi^{0}_{+} \\ \Phi^{0}_{-} \end{pmatrix} = \mathbf{\Phi}^{0}$$

and:

$$
\mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+\partial_0([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h) + \\
+\partial_1([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+\partial_2([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h) + \\
+\partial_3([-g_{11h}^3+g_{31h}^1]f_+^h+[-g_{01h}^2+g_{21h}^0]f_-^h) + \\
+(-g_{12h}^0+g_{02h}^1)\partial_0 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_0 f_-^h \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_1 f_+^h + \\
+(-g_{22h}^1+g_{12h}^2)\partial_2 f_+^h+(+g_{32h}^0-g_{02h}^3)\partial_2 f_-^h + \\
+(-g_{32h}^1+g_{12h}^3)\partial_3 f_+^h+(-g_{22h}^0+g_{02h}^2)\partial_3 f_-^h + \\
+([-g_{12k}^0-g_{02k}^1]g_{01h}^k+[+g_{02k}^0-g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{11h}^k+[-g_{22k}^1-g_{12k}^2]g_{21h}^k+[-g_{32k}^1-g_{12k}^3]g_{31h}^k)f_+^h + \\
+([+g_{32k}^2-g_{22k}^3]g_{01h}^k+[-g_{32k}^0+g_{02k}^3]g_{21h}^k+[+g_{22k}^0-g_{02k}^2]g_{31h}^k)f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+\partial_0([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h) + \\
+\partial_1([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_-^h) + \\
+\partial_2([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h) + \\
+\partial_3([-g_{21h}^0+g_{01h}^2]f_+^h+[-g_{31h}^1+g_{11h}^3]f_-^h) + \\
+(-g_{32h}^2+g_{22h}^3)\partial_0 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_0 f_-^h + \\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2-g_{32h}^3)\partial_1 f_-^h + \\
+(-g_{32h}^0+g_{02h}^3)\partial_2 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_2 f_-^h \\
+(+g_{22h}^0-g_{02h}^2)\partial_3 f_+^h+(+g_{32h}^1-g_{12h}^3)\partial_3 f_-^h + \\
+([+g_{32k}^2-g_{22k}^3]g_{01h}^k+[-g_{32k}^0+g_{02k}^3]g_{21h}^k+[+g_{22k}^0-g_{02k}^2]g_{31h}^k)f_+^h + \\
+([-g_{12k}^0-g_{02k}^1]g_{01h}^k+[+g_{02k}^0-g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{11h}^k+[-g_{22k}^1-g_{12k}^2]g_{21h}^k+[-g_{32k}^1-g_{12k}^3]g_{31h}^k)f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+\partial_0([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h) + \\
+\partial_1([-g_{21h}^1+g_{11h}^2]f_+^h+[+g_{31h}^0-g_{01h}^3]f_-^h) + \\
+\partial_2([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+\partial_3([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h) + \\
+(-g_{22h}^0+g_{02h}^2)\partial_0 f_+^h+(-g_{32h}^1+g_{12h}^3)\partial_0 f_-^h \\
+[+g_{22h}^1-g_{12h}^2]\partial_1 f_+^h+(-g_{32h}^0+g_{02h}^3)\partial_1 f_-^h \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_2 f_+^h + \\
+(-g_{32h}^2+g_{22h}^3)\partial_3 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_3 f_-^h + \\
+([-g_{02k}^2-g_{22k}^0]g_{01h}^k+[-g_{22k}^1-g_{12k}^2]g_{11h}^k+[+g_{02k}^0+g_{12k}^1-g_{22k}^2+g_{32k}^3]g_{21h}^k+[-g_{32k}^2-g_{22k}^3]g_{31h}^k)f_+^h + \\
+([-g_{32k}^1+g_{12k}^3]g_{01h}^k+[+g_{32k}^0-g_{02k}^3]g_{11h}^k+[-g_{12k}^0+g_{02k}^1]g_{31h}^k)f_-^h +
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
+\partial_0([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h) + \\
+\partial_1([-g_{31h}^0+g_{01h}^3]f_+^h+[+g_{21h}^1-g_{11h}^2]f_-^h) + \\
+\partial_3([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h) + \\
+\partial_2([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_-^h) + \\
(+g_{32h}^1-g_{12h}^3)\partial_0 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_0 f_-^h + \\
+(g_{32h}^0-g_{02h}^3)\partial_1 f_+^h+(g_{12h}^2-g_{22h}^1)\partial_1 f_-^1 + \\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2\partial_2 f_-^h-g_{32h}^3)\partial_2 f_-^h + \\
+(-g_{12h}^0+g_{02h}^1)\partial_3 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_3 f_-^h + \\
+([-g_{32k}^1+g_{12k}^3]g_{01h}^k+[+g_{32k}^0-g_{02k}^3]g_{11h}^k+[-g_{12k}^0+g_{02k}^1]g_{31h}^k)f_+^h + \\
+([-g_{22k}^0-g_{02k}^2]g_{01h}^k+[-g_{22k}^1-g_{12k}^2]g_{11h}^k+[+g_{02k}^0+g_{12k}^1-g_{22k}^2+g_{32k}^3]g_{21h}^k+[-g_{32k}^2-g_{22k}^3]g_{31h}^k)f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+\partial_0([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h) + \\
+\partial_1([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h) + \\
+\partial_2([-g_{31h}^2+g_{21h}^3]f_+^h+[+g_{11h}^0-g_{01h}^1]f_-^h) + \\
+\partial_3([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+(-g_{32h}^0\partial_0+g_{02h}^3)\partial_0 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_0 f_-^h + \\
+(+g_{32h}^1-g_{12h}^3)\partial_1 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_1 f_-^h + \\
+(+g_{32h}^2-g_{22h}^3)\partial_2 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_2 f_-^h + \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_3 f_+^h +
\end{pmatrix}
\end{pmatrix}
$$

Proof:

$$J(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} J^1_+ \\ J^1_- \end{pmatrix} \\ \begin{pmatrix} J^2_+ \\ J^2_- \end{pmatrix} \\ \begin{pmatrix} J^3_+ \\ J^3_- \end{pmatrix} \\ \begin{pmatrix} J^0_+ \\ J^0_- \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -D_{02} & D^{\rightleftarrows}_{32} & -D^{\rightleftarrows}_{22} & -D_{12} \\ -D^{\rightleftarrows}_{32} & -D_{02} & D^{\rightleftarrows}_{12} & -D_{22} \\ D^{\rightleftarrows}_{22} & -D^{\rightleftarrows}_{12} & -D_{02} & -D_{32} \\ -D^{\Updownarrow}_{12} & -D^{\Updownarrow}_{22} & -D^{\Updownarrow}_{32} & D^{\Updownarrow}_{02} \end{pmatrix} \begin{pmatrix} -D^{\Updownarrow}_{01} & -D^{\rightleftarrows}_{31} & D^{\rightleftarrows}_{21} & -D_{11} \\ D^{\rightleftarrows}_{31} & -D^{\Updownarrow}_{01} & -D^{\rightleftarrows}_{11} & -D_{21} \\ -D^{\rightleftarrows}_{21} & D^{\rightleftarrows}_{11} & -D^{\Updownarrow}_{01} & -D_{31} \\ -D^{\Updownarrow}_{11} & -D^{\Updownarrow}_{21} & -D^{\Updownarrow}_{31} & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} -D_{02} & D^{\rightleftarrows}_{32} & -D^{\rightleftarrows}_{22} & -D_{12} \\ -D^{\rightleftarrows}_{32} & -D_{02} & D^{\rightleftarrows}_{12} & -D_{22} \\ D^{\rightleftarrows}_{22} & -D^{\rightleftarrows}_{12} & -D_{02} & -D_{32} \\ -D^{\Updownarrow}_{12} & -D^{\Updownarrow}_{22} & -D^{\Updownarrow}_{32} & D^{\Updownarrow}_{02} \end{pmatrix} \begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0 \\ D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0 \\ -D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0 \\ -D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0 \end{pmatrix} \Rightarrow \begin{cases} \begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0 \\ D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0 \\ -D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0 \\ -D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^0 \end{pmatrix} \end{cases}$$

$$= \begin{pmatrix} -D_{02} & D^{\rightleftarrows}_{32} & -D^{\rightleftarrows}_{22} & -D_{12} \\ -D^{\rightleftarrows}_{32} & -D_{02} & D^{\rightleftarrows}_{12} & -D_{22} \\ D^{\rightleftarrows}_{22} & -D^{\rightleftarrows}_{12} & -D_{02} & -D_{32} \\ -D^{\Updownarrow}_{12} & -D^{\Updownarrow}_{22} & -D^{\Updownarrow}_{32} & D^{\Updownarrow}_{02} \end{pmatrix} \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} -D_{02}\Phi^1 + D^{\rightleftarrows}_{32}\Phi^2 - D^{\rightleftarrows}_{22}\Phi^3 - D_{12}\Phi^0 \\ -D^{\rightleftarrows}_{32}\Phi^1 - D_{02}\Phi^2 + D^{\rightleftarrows}_{12}\Phi^3 - D_{22}\Phi^0 \\ D^{\rightleftarrows}_{22}\Phi^1 - D^{\rightleftarrows}_{12}\Phi^2 - D_{02}\Phi^3 - D_{32}\Phi^0 \\ -D^{\Updownarrow}_{12}\Phi^1 - D^{\Updownarrow}_{22}\Phi^2 - D^{\Updownarrow}_{32}\Phi^3 + D^{\Updownarrow}_{02}\Phi^0 \end{pmatrix}$$

So, the linear PDE factoring equations are:

$$\begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} -D_{02}\Phi^1 + D^{\rightleftarrows}_{32}\Phi^2 - D^{\rightleftarrows}_{22}\Phi^3 - D_{12}\Phi^0 \\ -D^{\rightleftarrows}_{32}\Phi^1 - D_{02}\Phi^2 + D^{\rightleftarrows}_{12}\Phi^3 - D_{22}\Phi^0 \\ D^{\rightleftarrows}_{22}\Phi^1 - D^{\rightleftarrows}_{12}\Phi^2 - D_{02}\Phi^3 - D_{32}\Phi^0 \\ -D^{\Updownarrow}_{12}\Phi^1 - D^{\Updownarrow}_{22}\Phi^2 - D^{\Updownarrow}_{32}\Phi^3 + D^{\Updownarrow}_{02}\Phi^0 \end{pmatrix} \quad \& \quad \begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0 \\ D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0 \\ -D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0 \\ -D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^0 \end{pmatrix}$$

Continuing:

$$J(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} -D_{02}\Phi^1 + D^{\rightleftarrows}_{32}\Phi^2 - D^{\rightleftarrows}_{22}\Phi^3 - D_{12}\Phi^0 \\ -D^{\rightleftarrows}_{32}\Phi^1 - D_{02}\Phi^2 + D^{\rightleftarrows}_{12}\Phi^3 - D_{22}\Phi^0 \\ D^{\rightleftarrows}_{22}\Phi^1 - D^{\rightleftarrows}_{12}\Phi^2 - D_{02}\Phi^3 - D_{32}\Phi^0 \\ -D^{\Updownarrow}_{12}\Phi^1 - D^{\Updownarrow}_{22}\Phi^2 - D^{\Updownarrow}_{32}\Phi^3 + D^{\Updownarrow}_{02}\Phi^0 \end{pmatrix}$$

because of the generally non-associativity of the product:

$$= \begin{pmatrix} -D_{02} & D^{\rightleftarrows}_{32} & -D^{\rightleftarrows}_{22} & -D_{12} \\ -D^{\rightleftarrows}_{32} & -D_{02} & D^{\rightleftarrows}_{12} & -D_{22} \\ D^{\rightleftarrows}_{22} & -D^{\rightleftarrows}_{12} & -D_{02} & -D_{32} \\ -D^{\Updownarrow}_{12} & -D^{\Updownarrow}_{22} & -D^{\Updownarrow}_{32} & D^{\Updownarrow}_{02} \end{pmatrix} \begin{pmatrix} -D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0 \\ D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0 \\ -D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0 \\ -D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} -D_{02}\left[-D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0\right] + D^{\rightleftarrows}_{32}\left[D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0\right] + \\ \quad - D^{\rightleftarrows}_{22}\left[-D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0\right] - D_{12}\left[-D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0\right] \end{pmatrix} \\ \begin{pmatrix} -D^{\rightleftarrows}_{32}\left[-D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0\right] - D_{02}\left[D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0\right] + \\ \quad + D^{\rightleftarrows}_{12}\left[-D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0\right] - D_{22}\left[-D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0\right] \end{pmatrix} \\ \begin{pmatrix} D^{\rightleftarrows}_{22}\left[-D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0\right] - D^{\rightleftarrows}_{12}\left[D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0\right] + \\ \quad - D_{02}\left[-D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0\right] - D_{32}\left[-D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0\right] \end{pmatrix} \\ \begin{pmatrix} -D^{\Updownarrow}_{12}\left[-D^{\Updownarrow}_{01} f^1 - D^{\rightleftarrows}_{31} f^2 + D^{\rightleftarrows}_{21} f^3 - D_{11} f^0\right] - D^{\Updownarrow}_{22}\left[D^{\rightleftarrows}_{31} f^1 - D^{\Updownarrow}_{01} f^2 - D^{\rightleftarrows}_{11} f^3 - D_{21} f^0\right] + \\ \quad - D^{\Updownarrow}_{32}\left[-D^{\rightleftarrows}_{21} f^1 + D^{\rightleftarrows}_{11} f^2 - D^{\Updownarrow}_{01} f^3 - D_{31} f^0\right] + D^{\Updownarrow}_{02}\left[-D^{\Updownarrow}_{11} f^1 - D^{\Updownarrow}_{21} f^2 - D^{\Updownarrow}_{31} f^3 + D_{01} f^0\right] \end{pmatrix} \end{pmatrix}$$

$$
=\begin{pmatrix}
\begin{pmatrix}
-D_{02}\left[-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}+\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{11}^+ & 0 \\ 0 & D_{11}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
+D_{32}^{\Leftrightarrow}\left[\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{21}^+ & 0 \\ 0 & D_{21}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{22}^{\Leftrightarrow}\left[-\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}+\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{31}^+ & 0 \\ 0 & D_{31}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{12}\left[-\begin{pmatrix} D_{11}^- & 0 \\ 0 & D_{11}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{21}^- & 0 \\ 0 & D_{21}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{31}^- & 0 \\ 0 & D_{31}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}+\begin{pmatrix} D_{01}^+ & 0 \\ 0 & D_{01}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]
\end{pmatrix} \\
\begin{pmatrix}
-D_{32}^{\Leftrightarrow}\left[-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}+\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{11}^+ & 0 \\ 0 & D_{11}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{02}\left[\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{21}^+ & 0 \\ 0 & D_{21}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
+D_{12}^{\Leftrightarrow}\left[-\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}+\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{31}^+ & 0 \\ 0 & D_{31}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{22}\left[-\begin{pmatrix} D_{11}^- & 0 \\ 0 & D_{11}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{21}^- & 0 \\ 0 & D_{21}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{31}^- & 0 \\ 0 & D_{31}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}+\begin{pmatrix} D_{01}^+ & 0 \\ 0 & D_{01}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]
\end{pmatrix} \\
\begin{pmatrix}
D_{22}^{\Leftrightarrow}\left[-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}+\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{11}^+ & 0 \\ 0 & D_{11}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{12}^{\Leftrightarrow}\left[\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{21}^+ & 0 \\ 0 & D_{21}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{02}\left[-\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}+\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{31}^+ & 0 \\ 0 & D_{31}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{32}\left[-\begin{pmatrix} D_{11}^- & 0 \\ 0 & D_{11}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{21}^- & 0 \\ 0 & D_{21}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{31}^- & 0 \\ 0 & D_{31}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}+\begin{pmatrix} D_{01}^+ & 0 \\ 0 & D_{01}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]
\end{pmatrix} \\
\begin{pmatrix}
-D_{12}^{\Updownarrow}\left[-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}+\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{11}^+ & 0 \\ 0 & D_{11}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{22}^{\Updownarrow}\left[\begin{pmatrix} 0 & D_{31}^- \\ D_{31}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{21}^+ & 0 \\ 0 & D_{21}^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}\right]+ \\
-D_{32}^{\Updownarrow}\left[-\begin{pmatrix} 0 & D_{21}^- \\ D_{21}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}+\begin{pmatrix} 0 & D_{11}^- \\ D_{11}^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{01}^- & 0 \\ 0 & D_{01}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}-\begin{pmatrix} D_{31}^+ & 0 \\ 0 & D_{31}^- \end{pmatrix}f^0\right]+ \\
+D_{02}^{\Updownarrow}\left[-\begin{pmatrix} D_{11}^- & 0 \\ 0 & D_{11}^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix}-\begin{pmatrix} D_{21}^- & 0 \\ 0 & D_{21}^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix}-\begin{pmatrix} D_{31}^- & 0 \\ 0 & D_{31}^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix}+\begin{pmatrix} D_{01}^+ & 0 \\ 0 & D_{01}^- \end{pmatrix}f^0\right]
\end{pmatrix}
\end{pmatrix}
$$

$$
\begin{aligned}
&\left(\begin{array}{l}
\left(\begin{array}{l}
-D_{02}\left[-\begin{pmatrix} D^{-}_{01}f^{1}_{+} \\ D^{+}_{01}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{2}_{-} \\ D^{+}_{31}f^{2}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{21}f^{3}_{-} \\ D^{+}_{21}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{11}f^{0}_{+} \\ D^{-}_{11}f^{0}_{-} \end{pmatrix}\right]+\\
\quad+D^{\Leftrightarrow}_{32}\left[\begin{pmatrix} D^{-}_{31}f^{1}_{-} \\ D^{+}_{31}f^{1}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{2}_{+} \\ D^{+}_{01}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{11}f^{3}_{-} \\ D^{+}_{11}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{21}f^{0}_{+} \\ D^{-}_{21}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D^{\Leftrightarrow}_{22}\left[-\begin{pmatrix} D^{-}_{21}f^{1}_{-} \\ D^{+}_{21}f^{1}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{11}f^{2}_{-} \\ D^{+}_{11}f^{2}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{3}_{+} \\ D^{+}_{01}f^{3}_{-} \end{pmatrix}-\begin{pmatrix} D^{+}_{31}f^{0}_{+} \\ D^{-}_{31}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D_{12}\left[-\begin{pmatrix} D^{-}_{11}f^{1}_{+} \\ D^{+}_{11}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{21}f^{2}_{+} \\ D^{+}_{21}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{3}_{+} \\ D^{+}_{31}f^{3}_{-} \end{pmatrix}+\begin{pmatrix} D^{+}_{01}f^{0}_{+} \\ D^{-}_{01}f^{0}_{-} \end{pmatrix}\right]
\end{array}\right)\\
\left(\begin{array}{l}
-D^{\Leftrightarrow}_{32}\left[-\begin{pmatrix} D^{-}_{01}f^{1}_{+} \\ D^{+}_{01}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{2}_{-} \\ D^{+}_{31}f^{2}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{21}f^{3}_{-} \\ D^{+}_{21}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{11}f^{0}_{+} \\ D^{-}_{11}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D_{02}\left[\begin{pmatrix} D^{-}_{31}f^{1}_{-} \\ D^{+}_{31}f^{1}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{2}_{+} \\ D^{+}_{01}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{11}f^{3}_{-} \\ D^{+}_{11}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{21}f^{0}_{+} \\ D^{-}_{21}f^{0}_{-} \end{pmatrix}\right]+\\
\quad+D^{\Leftrightarrow}_{12}\left[-\begin{pmatrix} D^{-}_{21}f^{1}_{-} \\ D^{+}_{21}f^{1}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{11}f^{2}_{-} \\ D^{+}_{11}f^{2}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{3}_{+} \\ D^{+}_{01}f^{3}_{-} \end{pmatrix}-\begin{pmatrix} D^{+}_{31}f^{0}_{+} \\ D^{-}_{31}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D_{22}\left[-\begin{pmatrix} D^{-}_{11}f^{1}_{+} \\ D^{+}_{11}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{21}f^{2}_{+} \\ D^{+}_{21}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{3}_{+} \\ D^{+}_{31}f^{3}_{-} \end{pmatrix}+\begin{pmatrix} D^{+}_{01}f^{0}_{+} \\ D^{-}_{01}f^{0}_{-} \end{pmatrix}\right]
\end{array}\right)\\
=\left(\begin{array}{l}
\left(\begin{array}{l}
D^{\Leftrightarrow}_{22}\left[-\begin{pmatrix} D^{-}_{01}f^{1}_{+} \\ D^{+}_{01}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{2}_{-} \\ D^{+}_{31}f^{2}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{21}f^{3}_{-} \\ D^{+}_{21}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{11}f^{0}_{+} \\ D^{-}_{11}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D^{\Leftrightarrow}_{12}\left[\begin{pmatrix} D^{-}_{31}f^{1}_{-} \\ D^{+}_{31}f^{1}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{2}_{+} \\ D^{+}_{01}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{11}f^{3}_{-} \\ D^{+}_{11}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{21}f^{0}_{+} \\ D^{-}_{21}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D_{02}\left[-\begin{pmatrix} D^{-}_{21}f^{1}_{-} \\ D^{+}_{21}f^{1}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{11}f^{2}_{-} \\ D^{+}_{11}f^{2}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{3}_{+} \\ D^{+}_{01}f^{3}_{-} \end{pmatrix}-\begin{pmatrix} D^{+}_{31}f^{0}_{+} \\ D^{-}_{31}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D_{32}\left[-\begin{pmatrix} D^{-}_{11}f^{1}_{+} \\ D^{+}_{11}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{21}f^{2}_{+} \\ D^{+}_{21}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{3}_{+} \\ D^{+}_{31}f^{3}_{-} \end{pmatrix}+\begin{pmatrix} D^{+}_{01}f^{0}_{+} \\ D^{-}_{01}f^{0}_{-} \end{pmatrix}\right]
\end{array}\right)\\
\left(\begin{array}{l}
-D^{\Updownarrow}_{12}\left[-\begin{pmatrix} D^{-}_{01}f^{1}_{+} \\ D^{+}_{01}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{2}_{-} \\ D^{+}_{31}f^{2}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{21}f^{3}_{-} \\ D^{+}_{21}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{11}f^{0}_{+} \\ D^{-}_{11}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D^{\Updownarrow}_{22}\left[\begin{pmatrix} D^{-}_{31}f^{1}_{-} \\ D^{+}_{31}f^{1}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{2}_{+} \\ D^{+}_{01}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{11}f^{3}_{-} \\ D^{+}_{11}f^{3}_{+} \end{pmatrix}-\begin{pmatrix} D^{+}_{21}f^{0}_{+} \\ D^{-}_{21}f^{0}_{-} \end{pmatrix}\right]+\\
\quad-D^{\Updownarrow}_{32}\left[-\begin{pmatrix} D^{-}_{21}f^{1}_{-} \\ D^{+}_{21}f^{1}_{+} \end{pmatrix}+\begin{pmatrix} D^{-}_{11}f^{2}_{-} \\ D^{+}_{11}f^{2}_{+} \end{pmatrix}-\begin{pmatrix} D^{-}_{01}f^{3}_{+} \\ D^{+}_{01}f^{3}_{-} \end{pmatrix}-\begin{pmatrix} D^{+}_{31}f^{0}_{+} \\ D^{-}_{31}f^{0}_{-} \end{pmatrix}\right]+\\
\quad+D^{\Updownarrow}_{02}\left[-\begin{pmatrix} D^{-}_{11}f^{1}_{+} \\ D^{+}_{11}f^{1}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{21}f^{2}_{+} \\ D^{+}_{21}f^{2}_{-} \end{pmatrix}-\begin{pmatrix} D^{-}_{31}f^{3}_{+} \\ D^{+}_{31}f^{3}_{-} \end{pmatrix}+\begin{pmatrix} D^{+}_{01}f^{0}_{+} \\ D^{-}_{01}f^{0}_{-} \end{pmatrix}\right]
\end{array}\right)
\end{array}\right)
\end{aligned}
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=\begin{pmatrix}
\begin{pmatrix}
-D_{02}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} + D^{\Leftrightarrow}_{32}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- D^{\Leftrightarrow}_{22}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - D_{12}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
-D^{\Leftrightarrow}_{32}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - D_{02}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
+ D^{\Leftrightarrow}_{12}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - D_{22}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
D^{\Leftrightarrow}_{22}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - D^{\Leftrightarrow}_{12}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- D_{02}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - D_{32}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
-D^{\Updownarrow}_{12}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - D^{\Updownarrow}_{22}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- D^{\Updownarrow}_{32}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} + D^{\Updownarrow}_{02}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix}
\end{pmatrix} \\
=\begin{pmatrix}
\begin{pmatrix}
-\begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} + \begin{pmatrix} 0 & D^-_{32} \\ D^+_{32} & 0 \end{pmatrix}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- \begin{pmatrix} 0 & D^-_{22} \\ D^+_{22} & 0 \end{pmatrix}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - \begin{pmatrix} D^+_{12} & 0 \\ 0 & D^-_{12} \end{pmatrix}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
-\begin{pmatrix} 0 & D^-_{32} \\ D^+_{32} & 0 \end{pmatrix}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - \begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
+ \begin{pmatrix} 0 & D^-_{12} \\ D^+_{12} & 0 \end{pmatrix}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - \begin{pmatrix} D^+_{22} & 0 \\ 0 & D^-_{22} \end{pmatrix}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix} 0 & D^-_{22} \\ D^+_{22} & 0 \end{pmatrix}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - \begin{pmatrix} 0 & D^-_{12} \\ D^+_{12} & 0 \end{pmatrix}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- \begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} - \begin{pmatrix} D^+_{32} & 0 \\ 0 & D^-_{32} \end{pmatrix}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
-\begin{pmatrix} D^-_{12} & 0 \\ 0 & D^+_{12} \end{pmatrix}\begin{pmatrix} -D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ \\ -D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- \end{pmatrix} - \begin{pmatrix} D^-_{22} & 0 \\ 0 & D^+_{22} \end{pmatrix}\begin{pmatrix} D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ \\ D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- \end{pmatrix} + \\
- \begin{pmatrix} D^-_{32} & 0 \\ 0 & D^+_{32} \end{pmatrix}\begin{pmatrix} -D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ \\ -D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- \end{pmatrix} + \begin{pmatrix} D^-_{02} & 0 \\ 0 & D^+_{02} \end{pmatrix}\begin{pmatrix} -D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ \\ -D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- \end{pmatrix}
\end{pmatrix}
\end{pmatrix}
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=\left(\begin{array}{l}
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\left(\begin{array}{l}
-\begin{pmatrix} D^{+}_{02}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}] \\ D^{-}_{02}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}] \end{pmatrix}+\begin{pmatrix} D^{-}_{32}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \\ D^{+}_{32}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \end{pmatrix}+ \\
-\begin{pmatrix} D^{-}_{22}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}] \\ D^{+}_{22}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}] \end{pmatrix}-\begin{pmatrix} D^{+}_{12}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ D^{-}_{12}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
-\begin{pmatrix} D^{-}_{32}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}] \\ D^{+}_{32}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}] \end{pmatrix}-\begin{pmatrix} D^{+}_{02}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \\ D^{-}_{02}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \end{pmatrix}+ \\
+\begin{pmatrix} D^{-}_{12}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}] \\ D^{+}_{12}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}] \end{pmatrix}-\begin{pmatrix} D^{+}_{22}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ D^{-}_{22}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
\left(\begin{array}{l}
\begin{pmatrix} D^{-}_{22}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}] \\ D^{+}_{22}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}] \end{pmatrix}-\begin{pmatrix} D^{-}_{12}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \\ D^{+}_{12}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \end{pmatrix}+ \\
-\begin{pmatrix} D^{+}_{02}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}] \\ D^{-}_{02}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}] \end{pmatrix}-\begin{pmatrix} D^{+}_{32}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ D^{-}_{32}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
-\begin{pmatrix} D^{-}_{12}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}] \\ D^{+}_{12}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}] \end{pmatrix}-\begin{pmatrix} D^{-}_{22}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \\ D^{+}_{22}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \end{pmatrix}+ \\
-\begin{pmatrix} D^{-}_{32}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}] \\ D^{+}_{32}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}] \end{pmatrix}+\begin{pmatrix} D^{-}_{02}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ D^{+}_{02}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right)
\end{array}\right) \\
\left(\begin{array}{l}
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\left(\begin{array}{l}
\begin{pmatrix} -D^{+}_{02}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]+D^{-}_{32}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \\ -D^{-}_{02}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]+D^{+}_{32}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \end{pmatrix}+ \\
+\begin{pmatrix} -D^{-}_{22}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{+}_{12}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ -D^{+}_{22}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{-}_{12}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
\begin{pmatrix} -D^{-}_{32}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{+}_{02}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \\ -D^{+}_{32}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{-}_{02}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \end{pmatrix}+ \\
+\begin{pmatrix} D^{-}_{12}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{+}_{22}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ D^{+}_{12}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{-}_{22}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
\begin{pmatrix} D^{-}_{22}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{-}_{12}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \\ D^{+}_{22}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{+}_{12}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \end{pmatrix}+ \\
+\begin{pmatrix} -D^{+}_{02}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{+}_{32}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ -D^{-}_{02}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{-}_{32}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right) \\
\left(\begin{array}{l}
\left(\begin{array}{l}
\begin{pmatrix} -D^{-}_{12}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{-}_{22}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}] \\ -D^{+}_{12}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{+}_{22}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}] \end{pmatrix}+ \\
+\begin{pmatrix} -D^{-}_{32}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]+D^{-}_{02}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}] \\ -D^{+}_{32}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]+D^{+}_{02}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}] \end{pmatrix}
\end{array}\right)
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=\begin{pmatrix}
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\begin{pmatrix}
-D^{+}_{02}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]+D^{-}_{32}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}]+\\
-D^{-}_{22}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{+}_{12}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}]
\end{pmatrix}\\
\begin{pmatrix}
-D^{-}_{02}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]+D^{+}_{32}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}]+\\
-D^{+}_{22}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{-}_{12}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}]
\end{pmatrix}
\end{pmatrix}\\
\begin{pmatrix}
\begin{pmatrix}
-D^{-}_{32}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{+}_{02}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}]+\\
+D^{-}_{12}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{+}_{22}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}]
\end{pmatrix}\\
\begin{pmatrix}
-D^{+}_{32}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{-}_{02}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}]+\\
+D^{+}_{12}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{-}_{22}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}]
\end{pmatrix}
\end{pmatrix}\\
\begin{pmatrix}
\begin{pmatrix}
D^{-}_{22}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{-}_{12}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}]+\\
-D^{+}_{02}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]-D^{+}_{32}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}]
\end{pmatrix}\\
\begin{pmatrix}
D^{+}_{22}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{+}_{12}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}]+\\
-D^{-}_{02}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]-D^{-}_{32}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}]
\end{pmatrix}
\end{pmatrix}\\
\begin{pmatrix}
\begin{pmatrix}
-D^{-}_{12}[-D^{-}_{01}f^{1}_{+}-D^{-}_{31}f^{2}_{-}+D^{-}_{21}f^{3}_{-}-D^{+}_{11}f^{0}_{+}]-D^{-}_{22}[D^{-}_{31}f^{1}_{-}-D^{-}_{01}f^{2}_{+}-D^{-}_{11}f^{3}_{-}-D^{+}_{21}f^{0}_{+}]+\\
-D^{-}_{32}[-D^{-}_{21}f^{1}_{-}+D^{-}_{11}f^{2}_{-}-D^{-}_{01}f^{3}_{+}-D^{+}_{31}f^{0}_{+}]+D^{-}_{02}[-D^{-}_{11}f^{1}_{+}-D^{-}_{21}f^{2}_{+}-D^{-}_{31}f^{3}_{+}+D^{+}_{01}f^{0}_{+}]
\end{pmatrix}\\
\begin{pmatrix}
-D^{+}_{12}[-D^{+}_{01}f^{1}_{-}-D^{+}_{31}f^{2}_{+}+D^{+}_{21}f^{3}_{+}-D^{-}_{11}f^{0}_{-}]-D^{+}_{22}[D^{+}_{31}f^{1}_{+}-D^{+}_{01}f^{2}_{-}-D^{+}_{11}f^{3}_{+}-D^{-}_{21}f^{0}_{-}]+\\
-D^{+}_{32}[-D^{+}_{21}f^{1}_{+}+D^{+}_{11}f^{2}_{+}-D^{+}_{01}f^{3}_{-}-D^{-}_{31}f^{0}_{-}]+D^{+}_{02}[-D^{+}_{11}f^{1}_{-}-D^{+}_{21}f^{2}_{-}-D^{+}_{31}f^{3}_{-}+D^{-}_{01}f^{0}_{-}]
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

$$
\begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
-D^{+}_{02}[-(\partial_0 f^1_+ - g^1_{01h} f^h_+) - (\partial_3 f^2_- - g^2_{31h} f^h_-) + (\partial_2 f^3_- - g^3_{21h} f^h_-) - (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
+ D^{-}_{32}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
- D^{-}_{22}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
- D^{+}_{12}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix} \\
\begin{pmatrix}
-D^{-}_{02}[-(\partial_0 f^1_- + g^1_{01h} f^h_-) - (\partial_3 f^2_+ + g^2_{31h} f^h_+) + (\partial_2 f^3_+ + g^3_{21h} f^h_+) - (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
+ D^{+}_{32}[(\partial_3 f^1_- - g^1_{31h} f^h_-) - (\partial_0 f^2_+ - g^2_{01h} f^h_+) - (\partial_1 f^3_- - g^3_{11h} f^h_-) - (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
- D^{+}_{22}[-(\partial_2 f^1_- - g^1_{21h} f^h_-) + (\partial_1 f^2_- - g^2_{11h} f^h_-) - (\partial_0 f^3_+ - g^3_{01h} f^h_+) - (\partial_3 f^0_+ + g^0_{31h} f^h_+)] + \\
- D^{-}_{12}[-(\partial_1 f^1_- + g^1_{11h} f^h_-) - (\partial_2 f^2_- + g^2_{21h} f^h_-) - (\partial_3 f^3_- + g^3_{31h} f^h_-) + (\partial_0 f^0_- - g^0_{01h} f^h_-)]
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
-D^{-}_{32}[-(\partial_0 f^1_- + g^1_{01h} f^h_-) - (\partial_3 f^2_+ + g^2_{31h} f^h_+) + (\partial_2 f^3_+ + g^3_{21h} f^h_+) - (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
- D^{+}_{02}[(\partial_3 f^1_- - g^1_{31h} f^h_-) - (\partial_0 f^2_+ - g^2_{01h} f^h_+) - (\partial_1 f^3_- - g^3_{11h} f^h_-) - (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
+ D^{-}_{12}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
- D^{+}_{22}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix} \\
\begin{pmatrix}
-D^{+}_{32}[-(\partial_0 f^1_+ - g^1_{01h} f^h_+) - (\partial_3 f^2_- - g^2_{31h} f^h_-) + (\partial_2 f^3_- - g^3_{21h} f^h_-) - (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
- D^{-}_{02}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
+ D^{+}_{12}[-(\partial_2 f^1_- - g^1_{21h} f^h_-) + (\partial_1 f^2_- - g^2_{11h} f^h_-) - (\partial_0 f^3_+ - g^3_{01h} f^h_+) - (\partial_3 f^0_+ + g^0_{31h} f^h_+)] + \\
- D^{-}_{22}[-(\partial_1 f^1_- + g^1_{11h} f^h_-) - (\partial_2 f^2_- + g^2_{21h} f^h_-) - (\partial_3 f^3_- + g^3_{31h} f^h_-) + (\partial_0 f^0_- - g^0_{01h} f^h_-)]
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
D^{+}_{22}[-(\partial_0 f^1_+ - g^1_{01h} f^h_+) - (\partial_3 f^2_- - g^2_{31h} f^h_-) + (\partial_2 f^3_- - g^3_{21h} f^h_-) - (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
- D^{+}_{12}[+(\partial_3 f^1_- - g^1_{31h} f^h_-) - (\partial_0 f^2_+ - g^2_{01h} f^h_+) - (\partial_1 f^3_- - g^3_{11h} f^h_-) - (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
- D^{-}_{02}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
- D^{-}_{32}[-(\partial_1 f^1_- + g^1_{11h} f^h_-) - (\partial_2 f^2_- + g^2_{21h} f^h_-) - (\partial_3 f^3_- + g^3_{31h} f^h_-) + (\partial_0 f^0_- - g^0_{01h} f^h_-)]
\end{pmatrix} \\
\begin{pmatrix}
D^{-}_{22}[-(\partial_0 f^1_- + g^1_{01h} f^h_-) - (\partial_3 f^2_+ + g^2_{31h} f^h_+) + (\partial_2 f^3_+ + g^3_{21h} f^h_+) - (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
- D^{-}_{12}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
- D^{+}_{02}[-(\partial_2 f^1_- - g^1_{21h} f^h_-) + (\partial_1 f^2_- - g^2_{11h} f^h_-) - (\partial_0 f^3_+ - g^3_{01h} f^h_+) - (\partial_3 f^0_+ + g^0_{31h} f^h_+)] + \\
- D^{+}_{32}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
-D^{-}_{12}[-(\partial_0 f^1_+ - g^1_{01h} f^h_+) - (\partial_3 f^2_- - g^2_{31h} f^h_-) + (\partial_2 f^3_- - g^3_{21h} f^h_-) - (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
- D^{-}_{22}[(\partial_3 f^1_- - g^1_{31h} f^h_-) - (\partial_0 f^2_+ - g^2_{01h} f^h_+) - (\partial_1 f^3_- - g^3_{11h} f^h_-) - (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
- D^{-}_{32}[-(\partial_2 f^1_- - g^1_{21h} f^h_-) + (\partial_1 f^2_- - g^2_{11h} f^h_-) - (\partial_0 f^3_+ + g^3_{01h} f^h_+) - (\partial_3 f^0_+ + g^0_{31h} f^h_+)] + \\
+ D^{-}_{02}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix} \\
\begin{pmatrix}
-D^{+}_{12}[-(\partial_0 f^1_- + g^1_{01h} f^h_-) - (\partial_3 f^2_+ + g^2_{31h} f^h_+) + (\partial_2 f^3_+ + g^3_{21h} f^h_+) - (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
- D^{+}_{22}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
- D^{+}_{32}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
+ D^{+}_{02}[-(\partial_1 f^1_- + g^1_{11h} f^h_-) - (\partial_2 f^2_- + g^2_{21h} f^h_-) - (\partial_3 f^3_- + g^3_{31h} f^h_-) + (\partial_0 f^0_- - g^0_{01h} f^h_-)]
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

So: (*1+):

$$
\begin{pmatrix}
-D^{+}_{02}[-(\partial_0 f^1_+ - g^1_{01h} f^h_+) - (\partial_3 f^2_- - g^2_{31h} f^h_-) + (\partial_2 f^3_- - g^3_{21h} f^h_-) - (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
+ D^{-}_{32}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
- D^{-}_{22}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
- D^{+}_{12}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix} =
$$

$$
= \begin{pmatrix}
D^{+}_{02}[+(\partial_0 f^1_+ - g^1_{01h} f^h_+) + (\partial_3 f^2_- - g^2_{31h} f^h_-) - (\partial_2 f^3_- - g^3_{21h} f^h_-) + (\partial_1 f^0_+ + g^0_{11h} f^h_+)] + \\
+ D^{-}_{32}[(\partial_3 f^1_+ + g^1_{31h} f^h_+) - (\partial_0 f^2_- + g^2_{01h} f^h_-) - (\partial_1 f^3_+ + g^3_{11h} f^h_+) - (\partial_2 f^0_- - g^0_{21h} f^h_-)] + \\
+ D^{-}_{22}[+(\partial_2 f^1_+ + g^1_{21h} f^h_+) - (\partial_1 f^2_+ + g^2_{11h} f^h_+) + (\partial_0 f^3_- + g^3_{01h} f^h_-) + (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
+ D^{+}_{12}[+(\partial_1 f^1_+ - g^1_{11h} f^h_+) + (\partial_2 f^2_+ - g^2_{21h} f^h_+) + (\partial_3 f^3_+ - g^3_{31h} f^h_+) - (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{pmatrix}
$$

$$
= \begin{pmatrix}
D^{+}_{02}[\partial_0 f^1_+ + \partial_3 f^2_- - \partial_2 f^3_- + \partial_1 f^0_+ - g^1_{01h} f^h_+ - g^2_{31h} f^h_- + g^3_{21h} f^h_- + g^0_{11h} f^h_+] + \\
+ D^{-}_{32}[\partial_3 f^1_+ - \partial_0 f^2_- - \partial_1 f^3_+ - \partial_2 f^0_- + g^1_{31h} f^h_+ - g^2_{01h} f^h_- - g^3_{11h} f^h_+ + g^0_{21h} f^h_-] + \\
+ D^{-}_{22}[\partial_2 f^1_+ - \partial_1 f^2_+ + \partial_0 f^3_- + \partial_3 f^0_- + g^1_{21h} f^h_+ - g^2_{11h} f^h_+ + g^3_{01h} f^h_- - g^0_{31h} f^h_-] + \\
+ D^{+}_{12}[\partial_1 f^1_+ + \partial_2 f^2_+ + \partial_3 f^3_+ - \partial_0 f^0_+ - g^1_{11h} f^h_+ - g^2_{21h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+]
\end{pmatrix}
$$

$$
= \begin{pmatrix}
D^{+}_{02}(\partial_0 f^1_+ + \partial_3 f^2_- - \partial_2 f^3_- + \partial_1 f^0_+) + D^{+}_{02}(-g^1_{01h} f^h_+ - g^2_{31h} f^h_- + g^3_{21h} f^h_- + g^0_{11h} f^h_+) + \\
+ D^{-}_{32}(\partial_3 f^1_+ - \partial_0 f^2_- - \partial_1 f^3_+ - \partial_2 f^0_-) + D^{-}_{32}(+g^1_{31h} f^h_+ - g^2_{01h} f^h_- - g^3_{11h} f^h_+ + g^0_{21h} f^h_-) + \\
+ D^{-}_{22}(\partial_2 f^1_+ - \partial_1 f^2_+ + \partial_0 f^3_- + \partial_3 f^0_-) + D^{-}_{22}(+g^1_{21h} f^h_+ - g^2_{11h} f^h_+ + g^3_{01h} f^h_- - g^0_{31h} f^h_-) + \\
+ D^{+}_{12}(\partial_1 f^1_+ + \partial_2 f^2_+ + \partial_3 f^3_+ - \partial_0 f^0_+) + D^{+}_{12}(-g^1_{11h} f^h_+ - g^2_{21h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+)
\end{pmatrix}
$$

$$
=\left(\begin{array}{l}
\partial_0(\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+(g_{02h}^1\partial_0 f_+^h+g_{02h}^2\partial_3 f_-^h-g_{02h}^3\partial_2 f_-^h+g_{02h}^0\partial_1 f_+^h)+\\
+\partial_0(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+(-g_{02k}^1 g_{01h}^1 f_+^h-g_{02k}^2 g_{31h}^k f_-^h+g_{02k}^3 g_{21h}^k f_-^h+g_{02k}^0 g_{11h}^k f_+^h)+\\
\quad+\partial_3(\partial_3 f_+^1-\partial_0 f_-^2-\partial_1 f_+^3-\partial_2 f_-^0)+((-g_{32h}^1)\partial_3 f_+^h-(-g_{32h}^2)\partial_0 f_-^h-(-g_{32h}^3)\partial_1 f_+^h-(-g_{32h}^0)\partial_2 f_-^h)+\\
+\partial_3(+g_{31h}^1 f_+^h-g_{01h}^2 f_-^h-g_{11h}^3 f_+^h+g_{21h}^0 f_-^h)+(+(-g_{32k}^1)g_{31h}^k f_+^h-(-g_{32k}^2)g_{01h}^k f_-^h-(-g_{32k}^3)g_{11h}^k f_+^h+(-g_{32k}^0)g_{21h}^k f_-^h)+\\
\quad+\partial_2(\partial_2 f_+^1-\partial_1 f_+^2+\partial_0 f_-^3+\partial_3 f_-^0)+((-g_{22h}^1)\partial_2 f_+^h-(-g_{22h}^2)\partial_1 f_+^h+(-g_{22h}^3)\partial_0 f_-^h+(-g_{22h}^0)\partial_3 f_-^h)+\\
+\partial_2(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+(+(-g_{22k}^1)g_{21h}^k f_+^h-(-g_{22k}^2)g_{11h}^k f_+^h+(-g_{22k}^3)g_{01h}^k f_-^h-(-g_{22k}^0)g_{31h}^k f_-^h)+\\
\quad+\partial_1(\partial_1 f_+^1+\partial_2 f_+^2+\partial_3 f_+^3-\partial_0 f_+^0)+(g_{12h}^1\partial_1 f_+^h+g_{12h}^2\partial_2 f_+^h+g_{12h}^3\partial_3 f_+^h-g_{12h}^0\partial_0 f_+^h)+\\
+\partial_1(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+(-g_{12k}^1 g_{11h}^k f_+^h-g_{12k}^2 g_{21h}^k f_+^h-g_{12k}^3 g_{31h}^k f_+^h-g_{12k}^0 g_{01h}^k f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_0(\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+\partial_3(\partial_3 f_+^1-\partial_0 f_-^2-\partial_1 f_+^3-\partial_2 f_-^0)+\partial_2(\partial_2 f_+^1-\partial_1 f_+^2+\partial_0 f_-^3+\partial_3 f_-^0)+\partial_1(\partial_1 f_+^1+\partial_2 f_+^2+\partial_3 f_+^3-\partial_0 f_+^0)+\\
+(g_{02h}^1\partial_0 f_+^h+g_{02h}^2\partial_3 f_-^h-g_{02h}^3\partial_2 f_-^h+g_{02h}^0\partial_1 f_+^h)+((-g_{32h}^1)\partial_3 f_+^h-(-g_{32h}^2)\partial_0 f_-^h-(-g_{32h}^3)\partial_1 f_+^h-(-g_{32h}^0)\partial_2 f_-^h)+\\
+((-g_{22h}^1)\partial_2 f_+^h-(-g_{22h}^2)\partial_1 f_+^h+(-g_{22h}^3)\partial_0 f_-^h+(-g_{22h}^0)\partial_3 f_-^h)+(g_{12h}^1\partial_1 f_+^h+g_{12h}^2\partial_2 f_+^h+g_{12h}^3\partial_3 f_+^h-g_{12h}^0\partial_0 f_+^h)+\\
+\partial_0(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\partial_3(+g_{31h}^1 f_+^h-g_{01h}^2 f_-^h-g_{11h}^3 f_+^h+g_{21h}^0 f_-^h)+\\
+\partial_2(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+\partial_1(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+\\
+(-g_{02k}^1 g_{01h}^1 f_+^h-g_{02k}^2 g_{31h}^k f_-^h+g_{02k}^3 g_{21h}^k f_-^h+g_{02k}^0 g_{11h}^k f_+^h)+(+(-g_{32k}^1)g_{31h}^k f_+^h-(-g_{32k}^2)g_{01h}^k f_-^h-(-g_{32k}^3)g_{11h}^k f_+^h+(-g_{32k}^0)g_{21h}^k f_-^h)+\\
+(+(-g_{22k}^1)g_{21h}^k f_+^h-(-g_{22k}^2)g_{11h}^k f_+^h+(-g_{22k}^3)g_{01h}^k f_-^h-(-g_{22k}^0)g_{31h}^k f_-^h)+-g_{12k}^1 g_{11h}^k f_+^h-g_{12k}^2 g_{21h}^k f_+^h-g_{12k}^3 g_{31h}^k f_+^h-g_{12k}^0 g_{01h}^k f_+^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_0^2 f_+^1+\partial_3^2 f_+^1+\partial_2^2 f_+^1+\partial_1^2 f_+^1+\\
\partial_0(+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+\partial_3(-\partial_0 f_-^2-\partial_1 f_+^3-\partial_2 f_-^0)+\partial_2(-\partial_1 f_+^2+\partial_0 f_-^3+\partial_3 f_-^0)+\partial_1(+\partial_2 f_+^2+\partial_3 f_+^3-\partial_0 f_+^0)+\\
+\partial_0(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\\
+\partial_1(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+\\
+\partial_2(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+\\
+\partial_3(+g_{31h}^1 f_+^h-g_{01h}^2 f_-^h-g_{11h}^3 f_+^h+g_{21h}^0 f_-^h)+\\
+g_{02h}^1\partial_0 f_+^h+g_{02h}^2\partial_3 f_-^h-g_{02h}^3\partial_2 f_-^h+g_{02h}^0\partial_1 f_+^h-g_{32h}^1\partial_3 f_+^h+g_{32h}^2\partial_0 f_-^h+g_{32h}^3\partial_1 f_+^h+g_{32h}^0\partial_2 f_-^h+\\
-g_{22h}^1\partial_2 f_+^h+g_{22h}^2\partial_1 f_+^h-g_{22h}^3\partial_0 f_-^h-g_{22h}^0\partial_3 f_-^h+g_{12h}^1\partial_1 f_+^h+g_{12h}^2\partial_2 f_+^h+g_{12h}^3\partial_3 f_+^h-g_{12h}^0\partial_0 f_+^h+\\
-g_{02k}^1 g_{01h}^1 f_+^h-g_{02k}^2 g_{31h}^k f_-^h+g_{02k}^3 g_{21h}^k f_-^h+g_{02k}^0 g_{11h}^k f_+^h-g_{32k}^1 g_{31h}^k f_+^h+g_{32k}^2 g_{01h}^k f_-^h+g_{32k}^3 g_{11h}^k f_+^h-g_{32k}^0 g_{21h}^k f_-^h+\\
-g_{22k}^1 g_{21h}^k f_+^h+g_{22k}^2 g_{11h}^k f_+^h-g_{22k}^3 g_{01h}^k f_-^h+g_{22k}^0 g_{31h}^k f_-^h-g_{12k}^1 g_{11h}^k f_+^h-g_{12k}^2 g_{21h}^k f_+^h-g_{12k}^3 g_{31h}^k f_+^h-g_{12k}^0 g_{01h}^k f_+^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_0^2 f_+^1+\partial_3^2 f_+^1+\partial_2^2 f_+^1+\partial_1^2 f_+^1+\\
+\partial_0([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h)+\\
+\partial_1([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h)+\\
+\partial_2([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h)+\\
+\partial_3([-g_{11h}^3+g_{31h}^1]f_+^h+[-g_{01h}^2+g_{21h}^0]f_-^h)+\\
-g_{12h}^0\partial_0 f_+^h+g_{02h}^1\partial_0 f_+^h+g_{32h}^2\partial_0 f_-^h-g_{22h}^3\partial_0 f_-^h\\
+g_{02h}^0\partial_1 f_+^h+g_{12h}^1\partial_1 f_+^h+g_{22h}^2\partial_1 f_+^h+g_{32h}^3\partial_1 f_+^h+\\
-g_{22h}^1\partial_2 f_+^h+g_{12h}^2\partial_2 f_+^h-g_{02h}^3\partial_2 f_-^h+g_{32h}^0\partial_2 f_-^h+\\
-g_{22h}^0\partial_3 f_-^h+g_{02h}^2\partial_3 f_-^h-g_{32h}^1\partial_3 f_+^h+g_{12h}^3\partial_3 f_+^h+\\
-g_{02k}^1 g_{01h}^1 f_+^h-g_{02k}^2 g_{31h}^k f_-^h+g_{02k}^3 g_{21h}^k f_-^h+g_{02k}^0 g_{11h}^k f_+^h-g_{32k}^1 g_{31h}^k f_+^h+g_{32k}^2 g_{01h}^k f_-^h+g_{32k}^3 g_{11h}^k f_+^h-g_{32k}^0 g_{21h}^k f_-^h+\\
-g_{22k}^1 g_{21h}^k f_+^h+g_{22k}^2 g_{11h}^k f_+^h-g_{22k}^3 g_{01h}^k f_-^h+g_{22k}^0 g_{31h}^k f_-^h-g_{12k}^1 g_{11h}^k f_+^h-g_{12k}^2 g_{21h}^k f_+^h-g_{12k}^3 g_{31h}^k f_+^h-g_{12k}^0 g_{01h}^k f_+^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1+\\
+\partial_0([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h)+\\
+\partial_1([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h)+\\
+\partial_2([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h)+\\
+\partial_3([-g_{11h}^3+g_{31h}^1]f_+^h+[-g_{01h}^2+g_{21h}^0]f_-^h)+\\
+(-g_{12h}^0+g_{02h}^1)\partial_0 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_0 f_-^h\\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_1 f_+^h+\\
+(-g_{22h}^1+g_{12h}^2)\partial_2 f_+^h+(+g_{32h}^0-g_{02h}^3)\partial_2 f_-^h+\\
+(-g_{32h}^1+g_{12h}^3)\partial_3 f_+^h+(-g_{22h}^0+g_{02h}^2)\partial_3 f_-^h+\\
+([-g_{12k}^0-g_{02k}^1]g_{01h}^k+[+g_{02k}^0-g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{11h}^k+[-g_{22k}^1-g_{12k}^2]g_{21h}^k+[-g_{32k}^1-g_{12k}^3]g_{31h}^k)f_+^h+\\
+([+g_{32k}^2-g_{22k}^3]g_{01h}^k+[-g_{32k}^0+g_{02k}^3]g_{21h}^k+[+g_{22k}^0-g_{02k}^2]g_{31h}^k)f_-^h
\end{array}\right)
$$

and (*1-):

$$
\left(\begin{array}{l}
-D_{02}^-[-(\partial_0 f_-^1+g_{01h}^1 f_-^h)-(\partial_3 f_+^2+g_{31h}^2 f_+^h)+(\partial_2 f_+^3+g_{21h}^3 f_+^h)-(\partial_1 f_-^0-g_{11h}^0 f_-^h)]+\\
\quad+D_{32}^+[(\partial_3 f_-^1-g_{31h}^1 f_-^h)-(\partial_0 f_+^2-g_{01h}^2 f_+^h)-(\partial_1 f_-^3-g_{11h}^3 f_-^h)-(\partial_2 f_+^0+g_{21h}^0 f_+^h)]+\\
\quad-D_{22}^+[-(\partial_2 f_-^1-g_{21h}^1 f_-^h)+(\partial_1 f_-^2-g_{11h}^2 f_-^h)-(\partial_0 f_+^3-g_{01h}^3 f_+^h)-(\partial_3 f_+^0+g_{31h}^0 f_+^h)]+\\
\quad-D_{12}^-[-(\partial_1 f_-^1+g_{11h}^1 f_-^h)-(\partial_2 f_-^2+g_{21h}^2 f_-^h)-(\partial_3 f_-^3+g_{31h}^3 f_-^h)+(\partial_0 f_-^0-g_{01h}^0 f_-^h)]
\end{array}\right)=
$$

$$
\begin{aligned}
&= \left(\begin{array}{l}
D^-_{02}(\partial_0 f^1_- + \partial_3 f^2_+ - \partial_2 f^3_+ + \partial_1 f^0_-) + D^-_{02}(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + \\
\quad + D^+_{32}(\partial_3 f^1_- - \partial_0 f^2_+ - \partial_1 f^3_- - \partial_2 f^0_+) + D^+_{32}(-g^1_{31h} f^h_- + g^2_{01h} f^h_+ + g^3_{11h} f^h_- - g^0_{21h} f^h_+) + \\
\quad + D^+_{22}(\partial_2 f^1_- - \partial_1 f^2_- + \partial_0 f^3_+ + \partial_3 f^0_+) + D^+_{22}(-g^1_{21h} f^h_- + g^2_{11h} f^h_- - g^3_{01h} f^h_+ + g^0_{31h} f^h_+) + \\
\quad + D^-_{12}(\partial_1 f^1_- + \partial_2 f^2_- + \partial_3 f^3_- - \partial_0 f^0_-) + D^-_{12}(+g^1_{11h} f^h_- + g^2_{21h} f^h_- + g^3_{31h} f^h_- + g^0_{01h} f^h_-)
\end{array}\right) \\
&= \left(\begin{array}{l}
\partial_0(\partial_0 f^1_- + \partial_3 f^2_+ - \partial_2 f^3_+ + \partial_1 f^0_-) + ((-g^1_{02h})\partial_0 f^h_- + (-g^2_{02h})\partial_3 f^h_+ - (-g^3_{02h})\partial_2 f^h_+ + (-g^0_{02h})\partial_1 f^h_-) + \\
+\partial_0(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + (+(-g^1_{02k})g^k_{01h} f^h_- + (-g^2_{02k})g^k_{31h} f^h_+ - (-g^3_{02k})g^k_{21h} f^h_+ - (-g^0_{02k})g^k_{11h} f^h_-) + \\
\quad + \partial_3(\partial_3 f^1_- - \partial_0 f^2_+ - \partial_1 f^3_- - \partial_2 f^0_+) + ((+g^1_{32h})\partial_3 f^1_- - (+g^2_{32h})\partial_0 f^h_+ - (+g^3_{32h})\partial_1 f^h_- - (+g^0_{32h})\partial_2 f^h_+) + \\
+\partial_3(-g^1_{31h} f^h_- + g^2_{01h} f^h_+ + g^3_{11h} f^h_- - g^0_{21h} f^h_+) + (-(+g^1_{32k})g^k_{31h} f^h_- + (+g^2_{32k})g^k_{01h} f^h_+ + (+g^3_{32k})g^k_{11h} f^h_- - (+g^0_{32k})g^k_{21h} f^h_+) + \\
\quad + \partial_2(\partial_2 f^1_- - \partial_1 f^2_- + \partial_0 f^3_+ + \partial_3 f^0_+) + ((+g^1_{22h})\partial_2 f^h_- - (+g^2_{22h})\partial_1 f^h_- + (+g^3_{22h})\partial_0 f^h_+ + (+g^0_{22h})\partial_3 f^h_+) + \\
+\partial_2(-g^1_{21h} f^h_- + g^2_{11h} f^h_- - g^3_{01h} f^h_+ + g^0_{31h} f^h_+) + (-(+g^1_{22k})g^k_{21h} f^h_- + (+g^2_{22k})g^k_{11h} f^h_- - (+g^3_{22k})g^k_{01h} f^h_+ + (+g^0_{22k})g^k_{31h} f^h_+) + \\
\quad + \partial_1(\partial_1 f^1_- + \partial_2 f^2_- + \partial_3 f^3_- - \partial_0 f^0_-) + ((-g^1_{12h})\partial_1 f^h_- + (-g^2_{12h})\partial_2 f^h_- + (-g^3_{12h})\partial_3 f^h_- - (-g^0_{12h})\partial_0 f^h_-) + \\
+\partial_1(+g^1_{11h} f^h_- + g^2_{21h} f^h_- + g^3_{31h} f^h_- + g^0_{01h} f^h_-) + (+(-g^1_{12k})g^k_{11h} f^h_- + (-g^2_{12k})g^k_{21h} f^h_- + (-g^3_{12k})g^k_{31h} f^h_- + (-g^0_{12k})g^k_{01h} f^h_-)
\end{array}\right) \\
&= \left(\begin{array}{l}
\partial_0(\partial_0 f^1_- + \partial_3 f^2_+ - \partial_2 f^3_+ + \partial_1 f^0_-) + \partial_3(\partial_3 f^1_- - \partial_0 f^2_+ - \partial_1 f^3_- - \partial_2 f^0_+) + \partial_2(\partial_2 f^1_- - \partial_1 f^2_- + \partial_0 f^3_+ + \partial_3 f^0_+) + \partial_1(\partial_1 f^1_- + \partial_2 f^2_- + \partial_3 f^3_- - \partial_0 f^0_-) \\
+((-g^1_{02h})\partial_0 f^h_- + (-g^2_{02h})\partial_3 f^h_+ - (-g^3_{02h})\partial_2 f^h_+ + (-g^0_{02h})\partial_1 f^h_-) + ((+g^1_{32h})\partial_3 f^1_- - (+g^2_{32h})\partial_0 f^h_+ - (+g^3_{32h})\partial_1 f^h_- - (+g^0_{32h})\partial_2 f^h_+) + \\
+((+g^1_{22h})\partial_2 f^h_- - (+g^2_{22h})\partial_1 f^h_- + (+g^3_{22h})\partial_0 f^h_+ + (+g^0_{22h})\partial_3 f^h_+) + ((-g^1_{12h})\partial_1 f^h_- + (-g^2_{12h})\partial_2 f^h_- + (-g^3_{12h})\partial_3 f^h_- - (-g^0_{12h})\partial_0 f^h_-) + \\
+\partial_0(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + \partial_3(-g^1_{31h} f^h_- + g^2_{01h} f^h_+ + g^3_{11h} f^h_- - g^0_{21h} f^h_+) + \\
+\partial_2(-g^1_{21h} f^h_- + g^2_{11h} f^h_- - g^3_{01h} f^h_+ + g^0_{31h} f^h_+) + \partial_1(+g^1_{11h} f^h_- + g^2_{21h} f^h_- + g^3_{31h} f^h_- + g^0_{01h} f^h_-) + \\
+(+(-g^1_{02k})g^k_{01h} f^h_- + (-g^2_{02k})g^k_{31h} f^h_+ - (-g^3_{02k})g^k_{21h} f^h_+ - (-g^0_{02k})g^k_{11h} f^h_-) + (-(+g^1_{32k})g^k_{31h} f^h_- + (+g^2_{32k})g^k_{01h} f^h_+ + (+g^3_{32k})g^k_{11h} f^h_- - (+g^0_{32k})g^k_{21h} f^h_+) + \\
+(-(+g^1_{22k})g^k_{21h} f^h_- + (+g^2_{22k})g^k_{11h} f^h_- - (+g^3_{22k})g^k_{01h} f^h_+ + (+g^0_{22k})g^k_{31h} f^h_+) + (+(-g^1_{12k})g^k_{11h} f^h_- + (-g^2_{12k})g^k_{21h} f^h_- + (-g^3_{12k})g^k_{31h} f^h_- + (-g^0_{12k})g^k_{01h} f^h_-)
\end{array}\right) \\
&= \left(\begin{array}{l}
\partial^2_0 f^1_- + \partial^2_3 f^1_- + \partial^2_2 f^1_- + \partial^2_1 f^1_- + \\
\partial_0(+\partial_3 f^2_+ - \partial_2 f^3_+ + \partial_1 f^0_-) + \partial_3(-\partial_0 f^2_+ - \partial_1 f^3_- - \partial_2 f^0_+) + \partial_2(-\partial_1 f^2_- + \partial_0 f^3_+ + \partial_3 f^0_+) + \partial_1(+\partial_2 f^2_- + \partial_3 f^3_- - \partial_0 f^0_-) \\
+\partial_0(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + \\
+\partial_1(+g^1_{11h} f^h_- + g^2_{21h} f^h_- + g^3_{31h} f^h_- + g^0_{01h} f^h_-) + \\
+\partial_2(-g^1_{21h} f^h_- + g^2_{11h} f^h_- - g^3_{01h} f^h_+ + g^0_{31h} f^h_+) + \\
+\partial_3(-g^1_{31h} f^h_- + g^2_{01h} f^h_+ + g^3_{11h} f^h_- - g^0_{21h} f^h_+) + \\
-g^1_{02h}\partial_0 f^h_- - g^2_{02h}\partial_3 f^h_+ + g^3_{02h}\partial_2 f^h_+ - g^0_{02h}\partial_1 f^h_- + g^1_{32h}\partial_3 f^1_- - g^2_{32h}\partial_0 f^h_+ - g^3_{32h}\partial_1 f^h_- - g^0_{32h}\partial_2 f^h_+ + \\
+g^1_{22h}\partial_2 f^h_- - g^2_{22h}\partial_1 f^h_- + g^3_{22h}\partial_0 f^h_+ + g^0_{22h}\partial_3 f^h_+ - g^1_{12h}\partial_1 f^h_- - g^2_{12h}\partial_2 f^h_- - g^3_{12h}\partial_3 f^h_- + g^0_{12h}\partial_0 f^h_- + \\
-g^1_{02k}g^k_{01h} f^h_- - g^2_{02k}g^k_{31h} f^h_+ + g^3_{02k}g^k_{21h} f^h_+ + g^0_{02k}g^k_{11h} f^h_- - g^1_{32k}g^k_{31h} f^h_- + g^2_{32k}g^k_{01h} f^h_+ + g^3_{32k}g^k_{11h} f^h_- - g^0_{32k}g^k_{21h} f^h_+ + \\
-g^1_{22k}g^k_{21h} f^h_- + g^2_{22k}g^k_{11h} f^h_- - g^3_{22k}g^k_{01h} f^h_+ + g^0_{22k}g^k_{31h} f^h_+ - g^1_{12k}g^k_{11h} f^h_- - g^2_{12k}g^k_{21h} f^h_- - g^3_{12k}g^k_{31h} f^h_- - g^0_{12k}g^k_{01h} f^h_-
\end{array}\right) \\
&= \left(\begin{array}{l}
\partial^2_0 f^1_- + \partial^2_3 f^1_- + \partial^2_2 f^1_- + \partial^2_1 f^1_- + \\
+\partial_0([+g^2_{31h} - g^3_{21h}] f^h_+ + [-g^0_{11h} + g^1_{01h}] f^h_-) + \\
+\partial_1([+g^0_{01h} + g^1_{11h} + g^2_{21h} + g^3_{31h}] f^h_-) + \\
+\partial_2([+g^0_{31h} - g^3_{01h}] f^h_+ + [-g^1_{21h} + g^2_{11h}] f^h_-) + \\
+\partial_3([+g^0_{21h} + g^2_{01h}] f^h_+ + [-g^1_{31h} + g^3_{11h}] f^h_-) + \\
+g^0_{12h}\partial_0 f^h_- - g^1_{02h}\partial_0 f^h_- - g^2_{32h}\partial_0 f^h_+ + g^3_{22h}\partial_0 f^h_+ + \\
-g^0_{02h}\partial_1 f^h_- - g^1_{12h}\partial_1 f^h_- - g^2_{22h}\partial_1 f^h_- - g^3_{32h}\partial_1 f^h_- + \\
-g^0_{32h}\partial_2 f^h_+ + g^3_{02h}\partial_2 f^h_+ + g^1_{22h}\partial_2 f^h_- - g^2_{12h}\partial_2 f^h_- \\
+g^0_{22h}\partial_3 f^h_+ - g^2_{02h}\partial_3 f^h_+ + g^1_{32h}\partial_3 f^1_- - g^3_{12h}\partial_3 f^h_- + \\
-g^1_{02k}g^k_{01h} f^h_- - g^2_{02k}g^k_{31h} f^h_+ + g^3_{02k}g^k_{21h} f^h_+ + g^0_{02k}g^k_{11h} f^h_- - g^1_{32k}g^k_{31h} f^h_- + g^2_{32k}g^k_{01h} f^h_+ + g^3_{32k}g^k_{11h} f^h_- - g^0_{32k}g^k_{21h} f^h_+ + \\
-g^1_{22k}g^k_{21h} f^h_- + g^2_{22k}g^k_{11h} f^h_- - g^3_{22k}g^k_{01h} f^h_+ + g^0_{22k}g^k_{31h} f^h_+ - g^1_{12k}g^k_{11h} f^h_- - g^2_{12k}g^k_{21h} f^h_- - g^3_{12k}g^k_{31h} f^h_- - g^0_{12k}g^k_{01h} f^h_-
\end{array}\right) \\
&= \left(\begin{array}{l}
(\partial^2_0 + \partial^2_3 + \partial^2_2 + \partial^2_1) f^1_- + \\
+\partial_0([+g^2_{31h} - g^3_{21h}] f^h_+ + [-g^0_{11h} + g^1_{01h}] f^h_-) + \\
+\partial_1([+g^0_{01h} + g^1_{11h} + g^2_{21h} + g^3_{31h}] f^h_-) + \\
+\partial_2([+g^0_{31h} - g^3_{01h}] f^h_+ + [-g^1_{21h} + g^2_{11h}] f^h_-) + \\
+\partial_3([-g^0_{21h} + g^2_{01h}] f^h_+ + [-g^1_{31h} + g^3_{11h}] f^h_-) + \\
+(-g^2_{32h} + g^3_{22h})\partial_0 f^h_+ + (+g^0_{12h} - g^1_{02h})\partial_0 f^h_- + \\
+(-g^0_{02h} - g^1_{12h} - g^2_{22h} - g^3_{32h})\partial_1 f^h_- + \\
+(-g^0_{32h} + g^3_{02h})\partial_2 f^h_+ + (+g^1_{22h} - g^2_{12h})\partial_2 f^h_- \\
+(+g^0_{22h} - g^2_{02h})\partial_3 f^h_+ + (+g^1_{32h} - g^3_{12h})\partial_3 f^h_- + \\
+([+g^2_{32k} - g^3_{22k}]g^k_{01h} + [-g^0_{32k} + g^3_{02k}]g^k_{21h} + [+g^0_{22k} - g^2_{02k}]g^k_{31h}) f^h_+ \\
+([-g^0_{12k} - g^1_{02k}]g^k_{01h} + [+g^0_{02k} - g^1_{12k} + g^2_{22k} + g^3_{32k}]g^k_{11h} + [-g^1_{22k} - g^2_{12k}]g^k_{21h} + [-g^1_{32k} - g^3_{12k}]g^k_{31h}) f^h_-
\end{array}\right)
\end{aligned}
$$

and (*2+):

$$
\left(\begin{array}{l}
-D^-_{32}[-(\partial_0 f^1_- + g^1_{01h} f^h_-) - (\partial_3 f^2_+ + g^2_{31h} f^h_+) + (\partial_2 f^3_+ + g^3_{21h} f^h_+) - (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
\quad - D^+_{02}[(\partial_3 f^1_- - g^1_{31h} f^h_-) - (\partial_0 f^2_+ - g^2_{01h} f^h_+) - (\partial_1 f^3_- - g^3_{11h} f^h_-) - (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
\quad + D^-_{12}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
\quad - D^+_{22}[-(\partial_1 f^1_+ - g^1_{11h} f^h_+) - (\partial_2 f^2_+ - g^2_{21h} f^h_+) - (\partial_3 f^3_+ - g^3_{31h} f^h_+) + (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{array}\right) =
$$

$$
= \left(\begin{array}{l}
D^-_{32}[+(\partial_0 f^1_- + g^1_{01h} f^h_-) + (\partial_3 f^2_+ + g^2_{31h} f^h_+) - (\partial_2 f^3_+ + g^3_{21h} f^h_+) + (\partial_1 f^0_- - g^0_{11h} f^h_-)] + \\
\quad + D^+_{02}[-(\partial_3 f^1_- - g^1_{31h} f^h_-) + (\partial_0 f^2_+ - g^2_{01h} f^h_+) + (\partial_1 f^3_- - g^3_{11h} f^h_-) + (\partial_2 f^0_+ + g^0_{21h} f^h_+)] + \\
\quad + D^-_{12}[-(\partial_2 f^1_+ + g^1_{21h} f^h_+) + (\partial_1 f^2_+ + g^2_{11h} f^h_+) - (\partial_0 f^3_- + g^3_{01h} f^h_-) - (\partial_3 f^0_- - g^0_{31h} f^h_-)] + \\
\quad + D^+_{22}[+(\partial_1 f^1_+ - g^1_{11h} f^h_+) + (\partial_2 f^2_+ - g^2_{21h} f^h_+) + (\partial_3 f^3_+ - g^3_{31h} f^h_+) - (\partial_0 f^0_+ + g^0_{01h} f^h_+)]
\end{array}\right)
$$

$$
= \left(\begin{array}{l}
\partial_3(+\partial_0 f^1_- + \partial_3 f^2_+ - \partial_2 f^3_+ + \partial_1 f^0_-) + (+(-g^1_{32h})\partial_0 f^h_- + (-g^2_{32h})\partial_3 f^h_+ - (-g^3_{32h})\partial_2 f^h_+ + (-g^0_{32h})\partial_1 f^h_-) + \\
+\partial_3(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + (+(-g^1_{32k}) g^k_{01h} f^h_- + (-g^2_{32k}) g^k_{31h} f^h_+ - (-g^3_{32k}) g^k_{21h} f^h_+ - (-g^0_{32k}) g^k_{11h} f^h_-) + \\
\quad + \partial_0(-\partial_3 f^1_- + \partial_0 f^2_+ + \partial_1 f^3_- + \partial_2 f^0_+) + (-(+g^1_{02h})\partial_3 f^h_- + (+g^2_{02h})\partial_0 f^h_+ + (+g^3_{02h})\partial_1 f^h_- + (+g^0_{02h})\partial_2 f^h_+) + \\
+\partial_0(+g^1_{31h} f^h_- - g^2_{01h} f^h_+ - g^3_{11h} f^h_- + g^0_{21h} f^h_+) + (+(+g^1_{02k}) g^k_{31h} f^h_- - (+g^2_{02k}) g^k_{01h} f^h_+ - (+g^3_{02k}) g^k_{11h} f^h_- + (+g^0_{02k}) g^k_{21h} f^h_+) + \\
\quad + \partial_1(-\partial_2 f^1_+ + \partial_1 f^2_+ - \partial_0 f^3_- - \partial_3 f^0_-) + (-(-g^1_{12h})\partial_2 f^1_+ + (-g^2_{12h})\partial_1 f^h_+ - (-g^3_{12h})\partial_0 f^h_- - (-g^0_{12h})\partial_3 f^h_-) + \\
+\partial_1(-g^1_{21h} f^h_+ + g^2_{11h} f^h_+ - g^3_{01h} f^h_- + g^0_{31h} f^h_-) + (-(-g^1_{12k}) g^k_{21h} f^h_+ + (-g^2_{12k}) g^k_{11h} f^h_+ - (-g^3_{12k}) g^k_{01h} f^h_- + (-g^0_{12k}) g^k_{31h} f^h_-) + \\
\quad + \partial_2(+\partial_1 f^1_+ + \partial_2 f^2_+ + \partial_3 f^3_+ - \partial_0 f^0_+) + (+(+g^1_{22h})\partial_1 f^h_+ + (+g^2_{22h})\partial_2 f^h_+ + (+g^3_{22h})\partial_3 f^h_+ - (+g^0_{22h})\partial_0 f^h_+) + \\
+\partial_2(-g^1_{11h} f^h_+ - g^2_{21h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+) + (-(+g^1_{22k}) g^k_{11h} f^h_+ - (+g^2_{22k}) g^k_{21h} f^h_+ - (+g^3_{22k}) g^k_{31h} f^h_+ - (+g^0_{22k}) g^k_{01h} f^h_+) \\
+\partial_2(-g^1_{11h} f^h_+ - g^2_{21h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+) + (-(+g^1_{22k}) g^k_{11h} f^h_+ - (+g^2_{22k}) g^k_{21h} f^h_+ - (+g^3_{22k}) g^k_{31h} f^h_+ - (+g^0_{22k}) g^k_{01h} f^h_+)
\end{array}\right)
$$

$$
= \left(\begin{array}{l}
\partial_3^2 f^2_+ + \partial_0^2 f^2_+ + \partial_1^2 f^2_+ + \partial_2^2 f^2_+ + \\
+\partial_3(+\partial_0 f^1_- - \partial_2 f^3_+ + \partial_1 f^0_-) + \partial_0(-\partial_3 f^1_- + \partial_0 f^2_+ + \partial_1 f^3_- + \partial_2 f^0_+) + \partial_1(-\partial_2 f^1_+ - \partial_0 f^3_- - \partial_3 f^0_-) + \partial_2(+\partial_1 f^1_+ + \partial_3 f^3_+ - \partial_0 f^0_+) + \\
+\partial_0(+g^1_{31h} f^h_- - g^2_{01h} f^h_+ - g^3_{11h} f^h_- + g^0_{21h} f^h_+) + \\
+\partial_1(-g^1_{21h} f^h_+ + g^2_{11h} f^h_+ - g^3_{01h} f^h_- + g^0_{31h} f^h_-) + \\
+\partial_2(-g^1_{11h} f^h_+ - g^2_{21h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+) \\
+\partial_3(+g^1_{01h} f^h_- + g^2_{31h} f^h_+ - g^3_{21h} f^h_+ - g^0_{11h} f^h_-) + \\
+(+(-g^1_{32h})\partial_0 f^h_- + (-g^2_{32h})\partial_3 f^h_+ - (-g^3_{32h})\partial_2 f^h_+ + (-g^0_{32h})\partial_1 f^h_-) + \\
+(-(+g^1_{02h})\partial_3 f^h_- + (+g^2_{02h})\partial_0 f^h_+ + (+g^3_{02h})\partial_1 f^h_- + (+g^0_{02h})\partial_2 f^h_+) + \\
+(-(-g^1_{12h})\partial_2 f^1_+ + (-g^2_{12h})\partial_1 f^h_+ - (-g^3_{12h})\partial_0 f^h_- - (-g^0_{12h})\partial_3 f^h_-) + \\
+(+(+g^1_{22h})\partial_1 f^h_+ + (+g^2_{22h})\partial_2 f^h_+ + (+g^3_{22h})\partial_3 f^h_+ - (+g^0_{22h})\partial_0 f^h_+) + \\
+(+(-g^1_{32k}) g^k_{01h} f^h_- + (-g^2_{32k}) g^k_{31h} f^h_+ - (-g^3_{32k} + \partial_2(-g^1_{21h} f^h_+ - g^2_{11h} f^h_+ - g^3_{31h} f^h_+ - g^0_{01h} f^h_+)) g^k_{21h} f^h_+ - (-g^0_{32k}) g^k_{11h} f^h_-) + \\
+(+(+g^1_{02k}) g^k_{31h} f^h_- - (+g^2_{02k}) g^k_{01h} f^h_+ - (+g^3_{02k}) g^k_{11h} f^h_- + (+g^0_{02k}) g^k_{21h} f^h_+) + \\
+(-(-g^1_{12k}) g^k_{21h} f^h_+ + (-g^2_{12k}) g^k_{11h} f^h_+ - (-g^3_{12k}) g^k_{01h} f^h_- + (-g^0_{12k}) g^k_{31h} f^h_-) + \\
+(-(+g^1_{22k}) g^k_{11h} f^h_+ - (+g^2_{22k}) g^k_{21h} f^h_+ - (+g^3_{22k}) g^k_{31h} f^h_+ + (+g^0_{22k}) g^k_{01h} f^h_+)
\end{array}\right)
$$

$$
= \left(\begin{array}{l}
\partial_3^2 f^2_+ + \partial_0^2 f^2_+ + \partial_1^2 f^2_+ + \partial_2^2 f^2_+ + \\
+\partial_3(+\partial_0 f^1_- - \partial_2 f^3_+ + \partial_1 f^0_-) + \partial_0(-\partial_3 f^1_- + \partial_0 f^2_+ + \partial_1 f^3_- + \partial_2 f^0_+) + \partial_1(-\partial_2 f^1_+ - \partial_0 f^3_- - \partial_3 f^0_-) + \partial_2(+\partial_1 f^1_+ + \partial_3 f^3_+ - \partial_0 f^0_+) + \\
+\partial_0([+g^0_{21h} - g^2_{01h}] f^h_+ + [+g^1_{31h} - g^3_{11h}] f^h_-) + \\
+\partial_1([-g^1_{21h} + g^2_{11h}] f^h_+ + [+g^0_{31h} - g^3_{01h}] f^h_-) + \\
+\partial_2([-g^0_{01h} - g^1_{11h} - g^2_{21h} - g^3_{31h}] f^h_+) + \\
+\partial_3([+g^2_{31h} - g^3_{21h}] f^h_+ + [-g^0_{11h} + g^1_{01h}] f^h_-) + \\
-g^1_{32h}\partial_0 f^h_- - g^2_{32h}\partial_3 f^h_+ + g^3_{32h}\partial_2 f^h_+ - g^0_{32h}\partial_1 f^h_- + \\
-g^1_{02h}\partial_3 f^h_- + g^2_{02h}\partial_0 f^h_+ + g^3_{02h}\partial_1 f^h_- + g^0_{02h}\partial_2 f^h_+ + \\
+g^1_{12h}\partial_2 f^1_+ - g^2_{12h}\partial_1 f^h_+ + g^3_{12h}\partial_0 f^h_- + g^0_{12h}\partial_3 f^h_- + \\
+g^1_{22h}\partial_1 f^h_+ + g^2_{22h}\partial_2 f^h_+ + g^3_{22h}\partial_3 f^h_+ - g^0_{22h}\partial_0 f^h_+ + \\
-g^1_{32k} g^k_{01h} f^h_- - g^2_{32k} g^k_{31h} f^h_+ + g^3_{32k} g^k_{21h} f^h_+ + g^0_{32k} g^k_{11h} f^h_- + \\
+g^1_{02k} g^k_{31h} f^h_- - g^2_{02k} g^k_{01h} f^h_+ - g^3_{02k} g^k_{11h} f^h_- + g^0_{02k} g^k_{21h} f^h_+ + \\
+g^1_{12k} g^k_{21h} f^h_+ - g^2_{12k} g^k_{11h} f^h_+ + g^3_{12k} g^k_{01h} f^h_- - g^0_{12k} g^k_{31h} f^h_- + \\
-g^1_{22k} g^k_{11h} f^h_+ - g^2_{22k} g^k_{21h} f^h_+ - g^3_{22k} g^k_{31h} f^h_+ - g^0_{22k} g^k_{01h} f^h_+
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_3^2 f_+^2+\partial_0^2 f_+^2+\partial_1^2 f_+^2+\partial_2^2 f_+^2+\\
+\partial_0([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h)+\\
+\partial_1([-g_{21h}^1+g_{11h}^2]f_+^h+[+g_{31h}^0-g_{01h}^3]f_-^h)+\\
+\partial_2([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h)+\\
+\partial_3([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h)+\\
-g_{22h}^0\partial_0 f_+^h+g_{02h}^2\partial_0 f_+^h-g_{32h}^1\partial_0 f_-^h+g_{12h}^3\partial_0 f_-^h\\
-g_{32h}^0\partial_1 f_-^h+g_{02h}^3\partial_1 f_-^h+g_{22h}^1\partial_1 f_+^h-g_{12h}^2\partial_1 f_+^h\\
+g_{02h}^0\partial_2 f_+^h+g_{12h}^1\partial_2 f_+^1+g_{22h}^2\partial_2 f_+^h+g_{32h}^3\partial_2 f_+^h+\\
+g_{12h}^0\partial_3 f_-^h-g_{02h}^1\partial_3 f_-^h-g_{32h}^2\partial_3 f_+^h+g_{22h}^3\partial_3 f_+^h+\\
-g_{32k}^1 g_{01h}^k f_-^h-g_{32k}^2 g_{31h}^k f_+^h+g_{32k}^3 g_{21h}^k f_+^h+g_{32k}^0 g_{11h}^k f_-^h+\\
+g_{02k}^1 g_{31h}^k f_-^h-g_{02k}^2 g_{01h}^k f_+^h-g_{02k}^3 g_{11h}^k f_-^h+g_{02k}^0 g_{21h}^k f_+^h+\\
+g_{12k}^1 g_{21h}^k f_+^h-g_{12k}^2 g_{11h}^k f_+^h+g_{12k}^3 g_{01h}^k f_-^h-g_{12k}^0 g_{31h}^k f_-^h+\\
-g_{22k}^1 g_{11h}^k f_+^h-g_{22k}^2 g_{21h}^k f_+^h-g_{22k}^3 g_{31h}^k f_+^h-g_{22k}^0 g_{01h}^k f_+^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2+\\
+\partial_0([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h)+\\
+\partial_1([-g_{21h}^1+g_{11h}^2]f_+^h+[+g_{31h}^0-g_{01h}^3]f_-^h)+\\
+\partial_2([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h)+\\
+\partial_3([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h)+\\
+(-g_{22h}^0+g_{02h}^2)\partial_0 f_+^h+(-g_{32h}^1+g_{12h}^3)\partial_0 f_-^h\\
+[+g_{22h}^1-g_{12h}^2]\partial_1 f_+^h+(-g_{32h}^0+g_{02h}^3)\partial_1 f_-^h\\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_2 f_+^h+\\
+(-g_{32h}^2+g_{22h}^3)\partial_3 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_3 f_-^h+\\
+([-g_{02k}^2-g_{22k}^0]g_{01h}^k+[-g_{22k}^1-g_{12k}^2]g_{11h}^k+[+g_{02k}^0+g_{12k}^1-g_{22k}^2+g_{32k}^3]g_{21h}^k+[-g_{32k}^2-g_{22k}^3]g_{31h}^k)f_+^h\\
+([-g_{32k}^1+g_{12k}^3]g_{01h}^k+[+g_{32k}^0-g_{02k}^3]g_{11h}^k+[-g_{12k}^0+g_{02k}^1]g_{31h}^k)f_-^h+
\end{array}\right)
$$

and (*2-):

$$
\left(\begin{array}{l}
-D_{32}^+[-(\partial_0 f_+^1-g_{01h}^1 f_+^h)-(\partial_3 f_-^2-g_{31h}^2 f_-^h)+(\partial_2 f_-^3-g_{21h}^3 f_-^h)-(\partial_1 f_+^0+g_{11h}^0 f_+^h)]+\\
\quad -D_{02}^-[(\partial_3 f_+^1+g_{31h}^1 f_+^h)-(\partial_0 f_-^2+g_{01h}^2 f_-^h)-(\partial_1 f_+^3+g_{11h}^3 f_+^h)-(\partial_2 f_-^0-g_{21h}^0 f_-^h)]+\\
\quad +D_{12}^+[-(\partial_2 f_-^1-g_{21h}^1 f_-^h)+(\partial_1 f_-^2-g_{11h}^2 f_-^h)-(\partial_0 f_+^3-g_{01h}^3 f_+^h)-(\partial_3 f_+^0+g_{31h}^0 f_+^h)]+\\
\quad -D_{22}^-[-(\partial_1 f_-^1+g_{11h}^1 f_-^h)-(\partial_2 f_-^2+g_{21h}^2 f_-^h)-(\partial_3 f_-^3+g_{31h}^3 f_-^h)+(\partial_0 f_-^0-g_{01h}^0 f_-^h)]
\end{array}\right)=
$$

$$
=\left(\begin{array}{l}
D_{32}^+(+\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+D_{32}^+(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\\
\quad +D_{02}^-(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3+\partial_2 f_-^0)+D_{02}^-(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
\quad +D_{12}^+(-\partial_2 f_-^1+\partial_1 f_-^2-\partial_0 f_+^3-\partial_3 f_+^0)+D_{12}^+(+g_{21h}^1 f_-^h-g_{11h}^2 f_-^h+g_{01h}^3 f_+^h-g_{31h}^0 f_+^h)+\\
\quad +D_{22}^-(+\partial_1 f_-^1+\partial_2 f_-^2+\partial_3 f_-^3-\partial_0 f_-^0)+D_{22}^-(+g_{11h}^1 f_-^h+g_{21h}^2 f_-^h+g_{31h}^3 f_-^h+g_{01h}^0 f_-^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_3(+\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+(+(+g_{32h}^1)\partial_0 f_+^h+(+g_{32h}^2)\partial_3 f_-^h-(+g_{32h}^3)\partial_2 f_-^h+(+g_{32h}^0)\partial_1 f_+^h)+\\
+\partial_3(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+(-(+g_{32k}^1)g_{01h}^k f_+^h-(+g_{32k}^2)g_{31h}^k f_-^h+(+g_{32k}^3)g_{21h}^k f_-^h+(+g_{32k}^0)g_{11h}^k f_+^h)+\\
\quad +\partial_0(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3+\partial_2 f_-^0)+(-(-g_{02h}^1)\partial_3 f_+^h+(-g_{02h}^2)\partial_0 f_-^h+(-g_{02h}^3)\partial_1 f_+^h+(-g_{02h}^0)\partial_2 f_-^h)+\\
+\partial_0(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+(-(-g_{02k}^1)g_{31h}^k f_+^h+(-g_{02k}^2)g_{01h}^k f_-^h+(-g_{02k}^3)g_{11h}^k f_+^h-(-g_{02k}^0)g_{21h}^k f_-^h)+\\
\quad +\partial_1(-\partial_2 f_-^1+\partial_1 f_-^2-\partial_0 f_+^3-\partial_3 f_+^0)+(-(+g_{12h}^1)\partial_2 f_-^1+(+g_{12h}^2)\partial_1 f_-^h-(+g_{12h}^3)\partial_0 f_+^h-(+g_{12h}^0)\partial_3 f_+^h)+\\
+\partial_1(+g_{21h}^1 f_-^h-g_{11h}^2 f_-^h+g_{01h}^3 f_+^h-g_{31h}^0 f_+^h)+(+(+g_{12k}^1)g_{21h}^1 f_-^h-(+g_{12k}^2)g_{11h}^k f_-^h+(+g_{12k}^3)g_{01h}^k f_+^h-(+g_{12k}^0)g_{31h}^k f_+^h)+\\
\quad +\partial_2(+\partial_1 f_-^1+\partial_2 f_-^2+\partial_3 f_-^3-\partial_0 f_-^0)+(+(-g_{22h}^1)\partial_1 f_-^h+(-g_{22h}^2)\partial_2 f_-^h+(-g_{22h}^3)\partial_3 f_-^h-(-g_{22h}^0)\partial_0 f_-^h)+\\
+\partial_2(+g_{11h}^1 f_-^h+g_{21h}^2 f_-^h+g_{31h}^3 f_-^h+g_{01h}^0 f_-^h)+(+(-g_{22k}^1)g_{11h}^k f_-^h+(-g_{22k}^2)g_{21h}^k f_-^h+(-g_{22k}^3)g_{31h}^k f_-^h+(-g_{22k}^0)g_{01h}^k f_-^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_3^2 f_-^2+\partial_0^2 f_-^2+\partial_1^2 f_-^2+\partial_2^2 f_-^2+\\
\partial_3(+\partial_0 f_+^1-\partial_2 f_-^3+\partial_1 f_+^0)+\partial_0(-\partial_3 f_+^1+\partial_1 f_+^3+\partial_2 f_-^0)+\partial_1(-\partial_2 f_-^1-\partial_0 f_+^3-\partial_3 f_+^0)+\partial_2(+\partial_1 f_-^1+\partial_3 f_-^3-\partial_0 f_-^0)+\\
+\partial_0(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
+\partial_1(+g_{21h}^1 f_-^h-g_{11h}^2 f_-^h+g_{01h}^3 f_+^h-g_{31h}^0 f_+^h)+\\
+\partial_3(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\\
+\partial_2(+g_{11h}^1 f_-^h+g_{21h}^2 f_-^h+g_{31h}^3 f_-^h+g_{01h}^0 f_-^h)+\\
+(+(+g_{32h}^1)\partial_0 f_+^h+(+g_{32h}^2)\partial_3 f_-^h-(+g_{32h}^3)\partial_2 f_-^h+(+g_{32h}^0)\partial_1 f_+^h)+\\
+(-(-g_{02h}^1)\partial_3 f_+^h+(-g_{02h}^2)\partial_0 f_-^h+(-g_{02h}^3)\partial_1 f_+^h+(-g_{02h}^0)\partial_2 f_-^h)+\\
+(-(+g_{12h}^1)\partial_2 f_-^1+(+g_{12h}^2)\partial_1 f_-^h-(+g_{12h}^3)\partial_0 f_+^h-(+g_{12h}^0)\partial_3 f_+^h)+\\
+(+(-g_{22h}^1)\partial_1 f_-^1+(-g_{22h}^2)\partial_2 f_-^h+(-g_{22h}^3)\partial_3 f_-^h-(-g_{22h}^0)\partial_0 f_-^h)\\
+(-(+g_{32k}^1)g_{01h}^k f_+^h-(+g_{32k}^2)g_{31h}^k f_-^h+(+g_{32k}^3)g_{21h}^k f_-^h+(+g_{32k}^0)g_{11h}^k f_+^h)+\\
+(-(-g_{02k}^1)g_{31h}^k f_+^h+(-g_{02k}^2)g_{01h}^k f_-^h+(-g_{02k}^3)g_{11h}^k f_+^h-(-g_{02k}^0)g_{21h}^k f_-^h)+\\
+(+(+g_{12k}^1)g_{21h}^1 f_-^h-(+g_{12k}^2)g_{11h}^k f_-^h+(+g_{12k}^3)g_{01h}^k f_+^h-(+g_{12k}^0)g_{31h}^k f_+^h)+\\
+(+(-g_{22k}^1)g_{11h}^k f_-^h+(-g_{22k}^2)g_{21h}^k f_-^h+(-g_{22k}^3)g_{31h}^k f_-^h+(-g_{22k}^0)g_{01h}^k f_-^h)
\end{array}\right)
$$

$$
\begin{aligned}
&\partial_3^2 f_-^2 + \partial_0^2 f_-^2 + \partial_1 f_-^2 + \partial_2^2 f_-^2 + \\
&+\partial_0([-g_{31h}^1 + g_{11h}^3]f_+^h + [-g_{21h}^0 + g_{01h}^2]f_-^h) + \\
&+\partial_1([-g_{31h}^0 + g_{01h}^3]f_+^h + [+g_{21h}^1 - g_{11h}^2]f_-^h) + \\
&+\partial_3([+g_{11h}^0 - g_{01h}^1]f_+^h + [-g_{31h}^2 + g_{21h}^3]f_-^h) + \\
&+\partial_2([+g_{01h}^0 + g_{11h}^1 + g_{21h}^2 + g_{31h}^3]f_-^h) + \\
=\ &-(-g_{22h}^0)\partial_0 f_-^h + (-g_{02h}^2)\partial_0 f_-^h + (+g_{32h}^1)\partial_0 f_+^h - (+g_{12h}^3)\partial_0 f_+^h + \\
&+(+g_{32h}^0)\partial_1 f_+^h + (-g_{02h}^3)\partial_1 f_+^h + (+g_{12h}^2)\partial_1 f_-^h + (-g_{22h}^1)\partial_1 f_-^1 + \\
&+(-g_{02h}^0)\partial_2 f_-^h - (+g_{12h}^1)\partial_2 f_-^1 + (-g_{22h}^2)\partial_2 f_-^h - (+g_{32h}^3)\partial_2 f_-^h + \\
&-(+g_{12h}^0)\partial_3 f_+^h - (-g_{02h}^1)\partial_3 f_+^h + (+g_{32h}^2)\partial_3 f_-^h + (-g_{22h}^3)\partial_3 f_-^h + \\
&-(+g_{32k}^1)g_{01h}^k f_+^h + (+g_{12k}^3)g_{01h}^k f_+^h + (+g_{32k}^0)g_{11h}^k f_+^h + (-g_{02k}^3)g_{11h}^k f_+^h - (+g_{12k}^0)g_{31h}^k f_+^h - (-g_{02k}^1)g_{31h}^k f_+^h \\
&+(-g_{22k}^0)g_{01h}^k f_-^h + (-g_{02k}^2)g_{01h}^k f_-^h + (-g_{22k}^1)g_{11h}^k f_-^h - (+g_{12k}^2)g_{11h}^k f_-^h - (-g_{02k}^0)g_{21h}^k f_-^h + (+g_{12k}^1)g_{21h}^1 f_-^h + (-g_{22k}^2)g_{21h}^k f_-^h + (+g_{32k}^3)g_{21h}^k f_-^h - (+g_{32k}^2)g_{31h}^k f_-^h + +(-
\end{aligned}
$$

$$
\begin{aligned}
&(\partial_3^2 + \partial_0^2 + \partial_1 + \partial_2^2)f_-^2 + \\
&+\partial_0([-g_{31h}^1 + g_{11h}^3]f_+^h + [-g_{21h}^0 + g_{01h}^2]f_-^h) + \\
&+\partial_1([-g_{31h}^0 + g_{01h}^3]f_+^h + [+g_{21h}^1 - g_{11h}^2]f_-^h) + \\
&+\partial_3([+g_{11h}^0 - g_{01h}^1]f_+^h + [-g_{31h}^2 + g_{21h}^3]f_-^h) + \\
&+\partial_2([+g_{01h}^0 + g_{11h}^1 + g_{21h}^2 + g_{31h}^3]f_-^h) + \\
=\ &(+g_{32h}^1 - g_{12h}^3)\partial_0 f_+^h + (+g_{22h}^0 - g_{02h}^2)\partial_0 f_-^h + \\
&+(g_{32h}^0 - g_{02h}^3)\partial_1 f_+^h + (g_{12h}^2 - g_{22h}^1)\partial_1 f_-^1 + \\
&+(-g_{02h}^0 - g_{12h}^1 - g_{22h}^2 - g_{32h}^3)\partial_2 f_-^h + \\
&+(-g_{12h}^0 + g_{02h}^1)\partial_3 f_+^h + (+g_{32h}^2 - g_{22h}^3)\partial_3 f_-^h + \\
&+([-g_{32k}^1 + g_{12k}^3]g_{01h}^k + [+g_{32k}^0 - g_{02k}^3]g_{11h}^k + [-g_{12k}^0 + g_{02k}^1]g_{31h}^k)f_+^h \\
&+([-g_{22k}^0 - g_{02k}^2]g_{01h}^k + [-g_{22k}^1 - g_{12k}^2]g_{11h}^k + [+g_{02k}^0 + g_{12k}^1 - g_{22k}^2 + g_{32k}^3]g_{21h}^k + [-g_{32k}^2 - g_{22k}^3]g_{31h}^k)f_-^h
\end{aligned}
$$

and (*3+):

$$
\begin{aligned}
&D_{22}^+[-(\partial_0 f_+^1 - g_{01h}^1 f_+^h) - (\partial_3 f_-^2 - g_{31h}^2 f_-^h) + (\partial_2 f_-^3 - g_{21h}^3 f_-^h) - (\partial_1 f_+^0 + g_{11h}^0 f_+^h)] + \\
&\quad - D_{12}^+[+(\partial_3 f_-^1 - g_{31h}^1 f_-^h) - (\partial_0 f_+^2 - g_{01h}^2 f_+^h) - (\partial_1 f_-^3 - g_{11h}^3 f_-^h) - (\partial_2 f_+^0 + g_{21h}^0 f_+^h)] + \\
&\quad - D_{02}^-[-(\partial_2 f_+^1 + g_{21h}^1 f_+^h) + (\partial_1 f_+^2 + g_{11h}^2 f_+^h) - (\partial_0 f_-^3 + g_{01h}^3 f_-^h) - (\partial_3 f_-^0 - g_{31h}^0 f_-^h)] + \\
&\quad - D_{32}^-[-(\partial_1 f_-^1 + g_{11h}^1 f_-^h) - (\partial_2 f_-^2 + g_{21h}^2 f_-^h) - (\partial_3 f_-^3 + g_{31h}^3 f_-^h) + (\partial_0 f_-^0 - g_{01h}^0 f_-^h)]
\end{aligned}
=
$$

$$
\begin{aligned}
&D_{22}^+[(-\partial_0 f_+^1 - \partial_3 f_-^2 + \partial_2 f_-^3 - \partial_1 f_+^0) + (+g_{01h}^1 f_+^h + g_{31h}^2 f_-^h - g_{21h}^3 f_-^h - g_{11h}^0 f_+^h)] + \\
&\quad + D_{12}^+[(-\partial_3 f_-^1 + \partial_0 f_+^2 + \partial_1 f_-^3 + \partial_2 f_+^0) + (+g_{31h}^1 f_-^h - g_{01h}^2 f_+^h - g_{11h}^3 f_-^h + g_{21h}^0 f_+^h)] + \\
&\quad + D_{02}^-[(+\partial_2 f_+^1 - \partial_1 f_+^2 + \partial_0 f_-^3 + \partial_3 f_-^0) + (+g_{21h}^1 f_+^h - g_{11h}^2 f_+^h + g_{01h}^3 f_-^h - g_{31h}^0 f_-^h)] + \\
&\quad + D_{32}^-[(+\partial_1 f_-^1 + \partial_2 f_-^2 + \partial_3 f_-^3 - \partial_0 f_-^0) + (+g_{11h}^1 f_-^h + g_{21h}^2 f_-^h + g_{31h}^3 f_-^h + g_{01h}^0 f_-^h)]
\end{aligned}
=
$$

$$
\begin{aligned}
&\partial_2(-\partial_0 f_+^1 - \partial_3 f_-^2 + \partial_2 f_-^3 - \partial_1 f_+^0) + (-(+g_{22h}^1)\partial_0 f_+^h - (+g_{22h}^2)\partial_3 f_-^h + (+g_{22h}^3)\partial_2 f_-^h - (+g_{22h}^0)\partial_1 f_+^h) + \\
&+\partial_2(+g_{01h}^1 f_+^h + g_{31h}^2 f_-^h - g_{21h}^3 f_-^h - g_{11h}^0 f_+^h) + (+(+g_{22k}^1)g_{01h}^k f_+^h + (+g_{22k}^2)g_{31h}^k f_-^h - (+g_{22k}^3)g_{21h}^k f_-^h - (+g_{22k}^0)g_{11h}^k f_+^h) + \\
&\quad + \partial_1(-\partial_3 f_-^1 + \partial_0 f_+^2 + \partial_1 f_-^3 + \partial_2 f_+^0) + (-(+g_{12h}^1)\partial_3 f_-^h + (+g_{12h}^2)\partial_0 f_+^h + (+g_{12h}^3)\partial_1 f_-^h + (+g_{12h}^0)\partial_2 f_+^h) + \\
&+\partial_1(+g_{31h}^1 f_-^h - g_{01h}^2 f_+^h - g_{11h}^3 f_-^h + g_{21h}^0 f_+^h) + (+(+g_{12k}^1)g_{31h}^k f_-^h - (+g_{12k}^2)g_{01h}^k f_+^h - (+g_{12k}^3)g_{11h}^k f_-^h + (+g_{12k}^0)g_{21h}^k f_+^h) + \\
&\quad + \partial_0(+\partial_2 f_+^1 - \partial_1 f_+^2 + \partial_0 f_-^3 + \partial_3 f_-^0) + (+(-g_{02h}^1)\partial_2 f_+^h - (-g_{02h}^2)\partial_1 f_+^h + (-g_{02h}^3)\partial_0 f_-^h + (-g_{02h}^0)\partial_3 f_-^h) + \\
&+\partial_0(+g_{21h}^1 f_+^h - g_{11h}^2 f_+^h + g_{01h}^3 f_-^h - g_{31h}^0 f_-^h) + (+(-g_{02k}^1)g_{21h}^k f_+^h - (-g_{02k}^2)g_{11h}^k f_+^h + (-g_{02k}^3)g_{01h}^k f_-^h - (-g_{02k}^0)g_{31h}^k f_-^h) + \\
&\quad + \partial_3(+\partial_1 f_-^1 + \partial_2 f_-^2 + \partial_3 f_-^3 - \partial_0 f_-^0) + (+(-g_{32h}^1)\partial_1 f_-^h + (-g_{32h}^2)\partial_2 f_-^h + (-g_{32h}^3)\partial_3 f_-^h - (-g_{32h}^0)\partial_0 f_-^h) + \\
&+\partial_3(+g_{11h}^1 f_-^h + g_{21h}^2 f_-^h + g_{31h}^3 f_-^h + g_{01h}^0 f_-^h) + (+(-g_{32k}^1)g_{11h}^k f_-^h + (-g_{32k}^2)g_{21h}^k f_-^h + (-g_{32k}^3)g_{31h}^k f_-^h + (-g_{32k}^0)g_{01h}^k f_-^h)
\end{aligned}
=
$$

$$
\begin{aligned}
&\partial_2(-\partial_0 f_+^1 - \partial_3 f_-^2 + \partial_2 f_-^3 - \partial_1 f_+^0) + \partial_1(-\partial_3 f_-^1 + \partial_0 f_+^2 + \partial_1 f_-^3 + \partial_2 f_+^0) + \partial_0(+\partial_2 f_+^1 - \partial_1 f_+^2 + \partial_0 f_-^3 + \partial_3 f_-^0) + \partial_3(+\partial_1 f_-^1 + \partial_2 f_-^2 + \partial_3 f_-^3 - \partial_0 f_-^0) + \\
&+\partial_0(+g_{21h}^1 f_+^h - g_{11h}^2 f_+^h + g_{01h}^3 f_-^h - g_{31h}^0 f_-^h) + \partial_1(+g_{31h}^1 f_-^h - g_{01h}^2 f_+^h - g_{11h}^3 f_-^h + g_{21h}^0 f_+^h) + \\
&+\partial_2(+g_{01h}^1 f_+^h + g_{31h}^2 f_-^h - g_{21h}^3 f_-^h - g_{11h}^0 f_+^h) + \partial_3(+g_{11h}^1 f_-^h + g_{21h}^2 f_-^h + g_{31h}^3 f_-^h + g_{01h}^0 f_-^h) + \\
&+(+(-g_{02h}^1)\partial_2 f_+^h - (-g_{02h}^2)\partial_1 f_+^h + (-g_{02h}^3)\partial_0 f_-^h + (-g_{02h}^0)\partial_3 f_-^h) + \\
&+(-(+g_{12h}^1)\partial_3 f_-^h + (+g_{12h}^2)\partial_0 f_+^h + (+g_{12h}^3)\partial_1 f_-^h + (+g_{12h}^0)\partial_2 f_+^h) + \\
&+(-(+g_{22h}^1)\partial_0 f_+^h - (+g_{22h}^2)\partial_3 f_-^h + (+g_{22h}^3)\partial_2 f_-^h - (+g_{22h}^0)\partial_1 f_+^h) + \\
&+(+(-g_{32h}^1)\partial_1 f_-^h + (-g_{32h}^2)\partial_2 f_-^h + (-g_{32h}^3)\partial_3 f_-^{3h} - (-g_{32h}^0)\partial_0 f_-^h) \\
&+(+(-g_{02k}^1)g_{21h}^k f_+^h - (-g_{02k}^2)g_{11h}^k f_+^h + (-g_{02k}^3)g_{01h}^k f_-^h - (-g_{02k}^0)g_{31h}^k f_-^h) + \\
&+(+(+g_{12k}^1)g_{31h}^k f_-^h - (+g_{12k}^2)g_{01h}^k f_+^h - (+g_{12k}^3)g_{11h}^k f_-^h + (+g_{12k}^0)g_{21h}^k f_+^h) + \\
&+(+(+g_{22k}^1)g_{01h}^k f_+^h + (+g_{22k}^2)g_{31h}^k f_-^h - (+g_{22k}^3)g_{21h}^k f_-^h - (+g_{22k}^0)g_{11h}^k f_+^h) \\
&+(+(-g_{32k}^1)g_{11h}^k f_-^h + (-g_{32k}^2)g_{21h}^k f_-^h + (-g_{32k}^3)g_{31h}^k f_-^h + (-g_{32k}^0)g_{01h}^k f_-^h)
\end{aligned}
$$

$$
=\left(\begin{array}{l}
\partial_2^2 f_-^3+\partial_1^2 f_-^3+\partial_0^2 f_-^3+\partial_3^2 f_-^3+\\
\partial_2(-\partial_0 f_+^1-\partial_3 f_-^2-\partial_1 f_+^0)+\partial_1(-\partial_3 f_-^1+\partial_0 f_+^2+\partial_2 f_+^0)+\partial_0(+\partial_2 f_+^1-\partial_1 f_+^2+\partial_3 f_-^0)+\partial_3(+\partial_1 f_-^1+\partial_2 f_-^2-\partial_0 f_-^0)+\\
+\partial_0(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+\\
+\partial_1(+g_{31h}^1 f_-^h-g_{01h}^2 f_+^h-g_{11h}^3 f_-^h+g_{21h}^0 f_+^h)+\\
+\partial_2(+g_{01h}^1 f_+^h+g_{31h}^2 f_-^h-g_{21h}^3 f_-^h-g_{11h}^0 f_+^h)+\\
+\partial_3(+g_{11h}^1 f_-^h+g_{21h}^2 f_-^h+g_{31h}^3 f_-^h+g_{01h}^0 f_-^h)+\\
-(+g_{22h}^1)\partial_0 f_+^h+(+g_{12h}^2)\partial_0 f_+^h-(-g_{32h}^0)\partial_0 f_-^h+(-g_{02h}^3)\partial_0 f_-^h+\\
-(+g_{22h}^0)\partial_1 f_+^h-(-g_{02h}^2)\partial_1 f_+^h+(-g_{32h}^1)\partial_1 f_-^h+(+g_{12h}^3)\partial_1 f_-^h+\\
+(+g_{12h}^0)\partial_2 f_+^h+(-g_{02h}^1)\partial_2 f_+^h+(-g_{32h}^2)\partial_2 f_-^h+(+g_{22h}^3)\partial_2 f_-^h+\\
+(-g_{02h}^0)\partial_3 f_-^h-(+g_{12h}^1)\partial_3 f_-^h-(+g_{22h}^2)\partial_3 f_-^h+(-g_{32h}^3)\partial_3 f_-^h+\\
+(+g_{22k}^1)g_{01h}^k f_+^h-(+g_{12k}^2)g_{01h}^k f_+^h-(+g_{22k}^0)g_{11h}^k f_+^h-(-g_{02k}^2)g_{11h}^k f_+^h+(+g_{12k}^0)g_{21h}^k f_+^h+(-g_{02k}^1)g_{21h}^k f_+^h+\\
+(-g_{32k}^0)g_{01h}^k f_-^h+(-g_{02k}^3)g_{01h}^k f_-^h+(-g_{32k}^1)g_{11h}^k f_-^h-(+g_{12k}^3)g_{11h}^k f_-^h+(-g_{32k}^2)g_{21h}^k f_-^h-(+g_{22k}^3)g_{21h}^k f_-^h+\\
-(-g_{02k}^0)g_{31h}^k f_-^h+(+g_{12k}^1)g_{31h}^k f_-^h+(+g_{22k}^2)g_{31h}^k f_-^h+(-g_{32k}^3)g_{31h}^k f_-^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3+\\
+\partial_0([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h)+\\
+\partial_1([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h)+\\
+\partial_2([-g_{11h}^0+g_{01h}^1]f_+^h+[+g_{31h}^2-g_{21h}^3]f_-^h)+\\
+\partial_3([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_-^h)+\\
+(-g_{22h}^1+g_{12h}^2)\partial_0 f_+^h+(+g_{32h}^0-g_{02h}^3)\partial_0 f_-^h+\\
+(-g_{22h}^0+g_{02h}^2)\partial_1 f_+^h+(-g_{32h}^1+g_{12h}^3)\partial_1 f_-^h+\\
+(+g_{12h}^0-g_{02h}^1)\partial_2 f_+^h+(-g_{32h}^2+g_{22h}^3)\partial_2 f_-^h+\\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2-g_{32h}^3)\partial_3 f_-^h+\\
+([+g_{22k}^1-g_{12k}^2]g_{01h}^k+[-g_{22k}^0+g_{02k}^2]g_{11h}^k+[+g_{12k}^0-g_{02k}^1]g_{21h}^k)f_+^h+\\
+([-g_{32k}^0-g_{02k}^3]g_{01h}^k+[-g_{32k}^1-g_{12k}^3]g_{11h}^k+[-g_{32k}^2-g_{22k}^3]g_{21h}^k+[+g_{02k}^0+g_{12k}^1+g_{22k}^2-g_{32k}^3]g_{31h}^k)f_-^h
\end{array}\right)
$$

and (*3+):

$$
\left(\begin{array}{l}
D_{22}^-[-(\partial_0 f_-^1+g_{01h}^1 f_-^h)-(\partial_3 f_+^2+g_{31h}^2 f_+^h)+(\partial_2 f_+^3+g_{21h}^3 f_+^h)-(\partial_1 f_-^0-g_{11h}^0 f_-^h)]+\\
\quad -D_{12}^-[(\partial_3 f_+^1+g_{31h}^1 f_+^h)-(\partial_0 f_-^2+g_{01h}^2 f_-^h)-(\partial_1 f_+^3+g_{11h}^3 f_+^h)-(\partial_2 f_-^0-g_{21h}^0 f_-^h)]+\\
\quad -D_{02}^+[-(\partial_2 f_-^1-g_{21h}^1 f_-^h)+(\partial_1 f_-^2-g_{11h}^2 f_-^h)-(\partial_0 f_+^3-g_{01h}^3 f_+^h)-(\partial_3 f_+^0+g_{31h}^0 f_+^h)]+\\
\quad -D_{32}^+[-(\partial_1 f_+^1-g_{11h}^1 f_+^h)-(\partial_2 f_+^2-g_{21h}^2 f_+^h)-(\partial_3 f_+^3-g_{31h}^3 f_+^h)+(\partial_0 f_+^0+g_{01h}^0 f_+^h)]
\end{array}\right)=
$$

$$
=\left(\begin{array}{l}
D_{22}^-(-\partial_0 f_-^1-\partial_3 f_+^2+\partial_2 f_+^3-\partial_1 f_-^0)+D_{22}^-(-g_{01h}^1 f_-^h-g_{31h}^2 f_+^h+g_{21h}^3 f_+^h+g_{11h}^0 f_-^h)+\\
\quad +D_{12}^-(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3-\partial_2 f_-^0)+D_{12}^-(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
\quad +D_{02}^+(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_0 f_+^3+\partial_3 f_+^0)+D_{02}^+(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+\\
\quad +D_{32}^+(+\partial_1 f_+^1+\partial_2 f_+^2+\partial_3 f_+^3-\partial_0 f_+^0)+D_{32}^+(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_2(-\partial_0 f_-^1-\partial_3 f_+^2+\partial_2 f_+^3-\partial_1 f_-^0)+(-(-g_{22h}^1)\partial_0 f_-^h-(-g_{22h}^2)\partial_3 f_+^h+(-g_{22h}^3)\partial_2 f_+^h-(-g_{22h}^0)\partial_1 f_-^h)+\\
+\partial_2(-g_{01h}^1 f_-^h-g_{31h}^2 f_+^h+g_{21h}^3 f_+^h+g_{11h}^0 f_-^h)+(-(-g_{22k}^1)g_{01h}^k f_-^h-(-g_{22k}^2)g_{31h}^k f_+^h+(-g_{22k}^3)g_{21h}^k f_+^h+(-g_{22k}^0)g_{11h}^k f_-^h)+\\
\quad +\partial_1(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3-\partial_2 f_-^0)+(-(-g_{12h}^1)\partial_3 f_+^h+(-g_{12h}^2)\partial_0 f_-^h+(-g_{12h}^3)\partial_1 f_+^h-(-g_{12h}^0)\partial_2 f_-^h)+\\
+\partial_1(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+(-(-g_{12k}^1)g_{31h}^k f_+^h+(-g_{12k}^2)g_{01h}^k f_-^h+(-g_{12k}^3)g_{11h}^k f_+^h-(-g_{12k}^0)g_{21h}^k f_-^h)+\\
\quad +\partial_0(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_0 f_+^3+\partial_3 f_+^0)+(+(+g_{02h}^1)\partial_2 f_-^h-(+g_{02h}^2)\partial_1 f_-^h+(+g_{02h}^3)\partial_0 f_+^h+(+g_{02h}^0)\partial_3 f_+^h)+\\
+\partial_0(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+(-(+g_{02k}^1)g_{21h}^k f_-^h+(+g_{02k}^2)g_{11h}^k f_-^h-(+g_{02k}^3)g_{01h}^k f_+^h+(+g_{02k}^0)g_{31h}^k f_+^h)+\\
\quad +\partial_3(+\partial_1 f_+^1+\partial_2 f_+^2+\partial_3 f_+^3-\partial_0 f_+^0)+(+(+g_{32h}^1)\partial_1 f_+^h+(+g_{32h}^2)\partial_2 f_+^h+(+g_{32h}^3)\partial_3 f_+^h-(+g_{32h}^0)\partial_0 f_+^h)+\\
+\partial_3(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+(-(+g_{32k}^1)g_{11h}^k f_+^h-(+g_{32k}^2)g_{21h}^k f_+^h-(+g_{32k}^3)g_{31h}^k f_+^h-(+g_{32k}^0)g_{01h}^k f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_2^2 f_+^3+\partial_1^2 f_+^3+\partial_0^2 f_+^3+\partial_3 f_+^3+\\
\partial_2(-\partial_0 f_-^1-\partial_3 f_+^2-\partial_1 f_-^0)+\partial_1(-\partial_3 f_+^1+\partial_0 f_-^2-\partial_2 f_-^0)+\partial_0(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_3 f_+^0)+\partial_3(+\partial_1 f_+^1+\partial_2 f_+^2-\partial_0 f_+^0)+\\
+\partial_0(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+\partial_1(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
+\partial_2(-g_{01h}^1 f_-^h-g_{31h}^2 f_+^h+g_{21h}^3 f_+^h+g_{11h}^0 f_-^h)+\partial_3(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+\\
+(+(+g_{02h}^1)\partial_2 f_-^h-(+g_{02h}^2)\partial_1 f_-^h+(+g_{02h}^3)\partial_0 f_+^h+(+g_{02h}^0)\partial_3 f_+^h)+\\
+(-(-g_{12h}^1)\partial_3 f_+^h+(-g_{12h}^2)\partial_0 f_-^h+(-g_{12h}^3)\partial_1 f_+^h-(-g_{12h}^0)\partial_2 f_-^h)+\\
+(-(-g_{22h}^1)\partial_0 f_-^h-(-g_{22h}^2)\partial_3 f_+^h+(-g_{22h}^3)\partial_2 f_+^h-(-g_{22h}^0)\partial_1 f_-^h)+\\
+(+(+g_{32h}^1)\partial_1 f_+^h+(+g_{32h}^2)\partial_2 f_+^h+(+g_{32h}^3)\partial_3 f_+^h-(+g_{32h}^0)\partial_0 f_+^h)+\\
+(-(+g_{02k}^1)g_{21h}^k f_-^h+(+g_{02k}^2)g_{11h}^k f_-^h-(+g_{02k}^3)g_{01h}^k f_+^h+(+g_{02k}^0)g_{31h}^k f_+^h)+\\
+(-(-g_{12k}^1)g_{31h}^k f_+^h+(-g_{12k}^2)g_{01h}^k f_-^h+(-g_{12k}^3)g_{11h}^k f_+^h-(-g_{12k}^0)g_{21h}^k f_-^h)\\
+(-(-g_{22k}^1)g_{01h}^k f_-^h-(-g_{22k}^2)g_{31h}^k f_+^h+(-g_{22k}^3)g_{21h}^k f_+^h+(-g_{22k}^0)g_{11h}^k f_-^h)+\\
+(-(+g_{32k}^1)g_{11h}^k f_+^h-(+g_{32k}^2)g_{21h}^k f_+^h-(+g_{32k}^3)g_{31h}^k f_+^h-(+g_{32k}^0)g_{01h}^k f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_2^2 f_+^3+\partial_1^2 f_+^3+\partial_0^2 f_+^3+\partial_3 f_+^3+\\
+\partial_0(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+\partial_1(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
+\partial_2(-g_{01h}^1 f_-^h-g_{31h}^2 f_+^h+g_{21h}^3 f_+^h+g_{11h}^0 f_-^h)+\partial_3(-g_{11h}^1 f_+^h-g_{21h}^2 f_+^h-g_{31h}^3 f_+^h-g_{01h}^0 f_+^h)+\\
-(+g_{32h}^0)\partial_0 f_+^h+(+g_{02h}^3)\partial_0 f_+^h-(-g_{22h}^1)\partial_0 f_-^h+(-g_{12h}^2)\partial_0 f_-^h+\\
+(+g_{32h}^1)\partial_1 f_+^h+(-g_{12h}^3)\partial_1 f_+^h-(-g_{22h}^0)\partial_1 f_-^h-(+g_{02h}^2)\partial_1 f_-^h+\\
+(+g_{32h}^2)\partial_2 f_+^h+(-g_{22h}^3)\partial_2 f_+^h-(-g_{12h}^0)\partial_2 f_-^h+(+g_{02h}^1)\partial_2 f_-^h+\\
+(+g_{02h}^0)\partial_3 f_+^h+g_{12h}^1\partial_3 f_+^h+g_{22h}^2\partial_3 f_+^h+g_{32h}^3\partial_3 f_+^h+\\
-(+g_{02k}^3)g_{01h}^k f_+^h-(+g_{32h}^0)g_{01h}^k f_+^h-(+g_{32k}^1)g_{11h}^k f_+^h+(-g_{12k}^3)g_{11h}^k f_+^h-(+g_{32h}^2)g_{21h}^k f_+^h+(-g_{22k}^3)g_{21h}^k f_+^h+\\
+(+g_{02k}^0)g_{31h}^k f_+^h-(-g_{12k}^1)g_{31h}^k f_+^h-(-g_{22k}^2)g_{31h}^k f_+^h-(+g_{32h}^3)g_{31h}^k f_+^h+\\
-(-g_{22k}^1)g_{01h}^k f_-^h+(-g_{12k}^2)g_{01h}^k f_-^h+(-g_{22k}^0)g_{11h}^k f_-^h+(+g_{02k}^2)g_{11h}^k f_-^h-(-g_{12k}^0)g_{21h}^k f_-^h-(+g_{02k}^1)g_{21h}^k f_-^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3+\\
+\partial_0([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h)+\\
+\partial_1([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h)+\\
+\partial_2([-g_{31h}^2+g_{21h}^3]f_+^h+[+g_{11h}^0-g_{01h}^1]f_-^h)+\\
+\partial_3([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h)+\\
+(-g_{32h}^0+g_{02h}^3)\partial_0 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_0 f_-^h+\\
+(+g_{32h}^1-g_{12h}^3)\partial_1 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_1 f_-^h+\\
+(+g_{32h}^2-g_{22h}^3)\partial_2 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_2 f_-^h+\\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_3 f_+^h+\\
+([-g_{02k}^3-g_{32h}^0]g_{01h}^k+[-g_{32k}^1-g_{12k}^3]g_{11h}^k+[-g_{32h}^2-g_{22k}^3]g_{21h}^k+[g_{02k}^0+g_{12k}^1+g_{22k}^2-g_{32h}^3]g_{31h}^k)f_+^h+\\
+([+g_{22k}^1-g_{12k}^2]g_{01h}^k+[-g_{22k}^0+g_{02k}^2]g_{11h}^k+[+g_{12k}^0-g_{02k}^1]g_{21h}^k)f_-^h
\end{array}\right)
$$

and (*0+):

$$
\left(\begin{array}{l}
-D_{12}^-[-(\partial_0 f_+^1-g_{01h}^1 f_+^h)-(\partial_3 f_-^2-g_{31h}^2 f_-^h)+(\partial_2 f_-^3-g_{21h}^3 f_-^h)-(\partial_1 f_+^0+g_{11h}^0 f_+^h)]+\\
\quad -D_{22}^-[(\partial_3 f_-^1-g_{31h}^1 f_-^h)-(\partial_0 f_+^2-g_{01h}^2 f_+^h)-(\partial_1 f_-^3-g_{11h}^3 f_-^h)-(\partial_2 f_+^0+g_{21h}^0 f_+^h)]+\\
\quad -D_{32}^-[-(\partial_2 f_-^1-g_{21h}^1 f_-^h)+(\partial_1 f_-^2-g_{11h}^2 f_-^h)-(\partial_0 f_+^3-g_{01h}^3 f_+^h)-(\partial_3 f_+^0+g_{31h}^0 f_+^h)]+\\
\quad +D_{02}^-[-(\partial_1 f_+^1-g_{11h}^1 f_+^h)-(\partial_2 f_+^2-g_{21h}^2 f_+^h)-(\partial_3 f_+^3-g_{31h}^3 f_+^h)+(\partial_0 f_+^0+g_{01h}^0 f_+^h)]
\end{array}\right)=
$$

$$
=\left(\begin{array}{l}
D_{12}^-[+(\partial_0 f_+^1-g_{01h}^1 f_+^h)+(\partial_3 f_-^2-g_{31h}^2 f_-^h)-(\partial_2 f_-^3-g_{21h}^3 f_-^h)+(\partial_1 f_+^0+g_{11h}^0 f_+^h)]+\\
\quad +D_{22}^-[-(\partial_3 f_-^1-g_{31h}^1 f_-^h)+(\partial_0 f_+^2-g_{01h}^2 f_+^h)+(\partial_1 f_-^3-g_{11h}^3 f_-^h)+(\partial_2 f_+^0+g_{21h}^0 f_+^h)]+\\
\quad +D_{32}^-[+(\partial_2 f_-^1-g_{21h}^1 f_-^h)-(\partial_1 f_-^2-g_{11h}^2 f_-^h)+(\partial_0 f_+^3-g_{01h}^3 f_+^h)+(\partial_3 f_+^0+g_{31h}^0 f_+^h)]+\\
\quad +D_{02}^-[-(\partial_1 f_+^1-g_{11h}^1 f_+^h)-(\partial_2 f_+^2-g_{21h}^2 f_+^h)-(\partial_3 f_+^3-g_{31h}^3 f_+^h)+(\partial_0 f_+^0+g_{01h}^0 f_+^h)]
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
D_{12}^-(+\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+D_{12}^-(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\\
\quad +D_{22}^-(-\partial_3 f_-^1+\partial_0 f_+^2+\partial_1 f_-^3+\partial_2 f_+^0)+D_{22}^-(+g_{31h}^1 f_-^h-g_{01h}^2 f_+^h-g_{11h}^3 f_-^h+g_{21h}^0 f_+^h)+\\
\quad +D_{32}^-(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_0 f_+^3+\partial_3 f_+^0)+D_{32}^-(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+\\
\quad +D_{02}^-(-\partial_1 f_+^1-\partial_2 f_+^2-\partial_3 f_+^3+\partial_0 f_+^0)+D_{02}^-(+g_{11h}^1 f_+^h+g_{21h}^2 f_+^h+g_{31h}^3 f_+^h+g_{01h}^0 f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_1(+\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3+\partial_1 f_+^0)+(+(-g_{12h}^1)\partial_0 f_+^h+(-g_{12h}^2)\partial_3 f_-^h-(-g_{12h}^3)\partial_2 f_-^h+(-g_{12h}^0)\partial_1 f_+^h)+\\
+\partial_1(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+(-(-g_{12k}^1)g_{01h}^k f_+^h-(-g_{12k}^2)g_{31h}^k f_-^h+(-g_{12k}^3)g_{21h}^k f_-^h+(-g_{12k}^0)g_{11h}^k f_+^h)+\\
\quad +\partial_2(-\partial_3 f_-^1+\partial_0 f_+^2+\partial_1 f_-^3+\partial_2 f_+^0)+(-(-g_{22h}^1)\partial_3 f_-^h+(-g_{22h}^2)\partial_0 f_+^h+(-g_{22h}^3)\partial_1 f_-^h+(-g_{22h}^0)\partial_2 f_+^h)+\\
+\partial_2(+g_{31h}^1 f_-^h-g_{01h}^2 f_+^h-g_{11h}^3 f_-^h+g_{21h}^0 f_+^h)+(+(-g_{22k}^1)g_{31h}^k f_-^h-(-g_{22k}^2)g_{01h}^k f_+^h-(-g_{22k}^3)g_{11h}^k f_-^h+(-g_{22k}^0)g_{21h}^k f_+^h)+\\
\quad +\partial_3(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_0 f_+^3+\partial_3 f_+^0)+(+(-g_{32h}^1)\partial_2 f_-^h-(-g_{32h}^2)\partial_1 f_-^h+(-g_{32h}^3)\partial_0 f_+^h+(-g_{32h}^0)\partial_3 f_+^h)+\\
+\partial_3(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+(-(-g_{32k}^1)g_{21h}^k f_-^h+(-g_{32k}^2)g_{11h}^k f_-^h-(-g_{32k}^3)g_{01h}^k f_+^h+(-g_{32k}^0)g_{31h}^k f_+^h)+\\
\quad +\partial_0(-\partial_1 f_+^1-\partial_2 f_+^2-\partial_3 f_+^3+\partial_0 f_+^0)+(-(-g_{02h}^1)\partial_1 f_+^h-(-g_{02h}^2)\partial_2 f_+^h-(-g_{02h}^3)\partial_3 f_+^h+(-g_{02h}^0)\partial_0 f_+^h)+\\
+\partial_0(+g_{11h}^1 f_+^h+g_{21h}^2 f_+^h+g_{31h}^3 f_+^h+g_{01h}^0 f_+^h)+(+(-g_{02k}^1)g_{11h}^k f_+^h+(-g_{02k}^2)g_{21h}^k f_+^h+(-g_{02k}^3)g_{31h}^k f_+^h+(-g_{02k}^0)g_{01h}^k f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_1^2 f_+^0+\partial_2^2 f_+^0+\partial_3^2 f_+^0+\partial_0^2 f_+^0+\\
\partial_1(+\partial_0 f_+^1+\partial_3 f_-^2-\partial_2 f_-^3)+\partial_2(-\partial_3 f_-^1+\partial_0 f_+^2+\partial_1 f_-^3)+\partial_3(+\partial_2 f_-^1-\partial_1 f_-^2+\partial_0 f_+^3)+\partial_0(-\partial_1 f_+^1-\partial_2 f_+^2-\partial_3 f_+^3)+\\
+\partial_0(+g_{11h}^1 f_+^h+g_{21h}^2 f_+^h+g_{31h}^3 f_+^h+g_{01h}^0 f_+^h)+\\
+\partial_1(-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h+g_{11h}^0 f_+^h)+\\
+\partial_2(+g_{31h}^1 f_-^h-g_{01h}^2 f_+^h-g_{11h}^3 f_-^h+g_{21h}^0 f_+^h)+\\
+\partial_3(-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h-g_{01h}^3 f_+^h+g_{31h}^0 f_+^h)+\\
+(-(-g_{02h}^1)\partial_1 f_+^h-(-g_{02h}^2)\partial_2 f_+^h-(-g_{02h}^3)\partial_3 f_+^h+(-g_{02h}^0)\partial_0 f_+^h)+\\
+(+(-g_{12h}^1)\partial_0 f_+^h+(-g_{12h}^2)\partial_3 f_-^h-(-g_{12h}^3)\partial_2 f_-^h+(-g_{12h}^0)\partial_1 f_+^h)+\\
+(-(-g_{22h}^1)\partial_3 f_-^h+(-g_{22h}^2)\partial_0 f_+^h+(-g_{22h}^3)\partial_1 f_-^h+(-g_{22h}^0)\partial_2 f_+^h)+\\
+(+(-g_{32h}^1)\partial_2 f_-^h-(-g_{32h}^2)\partial_1 f_-^h+(-g_{32h}^3)\partial_0 f_+^h+(-g_{32h}^0)\partial_3 f_+^h)+\\
+(+(-g_{02k}^1)g_{11h}^k f_+^h+(-g_{02k}^2)g_{21h}^k f_+^h+(-g_{02k}^3)g_{31h}^k f_+^h+(-g_{02k}^0)g_{01h}^k f_+^h)+\\
+(-(-g_{12k}^1)g_{01h}^k f_+^h-(-g_{12k}^2)g_{31h}^k f_-^h+(-g_{12k}^3)g_{21h}^k f_-^h+(-g_{12k}^0)g_{11h}^k f_+^h)+\\
+(+(-g_{22k}^1)g_{31h}^k f_-^h-(-g_{22k}^2)g_{01h}^k f_+^h-(-g_{22k}^3)g_{11h}^k f_-^h+(-g_{22k}^0)g_{21h}^k f_+^h)+\\
+(-(-g_{32k}^1)g_{21h}^k f_-^h+(-g_{32k}^2)g_{11h}^k f_-^h-(-g_{32k}^3)g_{01h}^k f_+^h+(-g_{32k}^0)g_{31h}^k f_+^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_1^2 f_+^0+\partial_2^2 f_+^0+\partial_3^2 f_+^0+\partial_0^2 f_+^0+\\
+\partial_0([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_+^h)+\\
+\partial_1(+g_{11h}^0 f_+^h-g_{01h}^1 f_+^h-g_{31h}^2 f_-^h+g_{21h}^3 f_-^h)+\\
+\partial_2(+g_{21h}^0 f_+^h-g_{01h}^2 f_+^h+g_{31h}^1 f_-^h-g_{11h}^3 f_-^h)+\\
+\partial_3(+g_{31h}^0 f_+^h-g_{01h}^3 f_+^h-g_{21h}^1 f_-^h+g_{11h}^2 f_-^h)+\\
+(-g_{02h}^0)\partial_0 f_+^h+(-g_{12h}^1)\partial_0 f_+^h+(-g_{22h}^2)\partial_0 f_+^h+(-g_{32h}^3)\partial_0 f_+^h+\\
+(-g_{12h}^0)\partial_1 f_+^h-(-g_{02h}^1)\partial_1 f_+^h-(-g_{32h}^2)\partial_1 f_-^h+(-g_{22h}^3)\partial_1 f_-^h\\
+(-g_{22h}^0)\partial_2 f_+^h-(-g_{02h}^2)\partial_2 f_+^h+(-g_{32h}^1)\partial_2 f_-^h-(-g_{12h}^3)\partial_2 f_-^h+\\
-(-g_{22h}^1)\partial_3 f_-^h+(-g_{12h}^2)\partial_3 f_-^h+(-g_{02h}^0)\partial_3 f_+^h-(-g_{02h}^3)\partial_3 f_+^h+\\
+(-g_{02k}^0)g_{01h}^k f_+^h-(-g_{12k}^1)g_{01h}^k f_+^h-(-g_{22k}^2)g_{01h}^k f_+^h-(-g_{32k}^3)g_{01h}^k f_+^h\\
+(-g_{12k}^0)g_{11h}^k f_+^h+(-g_{02k}^1)g_{11h}^k f_+^h+(-g_{22k}^0)g_{21h}^k f_+^h+(-g_{02k}^2)g_{21h}^k f_+^h+(-g_{32k}^0)g_{31h}^k f_+^h+(-g_{02k}^3)g_{31h}^k f_+^h+\\
+(-g_{32k}^2)g_{11h}^k f_-^h-(-g_{22k}^3)g_{11h}^k f_-^h-(-g_{32k}^1)g_{21h}^k f_-^h+(-g_{12k}^3)g_{21h}^k f_-^h+(-g_{22k}^1)g_{31h}^k f_-^h-(-g_{12k}^2)g_{31h}^k f_-^h
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_+^0+\\
+\partial_0([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_+^h)+\\
+\partial_1([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h)+\\
+\partial_2([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h)+\\
+\partial_3([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h)+\\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2-g_{32h}^3)\partial_0 f_+^h+\\
+(-g_{12h}^0+g_{02h}^1)\partial_1 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_1 f_-^h+\\
+(-g_{22h}^0+g_{02h}^2)\partial_2 f_+^h+(-g_{32h}^1+g_{12h}^3)\partial_2 f_-^h+\\
+(-g_{32h}^0+g_{02h}^3)\partial_3 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_3 f_-^h+\\
+([-g_{02k}^0+g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{01h}^k+[-g_{12k}^0-g_{02k}^1]g_{11h}^k+[-g_{22k}^0-g_{02k}^2]g_{21h}^k+[-g_{32k}^0-g_{02k}^3]g_{31h}^k)f_+^h+\\
+([-g_{32k}^2+g_{22k}^3]g_{11h}^k+[+g_{32k}^1-g_{12k}^3]g_{21h}^k+[-g_{22k}^1+g_{12k}^2]g_{31h}^k)f_-^h
\end{array}\right)
$$

and (*0-):

$$
\left(\begin{array}{l}
-D_{12}^+[-(\partial_0 f_-^1+g_{01h}^1 f_-^h)-(\partial_3 f_+^2+g_{31h}^2 f_+^h)+(\partial_2 f_+^3+g_{21h}^3 f_+^h)-(\partial_1 f_-^0-g_{11h}^0 f_-^h)]+\\
\quad -D_{22}^+[(\partial_3 f_+^1+g_{31h}^1 f_+^h)-(\partial_0 f_-^2+g_{01h}^2 f_-^h)-(\partial_1 f_+^3+g_{11h}^3 f_+^h)-(\partial_2 f_-^0-g_{21h}^0 f_-^h)]+\\
\quad -D_{32}^+[-(\partial_2 f_+^1+g_{21h}^1 f_+^h)+(\partial_1 f_+^2+g_{11h}^2 f_+^h)-(\partial_0 f_-^3+g_{01h}^3 f_-^h)-(\partial_3 f_-^0-g_{31h}^0 f_-^h)]+\\
\quad +D_{02}^+[-(\partial_1 f_-^1+g_{11h}^1 f_-^h)-(\partial_2 f_-^2+g_{21h}^2 f_-^h)-(\partial_3 f_-^3+g_{31h}^3 f_-^h)+(\partial_0 f_-^0-g_{01h}^0 f_-^h)]
\end{array}\right)=
$$

$$
=\left(\begin{array}{l}
D_{12}^+(+\partial_0 f_-^1+\partial_3 f_+^2-\partial_2 f_+^3+\partial_1 f_-^0)+D_{12}^+(+g_{01h}^1 f_-^h+g_{31h}^2 f_+^h-g_{21h}^3 f_+^h-g_{11h}^0 f_-^h)+\\
\quad +D_{22}^+(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3+\partial_2 f_-^0)+D_{22}^+(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
\quad +D_{32}^+(+\partial_2 f_+^1-\partial_1 f_+^2+\partial_0 f_-^3+\partial_3 f_-^0)+D_{32}^+(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+\\
\quad +D_{02}^+(-\partial_1 f_-^1-\partial_2 f_-^2-\partial_3 f_-^3+\partial_0 f_-^0)+D_{02}^+(-g_{11h}^1 f_-^h-g_{21h}^2 f_-^h-g_{31h}^3 f_-^h-g_{01h}^0 f_-^h)
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\partial_1(+\partial_0 f_-^1+\partial_3 f_+^2-\partial_2 f_+^3+\partial_1 f_-^0)+(+(+g_{12h}^1)\partial_0 f_-^h+(+g_{12h}^2)\partial_3 f_+^h-(+g_{12h}^3)\partial_2 f_+^h+(+g_{12h}^0)\partial_1 f_-^h)+\\
+\partial_1(+g_{01h}^1 f_-^h+g_{31h}^2 f_+^h-g_{21h}^3 f_+^h-g_{11h}^0 f_-^h)+(+(+g_{12k}^1)g_{01h}^k f_-^h+(+g_{12k}^2)g_{31h}^k f_+^h-(+g_{12k}^3)g_{21h}^k f_+^h-(+g_{12k}^0)g_{11h}^k f_-^h)+\\
\quad +\partial_2(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3+\partial_2 f_-^0)+(-(+g_{22h}^1)\partial_3 f_+^h+(+g_{22h}^2)\partial_0 f_-^h+(+g_{22h}^3)\partial_1 f_+^h+(+g_{22h}^0)\partial_2 f_-^h)+\\
+\partial_2(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+(-(+g_{22k}^1)g_{31h}^k f_+^h+(+g_{22k}^2)g_{01h}^k f_-^h+(+g_{22k}^3)g_{11h}^k f_+^h-(+g_{22k}^0)g_{21h}^k f_-^h)+\\
\quad +\partial_3(+\partial_2 f_+^1-\partial_1 f_+^2+\partial_0 f_-^3+\partial_3 f_-^0)+(+(+g_{32h}^1)\partial_2 f_+^h-(+g_{32h}^2)\partial_1 f_+^h+(+g_{32h}^3)\partial_0 f_-^h+(+g_{32h}^0)\partial_3 f_-^h)+\\
+\partial_3(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+(+(+g_{32k}^1)g_{21h}^k f_+^h-(+g_{32k}^2)g_{11h}^k f_+^h+(+g_{32k}^3)g_{01h}^k f_-^h-(+g_{32k}^0)g_{31h}^k f_-^h)+\\
\quad +\partial_0(-\partial_1 f_-^1-\partial_2 f_-^2-\partial_3 f_-^3+\partial_0 f_-^0)+(-(+g_{02h}^1)\partial_1 f_-^h-(+g_{02h}^2)\partial_2 f_-^h-(+g_{02h}^3)\partial_3 f_-^h+(+g_{02h}^0)\partial_0 f_-^h)+\\
+\partial_0(-g_{11h}^1 f_-^h-g_{21h}^2 f_-^h-g_{31h}^3 f_-^h-g_{01h}^0 f_-^h)+(-(+g_{02k}^1)g_{11h}^k f_-^h-(+g_{02k}^2)g_{21h}^k f_-^h-(+g_{02k}^3)g_{31h}^k f_-^h-(+g_{02k}^0)g_{01h}^k f_-^h)
\end{array}\right)
$$

$$
\begin{aligned}
&+\partial_1^2 f_-^0+\partial_2^2 f_-^0+\partial_3^2 f_-^0+\partial_0^2 f_-^0+\\
&\partial_1(+\partial_0 f_-^1+\partial_3 f_+^2-\partial_2 f_+^3)+\partial_2(-\partial_3 f_+^1+\partial_0 f_-^2+\partial_1 f_+^3)+\partial_3(+\partial_2 f_+^1-\partial_1 f_+^2+\partial_0 f_-^3)+\partial_0(-\partial_1 f_-^1-\partial_2 f_-^2-\partial_3 f_-^3)+\\
&+\partial_0(-g_{11h}^1 f_-^h-g_{21h}^2 f_-^h-g_{31h}^3 f_-^h-g_{01h}^0 f_-^h)+\\
&+\partial_1(+g_{01h}^1 f_-^h+g_{31h}^2 f_+^h-g_{21h}^3 f_+^h-g_{11h}^0 f_-^h)+\\
&+\partial_2(-g_{31h}^1 f_+^h+g_{01h}^2 f_-^h+g_{11h}^3 f_+^h-g_{21h}^0 f_-^h)+\\
&+\partial_3(+g_{21h}^1 f_+^h-g_{11h}^2 f_+^h+g_{01h}^3 f_-^h-g_{31h}^0 f_-^h)+\\
&+(-(+g_{02h}^1)\partial_1 f_-^h-(+g_{02h}^2)\partial_2 f_-^h-(+g_{02h}^3)\partial_3 f_-^h+(+g_{02h}^0)\partial_0 f_-^h)+\\
&+(+(+g_{12h}^1)\partial_0 f_-^h++(+g_{12h}^2)\partial_3 f_+^h-(+g_{12h}^3)\partial_2 f_+^h+(+g_{12h}^0)\partial_1 f_-^h)+\\
&+(-(+g_{22h}^1)\partial_3 f_+^h+(+g_{22h}^2)\partial_0 f_-^h+(+g_{22h}^3)\partial_1 f_+^h+(+g_{22h}^0)\partial_2 f_-^h)+\\
&+(+(+g_{32h}^1)\partial_2 f_+^h-(+g_{32h}^2)\partial_1 f_+^h+(+g_{32h}^3)\partial_0 f_-^h+(+g_{32h}^0)\partial_3 f_-^h)+\\
&+(-(+g_{02k}^1)g_{11h}^k f_-^h-(+g_{02k}^2)g_{21h}^k f_-^h-(+g_{02k}^3)g_{31h}^k f_-^h-(+g_{02k}^0)g_{01h}^k f_-^h)+\\
&+(+(+g_{12k}^1)g_{01h}^k f_-^h+(+g_{12k}^2)g_{31h}^k f_+^h-(+g_{12k}^3)g_{21h}^k f_+^h-(+g_{12k}^0)g_{11h}^k f_-^h)+\\
&+(-(+g_{22k}^1)g_{31h}^k f_+^h+(+g_{22k}^2)g_{01h}^k f_-^h+(+g_{22k}^3)g_{11h}^k f_+^h-(+g_{22k}^0)g_{21h}^k f_-^h)+\\
&+(+(+g_{32k}^1)g_{21h}^1 f_+^k-(+g_{32k}^2)g_{11h}^k f_+^h+(+g_{32k}^3)g_{01h}^k f_-^h-(+g_{32k}^0)g_{31h}^k f_-^h)
\end{aligned}
$$

$$
\begin{aligned}
&+\partial_1^2 f_-^0+\partial_2^2 f_-^0+\partial_3^2 f_-^0+\partial_0^2 f_-^0+\\
&+\partial_0([-g_{01h}^0-g_{11h}^1 f_-^h-g_{21h}^2 f_-^h-g_{31h}^3 f_-^h]f_-^h)+\\
&+\partial_1([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h)+\\
&+\partial_2([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h)+\\
&+\partial_3([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h)+\\
&+(+g_{02h}^0)\partial_0 f_-^h+(+g_{12h}^1)\partial_0 f_-^h+(+g_{22h}^2)\partial_0 f_-^h+(+g_{32h}^3)\partial_0 f_-^h\\
&-(+g_{32h}^2)\partial_1 f_+^h+(+g_{22h}^3)\partial_1 f_+^h+(+g_{12h}^0)\partial_1 f_-^h-(+g_{02h}^1)\partial_1 f_-^h\\
&+(+g_{32h}^1)\partial_2 f_+^h-(+g_{12h}^3)\partial_2 f_+^h+(+g_{22h}^0)\partial_2 f_-^h-(+g_{02h}^2)\partial_2 f_-^h\\
&-(+g_{22h}^1)\partial_3 f_+^h++(+g_{12h}^2)\partial_3 f_+^h+(+g_{32h}^0)\partial_3 f_-^h-(+g_{02h}^3)\partial_3 f_-^h+\\
&+(+g_{22k}^3)g_{11h}^k f_+^h-(+g_{32k}^2)g_{11h}^k f_+^h+(+g_{32k}^1)g_{21h}^1 f_+^k-(+g_{12k}^3)g_{21h}^k f_+^h-(+g_{22k}^1)g_{31h}^k f_+^h+(+g_{12k}^2)g_{31h}^k f_+^h+\\
&-(+g_{02k}^0)g_{01h}^k f_-^h+(+g_{12k}^1)g_{01h}^k f_-^h+(+g_{22k}^2)g_{01h}^k f_-^h+(+g_{32k}^3)g_{01h}^k f_-^h+\\
&-(+g_{12k}^0)g_{11h}^k f_-^h-(+g_{02k}^1)g_{11h}^k f_-^h-(+g_{22k}^0)g_{21h}^k f_-^h-(+g_{02k}^2)g_{21h}^k f_-^h-(+g_{32k}^0)g_{31h}^k f_-^h-(+g_{02k}^3)g_{31h}^k f_-^h+
\end{aligned}
$$

$$
\begin{aligned}
&(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_-^0+\\
&+\partial_0([-g_{01h}^0-g_{11h}^1 f_-^h-g_{21h}^2 f_-^h-g_{31h}^3 f_-^h]f_-^h)+\\
&+\partial_1([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h)+\\
&+\partial_2([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h)+\\
&+\partial_3([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h)+\\
&+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_0 f_-^h+\\
&+(-g_{32h}^2+g_{22h}^3)\partial_1 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_1 f_-^h\\
&+(+g_{32h}^1-g_{12h}^3)\partial_2 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_2 f_-^h\\
&+(-g_{22h}^1+g_{12h}^2)\partial_3 f_+^h+(+g_{32h}^0-g_{02h}^3)\partial_3 f_-^h+\\
&+([+g_{22k}^3-g_{32k}^2]g_{11h}^k+[+g_{32k}^1-g_{12k}^3]g_{21h}^k+[-g_{22k}^1+g_{12k}^2]g_{31h}^k)f_+^h+\\
&+([-g_{02k}^0+g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{01h}^k+[-g_{12k}^0-g_{02k}^1]g_{11h}^k+[-g_{22k}^0-g_{02k}^2]g_{21h}^k+[-g_{32k}^0-g_{02k}^3]g_{31h}^k)f_-^h+
\end{aligned}
$$

So:

$$
\mathbf{J} = \left(\begin{array}{c}
\left(\begin{array}{l}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+\partial_0([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h) + \\
+\partial_1([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+\partial_2([+g_{21h}^1-g_{11h}^2]f_+^h+[-g_{31h}^0+g_{01h}^3]f_-^h) + \\
+\partial_3([-g_{11h}^3+g_{31h}^1]f_+^h+[-g_{01h}^2+g_{21h}^0]f_-^h) + \\
+(-g_{12h}^0+g_{02h}^1)\partial_0 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_0 f_-^h \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_1 f_+^h + \\
+(-g_{22h}^1+g_{12h}^2)\partial_2 f_+^h+(+g_{32h}^0-g_{02h}^3)\partial_2 f_-^h + \\
+(-g_{32h}^1+g_{12h}^3)\partial_3 f_+^h+(-g_{22h}^0+g_{02h}^2)\partial_3 f_-^h + \\
+([-g_{12k}^0-g_{02k}^1]g_{01h}^k+[+g_{02k}^0-g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{11h}^k+[-g_{22k}^1-g_{12k}^2]g_{21h}^k+[-g_{32k}^1-g_{12k}^3]g_{31h}^k)f_+^h + \\
+([+g_{32k}^2-g_{22k}^3]g_{01h}^k+[-g_{32k}^0+g_{02k}^3]g_{21h}^k+[+g_{22k}^0-g_{02k}^2]g_{31h}^k)f_-^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+\partial_0([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h) + \\
+\partial_1([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_-^h) + \\
+\partial_2([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h) + \\
+\partial_3([-g_{21h}^0+g_{01h}^2]f_+^h+[-g_{31h}^1+g_{11h}^3]f_-^h) + \\
+(-g_{32h}^2+g_{22h}^3)\partial_0 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_0 f_-^h + \\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2-g_{32h}^3)\partial_1 f_-^h + \\
+(-g_{32h}^0+g_{02h}^3)\partial_2 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_2 f_-^h \\
+(+g_{22h}^0-g_{02h}^2)\partial_3 f_+^h+(+g_{32h}^1-g_{12h}^3)\partial_3 f_-^h + \\
+([+g_{32k}^2-g_{22k}^3]g_{01h}^k+[-g_{32k}^0+g_{02k}^3]g_{21h}^k+[+g_{22k}^0-g_{02k}^2]g_{31h}^k)f_+^h + \\
+([-g_{12k}^0-g_{02k}^1]g_{01h}^k+[+g_{02k}^0-g_{12k}^1+g_{22k}^2+g_{32k}^3]g_{11h}^k+[-g_{22k}^1-g_{12k}^2]g_{21h}^k+[-g_{32k}^1-g_{12k}^3]g_{31h}^k)f_-^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+\partial_0([+g_{21h}^0-g_{01h}^2]f_+^h+[+g_{31h}^1-g_{11h}^3]f_-^h) + \\
+\partial_1([-g_{21h}^1+g_{11h}^2]f_+^h+[+g_{31h}^0-g_{01h}^3]f_-^h) + \\
+\partial_2([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+\partial_3([+g_{31h}^2-g_{21h}^3]f_+^h+[-g_{11h}^0+g_{01h}^1]f_-^h) + \\
+(-g_{22h}^0+g_{02h}^2)\partial_0 f_+^h+(-g_{32h}^1+g_{12h}^3)\partial_0 f_-^h \\
+[+g_{22h}^1-g_{12h}^2]\partial_1 f_+^h+(-g_{32h}^0+g_{02h}^3)\partial_1 f_-^h \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_2 f_+^h + \\
+(-g_{32h}^2+g_{22h}^3)\partial_3 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_3 f_-^h + \\
+([-g_{02k}^2-g_{22k}^0]g_{01h}^k+[-g_{22k}^1-g_{12k}^2]g_{11h}^k+[+g_{02k}^0+g_{12k}^1-g_{22k}^2+g_{32k}^3]g_{21h}^k+[-g_{32k}^2-g_{22k}^3]g_{31h}^k)f_+^h + \\
+([-g_{32k}^1+g_{12k}^3]g_{01h}^k+[+g_{32k}^0-g_{02k}^3]g_{11h}^k+[-g_{12k}^0+g_{02k}^1]g_{31h}^k)f_-^h +
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
+\partial_0([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h) + \\
+\partial_1([-g_{31h}^0+g_{01h}^3]f_+^h+[+g_{21h}^1-g_{11h}^2]f_-^h) + \\
+\partial_3([+g_{11h}^0-g_{01h}^1]f_+^h+[-g_{31h}^2+g_{21h}^3]f_-^h) + \\
+\partial_2([+g_{01h}^0+g_{11h}^1+g_{21h}^2+g_{31h}^3]f_-^h) + \\
(+g_{32h}^1-g_{12h}^3)\partial_0 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_0 f_-^h + \\
+(g_{32h}^0-g_{02h}^3)\partial_1 f_+^h+(g_{12h}^2-g_{22h}^1)\partial_1 f_-^1 + \\
+(-g_{02h}^0-g_{12h}^1-g_{22h}^2-g_{32h}^3)\partial_2 f_-^h + \\
+(-g_{12h}^0+g_{02h}^1)\partial_3 f_+^h+(+g_{32h}^2-g_{22h}^3)\partial_3 f_-^h + \\
+([-g_{32k}^1+g_{12k}^3]g_{01h}^k+[+g_{32k}^0-g_{02k}^3]g_{11h}^k+[-g_{12k}^0+g_{02k}^1]g_{31h}^k)f_+^h + \\
+([-g_{22k}^0-g_{02k}^2]g_{01h}^k+[-g_{22k}^1-g_{12k}^2]g_{11h}^k+[+g_{02k}^0+g_{12k}^1-g_{22k}^2+g_{32k}^3]g_{21h}^k+[-g_{32k}^2-g_{22k}^3]g_{31h}^k)f_-^h
\end{array}\right) \\
\left(\begin{array}{l}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+\partial_0([+g_{31h}^0-g_{01h}^3]f_+^h+[-g_{21h}^1+g_{11h}^2]f_-^h) + \\
+\partial_1([-g_{31h}^1+g_{11h}^3]f_+^h+[-g_{21h}^0+g_{01h}^2]f_-^h) + \\
+\partial_2([-g_{31h}^2+g_{21h}^3]f_+^h+[+g_{11h}^0-g_{01h}^1]f_-^h) + \\
+\partial_3([-g_{01h}^0-g_{11h}^1-g_{21h}^2-g_{31h}^3]f_+^h) + \\
+(-g_{32h}^0+g_{02h}^3)\partial_0 f_+^h+(+g_{22h}^1-g_{12h}^2)\partial_0 f_-^h + \\
+(+g_{32h}^1-g_{12h}^3)\partial_1 f_+^h+(+g_{22h}^0-g_{02h}^2)\partial_1 f_-^h + \\
+(+g_{32h}^2-g_{22h}^3)\partial_2 f_+^h+(+g_{12h}^0-g_{02h}^1)\partial_2 f_-^h + \\
+(+g_{02h}^0+g_{12h}^1+g_{22h}^2+g_{32h}^3)\partial_3 f_+^h +
\end{array}\right)
\end{array}\right)
$$

And:

$J^1 = -D_{02}\Phi^1 + D^{\rightleftarrows}_{32}\Phi^2 - D^{\rightleftarrows}_{22}\Phi^3 - D_{12}\Phi^0$	$-D^{\Updownarrow}_{01}f^1 - D^{\rightleftarrows}_{31}f^2 + D^{\rightleftarrows}_{21}f^3 - D_{11}f^0 = \Phi^1$
$J^2 = -D^{\rightleftarrows}_{32}\Phi^1 - D_{02}\Phi^2 + D^{\rightleftarrows}_{12}\Phi^3 - D_{22}\Phi^0$	$D^{\rightleftarrows}_{31}f^1 - D^{\Updownarrow}_{01}f^2 - D^{\rightleftarrows}_{11}f^3 - D_{21}f^0 = \Phi^2$
$J^3 = D^{\rightleftarrows}_{22}\Phi^1 - D^{\rightleftarrows}_{12}\Phi^2 - D_{02}\Phi^3 - D_{32}\Phi^0$	$-D^{\rightleftarrows}_{21}f^1 + D^{\rightleftarrows}_{11}f^2 - D^{\Updownarrow}_{01}f^3 - D_{31}f^0 = \Phi^3$
$J^0 = -D^{\Updownarrow}_{12}\Phi^1 - D^{\Updownarrow}_{22}\Phi^2 - D^{\Updownarrow}_{32}\Phi^3 + D^{\Updownarrow}_{02}\Phi^0$	$-D^{\Updownarrow}_{11}f^1 - D^{\Updownarrow}_{21}f^2 - D^{\Updownarrow}_{31}f^3 + D_{01}f^0 = \Phi^0$

$$\Downarrow$$

$$\begin{pmatrix} J^1_+ \\ J^1_- \end{pmatrix} = -\begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix} + \begin{pmatrix} 0 & D^-_{32} \\ D^+_{32} & 0 \end{pmatrix}\begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix} - \begin{pmatrix} 0 & D^-_{22} \\ D^+_{22} & 0 \end{pmatrix}\begin{pmatrix} \Phi^3_+ \\ \Phi^3_- \end{pmatrix} - \begin{pmatrix} D^+_{12} & 0 \\ 0 & D^-_{12} \end{pmatrix}\begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix}$$

$$\begin{pmatrix} J^2_+ \\ J^2_- \end{pmatrix} = -\begin{pmatrix} 0 & D^-_{32} \\ D^+_{32} & 0 \end{pmatrix}\begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix} - \begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix} + \begin{pmatrix} 0 & D^-_{12} \\ D^+_{12} & 0 \end{pmatrix}\begin{pmatrix} \Phi^3_+ \\ \Phi^3_- \end{pmatrix} - \begin{pmatrix} D^+_{22} & 0 \\ 0 & D^-_{22} \end{pmatrix}\begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix}$$

$$\begin{pmatrix} J^3_+ \\ J^3_- \end{pmatrix} = \begin{pmatrix} 0 & D^-_{22} \\ D^+_{22} & 0 \end{pmatrix}\begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix} - \begin{pmatrix} 0 & D^-_{12} \\ D^+_{12} & 0 \end{pmatrix}\begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix} - \begin{pmatrix} D^+_{02} & 0 \\ 0 & D^-_{02} \end{pmatrix}\begin{pmatrix} \Phi^3_+ \\ \Phi^3_- \end{pmatrix} - \begin{pmatrix} D^+_{32} & 0 \\ 0 & D^-_{32} \end{pmatrix}\begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix}$$

$$\begin{pmatrix} J^0_+ \\ J^0_- \end{pmatrix} = -\begin{pmatrix} D^-_{12} & 0 \\ 0 & D^+_{12} \end{pmatrix}\begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix} - \begin{pmatrix} D^-_{22} & 0 \\ 0 & D^+_{22} \end{pmatrix}\begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix} - \begin{pmatrix} D^-_{32} & 0 \\ 0 & D^+_{32} \end{pmatrix}\begin{pmatrix} \Phi^3_+ \\ \Phi^3_- \end{pmatrix} + \begin{pmatrix} D^-_{02} & 0 \\ 0 & D^+_{02} \end{pmatrix}\begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix}$$

$$-\begin{pmatrix} D^-_{01} & 0 \\ 0 & D^+_{01} \end{pmatrix}\begin{pmatrix} f^1_+ \\ f^1_- \end{pmatrix} - \begin{pmatrix} 0 & D^-_{31} \\ D^+_{31} & 0 \end{pmatrix}\begin{pmatrix} f^2_+ \\ f^2_- \end{pmatrix} + \begin{pmatrix} 0 & D^-_{21} \\ D^+_{21} & 0 \end{pmatrix}\begin{pmatrix} f^3_+ \\ f^3_- \end{pmatrix} - \begin{pmatrix} D^+_{11} & 0 \\ 0 & D^-_{11} \end{pmatrix}\begin{pmatrix} f^0_+ \\ f^0_- \end{pmatrix} = \begin{pmatrix} \Phi^1_+ \\ \Phi^1_- \end{pmatrix}$$

$$\begin{pmatrix} 0 & D^-_{31} \\ D^+_{31} & 0 \end{pmatrix}\begin{pmatrix} f^1_+ \\ f^1_- \end{pmatrix} - \begin{pmatrix} D^-_{01} & 0 \\ 0 & D^+_{01} \end{pmatrix}\begin{pmatrix} f^2_+ \\ f^2_- \end{pmatrix} - \begin{pmatrix} 0 & D^-_{11} \\ D^+_{11} & 0 \end{pmatrix}\begin{pmatrix} f^3_+ \\ f^3_- \end{pmatrix} - \begin{pmatrix} D^+_{21} & 0 \\ 0 & D^-_{21} \end{pmatrix}\begin{pmatrix} f^0_+ \\ f^0_- \end{pmatrix} = \begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix}$$

$$-\begin{pmatrix} 0 & D^-_{21} \\ D^+_{21} & 0 \end{pmatrix}\begin{pmatrix} f^1_+ \\ f^1_- \end{pmatrix} + \begin{pmatrix} 0 & D^-_{11} \\ D^+_{11} & 0 \end{pmatrix}\begin{pmatrix} f^2_+ \\ f^2_- \end{pmatrix} - \begin{pmatrix} D^-_{01} & 0 \\ 0 & D^+_{01} \end{pmatrix}\begin{pmatrix} f^3_+ \\ f^3_- \end{pmatrix} - \begin{pmatrix} D^+_{31} & 0 \\ 0 & D^-_{31} \end{pmatrix}\begin{pmatrix} f^0_+ \\ f^0_- \end{pmatrix} = \begin{pmatrix} \Phi^2_+ \\ \Phi^2_- \end{pmatrix}$$

$$-\begin{pmatrix} D^-_{11} & 0 \\ 0 & D^+_{11} \end{pmatrix}\begin{pmatrix} f^1_+ \\ f^1_- \end{pmatrix} - \begin{pmatrix} D^-_{21} & 0 \\ 0 & D^+_{21} \end{pmatrix}\begin{pmatrix} f^2_+ \\ f^2_- \end{pmatrix} - \begin{pmatrix} D^-_{31} & 0 \\ 0 & D^+_{31} \end{pmatrix}\begin{pmatrix} f^3_+ \\ f^3_- \end{pmatrix} + \begin{pmatrix} D^+_{01} & 0 \\ 0 & D^-_{01} \end{pmatrix}\begin{pmatrix} f^0_+ \\ f^0_- \end{pmatrix} = \begin{pmatrix} \Phi^0_+ \\ \Phi^0_- \end{pmatrix}$$

$$\Downarrow$$

$J^1_+ = -D^+_{02}\Phi^1_+ + D^-_{32}\Phi^2_- - D^-_{22}\Phi^3_- - D^+_{12}\Phi^0_+$	$-D^-_{01}f^1_+ - D^-_{31}f^2_- + D^-_{21}f^3_- - D^+_{11}f^0_+ = \Phi^1_+$
$J^1_- = -D^-_{02}\Phi^1_- + D^+_{32}\Phi^2_+ - D^+_{22}\Phi^3_+ - D^-_{12}\Phi^0_-$	$-D^+_{01}f^1_- - D^+_{31}f^2_+ + D^+_{21}f^3_+ - D^-_{11}f^0_- = \Phi^1_-$
$J^2_+ = D^-_{32}\Phi^1_- - D^+_{02}\Phi^2_+ + D^-_{12}\Phi^3_- - D^+_{22}\Phi^0_+$	$D^-_{31}f^1_- - D^-_{01}f^2_+ - D^-_{11}f^3_- - D^+_{21}f^0_+ = \Phi^2_+$
$J^2_- = D^+_{32}\Phi^1_+ - D^-_{02}\Phi^2_- + D^+_{12}\Phi^3_+ - D^-_{22}\Phi^0_-$	$D^+_{31}f^1_+ - D^+_{01}f^2_- - D^+_{11}f^3_+ - D^-_{21}f^0_- = \Phi^2_-$
$J^3_+ = D^-_{22}\Phi^1_- - D^-_{12}\Phi^2_- - D^+_{02}\Phi^3_+ - D^+_{32}\Phi^0_+$	$-D^-_{21}f^1_- + D^-_{11}f^2_- - D^-_{01}f^3_+ - D^+_{31}f^0_+ = \Phi^3_+$
$J^3_- = D^+_{22}\Phi^1_+ - D^+_{12}\Phi^2_+ - D^-_{02}\Phi^3_- - D^-_{32}\Phi^0_-$	$-D^+_{21}f^1_+ + D^+_{11}f^2_+ - D^+_{01}f^3_- - D^-_{31}f^0_- = \Phi^3_-$
$J^0_+ = -D^-_{12}\Phi^1_+ - D^-_{22}\Phi^2_+ - D^-_{32}\Phi^3_+ + D^-_{02}\Phi^0_+$	$-D^-_{11}f^1_+ - D^-_{21}f^2_+ - D^-_{31}f^3_+ + D^+_{01}f^0_+ = \Phi^0_+$
$J^0_- = -D^+_{12}\Phi^1_- - D^+_{22}\Phi^2_- - D^+_{32}\Phi^3_- + D^+_{02}\Phi^0_-$	$-D^+_{11}f^1_- - D^+_{21}f^2_- - D^+_{31}f^3_- + D^-_{01}f^0_- = \Phi^0_-$

$$\Downarrow$$

$J^1_+ = -\partial_0\Phi^1_+ - g^1_{0j2}\Phi^j_+ + \partial_3\Phi^2_- - g^2_{3j2}\Phi^j_- - \partial_2\Phi^3_- + g^3_{2j2}\Phi^j_- - \partial_1\Phi^0_+ - g^0_{1j2}\Phi^j_+$	$-\partial_0 f^1_+ - \partial_1 f^0_+ - \partial_3 f^2_- + \partial_2 f^3_- + g^1_{0j1}f^j_+ + g^2_{3j1}f^j_- - g^3_{2j1}f^j_- - g^0_{1j1}f^j_+ = \Phi^1_+$
$J^1_- = -\partial_0\Phi^1_- + g^1_{0j2}\Phi^j_- + \partial_3\Phi^2_+ + g^2_{3j2}\Phi^j_+ - \partial_2\Phi^3_+ - g^3_{2j2}\Phi^j_+ - \partial_1\Phi^0_- + g^0_{1j2}\Phi^j_-$	$-\partial_0 f^1_- - g^1_{0j1}f^j_- - \partial_3 f^2_+ - g^2_{3j1}f^j_+ + \partial_2 f^3_+ + g^3_{2j1}f^j_+ - \partial_1 f^0_- + g^0_{1j1}f^j_- = \Phi^1_-$
$J^2_+ = \partial_3\Phi^1_- - g^1_{3j2}\Phi^j_- - \partial_0\Phi^2_+ - g^2_{0j2}\Phi^j_+ + \partial_1\Phi^3_- - g^3_{1j2}\Phi^j_- - \partial_2\Phi^0_+ - g^0_{2j2}\Phi^j_+$	$\partial_3 f^1_- - g^1_{3j1}f^j_- - \partial_0 f^2_+ + g^2_{0j1}f^j_+ - \partial_1 f^3_- + g^3_{1j1}f^j_- - \partial_2 f^0_+ - g^0_{2j1}f^j_+ = \Phi^2_+$
$J^2_- = \partial_3\Phi^1_+ + g^1_{3j2}\Phi^j_+ - \partial_0\Phi^2_- + g^2_{0j2}\Phi^j_- + \partial_1\Phi^3_+ + g^3_{1j2}\Phi^j_+ - \partial_2\Phi^0_- + g^0_{2j2}\Phi^j_-$	$\partial_3 f^1_+ + g^1_{3j1}f^j_+ - \partial_0 f^2_- - g^2_{0j1}f^j_- - \partial_1 f^3_+ - g^3_{1j1}f^j_+ - \partial_2 f^0_- + g^0_{2j1}f^j_- = \Phi^2_-$
$J^3_+ = \partial_2\Phi^1_- - g^1_{2j2}\Phi^j_- - \partial_1\Phi^2_- + g^2_{1j2}\Phi^j_- - \partial_0\Phi^3_+ - g^3_{0j2}\Phi^j_+ - \partial_3\Phi^0_+ - g^0_{3j2}\Phi^j_+$	$-\partial_2 f^1_- + g^1_{2j1}f^j_- + \partial_1 f^2_- - g^2_{1j1}f^j_- - \partial_0 f^3_+ + g^3_{0j1}f^j_+ - \partial_3 f^0_+ - g^0_{3j1}f^j_+ = \Phi^3_+$
$J^3_- = \partial_2\Phi^1_+ + g^1_{2j2}\Phi^j_+ - \partial_1\Phi^2_+ - g^2_{1j2}\Phi^j_+ - \partial_0\Phi^3_- + g^3_{0j2}\Phi^j_- - \partial_3\Phi^0_- + g^0_{3j2}\Phi^j_-$	$-\partial_2 f^1_+ - g^1_{2j1}f^j_+ + \partial_1 f^2_+ + g^2_{1j1}f^j_+ - \partial_0 f^3_- - g^3_{0j1}f^j_- - \partial_3 f^0_- + g^0_{3j1}f^j_- = \Phi^3_-$
$J^0_+ = -\partial_1\Phi^1_+ + g^1_{1j2}\Phi^j_+ - \partial_2\Phi^2_+ + g^2_{2j2}\Phi^j_+ - \partial_3\Phi^3_+ + g^3_{3j2}\Phi^j_+ + \partial_0\Phi^0_+ - g^0_{0j2}\Phi^j_+$	$-\partial_1 f^1_+ + g^1_{1j1}f^j_+ - \partial_2 f^2_+ + g^2_{2j1}f^j_+ - \partial_3 f^3_+ + g^3_{3j1}f^j_+ + \partial_0 f^0_+ + g^0_{0j1}f^j_+ = \Phi^0_+$
$J^0_- = -\partial_1\Phi^1_- - g^1_{1j2}\Phi^j_- - \partial_2\Phi^2_- - g^2_{2j2}\Phi^j_- - \partial_3\Phi^3_- - g^3_{3j2}\Phi^j_- + \partial_0\Phi^0_- + g^0_{0j2}\Phi^j_-$	$-\partial_1 f^1_- - g^1_{1j1}f^j_- - \partial_2 f^2_- - g^2_{2j1}f^j_- - \partial_3 f^3_- - g^3_{3j1}f^j_- + \partial_0 f^0_- - g^0_{0j1}f^j_- = \Phi^0_-$

$$\Downarrow$$

$J^1_+ = -\partial_0\Phi^1_+ - \partial_1\Phi^0_+ + \partial_3\Phi^2_- - \partial_2\Phi^3_- + (-g^1_{0j2} - g^0_{1j2})\Phi^j_+ + (-g^2_{3j2} + g^3_{2j2})\Phi^j_-$	$-\partial_0 f^1_+ - \partial_1 f^0_+ - \partial_3 f^2_- + \partial_2 f^3_- + (+g^1_{0j1} - g^0_{1j1})f^j_+ + (+g^2_{3j1} - g^3_{2j1})f^j_- = \Phi^1_+$
$J^1_- = -\partial_0\Phi^1_- - \partial_1\Phi^0_- + \partial_3\Phi^2_+ - \partial_2\Phi^3_+ + (+g^2_{3j2} - g^3_{2j2})\Phi^j_+ + (+g^1_{0j2} + g^0_{1j2})\Phi^j_-$	$-\partial_0 f^1_- - \partial_1 f^0_- - \partial_3 f^2_+ + \partial_2 f^3_+ + (-g^2_{3j1} + g^3_{2j1})f^j_+ + (-g^1_{0j1} + g^0_{1j1})f^j_- = \Phi^1_-$
$J^2_+ = -\partial_0\Phi^2_+ - \partial_2\Phi^0_+ + \partial_3\Phi^1_- + \partial_1\Phi^3_- + (-g^2_{0j2} - g^0_{2j2})\Phi^j_+ + (-g^1_{3j2} - g^3_{1j2})\Phi^j_-$	$-\partial_0 f^2_+ - \partial_2 f^0_+ + \partial_3 f^1_- - \partial_1 f^3_- + (-g^0_{2j1} + g^2_{0j1})f^j_+ + (-g^1_{3j1} + g^3_{1j1})f^j_- = \Phi^2_+$
$J^2_- = -\partial_0\Phi^2_- - \partial_2\Phi^0_- + \partial_3\Phi^1_+ + \partial_1\Phi^3_+ + (+g^1_{3j2} + g^3_{1j2})\Phi^j_+ + (+g^2_{0j2} + g^0_{2j2})\Phi^j_-$	$-\partial_0 f^2_- - \partial_2 f^0_- + \partial_3 f^1_+ - \partial_1 f^3_+ + (+g^1_{3j1} - g^3_{1j1})f^j_+ + (-g^2_{0j1} + g^0_{2j1})f^j_- = \Phi^2_-$
$J^3_+ = -\partial_0\Phi^3_+ - \partial_3\Phi^0_+ + \partial_2\Phi^1_- - \partial_1\Phi^2_- + (-g^3_{0j2} - g^0_{3j2})\Phi^j_+ + (-g^1_{2j2} + g^2_{1j2})\Phi^j_-$	$-\partial_0 f^3_+ - \partial_3 f^0_+ - \partial_2 f^1_- + \partial_1 f^2_- + (-g^0_{3j1} + g^3_{0j1})f^j_+ + (+g^1_{2j1} - g^2_{1j1})f^j_- = \Phi^3_+$
$J^3_- = -\partial_0\Phi^3_- - \partial_3\Phi^0_- + \partial_2\Phi^1_+ - \partial_1\Phi^2_+ + (+g^1_{2j2} - g^2_{1j2})\Phi^j_+ + (+g^3_{0j2} + g^0_{3j2})\Phi^j_-$	$-\partial_0 f^3_- - \partial_3 f^0_- - \partial_2 f^1_+ + \partial_1 f^2_+ + (-g^1_{2j1} + g^2_{1j1})f^j_+ + (+g^0_{3j1} - g^3_{0j1})f^j_- = \Phi^3_-$
$J^0_+ = +\partial_0\Phi^0_+ - \partial_1\Phi^1_+ - \partial_2\Phi^2_+ - \partial_3\Phi^3_+ + (+g^1_{1j2} + g^2_{2j2} + g^3_{3j2} - g^0_{0j2})\Phi^j_+$	$+\partial_0 f^0_+ - \partial_1 f^1_+ - \partial_2 f^2_+ - \partial_3 f^3_+ + (+g^0_{0j1} + g^1_{1j1} + g^2_{2j1} + g^3_{3j1})f^j_+ = \Phi^0_+$
$J^0_- = +\partial_0\Phi^0_- - \partial_1\Phi^1_- - \partial_2\Phi^2_- - \partial_3\Phi^3_- + (-g^1_{1j2} - g^2_{2j2} - g^3_{3j2} + g^0_{0j2})\Phi^j_-$	$+\partial_0 f^0_- - \partial_1 f^1_- - \partial_2 f^2_- - \partial_3 f^3_- + (-g^0_{0j1} - g^1_{1j1} - g^2_{2j1} - g^3_{3j1})f^j_- = \Phi^0_-$

$$\Downarrow$$

$$\left(\begin{array}{c} J^1_+ \\ J^1_- \end{array}\right) = -\partial_0\left(\begin{array}{c} \Phi^1_+ \\ \Phi^1_- \end{array}\right) - \partial_1\left(\begin{array}{c} \Phi^0_+ \\ \Phi^0_- \end{array}\right) + \partial_3\left(\begin{array}{c} \Phi^2_+ \\ \Phi^2_- \end{array}\right) - \partial_2\left(\begin{array}{c} \Phi^3_+ \\ \Phi^3_- \end{array}\right) + \left(\begin{array}{cc} (-g^1_{0j2} - g^0_{1j2}) & (-g^2_{3j2} + g^3_{2j2}) \\ (+g^2_{3j2} - g^3_{2j2}) & (+g^1_{0j2} + g^0_{1j2}) \end{array}\right)\left(\begin{array}{c} \Phi^j_+ \\ \Phi^j_- \end{array}\right)$$

$$\left(\begin{array}{c} J^2_+ \\ J^2_- \end{array}\right) = -\partial_0\left(\begin{array}{c} \Phi^2_+ \\ \Phi^2_- \end{array}\right) - \partial_2\left(\begin{array}{c} \Phi^0_+ \\ \Phi^0_- \end{array}\right) + \partial_3\left(\begin{array}{c} \Phi^1_+ \\ \Phi^1_- \end{array}\right) + \partial_1\left(\begin{array}{c} \Phi^3_+ \\ \Phi^3_- \end{array}\right) + \left(\begin{array}{cc} (-g^2_{0j2} - g^0_{2j2}) & (-g^1_{3j2} - g^3_{1j2}) \\ (+g^1_{3j2} + g^3_{1j2}) & (+g^2_{0j2} + g^0_{2j2}) \end{array}\right)\left(\begin{array}{c} \Phi^j_+ \\ \Phi^j_- \end{array}\right)$$

$$\left(\begin{array}{c} J^3_+ \\ J^3_- \end{array}\right) = -\partial_0\left(\begin{array}{c} \Phi^3_+ \\ \Phi^3_- \end{array}\right) - \partial_3\left(\begin{array}{c} \Phi^0_+ \\ \Phi^0_- \end{array}\right) + \partial_2\left(\begin{array}{c} \Phi^1_+ \\ \Phi^1_- \end{array}\right) - \partial_1\left(\begin{array}{c} \Phi^2_+ \\ \Phi^2_- \end{array}\right) + \left(\begin{array}{cc} (-g^3_{0j2} - g^0_{3j2}) & (-g^1_{2j2} + g^2_{1j2}) \\ (+g^1_{2j2} - g^2_{1j2}) & (+g^3_{0j2} + g^0_{3j2}) \end{array}\right)\left(\begin{array}{c} \Phi^j_+ \\ \Phi^j_- \end{array}\right)$$

$$\left(\begin{array}{c} J^0_+ \\ J^0_- \end{array}\right) = +\partial_0\left(\begin{array}{c} \Phi^0_+ \\ \Phi^0_- \end{array}\right) - \partial_1\left(\begin{array}{c} \Phi^1_+ \\ \Phi^1_- \end{array}\right) - \partial_2\left(\begin{array}{c} \Phi^2_+ \\ \Phi^2_- \end{array}\right) - \partial_3\left(\begin{array}{c} \Phi^3_+ \\ \Phi^3_- \end{array}\right) + (+g^1_{1j2} + g^2_{2j2} + g^3_{3j2} - g^0_{0j2})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)\left(\begin{array}{c} \Phi^j_+ \\ \Phi^j_- \end{array}\right)$$

$$\Downarrow$$

$$\mathbf{J}^1 = \left(\begin{array}{c} J^1_+ \\ J^1_- \end{array}\right) = -\partial_0\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^0 + \partial_3\mathbf{\Phi}^2 - \partial_2\mathbf{\Phi}^3 + \left[(-g^1_{0j2} - g^0_{1j2})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (-g^2_{3j2} + g^3_{2j2})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{\Phi}^j$$

$$\mathbf{J}^2 = \left(\begin{array}{c} J^2_+ \\ J^2_- \end{array}\right) = -\partial_0\mathbf{\Phi}^2 - \partial_2\mathbf{\Phi}^0 + \partial_3\mathbf{\Phi}^1 + \partial_1\mathbf{\Phi}^3 + \left[(-g^2_{0j2} - g^0_{2j2})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (-g^1_{3j2} - g^3_{1j2})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{\Phi}^j$$

$$\mathbf{J}^3 = \left(\begin{array}{c} J^3_+ \\ J^3_- \end{array}\right) = -\partial_0\mathbf{\Phi}^3 - \partial_3\mathbf{\Phi}^0 + \partial_2\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^2 + \left[(-g^3_{0j2} - g^0_{3j2})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (-g^1_{2j2} + g^2_{1j2})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{\Phi}^j$$

$$\mathbf{J}^0 = \left(\begin{array}{c} J^0_+ \\ J^0_- \end{array}\right) = +\partial_0\mathbf{\Phi}^0 - \partial_1\mathbf{\Phi}^1 - \partial_2\mathbf{\Phi}^2 - \partial_3\mathbf{\Phi}^3 + (+g^1_{1j2} + g^2_{2j2} + g^3_{3j2} - g^0_{0j2})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)\mathbf{\Phi}^j$$

$$-\partial_0\mathbf{f}^1 - \partial_1\mathbf{f}^0 - \partial_3\mathbf{f}^2 + \partial_2\mathbf{f}^3 + \left[(+g^1_{0j1} - g^0_{1j1})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (+g^2_{3j1} - g^3_{2j1})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{f}^j = \left(\begin{array}{c} \Phi^1_+ \\ \Phi^1_- \end{array}\right) = \mathbf{\Phi}^1$$

$$-\partial_0\mathbf{f}^2 - \partial_2\mathbf{f}^0 + \partial_3\mathbf{f}^1 - \partial_1\mathbf{f}^3 + \left[(-g^0_{2j1} + g^2_{0j1})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (-g^1_{3j1} + g^3_{1j1})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{f}^j = \left(\begin{array}{c} \Phi^2_+ \\ \Phi^2_- \end{array}\right) = \mathbf{\Phi}^2$$

$$-\partial_0\mathbf{f}^3 - \partial_3\mathbf{f}^0 - \partial_2\mathbf{f}^1_- + \partial_1\mathbf{f}^2_- + \left[(-g^0_{3j1} + g^3_{0j1})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) + (+g^1_{2j1} - g^2_{1j1})\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)\right]\mathbf{f}^j = \left(\begin{array}{c} \Phi^3_+ \\ \Phi^3_- \end{array}\right) = \mathbf{\Phi}^3$$

$$-\partial_1\mathbf{f}^1 - \partial_2\mathbf{f}^2 - \partial_3\mathbf{f}^3 + \partial_0\mathbf{f}^0 + \left[(+g^0_{0j1} + g^1_{1j1} + g^2_{2j1} + g^3_{3j1})\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)\right]\mathbf{f}^j = \left(\begin{array}{c} \Phi^0_+ \\ \Phi^0_- \end{array}\right) = \mathbf{\Phi}^0$$

□

Corollary II.2 For differentiable functions $f^i_+, f^i_-, g^i_{jhk}; \forall i,j,h \in \{0,1,2,3\}$, $\forall k \in \{1,2\}$:

Given theorem II.1;

Whenever $g^i_{jhk} = \delta^i_h g_{jk}$ $; \forall i,j,h \in \{0,1,2,3\}$, $\forall k \in \{1,2\}$

$$
\left(\begin{array}{l}
\left(\begin{array}{l}
\left(\begin{array}{l}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1+\\
+(+g_{11}-g_{12})\partial_0 f_+^0+(-g_{01}+g_{02})\partial_0 f_+^1+(-g_{31}+g_{32})\partial_0 f_-^2+(+g_{21}-g_{22})\partial_0 f_-^3+\\
+(-g_{01}+g_{02})\partial_1 f_+^0+(-g_{11}+g_{12})\partial_1 f_+^1+(-g_{21}+g_{22})\partial_1 f_+^2+(-g_{31}+g_{32})\partial_1 f_+^3+\\
+(+g_{21}-g_{22})\partial_2 f_+^1+(-g_{11}+g_{12})\partial_2 f_+^2+(-g_{31}+g_{32})\partial_2 f_-^0+(+g_{01}-g_{02})\partial_2 f_-^3+\\
+(+g_{31}-g_{32})\partial_3 f_+^1+(-g_{11}+g_{12})\partial_3 f_+^3+(+g_{21}-g_{22})\partial_3 f_-^0+(-g_{01}+g_{02})\partial_3 f_-^2+\\
+(+\partial_0(g_{11})-\partial_1(g_{01})+g_{02}g_{11}-g_{12}g_{01})f_+^0+\\
+(-\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-g_{02}g_{01}-g_{12}g_{11}-g_{22}g_{21}-g_{32}g_{31})f_+^1+\\
+(-\partial_1(g_{21})-\partial_2(g_{11})-g_{12}g_{21}+g_{22}g_{11})f_+^2+(-\partial_1(g_{31})-\partial_3(g_{11})-g_{12}g_{31}+g_{32}g_{11})f_+^3+\\
+(-\partial_2(g_{31})+\partial_3(g_{21})+g_{22}g_{31}-g_{32}g_{21})f_-^0+(-\partial_0(g_{31})-\partial_3(g_{01})-g_{02}g_{31}+g_{32}g_{01})f_-^2+\\
+(+\partial_0(g_{21})+\partial_2(g_{01})+g_{02}g_{21}-g_{22}g_{01})f_-^3
\end{array}\right)\\
\left(\begin{array}{l}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1+\\
+(+g_{31}-g_{32})\partial_0 f_+^2+(-g_{21}+g_{22})\partial_0 f_+^3+(-g_{11}+g_{12})\partial_0 f_-^0+(+g_{01}-g_{02})\partial_0 f_-^1+\\
+(+g_{01}-g_{02})\partial_1 f_-^0+(+g_{11}-g_{12})\partial_1 f_-^1+(+g_{21}-g_{22})\partial_1 f_-^2+(+g_{31}-g_{32})\partial_1 f_-^3+\\
+(+g_{31}-g_{32})\partial_2 f_+^0+(-g_{01}+g_{02})\partial_2 f_+^3+(-g_{21}+g_{22})\partial_2 f_-^1+(+g_{11}-g_{12})\partial_2 f_-^2+\\
+(-g_{21}+g_{22})\partial_3 f_+^0+(+g_{01}-g_{02})\partial_3 f_+^2+(-g_{31}+g_{32})\partial_3 f_-^1+(+g_{11}-g_{12})\partial_3 f_-^3+\\
+(+\partial_2(g_{31})-\partial_3(g_{21})+g_{22}g_{31}-g_{32}g_{21})f_+^0+(+\partial_0(g_{31})+\partial_3(g_{01})-g_{02}g_{31}+g_{32}g_{01})f_+^2+\\
+(-\partial_0(g_{21})-\partial_2(g_{01})+g_{02}g_{21}-g_{22}g_{01})f_+^3+\\
+(-\partial_0(g_{11})+\partial_1(g_{01})+g_{02}g_{11}-g_{12}g_{01})f_-^0+\\
+(+\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})-\partial_3(g_{31})-g_{02}g_{01}-g_{12}g_{11}-g_{22}g_{21}-g_{32}g_{31})f_-^1+\\
+(+\partial_1(g_{21})+\partial_2(g_{11})-g_{12}g_{21}+g_{22}g_{11})f_-^2+(+\partial_{1\nu}(g_{31})+\partial_3(g_{11})-g_{12}g_{31}+g_{32}g_{11})f_-^3
\end{array}\right)
\end{array}\right)\\
\left(\begin{array}{l}
\left(\begin{array}{l}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2+\\
+(+g_{21}-g_{22})\partial_0 f_+^0+(-g_{01}+g_{02})\partial_0 f_+^2+(+g_{31}-g_{32})\partial_0 f_-^1+(-g_{11}+g_{12})\partial_0 f_-^3+\\
+(-g_{21}+g_{22})\partial_1 f_+^1+(+g_{11}-g_{12})\partial_1 f_+^2+(+g_{31}-g_{32})\partial_1 f_-^0+(-g_{01}+g_{02})\partial_1 f_-^3+\\
+(-g_{01}+g_{02})\partial_2 f_+^0+(-g_{11}+g_{12})\partial_2 f_+^1+(-g_{21}+g_{22})\partial_2 f_+^2+(-g_{31}+g_{32})\partial_2 f_+^3+\\
+(+g_{31}-g_{32})\partial_3 f_+^2+(-g_{21}+g_{22})\partial_3 f_+^3+(-g_{11}+g_{12})\partial_3 f_-^0+(+g_{01}-g_{02})\partial_3 f_-^1+\\
+(+\partial_0(g_{21})-\partial_2(g_{01})+g_{02}g_{21}-g_{22}g_{01})f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11})+g_{12}g_{21}-g_{22}g_{11})f_+^1+\\
+(-\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})+\partial_3(g_{31})-g_{02}g_{01}-g_{12}g_{11}-g_{22}g_{21}-g_{32}g_{31})f_+^2+\\
+(-\partial_3(g_{21})-\partial_2(g_{31})+g_{32}g_{21}-g_{22}g_{31})f_+^3+\\
+(+\partial_1(g_{31})-\partial_3(g_{11})-g_{12}g_{31}+g_{32}g_{11})f_-^0+(+\partial_0(g_{31})+\partial_3(g_{01})+g_{02}g_{31}-g_{32}g_{01})f_-^1+\\
+(-\partial_0(g_{11})-\partial_1(g_{01})-g_{02}g_{11}+g_{12}g_{01})f_-^3
\end{array}\right)\\
\left(\begin{array}{l}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2+\\
+(-g_{31}+g_{32})\partial_0 f_+^1+(+g_{11}-g_{12})\partial_0 f_+^3+(-g_{21}+g_{22})\partial_0 f_-^0+(+g_{01}-g_{02})\partial_0 f_-^2+\\
+(-g_{31}+g_{32})\partial_1 f_+^0+(+g_{01}-g_{02})\partial_1 f_+^3+(-g_{11}+g_{12})\partial_1 f_-^2+(+g_{21}-g_{22})\partial_1 f_-^1+\\
+(+g_{01}-g_{02})\partial_2 f_-^0+(+g_{11}-g_{12})\partial_2 f_-^1+(+g_{21}-g_{22})\partial_2 f_-^2+(+g_{31}-g_{32})\partial_2 f_-^3+\\
+(+g_{11}-g_{12})\partial_3 f_+^0+(-g_{01}+g_{02})\partial_3 f_+^1+(-g_{31}+g_{32})\partial_3 f_-^2+(+g_{21}-g_{22})\partial_3 f_-^3+\\
+(-\partial_1(g_{31})+\partial_3(g_{11})-g_{12}g_{31}+g_{32}g_{11})f_+^0+(-\partial_0(g_{31})-\partial_3(g_{01})+g_{02}g_{31}-g_{32}g_{01})f_+^1+\\
+(+\partial_0(g_{11})+\partial_1(g_{01})-g_{02}g_{11}+g_{12}g_{01})f_+^3+\\
+(-\partial_0(g_{21})+\partial_2(g_{01})+g_{02}g_{21}-g_{22}g_{01})f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11})+g_{12}g_{21}-g_{22}g_{11})f_-^1+\\
+(+\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-g_{02}g_{01}-g_{12}g_{11}-g_{22}g_{21}-g_{32}g_{31})f_-^2+\\
+(+\partial_2(g_{31})+\partial_3(g_{21})-g_{22}g_{31}+g_{32}g_{21})f_-^3
\end{array}\right)
\end{array}\right)\\
\Rightarrow \mathbf{J}=\left(\begin{array}{l}
\left(\begin{array}{l}
\left(\begin{array}{l}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3+\\
+(+g_{31}-g_{32})\partial_0 f_+^0+(-g_{01}+g_{02})\partial_0 f_+^3+(-g_{21}+g_{22})\partial_0 f_-^1+(+g_{11}-g_{12})\partial_0 f_-^2+\\
+(-g_{31}+g_{32})\partial_1 f_+^1+(+g_{11}-g_{12})\partial_1 f_+^3+(-g_{21}+g_{22})\partial_1 f_-^0+(+g_{01}-g_{02})\partial_1 f_-^2+\\
+(-g_{31}+g_{32})\partial_2 f_+^2+(+g_{21}-g_{22})\partial_2 f_+^3+(+g_{11}+g_{12})\partial_2 f_-^0+(-g_{01}-g_{02})\partial_2 f_-^1+\\
+(-g_{01}+g_{02})\partial_3 f_+^0+(-g_{11}+g_{12})\partial_3 f_+^1+(-g_{21}+g_{22})\partial_3 f_+^2+(-g_{31}+g_{32})\partial_3 f_+^3+\\
+(+\partial_0(g_{31})-\partial_3(g_{01})+g_{02}g_{31}-g_{32}g_{01})f_+^0+(-\partial_1(g_{31})-\partial_3(g_{11})+g_{12}g_{31}-g_{32}g_{11})f_+^1+\\
+(-\partial_2(g_{31})-\partial_3(g_{21})+g_{22}g_{31}-g_{32}g_{21})f_+^2+\\
+(-\partial_0(g_{01})+\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-g_{02}g_{01}-g_{12}g_{11}-g_{22}g_{21}-g_{32}g_{31})f_+^3+\\
+(-\partial_1(g_{21})+\partial_2(g_{11})+g_{12}g_{21}-g_{22}g_{11})f_-^0+(-\partial_0(g_{21})-\partial_2(g_{01})-g_{02}g_{21}+g_{22}g_{01})f_-^1+\\
+(+\partial_0(g_{11})+\partial_1(g_{01})+g_{02}g_{11}-g_{12}g_{01})f_-^2
\end{array}\right)\\
\left(\begin{array}{l}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3+\\
+(+g_{21}-g_{22})\partial_0 f_+^1+(-g_{11}+g_{12})\partial_0 f_+^2+(-g_{31}+g_{32})\partial_0 f_-^0+(+g_{01}-g_{02})\partial_0 f_-^3+\\
+(+g_{21}-g_{22})\partial_1 f_+^0+(-g_{01}+g_{02})\partial_1 f_+^2+(+g_{31}-g_{32})\partial_1 f_-^1+(-g_{11}+g_{12})\partial_1 f_-^3+\\
+(-g_{11}+g_{12})\partial_2 f_+^0+(+g_{01}-g_{02})\partial_2 f_+^1+(+g_{31}-g_{32})\partial_2 f_-^2+(-g_{21}+g_{22})\partial_2 f_-^3+
\end{array}\right)
\end{array}\right)
\end{array}\right)
$$

Proof:

Given theorem II.1;
Whenever $g^{i}_{jhk} = \delta^{i}_{h} g_{jk} \quad ; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$

$$
\mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
(\partial_0^2 + \partial_3^2 + \partial_2^2 + \partial_1^2) f_+^1 + \\
+\partial_0([+\delta_h^0 g_{11} - \delta_h^1 g_{01}] f_+^h + [-\delta_h^2 g_{31} + \delta_h^3 g_{21}] f_-^h) + \\
+\partial_1([-\delta_h^0 g_{01} - \delta_h^1 g_{11} - \delta_h^2 g_{21} - \delta_h^3 g_{31}] f_+^h) + \\
+\partial_2([+\delta_h^1 g_{21} - \delta_h^2 g_{11}] f_+^h + [-\delta_h^0 g_{31} + \delta_h^3 g_{01}] f_-^h) + \\
+\partial_3([-\delta_h^3 g_{11} + \delta_h^1 g_{31}] f_+^h + [-\delta_h^2 g_{01} + \delta_h^0 g_{21}] f_-^h) + \\
+(-\delta_h^0 g_{12} + \delta_h^1 g_{02}) \partial_0 f_+^h + (+\delta_h^2 g_{32} - \delta_h^3 g_{22}) \partial_0 f_-^h + \\
+(+\delta_h^0 g_{02} + \delta_h^1 g_{12} + \delta_h^2 g_{22} + \delta_h^3 g_{32}) \partial_1 f_+^h + \\
+(-\delta_h^1 g_{22} + \delta_h^2 g_{12}) \partial_2 f_+^h + (+\delta_h^0 g_{32} - \delta_h^3 g_{02}) \partial_2 f_-^h + \\
+(-\delta_h^1 g_{32} + \delta_h^3 g_{12}) \partial_3 f_+^h + (-\delta_h^0 g_{22} + \delta_h^2 g_{02}) \partial_3 f_-^h + \\
+([-\delta_k^0 g_{12} - \delta_k^1 g_{02}] \delta_h^k g_{01} + [+\delta_k^0 g_{02} - \delta_k^1 g_{12} + \delta_k^2 g_{22} + \delta_k^3 g_{32}] \delta_h^k g_{11} + [-\delta_k^1 g_{22} - \delta_k^2 g_{12}] \delta_h^k g_{21} + [-\delta_k^1 g_{32} - \delta_k^3 g_{12}] \delta_h^k g_{31}) f_+^h + \\
+([+\delta_k^2 g_{32} - \delta_k^3 g_{22}] \delta_h^k g_{01} + [-\delta_k^0 g_{32} + \delta_k^3 g_{02}] \delta_h^k g_{21} + [+\delta_k^0 g_{22} - \delta_k^2 g_{02}] \delta_h^k g_{31}) f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2 + \partial_3^2 + \partial_2^2 + \partial_1^2) f_-^1 + \\
+\partial_0([+\delta_h^2 g_{31} - \delta_h^3 g_{21}] f_+^h + [-\delta_h^0 g_{11} + \delta_h^1 g_{01}] f_-^h) + \\
+\partial_1([+\delta_h^0 g_{01} + \delta_h^1 g_{11} + \delta_h^2 g_{21} + \delta_h^3 g_{31}] f_-^h) + \\
+\partial_2([+\delta_h^0 g_{31} - \delta_h^3 g_{01}] f_+^h + [-\delta_h^1 g_{21} + \delta_h^2 g_{11}] f_-^h) + \\
+\partial_3([-\delta_h^0 g_{21} + \delta_h^2 g_{01}] f_+^h + [-\delta_h^1 g_{31} + \delta_h^3 g_{11}] f_-^h) + \\
+(-\delta_h^2 g_{32} + \delta_h^3 g_{22}) \partial_0 f_+^h + (+\delta_h^0 g_{12} - \delta_h^1 g_{02}) \partial_0 f_-^h + \\
+(-\delta_h^0 g_{02} - \delta_h^1 g_{12} - \delta_h^2 g_{22} - \delta_h^3 g_{32}) \partial_1 f_-^h + \\
+(-\delta_h^0 g_{32} + \delta_h^3 g_{02}) \partial_2 f_+^h + (+\delta_h^1 g_{22} - \delta_h^2 g_{12}) \partial_2 f_-^h + \\
+(+\delta_h^0 g_{22} - \delta_h^2 g_{02}) \partial_3 f_+^h + (+\delta_h^1 g_{32} - \delta_h^3 g_{12}) \partial_3 f_-^h + \\
+([+\delta_k^2 g_{32} - \delta_k^3 g_{22}] \delta_h^k g_{01} + [-\delta_k^0 g_{32} + \delta_k^3 g_{02}] \delta_h^k g_{21} + [+\delta_k^0 g_{22} - \delta_k^2 g_{02}] \delta_h^k g_{31}) f_+^h + \\
+([-\delta_k^0 g_{12} - \delta_k^1 g_{02}] \delta_h^k g_{01} + [+\delta_k^0 g_{02} - \delta_k^1 g_{12} + \delta_k^2 g_{22} + \delta_k^3 g_{32}] \delta_h^k g_{11} + [-\delta_k^1 g_{22} - \delta_k^2 g_{12}] \delta_h^k g_{21} + [-\delta_k^1 g_{32} - \delta_k^3 g_{12}] \delta_h^k g_{31}) f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2 + \partial_0^2 + \partial_1^2 + \partial_2^2) f_+^2 + \\
+\partial_0([+\delta_h^0 g_{21} - \delta_h^2 g_{01}] f_+^h + [+\delta_h^1 g_{31} - \delta_h^3 g_{11}] f_-^h) + \\
+\partial_1([-\delta_h^1 g_{21} + \delta_h^2 g_{11}] f_+^h + [+\delta_h^0 g_{31} - \delta_h^3 g_{01}] f_-^h) + \\
+\partial_2([-\delta_h^0 g_{01} - \delta_h^1 g_{11} - \delta_h^2 g_{21} - \delta_h^3 g_{31}] f_+^h) + \\
+\partial_3([+\delta_h^2 g_{31} - \delta_h^3 g_{21}] f_+^h + [-\delta_h^0 g_{11} + \delta_h^1 g_{01}] f_-^h) + \\
+(-\delta_h^0 g_{22} + \delta_h^2 g_{02}) \partial_0 f_+^h + (-\delta_h^1 g_{32} + \delta_h^3 g_{12}) \partial_0 f_-^h + \\
+[+\delta_h^1 g_{22} - \delta_h^2 g_{12}] \partial_1 f_+^h + (-\delta_h^0 g_{32} + \delta_h^3 g_{02}) \partial_1 f_-^h + \\
+(+\delta_h^0 g_{02} + \delta_h^1 g_{12} + \delta_h^2 g_{22} + \delta_h^3 g_{32}) \partial_2 f_+^h + \\
+(-\delta_h^2 g_{32} + \delta_h^3 g_{22}) \partial_3 f_+^h + (+\delta_h^0 g_{12} - \delta_h^1 g_{02}) \partial_3 f_-^h + \\
+([-\delta_k^2 g_{02} - \delta_k^0 g_{22}] \delta_h^k g_{01} + [-\delta_k^1 g_{22} - \delta_k^2 g_{12}] \delta_h^k g_{11} + [+\delta_k^0 g_{02} + \delta_k^1 g_{12} - \delta_k^2 g_{22} + \delta_k^3 g_{32}] \delta_h^k g_{21} + [-\delta_k^2 g_{32} - \delta_k^3 g_{22}] \delta_h^k g_{31}) f_+^h + \\
+([-\delta_k^1 g_{32} + \delta_k^3 g_{12}] \delta_h^k g_{01} + [+\delta_k^0 g_{32} - \delta_k^3 g_{02}] \delta_h^k g_{11} + [-\delta_k^0 g_{12} + \delta_k^1 g_{02}] \delta_h^k g_{31}) f_-^h + \\
\begin{pmatrix}
(\partial_3^2 + \partial_0^2 + \partial_1 + \partial_2^2) f_-^2 + \\
+\partial_0([-\delta_h^1 g_{31} + \delta_h^3 g_{11}] f_+^h + [-\delta_h^0 g_{21} + \delta_h^2 g_{01}] f_-^h) + \\
+\partial_1([-\delta_h^0 g_{31} + \delta_h^3 g_{01}] f_+^h + [+\delta_h^1 g_{21} - \delta_h^2 g_{11}] f_-^h) + \\
+\partial_3([+\delta_h^0 g_{11} - \delta_h^1 g_{01}] f_+^h + [-\delta_h^2 g_{31} + \delta_h^3 g_{21}] f_-^h) + \\
+\partial_2([+\delta_h^0 g_{01} + \delta_h^1 g_{11} + \delta_h^2 g_{21} + \delta_h^3 g_{31}] f_-^h) + \\
(+\delta_h^1 g_{32} - \delta_h^3 g_{12}) \partial_0 f_+^h + (+\delta_h^0 g_{22} - \delta_h^2 g_{02}) \partial_0 f_-^h + \\
+(\delta_h^0 g_{32} - \delta_h^3 g_{02}) \partial_1 f_+^h + (\delta_h^2 g_{12} - \delta_h^1 g_{22}) \partial_1 f_-^1 + \\
+(-\delta_h^0 g_{02} - \delta_h^1 g_{12} - \delta_h^2 g_{22} - \delta_h^3 g_{32}) \partial_2 f_-^h + \\
+(-\delta_h^0 g_{12} + \delta_h^1 g_{02}) \partial_3 f_+^h + (+\delta_h^2 g_{32} - \delta_h^3 g_{22}) \partial_3 f_-^h + \\
+([-\delta_k^1 g_{32} + \delta_k^3 g_{12}] \delta_h^k g_{01} + [+\delta_k^0 g_{32} - \delta_k^3 g_{02}] \delta_h^k g_{11} + [-\delta_k^0 g_{12} + \delta_k^1 g_{02}] \delta_h^k g_{31}) f_+^h + \\
+([-\delta_k^0 g_{22} - \delta_k^2 g_{02}] \delta_h^k g_{01} + [-\delta_k^1 g_{22} - \delta_k^2 g_{12}] \delta_h^k g_{11} + [+\delta_k^0 g_{02} + \delta_k^1 g_{12} - \delta_k^2 g_{22} + \delta_k^3 g_{32}] \delta_h^k g_{21} + [-\delta_k^2 g_{32} - \delta_k^3 g_{22}] \delta_h^k g_{31}) f_-^h
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2 + \partial_1^2 + \partial_0^2 + \partial_3) f_+^3 + \\
+\partial_0([+\delta_h^0 g_{31} - \delta_h^3 g_{01}] f_+^h + [-\delta_h^1 g_{21} + \delta_h^2 g_{11}] f_-^h) + \\
+\partial_1([-\delta_h^1 g_{31} + \delta_h^3 g_{11}] f_+^h + [-\delta_h^0 g_{21} + \delta_h^2 g_{01}] f_-^h) + \\
+\partial_2([-\delta_h^2 g_{31} + \delta_h^3 g_{21}] f_+^h + [+\delta_h^0 g_{11} - \delta_h^1 g_{01}] f_-^h) + \\
+\partial_3([-\delta_h^0 g_{01} - \delta_h^1 g_{11} - \delta_h^2 g_{21} - \delta_h^3 g_{31}] f_+^h) + \\
+(-\delta_h^0 g_{32} + \delta_h^3 g_{02}) \partial_0 f_+^h + (+\delta_h^1 g_{22} - \delta_h^2 g_{12}) \partial_0 f_-^h + \\
+(+\delta_h^1 g_{32} - \delta_h^3 g_{12}) \partial_1 f_+^h + (+\delta_h^0 g_{22} - \delta_h^2 g_{02}) \partial_1 f_-^h + \\
+(+\delta_h^2 g_{32} - \delta_h^3 g_{22}) \partial_2 f_+^h + (+\delta_h^0 g_{12} - \delta_h^1 g_{02}) \partial_2 f_-^h + \\
+(+\delta_h^0 g_{02} + \delta_h^1 g_{12} + \delta_h^2 g_{22} + \delta_h^3 g_{32}) \partial_3 f_+^h + \\
\end{pmatrix}
\end{pmatrix}
$$

$$
\mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+\partial_0(g_{11})f_+^0 + g_{11}\partial_0 f_+^0 - \partial_0(g_{01})f_+^1 - g_{01}\partial_0 f_+^1 - \partial_0(g_{31})f_-^2 - g_{31}\partial_0 f_-^2 + \partial_0(g_{21})f_-^3 + g_{21}\partial_0 f_-^3 + \\
-\partial_1(g_{01})f_+^0 - g_{01}\partial_1 f_+^0 - \partial_1(g_{11})f_+^1 - g_{11}\partial_1 f_+^1 - \partial_1(g_{21})f_+^2 - g_{21}\partial_1 f_+^2 - \partial_1(g_{31})f_+^3 - g_{31}\partial_1 f_+^3 + \\
+\partial_2(g_{21})f_+^1 + g_{21}\partial_2 f_+^1 - \partial_2(g_{11})f_+^2 - g_{11}\partial_2 f_+^2 - \partial_2(g_{31})f_-^0 - g_{31}\partial_2 f_-^0 + \partial_2(g_{01})f_-^3 + g_{01}\partial_2 f_-^3 + \\
-\partial_3(g_{11})f_+^3 - g_{11}\partial_3 f_+^3 + \partial_3(g_{31})f_+^1 + g_{31}\partial_3 f_+^1 - \partial_3(g_{01})f_-^2 - g_{01}\partial_3 f_-^2 + \partial_3(g_{21})f_-^0 + g_{21}\partial_3 f_-^0 + \\
-g_{12}\partial_0 f_+^0 + g_{02}\partial_0 f_+^1 + g_{32}\partial_0 f_-^2 - g_{22}\partial_0 f_-^3 + \\
+g_{02}\partial_1 f_+^0 + g_{12}\partial_1 f_+^1 + g_{22}\partial_1 f_+^2 + g_{32}\partial_1 f_+^3 + \\
-g_{22}\partial_2 f_+^1 + g_{12}\partial_2 f_+^2 + g_{32}\partial_2 f_-^0 - g_{02}\partial_2 f_-^3 + \\
-g_{32}\partial_3 f_+^1 + g_{12}\partial_3 f_+^3 - g_{22}\partial_3 f_-^0 + g_{02}\partial_3 f_-^2 + \\
-g_{12}g_{01}f_+^0 - g_{02}g_{01}f_+^1 + g_{02}g_{11}f_+^0 - g_{12}g_{11}f_+^1 + g_{22}g_{11}f_+^2 + g_{32}g_{11}f_+^3 - g_{22}g_{21}f_+^1 - g_{12}g_{21}f_+^2 - g_{32}g_{31}f_+^1 - g_{12}g_{31}f_+^3 + \\
+g_{32}g_{01}f_-^2 - g_{22}g_{01}f_-^3 - g_{32}g_{21}f_-^0 + g_{02}g_{21}f_-^3 + g_{22}g_{31}f_-^0 - g_{02}g_{31}f_-^2
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+\partial_0(g_{31})f_+^2 + g_{31}\partial_0 f_+^2 - \partial_0(g_{21})f_+^3 - g_{21}\partial_0 f_+^3 - \partial_0(g_{11})f_-^0 - g_{11}\partial_0 f_-^0 + \partial_0(g_{01})f_-^1 + g_{01}\partial_0 f_-^1 + \\
+\partial_1(g_{01})f_-^0 + g_{01}\partial_1 f_-^0 + \partial_1(g_{11})f_-^1 + g_{11}\partial_1 f_-^1 + \partial_1(g_{21})f_-^2 + g_{21}\partial_1 f_-^2 + \partial_{1v}(g_{31})f_-^3 + g_{31}\partial_1 f_-^3 + \\
+\partial_2(g_{31})f_+^0 + g_{31}\partial_2 f_+^0 - \partial_2(g_{01})f_+^3 - g_{01}\partial_2 f_+^3 - \partial_2(g_{21})f_-^1 - g_{21}\partial_2 f_-^1 + \partial_2(g_{11})f_-^2 + g_{11}\partial_2 f_-^2 + \\
-\partial_3(g_{21})f_+^0 - g_{21}\partial_3 f_+^0 + \partial_3(g_{01})f_+^2 + g_{01}\partial_3 f_+^2 - \partial_3(g_{31})f_-^1 - g_{31}\partial_3 f_-^1 + \partial_3(g_{11})f_-^3 + g_{11}\partial_3 f_-^3 + \\
-g_{32}\partial_0 f_+^2 + g_{22}\partial_0 f_+^3 + g_{12}\partial_0 f_-^0 - g_{02}\partial_0 f_-^1 + \\
-g_{02}\partial_1 f_-^0 - g_{12}\partial_1 f_-^1 - g_{22}\partial_1 f_-^2 - g_{32}\partial_1 f_-^3 + \\
-g_{32}\partial_2 f_+^0 + g_{02}\partial_2 f_+^3 + g_{22}\partial_2 f_-^1 - g_{12}\partial_2 f_-^2 + \\
+g_{22}\partial_3 f_+^0 - g_{02}\partial_3 f_+^2 + g_{32}\partial_3 f_-^1 - g_{12}\partial_3 f_-^3 + \\
+g_{32}g_{01}f_+^2 - g_{22}g_{01}f_+^3 - g_{32}g_{21}f_+^0 + g_{02}g_{21}f_+^3 + g_{22}g_{31}f_+^0 - g_{02}g_{31}f_+^2 + \\
-g_{12}g_{01}f_-^0 - g_{02}g_{01}f_-^1 + g_{02}g_{11}f_-^0 - g_{12}g_{11}f_-^1 + g_{22}g_{11}f_-^2 + g_{32}g_{11}f_-^3 - g_{22}g_{21}f_-^1 - g_{12}g_{21}f_-^2 - g_{32}g_{31}f_-^1 - g_{12}g_{31}f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+\partial_0(g_{21})f_+^0 + g_{21}\partial_0 f_+^0 - \partial_0(g_{01})f_+^2 - g_{01}\partial_0 f_+^2 + \partial_0(g_{31})f_-^1 + g_{31}\partial_0 f_-^1 - \partial_0(g_{11})f_-^3 - g_{11}\partial_0 f_-^3 + \\
-\partial_1(g_{21})f_+^1 - g_{21}\partial_1 f_+^1 + \partial_1(g_{11})f_+^2 + g_{11}\partial_1 f_+^2 + \partial_1(g_{31})f_-^0 + g_{31}\partial_1 f_-^0 - \partial_1(g_{01})f_-^3 - g_{01}\partial_1 f_-^3 + \\
-\partial_2(g_{01})f_+^0 - g_{01}\partial_2 f_+^0 - \partial_2(g_{11})f_+^1 - g_{11}\partial_2 f_+^1 - \partial_2(g_{21})f_+^2 - g_{21}\partial_2 f_+^2 - \partial_2(g_{31})f_+^3 - g_{31}\partial_2 f_+^3 + \\
+\partial_3(g_{31})f_+^2 + g_{31}\partial_3 f_+^2 - \partial_3(g_{21})f_+^3 - g_{21}\partial_3 f_+^3 - \partial_3(g_{11})f_-^0 - g_{11}\partial_3 f_-^0 + \partial_3(g_{01})f_-^1 + g_{01}\partial_3 f_-^1 + \\
-g_{22}\partial_0 f_+^0 + g_{02}\partial_0 f_+^2 - g_{32}\partial_0 f_-^1 + g_{12}\partial_0 f_-^3 + \\
+g_{22}\partial_1 f_+^1 - g_{12}\partial_1 f_+^2 - g_{32}\partial_1 f_-^0 + g_{02}\partial_1 f_-^3 + \\
+g_{02}\partial_2 f_+^0 + g_{12}\partial_2 f_+^1 + g_{22}\partial_2 f_+^2 + g_{32}\partial_2 f_+^3 + \\
-g_{32}\partial_3 f_+^2 + g_{22}\partial_3 f_+^3 + g_{12}\partial_3 f_-^0 - g_{02}\partial_3 f_-^1 + \\
-g_{02}g_{01}f_+^2 - g_{22}g_{01}f_+^0 - g_{22}g_{11}f_+^1 - g_{12}g_{11}f_+^2 + g_{02}g_{21}f_+^0 + g_{12}g_{21}f_+^1 - g_{22}g_{21}f_+^2 + g_{32}g_{21}f_+^3 - g_{32}g_{31}f_+^2 - g_{22}g_{31}f_+^3 + \\
-g_{32}g_{01}f_-^1 + g_{12}g_{01}f_-^3 + g_{32}g_{11}f_-^0 - g_{02}g_{11}f_-^3 - g_{12}g_{31}f_-^0 + \delta_k^1 g_{02}g_{31}f_-^1
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
-\partial_0(g_{31})f_+^1 - g_{31}\partial_0 f_+^1 + \partial_0(g_{11})f_+^3 + g_{11}\partial_0 f_+^3 - \partial_0(g_{21})f_-^0 - g_{21}\partial_0 f_-^0 + \partial_0(g_{01})f_-^2 + g_{01}\partial_0 f_-^2 + \\
-\partial_1(g_{31})f_+^0 - g_{31}\partial_1 f_+^0 + \partial_1(g_{01})f_+^3 + g_{01}\partial_1 f_+^3 + \partial_1(g_{21})f_-^1 + g_{21}\partial_1 f_-^1 - \partial_1(g_{11})f_-^2 - g_{11}\partial_1 f_-^2 + \\
+\partial_2(g_{01})f_-^0 + g_{01}\partial_2 f_-^0 + \partial_2(g_{11})f_-^1 + g_{11}\partial_2 f_-^1 + \partial_2(g_{21})f_-^2 + g_{21}\partial_2 f_-^2 + \partial_2(g_{31})f_-^3 + g_{31}\partial_2 f_-^3 + \\
+\partial_3(g_{11})f_+^0 + g_{11}\partial_3 f_+^0 - \partial_3(g_{01})f_+^1 - g_{01}\partial_3 f_+^1 - \partial_3(g_{31})f_-^2 - g_{31}\partial_3 f_-^2 + \partial_3(g_{21})f_-^3 + g_{21}\partial_3 f_-^3 + \\
+g_{32}\partial_0 f_+^1 - g_{12}\partial_0 f_+^3 + g_{22}\partial_0 f_-^0 - g_{02}\partial_0 f_-^2 + \\
+g_{32}\partial_1 f_+^0 - g_{02}\partial_1 f_+^3 + g_{12}\partial_1 f_-^2 - g_{22}\partial_1 f_-^1 + \\
-g_{02}\partial_2 f_-^0 - g_{12}\partial_2 f_-^1 - g_{22}\partial_2 f_-^2 - g_{32}\partial_2 f_-^3 + \\
-g_{12}\partial_3 f_+^0 + g_{02}\partial_3 f_+^1 + g_{32}\partial_3 f_-^2 - g_{22}\partial_3 f_-^3 + \\
-g_{32}g_{01}f_+^1 + g_{12}g_{01}f_+^3 + g_{32}g_{11}f_+^0 - g_{02}g_{11}f_+^3 - g_{12}g_{31}f_+^0 + g_{02}g_{31}f_+^1 + \\
-g_{22}g_{01}f_-^0 - g_{02}g_{01}f_-^2 - g_{22}g_{11}f_-^1 - g_{12}g_{11}f_-^2 + g_{02}g_{21}f_-^0 + g_{12}g_{21}f_-^1 - g_{22}g_{21}f_-^2 + g_{32}g_{21}f_-^3 - g_{32}g_{31}f_-^2 - g_{22}g_{31}f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+\partial_0(g_{31})f_+^0 + g_{31}\partial_0 f_+^0 - \partial_0(g_{01})f_+^3 - g_{01}\partial_0 f_+^3 - \partial_0(g_{21})f_-^1 - g_{21}\partial_0 f_-^1 + \partial_0(g_{11})f_-^2 + g_{11}\partial_0 f_-^2 + \\
-\partial_1(g_{31})f_+^1 - g_{31}\partial_1 f_+^1 + \partial_1(g_{11})f_+^3 + g_{11}\partial_1 f_+^3 - \partial_1(g_{21})f_-^0 - g_{21}\partial_1 f_-^0 + \partial_1(g_{01})f_-^2 + g_{01}\partial_1 f_-^2 + \\
-\partial_2(g_{31})f_+^2 - g_{31}\partial_2 f_+^2 + \partial_2(g_{21})f_+^3 + g_{21}\partial_2 f_+^3 + \partial_2(g_{11})f_-^0 + g_{11}\partial_2 f_-^0 - \partial_2(g_{01})f_-^1 - g_{01}\partial_2 f_-^1 + \\
-\partial_3(g_{01})f_+^0 - g_{01}\partial_3 f_+^0 - \partial_3(g_{11})f_+^1 - g_{11}\partial_3 f_+^1 - \partial_3(g_{21})f_+^2 - g_{21}\partial_3 f_+^2 - \partial_3(g_{31})f_+^3 - g_{31}\partial_3 f_+^3 + \\
-g_{32}\partial_0 f_+^0 + g_{02}\partial_0 f_+^3 + g_{22}\partial_0 f_-^1 - g_{12}\partial_0 f_-^2 + \\
+g_{32}\partial_1 f_+^1 - g_{12}\partial_1 f_+^3 + g_{22}\partial_1 f_-^0 - g_{02}\partial_1 f_-^2 + \\
+g_{32}\partial_2 f_+^2 - g_{22}\partial_2 f_+^3 + g_{12}\partial_2 f_-^0 - g_{02}\partial_2 f_-^1 + \\
+g_{02}\partial_3 f_+^0 + g_{12}\partial_3 f_+^1 + g_{22}\partial_3 f_+^2 + g_{32}\partial_3 f_+^3 + \\
-g_{02}g_{01}f_+^3 - g_{32}g_{01}f_+^0 - g_{32}g_{11}f_+^1 - g_{12}g_{11}f_+^3 - g_{32}g_{21}f_+^2 - g_{22}g_{21}f_+^3 + g_{02}g_{31}f_+^0 + g_{12}g_{31}f_+^1 + g_{22}g_{31}f_+^2 - g_{32}g_{31}f_+^3 + \\
+g_{32}g_{01}f_-^1 - g_{12}g_{01}f_-^2 - g_{32}g_{11}f_-^0 + g_{02}g_{11}f_-^2 + g_{12}g_{21}f_-^0 - g_{02}g_{21}f_-^1
\end{pmatrix}
\end{pmatrix}
$$

$$
\mathbf{J} = \left(\begin{array}{c}
\left(\begin{array}{c}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+(+g_{11}-g_{12})\partial_0 f_+^0 + (-g_{01}+g_{02})\partial_0 f_+^1 + (-g_{31}+g_{32})\partial_0 f_-^2 + (+g_{21}-g_{22})\partial_0 f_-^3 + \\
+(-g_{01}+g_{02})\partial_1 f_+^0 + (-g_{11}+g_{12})\partial_1 f_+^1 + (-g_{21}+g_{22})\partial_1 f_+^2 + (-g_{31}+g_{32})\partial_1 f_+^3 + \\
+(+g_{21}-g_{22})\partial_2 f_+^1 + (-g_{11}+g_{12})\partial_2 f_+^2 + (-g_{31}+g_{32})\partial_2 f_-^0 + (+g_{01}-g_{02})\partial_2 f_-^3 + \\
+(+g_{31}-g_{32})\partial_3 f_+^1 + (-g_{11}+g_{12})\partial_3 f_+^3 + (+g_{21}-g_{22})\partial_3 f_-^0 + (-g_{01}+g_{02})\partial_3 f_-^2 + \\
+(+\partial_0(g_{11}) - \partial_1(g_{01}) + g_{02}g_{11} - g_{12}g_{01})f_+^0 + \\
+(-\partial_0(g_{01}) - \partial_1(g_{11}) + \partial_2(g_{21}) + \partial_3(g_{31}) - g_{02}g_{01} - g_{12}g_{11} - g_{22}g_{21} - g_{32}g_{31})f_+^1 + \\
+(-\partial_1(g_{21}) - \partial_2(g_{11}) - g_{12}g_{21} + g_{22}g_{11})f_+^2 + (-\partial_1(g_{31}) - \partial_3(g_{11}) - g_{12}g_{31} + g_{32}g_{11})f_+^3 + \\
+(-\partial_2(g_{31}) + \partial_3(g_{21}) + g_{22}g_{31} - g_{32}g_{21})f_-^0 + (-\partial_0(g_{31}) - \partial_3(g_{01}) - g_{02}g_{31} + g_{32}g_{01})f_-^2 + \\
+(+\partial_0(g_{21}) + \partial_2(g_{01}) + g_{02}g_{21} - g_{22}g_{01})f_-^3
\end{array}\right) \\
\left(\begin{array}{c}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+(+g_{31}-g_{32})\partial_0 f_+^2 + (-g_{21}+g_{22})\partial_0 f_+^3 + (-g_{11}+g_{12})\partial_0 f_-^0 + (+g_{01}-g_{02})\partial_0 f_-^1 + \\
+(+g_{01}-g_{02})\partial_1 f_-^0 + (+g_{11}-g_{12})\partial_1 f_-^1 + (+g_{21}-g_{22})\partial_1 f_-^2 + (+g_{31}-g_{32})\partial_1 f_-^3 + \\
+(+g_{31}-g_{32})\partial_2 f_+^0 + (-g_{01}+g_{02})\partial_2 f_+^3 + (-g_{21}+g_{22})\partial_2 f_-^1 + (+g_{11}-g_{12})\partial_2 f_-^2 + \\
+(-g_{21}+g_{22})\partial_3 f_+^0 + (+g_{01}-g_{02})\partial_3 f_+^2 + (-g_{31}+g_{32})\partial_3 f_-^1 + (+g_{11}-g_{12})\partial_3 f_-^3 + \\
+(+\partial_2(g_{31}) - \partial_3(g_{21}) + g_{22}g_{31} - g_{32}g_{21})f_+^0 + (+\partial_0(g_{31}) + \partial_3(g_{01}) - g_{02}g_{31} + g_{32}g_{01})f_+^2 + \\
+(-\partial_0(g_{21}) - \partial_2(g_{01}) + g_{02}g_{21} - g_{22}g_{01})f_+^3 + \\
+(-\partial_0(g_{11}) + \partial_1(g_{01}) + g_{02}g_{11} - g_{12}g_{01})f_-^0 + \\
+(+\partial_0(g_{01}) + \partial_1(g_{11}) - \partial_2(g_{21}) - \partial_3(g_{31}) - g_{02}g_{01} - g_{12}g_{11} - g_{22}g_{21} - g_{32}g_{31})f_-^1 + \\
+(+\partial_1(g_{21}) + \partial_2(g_{11}) - g_{12}g_{21} + g_{22}g_{11})f_-^2 + (+\partial_{1v}(g_{31}) + \partial_3(g_{11}) - g_{12}g_{31} + g_{32}g_{11})f_-^3
\end{array}\right)
\end{array}\right.
$$

$$
\left.\begin{array}{c}
\left(\begin{array}{c}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+(+g_{21}-g_{22})\partial_0 f_+^0 + (-g_{01}+g_{02})\partial_0 f_+^2 + (+g_{31}-g_{32})\partial_0 f_-^1 + (-g_{11}+g_{12})\partial_0 f_-^3 + \\
+(-g_{21}+g_{22})\partial_1 f_+^1 + (+g_{11}-g_{12})\partial_1 f_+^2 + (+g_{31}-g_{32})\partial_1 f_-^0 + (-g_{01}+g_{02})\partial_1 f_-^3 + \\
+(-g_{01}+g_{02})\partial_2 f_+^0 + (-g_{11}+g_{12})\partial_2 f_+^1 + (-g_{21}+g_{22})\partial_2 f_+^2 + (-g_{31}+g_{32})\partial_2 f_+^3 + \\
+(+g_{31}-g_{32})\partial_3 f_+^2 + (-g_{21}+g_{22})\partial_3 f_+^3 + (-g_{11}+g_{12})\partial_3 f_-^0 + (+g_{01}-g_{02})\partial_3 f_-^1 + \\
+(+\partial_0(g_{21}) - \partial_2(g_{01}) + g_{02}g_{21} - g_{22}g_{01})f_+^0 + (-\partial_1(g_{21}) - \partial_2(g_{11}) + g_{12}g_{21} - g_{22}g_{11})f_+^1 + \\
+(-\partial_0(g_{01}) + \partial_1(g_{11}) - \partial_2(g_{21}) + \partial_3(g_{31}) - g_{02}g_{01} - g_{12}g_{11} - g_{22}g_{21} - g_{32}g_{31})f_+^2 + \\
+(-\partial_3(g_{21}) - \partial_2(g_{31}) + g_{32}g_{21} - g_{22}g_{31})f_+^3 + \\
+(+\partial_1(g_{31}) - \partial_3(g_{11}) - g_{12}g_{31} + g_{32}g_{11})f_-^0 + (+\partial_0(g_{31}) + \partial_3(g_{01}) + g_{02}g_{31} - g_{32}g_{01})f_-^1 + \\
+(-\partial_0(g_{11}) - \partial_1(g_{01}) - g_{02}g_{11} + g_{12}g_{01})f_-^3
\end{array}\right) \\
\left(\begin{array}{c}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
+(-g_{31}+g_{32})\partial_0 f_+^1 + (+g_{11}-g_{12})\partial_0 f_+^3 + (-g_{21}+g_{22})\partial_0 f_-^0 + (+g_{01}-g_{02})\partial_0 f_-^2 + \\
+(-g_{31}+g_{32})\partial_1 f_+^0 + (+g_{01}-g_{02})\partial_1 f_+^3 + (-g_{11}+g_{12})\partial_1 f_-^2 + (+g_{21}-g_{22})\partial_1 f_-^1 + \\
+(+g_{01}-g_{02})\partial_2 f_-^0 + (+g_{11}-g_{12})\partial_2 f_-^1 + (+g_{21}-g_{22})\partial_2 f_-^2 + (+g_{31}-g_{32})\partial_2 f_-^3 + \\
+(+g_{11}-g_{12})\partial_3 f_+^0 + (-g_{01}+g_{02})\partial_3 f_+^1 + (-g_{31}+g_{32})\partial_3 f_-^2 + (+g_{21}-g_{22})\partial_3 f_-^3 + \\
+(-\partial_1(g_{31}) + \partial_3(g_{11}) - g_{12}g_{31} + g_{32}g_{11})f_+^0 + (-\partial_0(g_{31}) - \partial_3(g_{01}) + g_{02}g_{31} - g_{32}g_{01})f_+^1 + \\
+(+\partial_0(g_{11}) + \partial_1(g_{01}) - g_{02}g_{11} + g_{12}g_{01})f_+^3 + \\
+(-\partial_0(g_{21}) + \partial_2(g_{01}) + g_{02}g_{21} - g_{22}g_{01})f_-^0 + (+\partial_1(g_{21}) + \partial_2(g_{11}) + g_{12}g_{21} - g_{22}g_{11})f_-^1 + \\
+(+\partial_0(g_{01}) - \partial_1(g_{11}) + \partial_2(g_{21}) - \partial_3(g_{31}) - g_{02}g_{01} - g_{12}g_{11} - g_{22}g_{21} - g_{32}g_{31})f_-^2 + \\
+(+\partial_2(g_{31}) + \partial_3(g_{21}) - g_{22}g_{31} + g_{32}g_{21})f_-^3
\end{array}\right)
\end{array}\right.
$$

$$
\left.\begin{array}{c}
\left(\begin{array}{c}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+(+g_{31}-g_{32})\partial_0 f_+^0 + (-g_{01}+g_{02})\partial_0 f_+^3 + (-g_{21}+g_{22})\partial_0 f_-^1 + (+g_{11}-g_{12})\partial_0 f_-^2 + \\
+(-g_{31}+g_{32})\partial_1 f_+^1 + (+g_{11}-g_{12})\partial_1 f_+^3 + (-g_{21}+g_{22})\partial_1 f_-^0 + (+g_{01}-g_{02})\partial_1 f_-^2 + \\
+(-g_{31}+g_{32})\partial_2 f_+^2 + (+g_{21}-g_{22})\partial_2 f_+^3 + (+g_{11}+g_{12})\partial_2 f_-^0 + (-g_{01}-g_{02})\partial_2 f_-^1 + \\
+(-g_{01}+g_{02})\partial_3 f_+^0 + (-g_{11}+g_{12})\partial_3 f_+^1 + (-g_{21}+g_{22})\partial_3 f_+^2 + (-g_{31}+g_{32})\partial_3 f_+^3 + \\
+(+\partial_0(g_{31}) - \partial_3(g_{01}) + g_{02}g_{31} - g_{32}g_{01})f_+^0 + (-\partial_1(g_{31}) - \partial_3(g_{11}) + g_{12}g_{31} - g_{32}g_{11})f_+^1 + \\
+(-\partial_2(g_{31}) - \partial_3(g_{21}) + g_{22}g_{31} - g_{32}g_{21})f_+^2 + \\
+(-\partial_0(g_{01}) + \partial_1(g_{11}) + \partial_2(g_{21}) - \partial_3(g_{31}) - g_{02}g_{01} - g_{12}g_{11} - g_{22}g_{21} - g_{32}g_{31})f_+^3 + \\
+(-\partial_1(g_{21}) + \partial_2(g_{11}) + g_{12}g_{21} - g_{22}g_{11})f_-^0 + (-\partial_0(g_{21}) - \partial_2(g_{01}) - g_{02}g_{21} + g_{22}g_{01})f_-^1 + \\
+(+\partial_0(g_{11}) + \partial_1(g_{01}) + g_{02}g_{11} - g_{12}g_{01})f_-^2
\end{array}\right) \\
\left(\begin{array}{c}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3 + \\
+(+g_{21}-g_{22})\partial_0 f_+^1 + (-g_{11}+g_{12})\partial_0 f_+^2 + (-g_{31}+g_{32})\partial_0 f_-^0 + (+g_{01}-g_{02})\partial_0 f_-^3 + \\
+(+g_{21}-g_{22})\partial_1 f_+^0 + (-g_{01}+g_{02})\partial_1 f_+^2 + (+g_{31}-g_{32})\partial_1 f_-^1 + (-g_{11}+g_{12})\partial_1 f_-^3 + \\
+(-g_{11}+g_{12})\partial_2 f_+^0 + (+g_{01}-g_{02})\partial_2 f_+^1 + (+g_{31}-g_{32})\partial_2 f_-^2 + (-g_{21}+g_{22})\partial_2 f_-^3 +
\end{array}\right)
\end{array}\right.
$$

□

Corollary II.3 For differentiable functions $f^i_+, f^i_-, g^i_{jhk}; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$:

Given theorem II.1;

Whenever $g^i_{jhk} = \delta^i_h g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$

$$\Rightarrow \mathbf{J} = \begin{pmatrix} \begin{pmatrix} \begin{pmatrix} (\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f^1_+ + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^1_+ + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f^0_+ + (-\partial_1(g_{21})-\partial_2(g_{11}))f^2_+ + (-\partial_1(g_{31})-\partial_3(g_{11}))f^3_+ + \\ +(-\partial_2(g_{31})+\partial_3(g_{21}))f^0_- + (-\partial_0(g_{31})-\partial_3(g_{01}))f^2_- + (+\partial_0(g_{21})+\partial_2(g_{01}))f^3_- \end{pmatrix} \\ \begin{pmatrix} (\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f^1_- + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^1_- + \\ +(+\partial_2(g_{31})-\partial_3(g_{21}))f^0_+ + (+\partial_0(g_{31})+\partial_3(g_{01}))f^2_+ + (-\partial_0(g_{21})-\partial_2(g_{01}))f^3_+ + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f^0_- + (+\partial_1(g_{21})+\partial_2(g_{11}))f^2_- + (+\partial_1(g_{31})+\partial_3(g_{11}))f^3_- \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} (\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f^2_+ + \\ +(-\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^2_+ + \\ +(+\partial_0(g_{21})-\partial_2(g_{01}))f^0_+ + (-\partial_1(g_{21})-\partial_2(g_{11}))f^1_+ + (-\partial_3(g_{21})-\partial_2(g_{31}))f^3_+ + \\ +(+\partial_1(g_{31})-\partial_3(g_{11}))f^0_- + (+\partial_0(g_{31})+\partial_3(g_{01}))f^1_- + (-\partial_0(g_{11})-\partial_1(g_{01}))f^3_- \end{pmatrix} \\ \begin{pmatrix} (\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f^2_- + \\ +(+\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^2_- + \\ +(-\partial_1(g_{31})+\partial_3(g_{11}))f^0_+ + (-\partial_0(g_{31})-\partial_3(g_{01}))f^1_+ + (+\partial_0(g_{11})+\partial_1(g_{01}))f^3_+ + \\ +(-\partial_0(g_{21})+\partial_2(g_{01}))f^0_- + (+\partial_1(g_{21})+\partial_2(g_{11}))f^1_- + (+\partial_2(g_{31})+\partial_3(g_{21}))f^3_- + \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} (\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f^3_+ + \\ +(-\partial_0(g_{01})+\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^3_+ + \\ +(+\partial_0(g_{31})-\partial_3(g_{01}))f^0_+ + (-\partial_1(g_{31})-\partial_3(g_{11}))f^1_+ + (-\partial_2(g_{31})-\partial_3(g_{21}))f^2_+ + \\ +(-\partial_1(g_{21})+\partial_2(g_{11}))f^0_- + (-\partial_0(g_{21})-\partial_2(g_{01}))f^1_- + (+\partial_0(g_{11})+\partial_1(g_{01}))f^2_- \end{pmatrix} \\ \begin{pmatrix} (\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f^3_- + \\ +(+\partial_0(g_{01})-\partial_1(g_{11})-\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^3_- + \\ +(+\partial_1(g_{21})-\partial_2(g_{11}))f^0_+ + (+\partial_0(g_{21})+\partial_2(g_{01}))f^1_+ + (-\partial_0(g_{11})-\partial_1(g_{01}))f^2_+ + \\ +(-\partial_0(g_{31})+\partial_3(g_{01}))f^0_- + (+\partial_1(g_{31})+\partial_3(g_{11}))f^1_- + (+\partial_2(g_{31})+\partial_3(g_{21}))f^2_- + \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} (\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f^0_+ + \\ +(+\partial_0(g_{01})+\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^0_+ + \\ +(+\partial_0(g_{11})-\partial_1(g_{01}))f^1_+ + (+\partial_0(g_{21})-\partial_2(g_{01}))f^2_+ + (+\partial_0(g_{31})-\partial_3(g_{01}))f^3_+ + \\ +(+\partial_2(g_{31})-\partial_3(g_{21}))f^1_- + (-\partial_1(g_{31})+\partial_3(g_{11}))f^2_- + (+\partial_1(g_{21})-\partial_2(g_{11}))f^3_- \end{pmatrix} \\ \begin{pmatrix} (\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f^0_- + \\ +(-\partial_0(g_{01})-\partial_1(g_{11})-\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f^0_- + \\ +(-\partial_2(g_{31})+\partial_3(g_{21}))f^1_+ + (+\partial_1(g_{31})-\partial_3(g_{11}))f^2_+ + (-\partial_1(g_{21})+\partial_2(g_{11}))f^3_+ + \\ +(-\partial_0(g_{11})+\partial_1(g_{01}))f^1_- + (-\partial_0(g_{21})+\partial_2(g_{01}))f^2_- + (-\partial_0(g_{31})+\partial_3(g_{01}))f^3_- \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

Proof:

Given theorem II.1;

Whenever $g^i_{jhk} = \delta^i_h g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$

$$
\mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+(-\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_+^1 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^2+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^3 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_-^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_-^2+(+\partial_0(g_{21})+\partial_2(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+(+\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_-^1 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_+^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_+^2+(-\partial_0(g_{21})-\partial_2(g_{01}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^2+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^3
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+(-\partial_0(g_{01})+\partial_1(g_{11})-\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_+^2 + \\
+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^1+(-\partial_3(g_{21})-\partial_2(g_{31}))f_+^3 + \\
+(+\partial_1(g_{31})-\partial_3(g_{11}))f_-^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_-^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
(+\partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_-^2 + \\
+(-\partial_1(g_{31})+\partial_3(g_{11}))f_+^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_+^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_+^3 + \\
+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^3 +
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+(-\partial_0(g_{01})+\partial_1(g_{11})+\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_+^3 + \\
+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^0+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^1+(-\partial_2(g_{31})-\partial_3(g_{21}))f_+^2 + \\
+(-\partial_1(g_{21})+\partial_2(g_{11}))f_-^0+(-\partial_0(g_{21})-\partial_2(g_{01}))f_-^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_-^2
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3 + \\
+(+\partial_0(g_{01})-\partial_1(g_{11})-\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_-^3 + \\
+(+\partial_1(g_{21})-\partial_2(g_{11}))f_+^0+(+\partial_0(g_{21})+\partial_2(g_{01}))f_+^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_+^2 + \\
+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^0+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^2 +
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_+^0 + \\
+(+\partial_0(g_{01})+\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_+^0 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^1+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^2+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^3 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_-^1+(-\partial_1(g_{31})+\partial_3(g_{11}))f_-^2+(+\partial_1(g_{21})-\partial_2(g_{11}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_-^0 + \\
+(-\partial_0(g_{01})-\partial_1(g_{11})-\partial_2(g_{21})-\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2])f_-^0 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_+^1+(+\partial_1(g_{31})-\partial_3(g_{11}))f_+^2+(-\partial_1(g_{21})+\partial_2(g_{11}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^1+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^2+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^3
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

□

Corollary II.4 For differentiable functions $f_+^i, f_-^i, g_{jhk}^i; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$:
Given corollary II.3;
Whenever $g_{jhk}^i = \delta_h^i g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$

$$
\Rightarrow \mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right)\right]+\sum_{j=2}^{3}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)\right]\right)f_+^1 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^2+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^3 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_-^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_-^2+(+\partial_0(g_{21})+\partial_2(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)+e^{-\int g_{(j+1)1}\partial x_{(j+1)}}\frac{\partial^2}{\partial x_{(j+1)}^2}\left(e^{\int g_{(j+1)1}\partial x_{(j+1)}}\right)\right]\right)f_-^1 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_+^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_+^2+(-\partial_0(g_{21})-\partial_2(g_{01}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^2+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^3
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{2j1}\partial x_j}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{2j1}\partial x_{2j}}\right)+e^{\int g_{(2j+1)1}\partial x_{(2j+1)}}\frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]\right)f_+^2 + \\
+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^1+(-\partial_3(g_{21})-\partial_2(g_{31}))f_+^3 + \\
+(+\partial_1(g_{31})-\partial_3(g_{11}))f_-^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_-^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{2j1}\partial x_j}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{2j1}\partial x_{2j}}\right)+e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}}\frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]\right)f_-^2 + \\
+(-\partial_1(g_{31})+\partial_3(g_{11}))f_+^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_+^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_+^3 + \\
+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^3+
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{(3j)1}\partial x_{3j}}\frac{\partial^2}{\partial x_{3j}^2}\left(e^{\int g_{(3j)1}\partial x_{3j}}\right)+e^{\int g_{(2j)1}\partial x_{2j}}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{(2j)1}\partial x_{2j}}\right)\right]\right)f_+^3 + \\
+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^0+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^1+(-\partial_2(g_{31})-\partial_3(g_{21}))f_+^2 + \\
+(-\partial_1(g_{21})+\partial_2(g_{11}))f_-^0+(-\partial_0(g_{21})-\partial_2(g_{01}))f_-^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_-^2
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{(3j)1}\partial x_{3j}}\frac{\partial^2}{\partial x_{3j}^2}\left(e^{-\int g_{(3j)1}\partial x_{3j}}\right)+e^{-\int g_{(2j)1}\partial x_{2j}}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{(2j)1}\partial x_{2j}}\right)\right]\right)f_-^3 + \\
+(+\partial_0(g_{01})-g_{02}g_{01}-\partial_1(g_{11})-g_{12}g_{11}-\partial_2(g_{21})-g_{22}g_{21}+\partial_3(g_{31})-g_{32}g_{31})f_-^3 + \\
+(+\partial_1(g_{21})-\partial_2(g_{11}))f_+^0+(+\partial_0(g_{21})+\partial_2(g_{01}))f_+^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_+^2 + \\
+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^0+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^2+
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_+^0 + \\
+\left(-\sum_{j=0}^{3}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)\right]\right)f_+^0 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^1+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^2+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^3 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_-^1+(-\partial_1(g_{31})+\partial_3(g_{11}))f_-^2+(+\partial_1(g_{21})-\partial_2(g_{11}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_-^0 + \\
+\left(-\sum_{j=0}^{3}\left[e^{-\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right)\right]\right)f_-^0 + \\
+(-\partial_0(g_{01})-g_{02}g_{01}-\partial_1(g_{11})-g_{12}g_{11}-\partial_2(g_{21})-g_{22}g_{21}-\partial_3(g_{31})-g_{32}g_{31})f_-^0 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_+^1+(+\partial_1(g_{31})-\partial_3(g_{11}))f_+^2+(-\partial_1(g_{21})+\partial_2(g_{11}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^1+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^2+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^3
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

Proof:

$$
\left(e^{\int ydx}\right)' = ye^{\int ydx} \quad || \quad -\left(e^{-\int ydx}\right)' = ye^{-\int ydx}
$$

$$
\left(ye^{\int ydx}\right)' = e^{\int ydx}(y'+y^2) \quad || \quad \left(ye^{-\int ydx}\right)' = e^{-\int ydx}(y'-y^2)
$$

$$
\Rightarrow \left(e^{\int ydx}\right)'' = e^{\int ydx}(y'+y^2) \quad || \quad \Rightarrow \left(e^{-\int ydx}\right)'' = -\left(ye^{-\int ydx}\right)' = -e^{-\int ydx}(y'-y^2)
$$

f_+^1 :

$$
\begin{aligned}
\Rightarrow \partial_0(g_{01})-\partial_1(g_{11})+\partial_2(g_{21})+\partial_3(g_{31})-[g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2] = \\
= -\partial_0(g_{01})-g_{01}^2-\partial_1(g_{11})-g_{11}^2+\partial_2(g_{21})-g_{21}^2+\partial_3(g_{31})-g_{31}^2 \\
= -([\partial_0(g_{01})+g_{01}^2]+[\partial_1(g_{11})+g_{11}^2])+([\partial_2(g_{21})-g_{21}^2]+[\partial_3(g_{31})-g_{31}^2])
\end{aligned}
$$

$$= -\left[e^{-\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{\int g_{01}\partial x_0}\right) + e^{-\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{\int g_{11}\partial x_1}\right)\right] +$$
$$-\left[e^{\int g_{21}\partial x_2} \frac{\partial^2}{\partial x_2^2}\left(e^{-\int g_{21}\partial x_2}\right) + e^{\int g_{31}\partial x_3} \frac{\partial^2}{\partial x_3^2}\left(e^{-\int g_{31}\partial x_3}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{-\int g_{j1}\partial x_j} \frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right) + e^{\int g_{(j+1)1}\partial x_{(j+1)}} \frac{\partial^2}{\partial x_{(j+1)}^2}\left(e^{-\int g_{(j+1)1}\partial x_{(j+1)}}\right)\right]$$

f_{-}^{1} :

$$\Rightarrow \partial_0(g_{01}) + \partial_1(g_{11}) - \partial_2(g_{21}) - \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2] =$$
$$= \partial_0(g_{01}) - g_{01}^2 + \partial_1(g_{11}) - g_{11}^2 - \partial_2(g_{21}) - g_{21}^2 - \partial_3(g_{31}) - g_{31}^2$$
$$= ([\partial_0(g_{01}) - g_{01}^2] + [\partial_1(g_{11}) - g_{11}^2]) - ([\partial_2(g_{21}) + g_{21}^2] + [\partial_3(g_{31}) + g_{31}^2])$$
$$= -\left[e^{\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{-\int g_{01}\partial x_0}\right) + e^{\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{-\int g_{11}\partial x_3}\right)\right] +$$
$$-\left[e^{-\int g_{21}\partial x_0} \frac{\partial^2}{\partial x_2^2}\left(e^{\int g_{21}\partial x_2}\right) + e^{-\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{\int g_{11}\partial x_1}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{\int g_{j1}\partial x_j} \frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right) + e^{-\int g_{(j+1)1}\partial x_{(j+1)}} \frac{\partial^2}{\partial x_{(j+1)}^2}\left(e^{\int g_{(j+1)1}\partial x_{(j+1)}}\right)\right]$$

f_{+}^{2} :

$$\Rightarrow -\partial_0(g_{01}) + \partial_1(g_{11}) - \partial_2(g_{21}) + \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2] =$$
$$= -\partial_0(g_{01}) - g_{01}^2 + \partial_1(g_{11}) - g_{11}^2 - \partial_2(g_{21}) - g_{21}^2 + \partial_3(g_{31}) - g_{31}^2$$
$$= -\left([\partial_0(g_{01}) + g_{01}^2] + \left[\partial_2\left(g_{21}\right) + g_{21}^2\right]\right) + ([\partial_1(g_{11}) - g_{11}^2] + [\partial_3(g_{31}) - g_{31}^2])$$
$$= -\left[e^{-\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{\int g_{01}\partial x_0}\right) + e^{-\int g_{21}\partial x_2} \frac{\partial^2}{\partial x_2^2}\left(e^{\int g_{21}\partial x_2}\right)\right] +$$
$$-\left[e^{\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{-\int g_{11}\partial x_2}\right) + e^{\int g_{31}\partial x_3} \frac{\partial^2}{\partial x_3^2}\left(e^{-\int g_{31}\partial x_3}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{-\int g_{2j1}\partial x_j} \frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{2j1}\partial x_{2j}}\right) + e^{\int g_{(2j+1)1}\partial x_{(2j+1)}} \frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]$$

f_{-}^{2} :

$$\Rightarrow \partial_0(g_{01}) - \partial_1(g_{11}) + \partial_2(g_{21}) - \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2] =$$
$$= ([\partial_0(g_{01}) - g_{01}^2] + [\partial_2(g_{21}) - g_{21}^2]) - ([\partial_1(g_{11}) + g_{11}^2] + [\partial_3(g_{31}) + g_{31}^2])$$
$$= -\left[e^{\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{-\int g_{01}\partial x_0}\right) + e^{\int g_{21}\partial x_2} \frac{\partial^2}{\partial x_2^2}\left(e^{-\int g_{21}\partial x_2}\right)\right] +$$
$$-\left[e^{-\int g_{11}\partial x_0} \frac{\partial^2}{\partial x_1^2}\left(e^{\int g_{11}\partial x_1}\right) + e^{-\int g_{31}\partial x_3} \frac{\partial^2}{\partial x_3^2}\left(e^{\int g_{31}\partial x_3}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{\int g_{2j1}\partial x_j} \frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{2j1}\partial x_{2j}}\right) + e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}} \frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]$$

f_{+}^{3} :

$$\Rightarrow -\partial_0(g_{01}) + \partial_1(g_{11}) + \partial_2(g_{21}) - \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2]$$
$$= -([\partial_0(g_{01}) + g_{01}^2] + [\partial_3(g_{31}) + g_{31}^2]) + [\partial_1(g_{11}) - g_{11}^2] + [\partial_2(g_{21}) - g_{21}^2]$$
$$= -\left[e^{-\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{\int g_{01}\partial x_0}\right) + e^{-\int g_{31}\partial x_3} \frac{\partial^2}{\partial x_3^2}\left(e^{\int g_{31}\partial x_3}\right)\right] +$$
$$-\left[e^{\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{-\int g_{11}\partial x_1}\right) + e^{\int g_{21}\partial x_2} \frac{\partial^2}{\partial x_2^2}\left(e^{-\int g_{21}\partial x_2}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{-\int g_{(3j)1}\partial x_{3j}} \frac{\partial^2}{\partial x_{3j}^2}\left(e^{\int g_{(3j)1}\partial x_{3j}}\right) + e^{\int g_{(2j)1}\partial x_{2j}} \frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{(2j)1}\partial x_{2j}}\right)\right]$$

f_{-}^{3} :

$$\Rightarrow \partial_0(g_{01}) - \partial_1(g_{11}) - \partial_2(g_{21}) + \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2]$$
$$= ([\partial_0(g_{01}) - g_{01}^2] + [\partial_3(g_{31}) - g_{21}^2]) - ([\partial_1(g_{11}) + g_{11}^2] + [\partial_2(g_{21}) + g_{21}^2])$$
$$= -\left[e^{\int g_{01}\partial x_0} \frac{\partial^2}{\partial x_0^2}\left(e^{-\int g_{01}\partial x_0}\right) + e^{\int g_{31}\partial x_3} \frac{\partial^2}{\partial x_3^2}\left(e^{-\int g_{31}\partial x_3}\right)\right] +$$
$$= -\left[e^{-\int g_{11}\partial x_1} \frac{\partial^2}{\partial x_1^2}\left(e^{\int g_{11}\partial x_1}\right) + e^{-\int g_{21}\partial x_2} \frac{\partial^2}{\partial x_2^2}\left(e^{\int g_{21}\partial x_2}\right)\right]$$
$$= -\sum_{j=0}^{1}\left[e^{\int g_{(3j)1}\partial x_{3j}} \frac{\partial^2}{\partial x_{3j}^2}\left(e^{-\int g_{(3j)1}\partial x_{3j}}\right) + e^{-\int g_{(2j)1}\partial x_{2j}} \frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{(2j)1}\partial x_{2j}}\right)\right]$$

f_{+}^{0} :

$$\Rightarrow \partial_0(g_{01}) + \partial_1(g_{11}) + \partial_2(g_{21}) + \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2]$$
$$= [\partial_0(g_{01}) - g_{01}^2] + [\partial_1(g_{11}) - g_{11}^2] + [\partial_2(g_{21}) - g_{21}^2] + [\partial_3(g_{31}) - g_{31}^2]$$
$$= -\sum_{j=0}^{3}\left[e^{\int g_{j1}\partial x_j} \frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)\right]$$

f_{-}^{0} :

$$\Rightarrow -\partial_0(g_{01}) - \partial_1(g_{11}) - \partial_2(g_{21}) - \partial_3(g_{31}) - [g_{01}^2 + g_{11}^2 + g_{21}^2 + g_{31}^2]$$
$$= -[\partial_0(g_{01}) + g_{01}^2] - [\partial_1(g_{11}) + g_{11}^2] - [\partial_2(g_{21}) + g_{21}^2] - [\partial_3(g_{31}) + g_{31}^2]$$
$$= -\sum_{j=0}^{3}\left[e^{-\int g_{j1}\partial x_j} \frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right)\right]$$

So::

Given theorem II.1;

Whenever $g_{jhk}^{i} = \delta_{h}^{i} g_{jk} \quad AND \quad g_{j2} = g_{j1} \quad ; \forall i,j,h \in \{0,1,2,3\} \quad , \quad \forall k \in \{1,2\}$

$$
\Rightarrow \mathbf{J} = \begin{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_+^1 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right)\right]+\sum_{j=2}^{3}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)\right]\right)f_+^1 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^2+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^3 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_-^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_-^2+(+\partial_0(g_{21})+\partial_2(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)f_-^1 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)+e^{-\int g_{(j+1)1}\partial x_{(j+1)}}\frac{\partial^2}{\partial x_{(j+1)}^2}\left(e^{\int g_{(j+1)1}\partial x_{(j+1)}}\right)\right]\right)f_-^1 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_+^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_+^2+(-\partial_0(g_{21})-\partial_2(g_{01}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^2+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^3
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1^2+\partial_2^2)f_+^2 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{2j1}\partial x_j}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{2j1}\partial x_{2j}}\right)+e^{\int g_{(2j+1)1}\partial x_{(2j+1)}}\frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]\right)f_+^2 + \\
+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^0+(-\partial_1(g_{21})-\partial_2(g_{11}))f_+^1+(-\partial_3(g_{21})-\partial_2(g_{31}))f_+^3 + \\
+(+\partial_1(g_{31})-\partial_3(g_{11}))f_-^0+(+\partial_0(g_{31})+\partial_3(g_{01}))f_-^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_3^2+\partial_0^2+\partial_1+\partial_2^2)f_-^2 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{2j1}\partial x_j}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{2j1}\partial x_{2j}}\right)+e^{-\int g_{(2j+1)1}\partial x_{(2j+1)}}\frac{\partial^2}{\partial x_{(2j+1)}^2}\left(e^{\int g_{(2j+1)1}\partial x_{(2j+1)}}\right)\right]\right)f_-^2 + \\
+(-\partial_1(g_{31})+\partial_3(g_{11}))f_+^0+(-\partial_0(g_{31})-\partial_3(g_{01}))f_+^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_+^3 + \\
+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^0+(+\partial_1(g_{21})+\partial_2(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^3+
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3)f_+^3 + \\
+\left(-\sum_{j=0}^{1}\left[e^{-\int g_{(3j)1}\partial x_{3j}}\frac{\partial^2}{\partial x_{3j}^2}\left(e^{\int g_{(3j)1}\partial x_{3j}}\right)+e^{\int g_{(2j)1}\partial x_{2j}}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{-\int g_{(2j)1}\partial x_{2j}}\right)\right]\right)f_+^3 + \\
+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^0+(-\partial_1(g_{31})-\partial_3(g_{11}))f_+^1+(-\partial_2(g_{31})-\partial_3(g_{21}))f_+^2 + \\
+(-\partial_1(g_{21})+\partial_2(g_{11}))f_-^0+(-\partial_0(g_{21})-\partial_2(g_{01}))f_-^1+(+\partial_0(g_{11})+\partial_1(g_{01}))f_-^2
\end{pmatrix} \\
\begin{pmatrix}
(\partial_2^2+\partial_1^2+\partial_0^2+\partial_3^2)f_-^3 + \\
+\left(-\sum_{j=0}^{1}\left[e^{\int g_{(3j)1}\partial x_{3j}}\frac{\partial^2}{\partial x_{3j}^2}\left(e^{-\int g_{(3j)1}\partial x_{3j}}\right)+e^{-\int g_{(2j)1}\partial x_{2j}}\frac{\partial^2}{\partial x_{2j}^2}\left(e^{\int g_{(2j)1}\partial x_{2j}}\right)\right]\right)f_-^3 + \\
+(+\partial_0(g_{01})-g_{02}g_{01}-\partial_1(g_{11})-g_{12}g_{11}-\partial_2(g_{21})-g_{22}g_{21}+\partial_3(g_{31})-g_{32}g_{31})f_-^3 + \\
+(+\partial_1(g_{21})-\partial_2(g_{11}))f_+^0+(+\partial_0(g_{21})+\partial_2(g_{01}))f_+^1+(-\partial_0(g_{11})-\partial_1(g_{01}))f_+^2 + \\
+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^0+(+\partial_1(g_{31})+\partial_3(g_{11}))f_-^1+(+\partial_2(g_{31})+\partial_3(g_{21}))f_-^2+
\end{pmatrix}
\end{pmatrix} \\
\begin{pmatrix}
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_+^0 + \\
+\left(-\sum_{j=0}^{3}\left[e^{\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{-\int g_{j1}\partial x_j}\right)\right]\right)f_+^0 + \\
+(+\partial_0(g_{11})-\partial_1(g_{01}))f_+^1+(+\partial_0(g_{21})-\partial_2(g_{01}))f_+^2+(+\partial_0(g_{31})-\partial_3(g_{01}))f_+^3 + \\
+(+\partial_2(g_{31})-\partial_3(g_{21}))f_-^1+(-\partial_1(g_{31})+\partial_3(g_{11}))f_-^2+(+\partial_1(g_{21})-\partial_2(g_{11}))f_-^3
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1^2+\partial_2^2+\partial_3^2+\partial_0^2)f_-^0 + \\
+\left(-\sum_{j=0}^{3}\left[e^{-\int g_{j1}\partial x_j}\frac{\partial^2}{\partial x_j^2}\left(e^{\int g_{j1}\partial x_j}\right)\right]\right)f_-^0 + \\
+(-\partial_0(g_{01})-g_{02}g_{01}-\partial_1(g_{11})-g_{12}g_{11}-\partial_2(g_{21})-g_{22}g_{21}-\partial_3(g_{31})-g_{32}g_{31})f_-^0 + \\
+(-\partial_2(g_{31})+\partial_3(g_{21}))f_+^1+(+\partial_1(g_{31})-\partial_3(g_{11}))f_+^2+(-\partial_1(g_{21})+\partial_2(g_{11}))f_+^3 + \\
+(-\partial_0(g_{11})+\partial_1(g_{01}))f_-^1+(-\partial_0(g_{21})+\partial_2(g_{01}))f_-^2+(-\partial_0(g_{31})+\partial_3(g_{01}))f_-^3
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
$$

□

Corollary II.5 For differentiable functions $f_+^i, f_-^i, g_{jhk}^i; \forall i,j,h \in \{0,1,2,3\}$, $\forall k \in \{1,2\}$:

Given corollary II.3;

Whenever:

$g_{jhk}^i = \delta_h^i g_{jk}$ *AND* $g_{j2} = g_{j1}$ *AND* g_{j1} are constants ; $\forall i,j,h \in \{0,1,2,3\}$, $\forall k \in \{1,2\}$

$$\mathbf{J} = \begin{pmatrix} \begin{pmatrix} [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_+^1 \\ [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_-^1 \end{pmatrix} \\ \begin{pmatrix} [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_+^2 \\ [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_-^2 \end{pmatrix} \\ \begin{pmatrix} [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_+^3 \\ [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_-^3 \end{pmatrix} \\ \begin{pmatrix} [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_+^0 \\ [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]f_-^0 \end{pmatrix} \end{pmatrix}$$

$$\Downarrow$$

$$\mathbf{J}^h = [(\partial_0^2+\partial_3^2+\partial_2^2+\partial_1^2)-(g_{01}^2+g_{11}^2+g_{21}^2+g_{31}^2)]\mathbf{f}^h$$

□

Definition II.6: Given a set λ_j , $j \in \mathbb{N}$:

For differentiable functions $f^l, f_+^l, f_-^l, g_{ij}, \mathbf{J}(x_3,x_2,x_1,x_0), \Phi(x_3,x_2,x_1,x_0); \forall i,j \in \{0,1,2,3\}$:

$$\mathbf{J}^n(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix} -D_{02} & D_{32}^{\Leftrightarrows} & -D_{22}^{\Leftrightarrows} & -D_{12} \\ -D_{32}^{\Leftrightarrows} & -D_{02} & D_{12}^{\Leftrightarrows} & -D_{22} \\ D_{22}^{\Leftrightarrows} & -D_{12}^{\Leftrightarrows} & -D_{02} & -D_{32} \\ -D_{12}^{\Updownarrow} & -D_{22}^{\Updownarrow} & -D_{32}^{\Updownarrow} & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & -D_{31}^{\Leftrightarrows} & D_{21}^{\Leftrightarrows} & -D_{11} \\ D_{31}^{\Leftrightarrows} & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrows} & -D_{21} \\ -D_{21}^{\Leftrightarrows} & D_{11}^{\Leftrightarrows} & -D_{01}^{\Updownarrow} & -D_{31} \\ -D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01} \end{pmatrix} \mathbf{f}^n$$

$(\mathbf{J})^\lambda \equiv \{\mathbf{J}^n \mid \forall n \in \mathbb{N} : 0 \le n \le \lambda\}$

then for a set $\{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_\kappa\}$ and index $m \in \{\mathbb{N} \mid m \le \kappa \in \mathbb{N}\}$:

$\bigcup_{j=1}^{m} (\mathbf{J})^{\lambda_j}$ defines the design of the fundamental second order linear partial differential field wave equations of a universe.

Thus, a semigroup algebra vector space design that generates the generalized Covariant Helmholtzian oprator factorization and generalized Maxwell-Cassano equations , as it's suitably smooth finctions with the specifics of a $\bigcup_{j=1}^{m} (\mathbf{J})^{\lambda_j}$, specifies a reality.