On the study of two Ramanujan's equations of the "Ramanujan's first letter to Hardy". Mathematical connections with various sectors of String Theory (Supersymmetry Breaking).

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Abstract

In this research thesis, we analyze two Ramanujan's equations of the "Ramanujan's first letter to Hardy". We describe new possible mathematical connections with various sectors of String Theory (Supersymmetry Breaking).

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From:

Collected Papers of Srinivasa Ramanujan - 2000 *of Srinivasa Ramanujan* (Author), G. H. Hardy, P. V. Seshu Aiyar, B. M. Wilson , Bruce Berndt

We analyze the following equation:

$$\int_0^\infty \frac{1+\frac{x^2}{(b+1)^2}}{1+\frac{x^2}{a^2}} \times \frac{1+\frac{x^2}{(b+2)^2}}{1+\frac{x^2}{(a+1)^2}} \times \cdots dx = \frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\Gamma(b-a+1)}{\Gamma(a)\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}.$$

Integrate((((((1+x^2/((b+1)^2)))) / (((1+(x^2)/(a^2)))))))*((((((1+x^2/((b+2)^2)))) / ((((((1+x^2/((a+1)^2))))

sqrt(Pi)/2 * gamma(a+1/2) gamma(b+1) gamma(b-a+1) / gamma(a) gamma(b+1/2) gamma(b-a+1/2)

We have:

Integrate((((((1+x^2/((b+1)^2)))) / (((1+(x^2)/(a^2)))))))*((((((1+x^2/((b+2)^2))))))) / ((((((1+x^2/((a+1)^2)))))))x

Indefinite integral

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx =$$

$$\Big(a^2(a+1)^2\Big(\big(a^4-a^2\left(2b^2+6b+5\right)+\left(b^2+3b+2\right)^2\big)\log\left(a^2+x^2\right) - \big(a^4+4a^3+a^2\left(-2b^2-6b+1\right)-2a\left(2b^2+6b+3\right) + b\left(b^3+6b^2+11b+6\right)\big)\log\left(a^2+2a+x^2+1\right)\Big)+(2a+1)x^2\Big)\Big/ \left(2(2a+1)\left(b^2+3b+2\right)^2\right) + \text{constant}$$

$\log(x)$ is the natural logarithm

Alternate forms of the integral

$$\left(a^2 (a+1)^2 \left((a-b-1)(a+b+2)\left((a-b-2)(a+b+1)\log\left(a^2+x^2\right)-(a-b)(a+b+3)\log\left((a+1)^2+x^2\right)\right)+(2a+1)x^2\right)\right)\Big/\left(2(2a+1)(b+1)^2(b+2)^2\right) + \text{constant}$$

$$\frac{a^2 (a+1)^2 x^2}{2(b+1)^2(b+2)^2} + \frac{a^2 (a+1)^2 (a-b-2)(a-b-1)(a+b+1)(a+b+2)\log\left(a^2+x^2\right)}{2(2a+1)(b+1)^2(b+2)^2} - \frac{a^2 (a+1)^2 (a-b-1)(a-b)(a+b+2)(a+b+3)\log\left(a^2+2a+x^2+1\right)}{2(2a+1)(b+1)^2(b+2)^2} + \text{constant}$$

$$\left(a^2 (a+1)^2 \left(2a\left(\left(2b^2+6b+3\right)\log\left(a^2+2a+x^2+1\right)+x^2\right)+\left(a^4+a^2\left(-2b^2-6b-5\right)+b^4+6b^3+13b^2+12b+4\right)\log\left(a^2+x^2\right)+\left(-a^4-4a^3+a^2\left(2b^2+6b-1\right)-b^4-6b^3-11b^2-6b\right)\log\left(a^2+2a+x^2+1\right)+x^2\right)\right)\Big/\left(2(2a+1)\left(b^2+3b+2\right)^2\right) + \text{constant}$$

Expanded form of the integral

$$\frac{\log(a^2+x^2)\,a^8}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{\log(a^2+2\,a+x^2+1)\,a^8}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{\log(a^2+x^2)\,a^7}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{3\log(a^2+2\,a+x^2+1)\,a^7}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{b^2\log(a^2+x^2)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{3\,b\log(a^2+x^2)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{2\log(a^2+x^2)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{b^2\log(a^2+2\,a+x^2+1)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{3\,b\log(a^2+2\,a+x^2+1)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{5\log(a^2+2\,a+x^2+1)\,a^6}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{x^2\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{2\,b^2\log(a^2+x^2)\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{6\,b\log(a^2+x^2)\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{5\log(a^2+x^2)\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{4\,b^2\log(a^2+2\,a+x^2+1)\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{12\,b\log(a^2+2\,a+x^2+1)\,a^5}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{5\,x^2\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{b^4\log(a^2+x^2)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{3\,b^3\log(a^2+x^2)\,a^4}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{11\,b^2\log(a^2+x^2)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{3\,b\log(a^2+x^2)\,a^4}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{\log(a^2+x^2)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{b^4\log(a^2+2\,a+x^2+1)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{3\,b^3\log(a^2+2\,a+x^2+1)\,a^4}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{b^2\log(a^2+2\,a+x^2+1)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{12\,b\log(a^2+2\,a+x^2+1)\,a^4}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{11\log(a^2+2\,a+x^2+1)\,a^4}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{2\,x^2\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{b^4\log(a^2+x^2)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{6\,b^3\log(a^2+x^2)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{13\,b^2\log(a^2+x^2)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{12\,b\log(a^2+x^2)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{4\log(a^2+x^2)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{b^4\log(a^2+2\,a+x^2+1)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{6\,b^3\log(a^2+2\,a+x^2+1)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{9\,b^2\log(a^2+2\,a+x^2+1)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{3\log(a^2+2\,a+x^2+1)\,a^3}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{x^2\,a^2}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{b^4\log(a^2+x^2)\,a^2}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{3\,b^3\log(a^2+x^2)\,a^2}{(2\,a+1)\,(b^2+3\,b+2)^2}+$$
$$\frac{13\,b^2\log(a^2+x^2)\,a^2}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{6\,b\log(a^2+x^2)\,a^2}{(2\,a+1)\,(b^2+3\,b+2)^2}+\frac{2\log(a^2+x^2)\,a^2}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{b^4\log(a^2+2\,a+x^2+1)\,a^2}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{3\,b^3\log(a^2+2\,a+x^2+1)\,a^2}{(2\,a+1)\,(b^2+3\,b+2)^2}-$$
$$\frac{11\,b^2\log(a^2+2\,a+x^2+1)\,a^2}{2\,(2\,a+1)\,(b^2+3\,b+2)^2}-\frac{3\,b\log(a^2+2\,a+x^2+1)\,a^2}{(2\,a+1)\,(b^2+3\,b+2)^2}+\text{constant}$$

Series expansion of the integral at x=0

$$\left(a^2 (a+1)^2 \left(a^2+a-b^2-3b-2\right) \left(\left(a^2-a-b^2-3b-2\right) \log\left(a^2\right) + \left(-a^2-3a+b(b+3)\right) \log\left((a+1)^2\right)\right)\right) / \left(2(2a+1)\left(b^2+3b+2\right)^2\right) + \frac{x^2}{2} + O\left(x^4\right)$$

(Taylor series)

Series expansion of the integral at x=∞

$$\frac{a^2 (a+1)^2 x^2}{2\left(b^2+3b+2\right)^2} - \frac{2\left(a^2 (a+1)^2 \left(a^2+a-b^2-3b-2\right) \log(x)\right)}{\left(b^2+3b+2\right)^2} - \frac{1}{2\left(b^2+3b+2\right)^2 x^2} a^2 (a+1)^2 \left(3a^4+6a^3-a^2\left(4b^2+12b+3\right) - 2a\left(2b^2+6b+3\right) + b\left(b^3+6b^2+11b+6\right)\right) + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Puiseux series)

sqrt(Pi)/2 * (((gamma(a+1/2) gamma(b+1) gamma(b-a+1)))) / (((gamma(a) gamma(b+1/2) gamma(b-a+1/2))))

Input

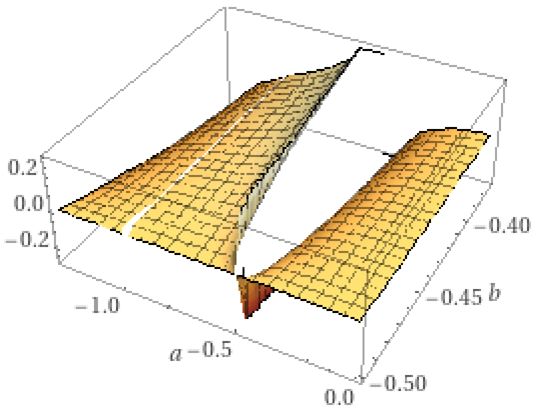
$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a+\frac{1}{2}\right) \Gamma(b+1) \Gamma(b-a+1)}{\Gamma(a) \Gamma\left(b+\frac{1}{2}\right) \Gamma\left(b-a+\frac{1}{2}\right)}$$

$\Gamma(x)$ is the gamma function

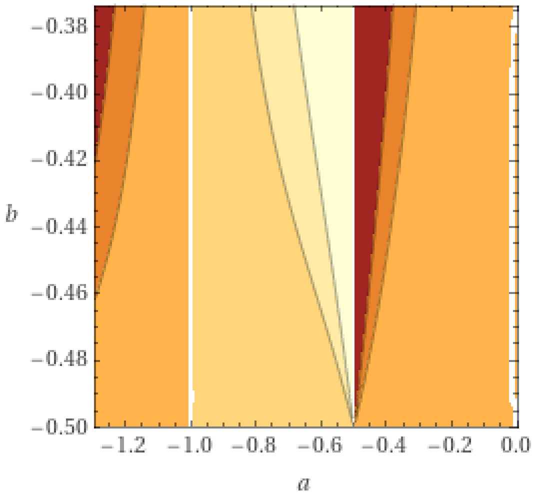
Exact result

$$\frac{\sqrt{\pi}\, \Gamma\left(a+\frac{1}{2}\right) \Gamma(b+1) \Gamma(-a+b+1)}{2\, \Gamma(a) \Gamma\left(b+\frac{1}{2}\right) \Gamma\left(-a+b+\frac{1}{2}\right)}$$

3D plot **(figure that can be related to a D-brane)**



Contour plot



Roots

(no roots exist)

Series expansion at a=∞

$$\cos(\pi(a-b))\csc(\pi(-a+b+1))\left(\frac{\sqrt{\pi}\,\Gamma(b+1)\,a}{2\,\Gamma\left(b+\frac{1}{2}\right)}-\frac{(2\,b+1)\sqrt{\pi}\,\Gamma(b+1)}{8\,\Gamma\left(b+\frac{1}{2}\right)}-\frac{\left(4\,b^2-1\right)\sqrt{\pi}\,\Gamma(b+1)}{64\,\Gamma\left(b+\frac{1}{2}\right)a}+O\left(\left(\frac{1}{a}\right)^2\right)\right)$$

Derivative

$$\frac{\partial}{\partial a}\left(\frac{\sqrt{\pi}\,\left(\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma(b-a+1)\right)}{2\left(\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)\right)}\right)=$$
$$\left(\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma(-a+b+1)\left(\psi^{(0)}\left(-a+b+\frac{1}{2}\right)-\psi^{(0)}(-a+b+1)-\psi^{(0)}(a)+\psi^{(0)}\left(a+\frac{1}{2}\right)\right)\right)\Big/\left(2\,\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(-a+b+\frac{1}{2}\right)\right)$$

From:

$$\frac{\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma(-a+b+1)}{2\,\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(-a+b+\frac{1}{2}\right)}$$

For a = 2 , b = 3, we obtain :

(sqrt(π) Γ(2 + 1/2) Γ(3 + 1) Γ(-2 + 3 + 1))/(2 Γ(2) Γ(3 + 1/2) Γ(-2 + 3 + 1/2))

Input

$$\frac{\sqrt{\pi}\,\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)}$$

$\Gamma(x)$ is the gamma function

Exact result

$$\frac{12}{5}$$

Decimal form

2.4

2.4

The study of this function provides the following representations:

Alternative representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{1!\times\frac{3}{2}!\times 3!\,\sqrt{\pi}}{2\times\frac{1}{2}!\times 1!\times\frac{5}{2}!}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{e^{0}\,e^{-\log(2)+\log(12)}\,e^{-\log G(5/2)+\log G(7/2)}\,\sqrt{\pi}}{2\,e^{0}\,e^{-\log G(3/2)+\log G(5/2)}\,e^{-\log G(7/2)+\log G(9/2)}}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{\Gamma(2,0)\,\Gamma\left(\frac{5}{2},0\right)\Gamma(4,0)\,\sqrt{\pi}}{2\,\Gamma\left(\frac{3}{2},0\right)\Gamma(2,0)\,\Gamma\left(\frac{7}{2},0\right)}$$

Series representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\Gamma(4)\,\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\,\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{7}{2}\right)} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{\Gamma\left(\frac{5}{2}\right)\Gamma(4)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(\pi-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k\,z_0^{-k}}{k!}}{2\,\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{7}{2}\right)}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{\sqrt{-1+\pi}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{(-1+\pi)^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{5}{2}-z_0\right)^{k_2}(4-z_0)^{k_3}\,\Gamma^{(k_2)}(z_0)\,\Gamma^{(k_3)}(z_0)}{k_2!\,k_3!}}{2\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{7}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

Integral representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \int_0^1\int_0^1 \log^{3/2}\left(\frac{1}{t_1}\right)\log^3\left(\frac{1}{t_2}\right)dt_2\,dt_1$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{1}{2}\exp\left(\int_0^1 \frac{-3-3\sqrt{x}+2x^{3/2}+2x^2+2x^{7/2}}{2\left(1+\sqrt{x}\right)\log(x)}\,dx\right)\sqrt{\pi}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)} = \frac{1}{2}\exp\left(-\frac{3\gamma}{2}+\int_0^1 \frac{x^{3/2}-x^{5/2}+x^{7/2}-x^4-\log\left(x^{3/2}\right)+\log\left(x^{5/2}\right)-\log\left(x^{7/2}\right)+\log\left(x^4\right)}{\log(x)-x\log(x)}\,dx\right)\sqrt{\pi}$$

γ is the Euler-Mascheroni constant

From:

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx = \Big(a^2\,(a+1)^2\,\big(\big(a^4-a^2\,(2\,b^2+6\,b+5)+(b^2+3\,b+2)^2\big)\log(a^2+x^2) - \big(a^4+4\,a^3+a^2\,(-2\,b^2-6\,b+1)-2\,a\,(2\,b^2+6\,b+3)+b\,(b^3+6\,b^2+11\,b+6)\big)\log(a^2+2\,a+x^2+1)\big)+(2\,a+1)\,x^2\Big)\Big/\big(2\,(2\,a+1)\,(b^2+3\,b+2)^2\big) + \text{constant}$$

log(x) is the natural logarithm

For:

$$\Big(a^2\,(a+1)^2\,\big((a-b-1)\,(a+b+2)\,\big((a-b-2)\,(a+b+1)\log(a^2+x^2)-(a-b)\,(a+b+3)\log\big((a+1)^2+x^2\big)\big)+(2\,a+1)\,x^2\big)\Big)\Big/\big(2\,(2\,a+1)\,(b+1)^2\,(b+2)^2\big) + \text{constant}$$

If we consider x = 1, we obtain:

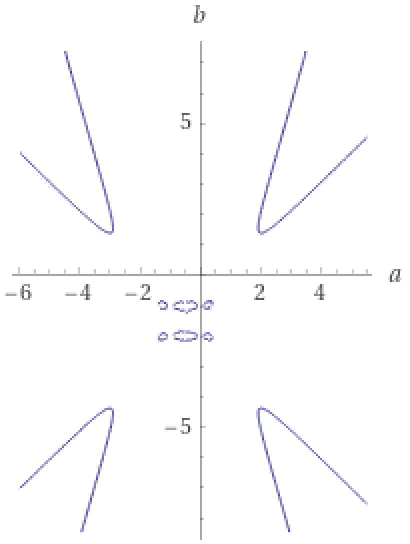
(a^2 (a+1)^2 ((a-b-1)(a+b+2)((a-b-2) (a+b+1) log(a^2+1)-(a-b) (a+b+3) log((a+1)^2+1)) + (2a+1)))/(2 (2a+1) (b+1)^2 (b+2)^2) = 2.4

Input

$$\Big(a^2\,(a+1)^2\,\big((a-b-1)\,(a+b+2)\,\big((a-b-2)\,(a+b+1)\log(a^2+1) - (a-b)\,(a+b+3)\log\big((a+1)^2+1\big)\big)+(2\,a+1)\big)\Big)\Big/\big(2\,(2\,a+1)\,(b+1)^2\,(b+2)^2\big) = 2.4$$

log(x) is the natural logarithm

Implicit plot



The study of this function provides the following representations:

Solutions for the variable b

$$b \approx 0.5 \left(-\sqrt{\left(9 - \left(2 \left(10 a^6 \log\left((a+1)^2+1\right) + 40 a^5 \log\left((a+1)^2+1\right) + 40 a^4 \log\left((a+1)^2+1\right) + 20 a^2 \log\left(a^2+1\right) - 10 a^2 \log\left((a+1)^2+1\right) - 10 a^6 \log\left(a^2+1\right) - 20 a^5 \log\left(a^2+1\right) + 10 a^4 \log\left(a^2+1\right) + 40 a^3 \log\left(a^2+1\right) - \sqrt{\left(\left(-10 a^6 \log\left((a+1)^2+1\right) - 40 a^5 \log\left((a+1)^2+1\right) - 40 a^4 \log\left((a+1)^2+1\right) - 20 a^2 \log\left(a^2+1\right) + 10 a^2 \log\left((a+1)^2+1\right) + 10 a^6 \log\left(a^2+1\right) + 20 a^5 \log\left(a^2+1\right) - 10 a^4 \log\left(a^2+1\right) - 40 a^3 \log\left(a^2+1\right) + 192 a + 96\right)^2 - 4 \left(-5 a^4 \log\left((a+1)^2+1\right) - 10 a^3 \log\left((a+1)^2+1\right) + 5 a^2 \log\left(a^2+1\right) - 5 a^2 \log\left((a+1)^2+1\right) + 5 a^4 \log\left(a^2+1\right) + 10 a^3 \log\left(a^2+1\right) - 48 a - 24\right) \left(-5 a^8 \log\left((a+1)^2+1\right) - 30 a^7 \log\left((a+1)^2+1\right) - 50 a^6 \log\left((a+1)^2+1\right) + 10 a^5 + 25 a^4 + 55 a^4 \log\left((a+1)^2+1\right) + 20 a^3 + 30 a^3 \log\left((a+1)^2+1\right) + 5 a^2 + 20 a^2 \log\left(a^2+1\right) + 5 a^8 \log\left(a^2+1\right) + 10 a^7 \log\left(a^2+1\right) - 20 a^6 \log\left(a^2+1\right) - 50 a^5 \log\left(a^2+1\right) - 5 a^4 \log\left(a^2+1\right) + 40 a^3 \log\left(a^2+1\right) - 192 a - 96\right)\right)} - 192 a - 96\right)\right) \Big/ \left(-5 a^4 \log\left((a+1)^2+1\right) - 10 a^3 \log\left((a+1)^2+1\right) + 5 a^2 \log\left(a^2+1\right) - 5 a^2 \log\left((a+1)^2+1\right) + 5 a^4 \log\left(a^2+1\right) + 10 a^3 \log\left(a^2+1\right) - 48 a - 24\right)\right)} - 3\right)$$

$$
\begin{aligned}
b \approx {} & \\
& 0.5 \Big(\sqrt{\;} \Big(9 - \Big(2 \Big(10 a^6 \log\big((a+1)^2 + 1\big) + 40 a^5 \log\big((a+1)^2 + 1\big) + 40 a^4 \log\big((a+1)^2 + {} \\
& \qquad 1\big) + 20 a^2 \log\big(a^2 + 1\big) - {} \\
& \qquad 10 a^2 \log\big((a+1)^2 + 1\big) - 10 a^6 \log\big(a^2 + 1\big) - {} \\
& \qquad 20 a^5 \log\big(a^2 + 1\big) + 10 a^4 \log\big(a^2 + 1\big) + 40 a^3 \log\big(a^2 + 1\big) - {} \\
& \qquad \sqrt{\;} \Big(\Big(-10 a^6 \log\big((a+1)^2 + 1\big) - 40 a^5 \log\big((a+1)^2 + 1\big) - {} \\
& \qquad\qquad 40 a^4 \log\big((a+1)^2 + 1\big) - 20 a^2 \log\big(a^2 + 1\big) + {} \\
& \qquad\qquad 10 a^2 \log\big((a+1)^2 + 1\big) + 10 a^6 \log\big(a^2 + 1\big) + {} \\
& \qquad\qquad 20 a^5 \log\big(a^2 + 1\big) - 10 a^4 \log\big(a^2 + 1\big) - {} \\
& \qquad\qquad 40 a^3 \log\big(a^2 + 1\big) + 192 a + 96 \Big)^2 - 4 \\
& \qquad\quad \Big(-5 a^4 \log\big((a+1)^2 + 1\big) - 10 a^3 \log\big((a+1)^2 + 1\big) + {} \\
& \qquad\qquad 5 a^2 \log\big(a^2 + 1\big) - 5 a^2 \log\big((a+1)^2 + 1\big) + {} \\
& \qquad\qquad 5 a^4 \log\big(a^2 + 1\big) + 10 a^3 \log\big(a^2 + 1\big) - 48 a - 24 \Big) \\
& \qquad\quad \Big(-5 a^8 \log\big((a+1)^2 + 1\big) - 30 a^7 \log\big((a+1)^2 + 1\big) - {} \\
& \qquad\qquad 50 a^6 \log\big((a+1)^2 + 1\big) + 10 a^5 + {} \\
& \qquad\qquad 25 a^4 + 55 a^4 \log\big((a+1)^2 + 1\big) + {} \\
& \qquad\qquad 20 a^3 + 30 a^3 \log\big((a+1)^2 + 1\big) + 5 a^2 + {} \\
& \qquad\qquad 20 a^2 \log\big(a^2 + 1\big) + 5 a^8 \log\big(a^2 + 1\big) + {} \\
& \qquad\qquad 10 a^7 \log\big(a^2 + 1\big) - 20 a^6 \log\big(a^2 + 1\big) - {} \\
& \qquad\qquad 50 a^5 \log\big(a^2 + 1\big) - 5 a^4 \log\big(a^2 + 1\big) + 40 a^3 \\
& \qquad\qquad\quad \log\big(a^2 + 1\big) - 192 a - 96 \Big) \Big) - 192 a - 96 \Big) \Big) \Big/ \\
& \quad \Big(-5 a^4 \log\big((a+1)^2 + 1\big) - 10 a^3 \log\big((a+1)^2 + 1\big) + {} \\
& \qquad 5 a^2 \log\big(a^2 + 1\big) - {} \\
& \qquad 5 a^2 \log\big((a+1)^2 + 1\big) + {} \\
& \qquad 5 a^4 \log\big(a^2 + 1\big) + {} \\
& \qquad 10 a^3 \log\big(a^2 + 1\big) - {} \\
& \qquad 48 a - 24 \Big) \Big) - 3 \Big)
\end{aligned}
$$

$$
\begin{aligned}
b \approx 0.5 & \Big(-\sqrt{\Big(9-\Big(2\Big(10\,a^6 \log\big((a+1)^2+1\big)+40\,a^5 \log\big((a+1)^2+1\big)+40\,a^4 \log\big((a+1)^2+{}} \\
& \quad 1\big)+20\,a^2 \log\big(a^2+1\big)-10\,a^2 \log\big((a+1)^2+1\big)- \\
& \quad 10\,a^6 \log\big(a^2+1\big)-20\,a^5 \log\big(a^2+1\big)+ \\
& \quad 10\,a^4 \log\big(a^2+1\big)+40\,a^3 \log\big(a^2+1\big)+ \\
& \quad \sqrt{\Big(\Big(-10\,a^6 \log\big((a+1)^2+1\big)-40\,a^5 \log\big((a+1)^2+1\big)-} \\
& \qquad 40\,a^4 \log\big((a+1)^2+1\big)-20\,a^2 \log\big(a^2+1\big)+ \\
& \qquad 10\,a^2 \log\big((a+1)^2+1\big)+10\,a^6 \log\big(a^2+1\big)+ \\
& \qquad 20\,a^5 \log\big(a^2+1\big)-10\,a^4 \log\big(a^2+1\big)- \\
& \qquad 40\,a^3 \log\big(a^2+1\big)+192\,a+96\Big)^2- \\
& \qquad 4\,\Big(-5\,a^4 \log\big((a+1)^2+1\big)-10\,a^3 \log\big((a+1)^2+1\big)+ \\
& \qquad 5\,a^2 \log\big(a^2+1\big)-5\,a^2 \log\big((a+1)^2+1\big)+5\,a^4 \\
& \qquad \log\big(a^2+1\big)+10\,a^3 \log\big(a^2+1\big)-48\,a-24\Big) \\
& \qquad \Big(-5\,a^8 \log\big((a+1)^2+1\big)-30\,a^7 \log\big((a+1)^2+1\big)- \\
& \qquad 50\,a^6 \log\big((a+1)^2+1\big)+10\,a^5+ \\
& \qquad 25\,a^4+55\,a^4 \log\big((a+1)^2+1\big)+ \\
& \qquad 20\,a^3+30\,a^3 \log\big((a+1)^2+1\big)+5\,a^2+ \\
& \qquad 20\,a^2 \log\big(a^2+1\big)+5\,a^8 \log\big(a^2+1\big)+ \\
& \qquad 10\,a^7 \log\big(a^2+1\big)-20\,a^6 \log\big(a^2+1\big)- \\
& \qquad 50\,a^5 \log\big(a^2+1\big)-5\,a^4 \log\big(a^2+1\big)+40\,a^3 \\
& \qquad \log\big(a^2+1\big)-192\,a-96\Big)\Big)-192\,a-96\Big)\Big)\Big/ \\
& \quad \Big(-5\,a^4 \log\big((a+1)^2+1\big)-10\,a^3 \log\big((a+1)^2+1\big)+ \\
& \quad 5\,a^2 \log\big(a^2+1\big)- \\
& \quad 5\,a^2 \log\big((a+1)^2+1\big)+ \\
& \quad 5\,a^4 \log\big(a^2+1\big)+ \\
& \quad 10\,a^3 \log\big(a^2+1\big)- \\
& \quad 48\,a-24\Big)\Big)-3\Big)
\end{aligned}
$$

$$
\begin{aligned}
b \approx{} & \\
& 0.5 \Bigg(\sqrt{\Big(9 - \Big(2 \Big(10\, a^6 \log\big((a+1)^2+1\big) + 40\, a^5 \log\big((a+1)^2+1\big) + 40\, a^4 \log\big((a+1)^2 +}\\
& \quad 1\big) + 20\, a^2 \log\big(a^2+1\big) - \\
& \quad 10\, a^2 \log\big((a+1)^2+1\big) - 10\, a^6 \log\big(a^2+1\big) - \\
& \quad 20\, a^5 \log\big(a^2+1\big) + 10\, a^4 \log\big(a^2+1\big) + 40\, a^3 \log\big(a^2+1\big) + \\
& \quad \sqrt{\Big(\big(-10\, a^6 \log\big((a+1)^2+1\big) - 40\, a^5 \log\big((a+1)^2+1\big) -} \\
& \qquad 40\, a^4 \log\big((a+1)^2+1\big) - 20\, a^2 \log\big(a^2+1\big) + \\
& \qquad 10\, a^2 \log\big((a+1)^2+1\big) + 10\, a^6 \log\big(a^2+1\big) + \\
& \qquad 20\, a^5 \log\big(a^2+1\big) - 10\, a^4 \log\big(a^2+1\big) - \\
& \qquad 40\, a^3 \log\big(a^2+1\big) + 192\, a + 96\big)^2 - 4 \\
& \qquad \big(-5\, a^4 \log\big((a+1)^2+1\big) - 10\, a^3 \log\big((a+1)^2+1\big) + \\
& \qquad 5\, a^2 \log\big(a^2+1\big) - 5\, a^2 \log\big((a+1)^2+1\big) + \\
& \qquad 5\, a^4 \log\big(a^2+1\big) + 10\, a^3 \log\big(a^2+1\big) - 48\, a - 24\big) \\
& \qquad \big(-5\, a^8 \log\big((a+1)^2+1\big) - 30\, a^7 \log\big((a+1)^2+1\big) - \\
& \qquad 50\, a^6 \log\big((a+1)^2+1\big) + 10\, a^5 + \\
& \qquad 25\, a^4 + 55\, a^4 \log\big((a+1)^2+1\big) + \\
& \qquad 20\, a^3 + 30\, a^3 \log\big((a+1)^2+1\big) + 5\, a^2 + \\
& \qquad 20\, a^2 \log\big(a^2+1\big) + 5\, a^8 \log\big(a^2+1\big) + \\
& \qquad 10\, a^7 \log\big(a^2+1\big) - 20\, a^6 \log\big(a^2+1\big) - \\
& \qquad 50\, a^5 \log\big(a^2+1\big) - 5\, a^4 \log\big(a^2+1\big) + 40\, a^3 \\
& \qquad \log\big(a^2+1\big) - 192\, a - 96)\Big) - 192\, a - 96\Big)\Big) \Big/ \\
& \big(-5\, a^4 \log\big((a+1)^2+1\big) - 10\, a^3 \log\big((a+1)^2+1\big) + \\
& \quad 5\, a^2 \log\big(a^2+1\big) - \\
& \quad 5\, a^2 \log\big((a+1)^2+1\big) + \\
& \quad 5\, a^4 \log\big(a^2+1\big) + \\
& \quad 10\, a^3 \log\big(a^2+1\big) - \\
& \quad 48\, a - 24)\big)\Big) - 3\Bigg)
\end{aligned}
$$

For b = 5, we obtain :

(a^2 (a+1)^2 ((a-5-1)(a+5+2)((a-5-2) (a+5+1) log(a^2+1)-(a-5) (a+5+3) log((a+1)^2+1)) + (2a+1)))/(2 (2a+1) (5+1)^2 (5+2)^2) = 2.4

Input

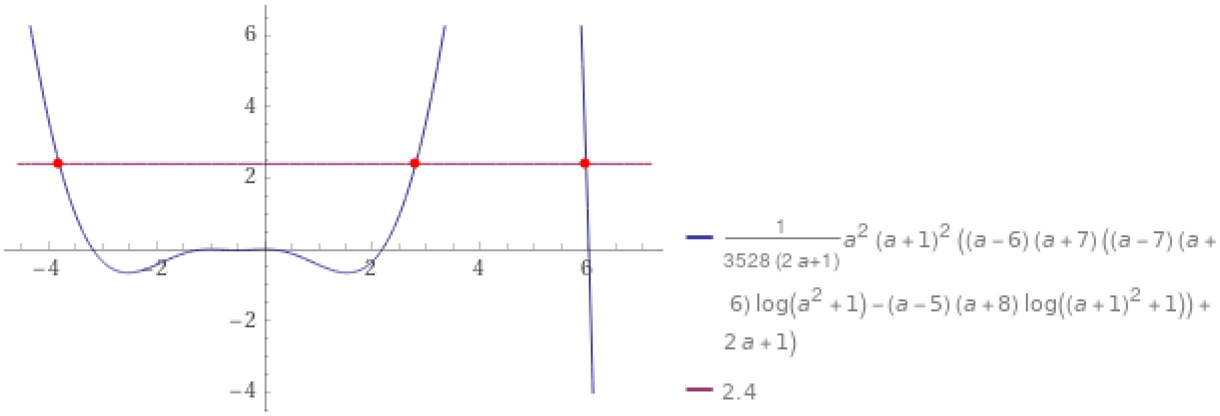
$$\left(a^2 (a+1)^2 \left((a-5-1)(a+5+2)\left((a-5-2)(a+5+1)\log\left(a^2+1\right) - (a-5)(a+5+3)\log\left((a+1)^2+1\right)\right) + (2a+1)\right)\right) / \left(2 (2a+1)(5+1)^2 (5+2)^2\right) = 2.4$$

log(x) is the natural logarithm

Result

$$\frac{1}{3528 (2a+1)} a^2 (a+1)^2 \left((a-6)(a+7)\left((a-7)(a+6)\log\left(a^2+1\right) - (a-5)(a+8)\log\left((a+1)^2+1\right)\right) + 2a+1\right) = 2.4$$

Plot



Solutions

$a = -6.95971$

$a = -3.80038$

$a = 2.80038$

$a = 5.95971$

Numerical solutions

$a \approx -6.95970536173634\ldots$

$a \approx -3.80038113094299\ldots$

$a \approx 2.80038113094299\ldots$

$a \approx 5.95970536173634\ldots$

For a = 5.9597053, we obtain :

(5.9597053^2(5.9597053+1)^2 ((5.9597053-6)(5.9597053+7)((5.9597053-7)(5.9597053+6) log(5.9597053^2+1)-(5.9597053-5)(5.9597053+8) ln((5.9597053+1)^2+1))+(2*5.9597053+1)))/(2(2*5.9597053+1)36*49)

Input interpretation

$$\left(5.9597053^2 (5.9597053+1)^2 \left((5.9597053-6)(5.9597053+7)\left((5.9597053-7)(5.9597053+6)\log\left(5.9597053^2+1\right) - ((5.9597053-5)(5.9597053+8))\log\left((5.9597053+1)^2+1\right)\right) + (2\times 5.9597053+1)\right)\right) / (2(2\times 5.9597053+1)(36\times 49))$$

$\log(x)$ is the natural logarithm

Result

$2.40000\ldots$

2.4

The study of this function provides the following representations:

Alternative representations

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7) \left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right) - ((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right) + (2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = \frac{1}{45\,579.7}\left(12.9194 - 0.522207\left(-12.4416\log(a)\log_a\left(1+5.95971^2\right) - 13.3972\log(a)\log_a\left(1+6.95971^2\right)\right)\right) 5.95971^2 \times 6.95971^2$$

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7) \left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right) - ((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right) + (2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = \frac{1}{45\,579.7}\left(12.9194 - 0.522207\left(-12.4416\log_e\left(1+5.95971^2\right) - 13.3972\log_e\left(1+6.95971^2\right)\right)\right) 5.95971^2 \times 6.95971^2$$

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7) \left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right) - ((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right) + (2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = \frac{1}{45\,579.7}\left(12.9194 - 0.522207\left(12.4416\,\mathrm{Li}_1\left(-5.95971^2\right) + 13.3972\,\mathrm{Li}_1\left(-6.95971^2\right)\right)\right) 5.95971^2 \times 6.95971^2$$

Series representations

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7)\left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right)-((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right)+(2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = 0.487644 + 0.245234 \log(35.5181) + 0.264069 \log(48.4375) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-0.264069\, e^{-3.88027 k} - 0.245234\, e^{-3.57004 k}\right)}{k}$$

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7)\left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right)-((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right)+(2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = 0.487644 + 0.490467\, i\, \pi \left\lfloor \frac{\arg(36.5181-x)}{2\pi} \right\rfloor + 0.528138\, i\, \pi \left\lfloor \frac{\arg(49.4375-x)}{2\pi} \right\rfloor + 0.509302 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-0.245234\,(36.5181-x)^k - 0.264069\,(49.4375-x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7)\left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right)-((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right)+(2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = 0.487644 + 0.245234 \left\lfloor \frac{\arg(36.5181-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 0.264069 \left\lfloor \frac{\arg(49.4375-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 0.509302 \log(z_0) + 0.245234 \left\lfloor \frac{\arg(36.5181-z_0)}{2\pi} \right\rfloor \log(z_0) + 0.264069 \left\lfloor \frac{\arg(49.4375-z_0)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-0.245234(36.5181-z_0)^k - 0.264069(49.4375-z_0)^k\right) z_0^{-k}}{k}$$

Integral representation

$$\left(5.95971^2 \left((5.95971+1)^2 \left((5.95971-6)(5.95971+7)\left((5.95971-7)(5.95971+6)\log\left(5.95971^2+1\right)-((5.95971-5)(5.95971+8))\log\left((5.95971+1)^2+1\right)\right)+(2\times 5.95971+1)\right)\right)\right) / (2(2\times 5.95971+1)(36\times 49)) = 0.487644 + \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{0.132034\, e^{-7.45032 s}\left(e^{3.57004 s}+0.928673\, e^{3.88027 s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{i\pi\,\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

Thence, we obtain, in conclusion:

$$\frac{\sqrt{\pi}\,\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma(-2+3+1)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma\left(-2+3+\frac{1}{2}\right)}$$

$\Gamma(x)$ is the gamma function

$$= \frac{12}{5}$$

$$= 2.4$$

is equal to :

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx = \left(a^2 (a+1)^2 \left(\left(a^4 - a^2\left(2b^2+6b+5\right)+\left(b^2+3b+2\right)^2\right)\log\left(a^2+x^2\right) - \left(a^4+4a^3+a^2\left(-2b^2-6b+1\right)-2a\left(2b^2+6b+3\right)+ b\left(b^3+6b^2+11b+6\right)\right)\log\left(a^2+2a+x^2+1\right)+(2a+1)x^2\right)\right) \Big/ \left(2(2a+1)\left(b^2+3b+2\right)^2\right) + \text{constant}$$

From:

$$\left(a^2 (a+1)^2 \left((a-b-1)(a+b+2)\left((a-b-2)(a+b+1)\log\left(a^2+x^2\right)-(a-b)(a+b+3)\log\left((a+1)^2+x^2\right)\right)+(2a+1)x^2\right)\right)\Big/\left(2(2a+1)(b+1)^2(b+2)^2\right) + \text{constant}$$

for b = 5, we obtain :

(a^2 (a+1)^2 ((a-5-1)(a+5+2)((a-5-2) (a+5+1) log(a^2+1)-(a-5) (a+5+3) log((a+1)^2+1)) + (2a+1)))/(2 (2a+1) (5+1)^2 (5+2)^2) = 2.4

$$\left(a^2 (a+1)^2 \left((a-5-1)(a+5+2)\left((a-5-2)(a+5+1)\log\left(a^2+1\right) - (a-5)(a+5+3)\log\left((a+1)^2+1\right)\right)+(2a+1)\right)\right)\Big/\left(2(2a+1)(5+1)^2(5+2)^2\right) = 2.4$$

and for a = 5.9597053, we obtain :

$$(5.9597053^2 (5.9597053 + 1)^2 ((5.9597053 - 6) (5.9597053 + 7) ((5.9597053 - 7) (5.9597053 + 6) \log(5.9597053^2 + 1) - ((5.9597053 - 5) (5.9597053 + 8)) \log((5.9597053 + 1)^2 + 1)) + (2\times 5.9597053 + 1))) / (2 (2\times 5.9597053 + 1) (36\times 49))$$

log(x) is the natural logarithm

= 2.40000...

Now, for a = 8 and b = 64 ,

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma(b-a+1)}{\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}$$

we obtain:

(sqrt(π) Γ(8 + 1/2) Γ(64 + 1) Γ(-8 + 64 + 1))/(2 Γ(8) Γ(64 + 1/2) Γ(-8 + 64 + 1/2))

Input

$$\frac{\sqrt{\pi}\,\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)}$$

Γ(x) is the gamma function

Exact result

13 479 973 333 575 319 897 333 507 543 509 815 336 818 572 211 270 286 240 551 805 124 608 / 90 861 297 665 263 806 397 852 504 259 184 867 012 180 701 150 408 708 366 012 722 575

Decimal approximation

148.35770212347189226490825070847834610348244898384466234402961177...

148.35770212….

The study of this function provides the following representations:

Alternative representations

$$\frac{\sqrt{\pi}\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{\frac{15}{2}!\times 56!\times 64!\,\sqrt{\pi}}{2\times 7!\times \frac{111}{2}!\times \frac{127}{2}!}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{\Gamma\left(\frac{17}{2},0\right)\Gamma(57,0)\,\Gamma(65,0)\,\sqrt{\pi}}{2\,\Gamma(8,0)\,\Gamma\left(\frac{113}{2},0\right)\Gamma\left(\frac{129}{2},0\right)}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{(1)_{\frac{15}{2}}\,(1)_{56}\,(1)_{64}\,\sqrt{\pi}}{2\,(1)_7\,(1)_{\frac{111}{2}}\,(1)_{\frac{127}{2}}}$$

Series representations

$$\frac{\sqrt{\pi}\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{17}{2}\right)\Gamma(57)\,\Gamma(65)\,\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\,\Gamma(8)\,\Gamma\left(\frac{113}{2}\right)\Gamma\left(\frac{129}{2}\right)}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{1}{2\,\Gamma(8)\,\Gamma\left(\frac{113}{2}\right)\Gamma\left(\frac{129}{2}\right)}\,\Gamma\left(\frac{17}{2}\right)\Gamma(57)\,\Gamma(65)$$
$$\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(\pi-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (\pi-z_0)^k\, z_0^{-k}}{k!}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} =$$
$$\left(\sqrt{-1+\pi}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}\frac{1}{k_2!\,k_3!\,k_4!}\,(-1+\pi)^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{17}{2}-z_0\right)^{k_2}\right.$$
$$\left.(57-z_0)^{k_3}\,(65-z_0)^{k_4}\,\Gamma^{(k_2)}(z_0)\,\Gamma^{(k_3)}(z_0)\,\Gamma^{(k_4)}(z_0)\right)\Big/$$
$$\left(2\left(\sum_{k=0}^{\infty}\frac{(8-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)\left(\sum_{k=0}^{\infty}\frac{\left(\frac{113}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{129}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)$$
for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

Integral representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} =$$
$$\int_0^1\int_0^1\int_0^1 \log^{15/2}\left(\frac{1}{t_1}\right)\log^{56}\left(\frac{1}{t_2}\right)\log^{64}\left(\frac{1}{t_3}\right)dt_3\,dt_2\,dt_1$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} =$$
$$\frac{1}{2}\exp\left(\int_0^1\frac{-3-3\sqrt{x}+2x^8+2x^{113/2}+2x^{129/2}}{2\left(1+\sqrt{x}\right)\log(x)}\,dx\right)\sqrt{\pi}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)\right)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} = \frac{1}{2}\exp\left(-\frac{3\gamma}{2} + \int_0^1 \frac{1}{\log(x) - x\log(x)}\left(x^8 - x^{17/2} + x^{113/2} - x^{57} + x^{129/2} - x^{65} - \log\left(x^8\right) + \log\left(x^{17/2}\right) - \log\left(x^{113/2}\right) + \log\left(x^{57}\right) - \log\left(x^{129/2}\right) + \log\left(x^{65}\right)\right) dx\right)\sqrt{\pi}$$

$\log(x)$ is the natural logarithm

γ is the Euler-Mascheroni constant

We obtain also:

(sqrt(π) Γ(8 + 1/2) Γ(64 + 1) Γ(-8 + 64 + 1))/(2 Γ(8) Γ(64 + 1/2) Γ(-8 + 64 + 1/2)) – 29 – Φ

Input

$$\frac{\sqrt{\pi}\,\Gamma\left(8+\frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)} - 29 - \Phi$$

$\Gamma(x)$ is the gamma function

Φ is the golden ratio conjugate

Exact result

$$10\,844\,995\,701\,282\,669\,511\,795\,784\,919\,993\,454\,193\,465\,331\,877\,908\,433\,697\,937\,436\,169\,933 / 90\,861\,297\,665\,263\,806\,397\,852\,504\,259\,184\,867\,012\,180\,701\,150\,408\,708\,366\,012\,722\,575 - \Phi$$

Exact form

$$10\,935\,856\,998\,947\,933\,318\,193\,637\,424\,252\,639\,060\,477\,512\,579\,058\,842\,406\,303\,448\,892\,508 / 90\,861\,297\,665\,263\,806\,397\,852\,504\,259\,184\,867\,012\,180\,701\,150\,408\,708\,366\,012\,722\,575 - \phi$$

ϕ is the golden ratio

Decimal approximation

118.73966813472199741670366387411270798576213980403889948189416314…

118.73966813…. result very near to the value of the following soliton mass, deriving from:

The total energy or the soliton mass for a single soliton becomes.

$$E = \int dx 2U(\phi) = \int dx \left(\frac{\lambda}{2}(\phi^2 - v^2)^2 \right) = \mp \frac{2\lambda v}{\sqrt{2m}} \int_0^{\pm v} d\phi \left(\phi^2 - v^2\right)$$
$$= \mp \frac{2\lambda v}{\sqrt{2m}} \left(\mp \frac{2v^3}{3} \right) = \frac{2\sqrt{2}m^3}{3\lambda}$$

(2*sqrt2*125.35^3)/(3*125.35^2)

Input interpretation

$$\frac{2\sqrt{2} \times 125.35^3}{3 \times 125.35^2}$$

Result

118.18111336231164291152778771979043609913891305233362731513120343
…

118.18111336…..

The study of this function provides the following representations:

Alternate form

(10 844 995 701 282 669 511 795 784 919 993 454 193 465 331 877 908 433 697 937 436 169 933 − 90 861 297 665 263 806 397 852 504 259 184 867 012 180 701 150 408 708 366 012 722 575 Φ) / 90 861 297 665 263 806 397 852 504 259 184 867 012 180 701 150 408 708 366 012 722 575

From:

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx = \Big(a^2 (a+1)^2 \big((a^4 - a^2 (2b^2 + 6b + 5) + (b^2 + 3b + 2)^2) \log(a^2 + x^2) - (a^4 + 4a^3 + a^2 (-2b^2 - 6b + 1) - 2a (2b^2 + 6b + 3) + b (b^3 + 6b^2 + 11b + 6)) \log(a^2 + 2a + x^2 + 1) + (2a + 1) x^2\big)\Big) \Big/ \left(2 (2a + 1) (b^2 + 3b + 2)^2\right) + \text{constant}$$

$$\Big(a^2 (a+1)^2 \big((a - b - 1)(a + b + 2)((a - b - 2)(a + b + 1) \log(a^2 + x^2) - (a - b)(a + b + 3) \log((a+1)^2 + x^2)) + (2a + 1) x^2\big)\Big) \Big/ \left(2 (2a + 1) (b + 1)^2 (b + 2)^2\right) + \text{constant}$$

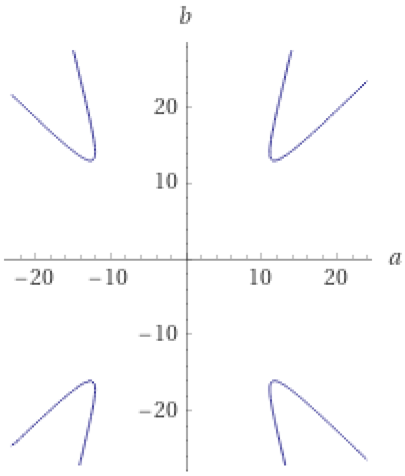
(a^2 (a + 1)^2 ((a - b - 1) (a + b + 2) ((a - b - 2) (a + b + 1) log(a^2 + 1) - (a - b) (a + b + 3) log((a + 1)^2 + 1)) + (2 a + 1)))/(2 (2 a + 1) (b + 1)^2 (b + 2)^2) = 148.357702

Input interpretation

$$\Big(a^2 (a+1)^2 \big((a - b - 1)(a + b + 2)((a - b - 2)(a + b + 1) \log(a^2 + 1) - (a - b)(a + b + 3) \log((a+1)^2 + 1)) + (2a + 1)\big)\Big) \Big/ \left(2 (2a + 1) (b + 1)^2 (b + 2)^2\right) = 148.357702$$

log(x) is the natural logarithm

Implicit plot



Solutions for the variable b:

$$
\begin{aligned}
b \approx 0.5 \Big(-\sqrt{\Big(9 - \Big(2 \Big(} & 500000\, a^6 \log\big((a+1)^2+1\big) + 2000000\, a^5 \log\big((a+1)^2+1\big) + \\
& 2000000\, a^4 \log\big((a+1)^2+1\big) + \\
& 1000000\, a^2 \log\big(a^2+1\big) - 500000\, a^2 \log\big((a+1)^2+1\big) - \\
& 500000\, a^6 \log\big(a^2+1\big) - 1000000\, a^5 \log\big(a^2+1\big) + \\
& 500000\, a^4 \log\big(a^2+1\big) + 2000000\, a^3 \log\big(a^2+1\big) - \\
& \sqrt{\Big(\Big(} -500000\, a^6 \log\big((a+1)^2+1\big) - 2000000\, a^5 \\
& \quad \log\big((a+1)^2+1\big) - 2000000\, a^4 \log\big(\\
& \quad (a+1)^2+1\big) - 1000000\, a^2 \log\big(a^2+1\big) + \\
& \quad 500000\, a^2 \log\big((a+1)^2+1\big) + 500000 \\
& \quad a^6 \log\big(a^2+1\big) + 1000000\, a^5 \log\big(a^2+1\big) - \\
& \quad 500000\, a^4 \log\big(a^2+1\big) - 2000000\, a^3 \log\big(\\
& \quad a^2+1\big) + 593430808\, a + 296715404\Big)^2 - \\
& \quad 4\, \Big(-250000\, a^4 \log\big((a+1)^2+1\big) - 500000\, a^3 \\
& \quad \log\big((a+1)^2+1\big) + 250000\, a^2 \log\big(a^2+1\big) - \\
& \quad 250000\, a^2 \log\big((a+1)^2+1\big) + \\
& \quad 250000\, a^4 \log\big(a^2+1\big) + 500000\, a^3 \\
& \quad \log\big(a^2+1\big) - 148357702\, a - 74178851\Big) \\
& \quad \Big(-250000\, a^8 \log\big((a+1)^2+1\big) - \\
& \quad 1500000\, a^7 \log\big((a+1)^2+1\big) - \\
& \quad 2500000\, a^6 \log\big((a+1)^2+1\big) + 500000\, a^5 + \\
& \quad 1250000\, a^4 + 2750000\, a^4 \log\big((a+1)^2+1\big) + \\
& \quad 1000000\, a^3 + 1500000\, a^3 \log\big((a+1)^2+1\big) + \\
& \quad 250000\, a^2 + 1000000\, a^2 \log\big(a^2+1\big) + \\
& \quad 250000\, a^8 \log\big(a^2+1\big) + 500000\, a^7 \\
& \quad \log\big(a^2+1\big) - 1000000\, a^6 \log\big(a^2+1\big) - \\
& \quad 2500000\, a^5 \log\big(a^2+1\big) - 250000\, a^4 \\
& \quad \log\big(a^2+1\big) + 2000000\, a^3 \log\big(a^2+1\big) - \\
& \quad 593430808\, a - 296715404\Big)\Big) - \\
& 593430808\, a - 296715404\Big)\Big) \Big/ \\
& \Big(-250000\, a^4 \log\big((a+1)^2+1\big) - 500000\, a^3 \\
& \quad \log\big((a+1)^2+1\big) + \\
& \quad 250000\, a^2 \log\big(a^2+1\big) - \\
& \quad 250000\, a^2 \log\big((a+1)^2+1\big) + \\
& \quad 250000\, a^4 \log\big(a^2+1\big) + \\
& \quad 500000\, a^3 \log\big(a^2+1\big) - \\
& \quad 148357702\, a - \\
& \quad 74178851\Big)\Big) - 3\Big)
\end{aligned}
$$

$$
\begin{aligned}
b \approx 0.5\Big(\sqrt{}\Big(9 - \Big(2\Big(&500000\,a^6 \log\big((a+1)^2+1\big) + 2000000\,a^5 \log\big((a+1)^2+1\big) + \\
&2000000\,a^4 \log\big((a+1)^2+1\big) + \\
&1000000\,a^2 \log\big(a^2+1\big) - 500000\,a^2 \log\big((a+1)^2+1\big) - \\
&500000\,a^6 \log\big(a^2+1\big) - 1000000\,a^5 \log\big(a^2+1\big) + \\
&500000\,a^4 \log\big(a^2+1\big) + 2000000\,a^3 \log\big(a^2+1\big) - \\
&\sqrt{}\Big(\Big(-500000\,a^6 \log\big((a+1)^2+1\big) - 2000000\,a^5 \\
&\quad \log\big((a+1)^2+1\big) - 2000000\,a^4 \log\big(\\
&\quad (a+1)^2+1\big) - 1000000\,a^2 \log\big(a^2+1\big) + \\
&\quad 500000\,a^2 \log\big((a+1)^2+1\big) + 500000 \\
&\quad a^6 \log\big(a^2+1\big) + 1000000\,a^5 \log\big(a^2+1\big) - \\
&\quad 500000\,a^4 \log\big(a^2+1\big) - 2000000\,a^3 \log\big(\\
&\quad a^2+1\big) + 593430808\,a + 296715404\Big)^2 - \\
&\quad 4\,\Big(-250000\,a^4 \log\big((a+1)^2+1\big) - 500000 \\
&\quad a^3 \log\big((a+1)^2+1\big) + 250000\,a^2 \log\big(a^2+1\big) - \\
&\quad 250000\,a^2 \log\big((a+1)^2+1\big) + \\
&\quad 250000\,a^4 \log\big(a^2+1\big) + 500000\,a^3 \\
&\quad \log\big(a^2+1\big) - 148357702\,a - 74178851\Big) \\
&\quad \Big(-250000\,a^8 \log\big((a+1)^2+1\big) - \\
&\quad 1500000\,a^7 \log\big((a+1)^2+1\big) - \\
&\quad 2500000\,a^6 \log\big((a+1)^2+1\big) + 500000\,a^5 + \\
&\quad 1250000\,a^4 + 2750000\,a^4 \log\big((a+1)^2+1\big) + \\
&\quad 1000000\,a^3 + 1500000\,a^3 \log\big((a+1)^2+1\big) + \\
&\quad 250000\,a^2 + 1000000\,a^2 \log\big(a^2+1\big) + \\
&\quad 250000\,a^8 \log\big(a^2+1\big) + 500000\,a^7 \\
&\quad \log\big(a^2+1\big) - 1000000\,a^6 \log\big(a^2+1\big) - \\
&\quad 2500000\,a^5 \log\big(a^2+1\big) - 250000\,a^4 \\
&\quad \log\big(a^2+1\big) + 2000000\,a^3 \log\big(a^2+1\big) - \\
&\quad 593430808\,a - 296715404\Big)\Big) - \\
&593430808\,a - 296715404\Big)\Big) \Big/ \\
\Big(&-250000\,a^4 \log\big((a+1)^2+1\big) - \\
&500000 \\
&a^3 \\
&\log\big((a+1)^2+1\big) + \\
&250000\,a^2 \log\big(a^2+1\big) - \\
&250000\,a^2 \\
&\log\big((a+1)^2+1\big) + \\
&250000\,a^4 \log\big(a^2+1\big) + \\
&500000\,a^3 \log\big(a^2+1\big) - \\
&148357702\,a - \\
&74178851\Big)\Big) - 3\Big)
\end{aligned}
$$

$$
\begin{aligned}
b \approx 0.5 \Big(-\sqrt{\Big(} 9 - \Big(2 \Big(& 500\,000\, a^6 \log\big((a+1)^2+1\big) + 2\,000\,000\, a^5 \log\big((a+1)^2+1\big) + \\
& 2\,000\,000\, a^4 \log\big((a+1)^2+1\big) + \\
& 1\,000\,000\, a^2 \log\big(a^2+1\big) - 500\,000\, a^2 \log\big((a+1)^2+1\big) - \\
& 500\,000\, a^6 \log\big(a^2+1\big) - 1\,000\,000\, a^5 \log\big(a^2+1\big) + \\
& 500\,000\, a^4 \log\big(a^2+1\big) + 2\,000\,000\, a^3 \log\big(a^2+1\big) + \\
& \sqrt{\big(}\big(\big(-500\,000\, a^6 \log\big((a+1)^2+1\big) - 2\,000\,000\, a^5 \\
& \quad \log\big((a+1)^2+1\big) - 2\,000\,000\, a^4 \log\big(\\
& \quad (a+1)^2+1\big) - 1\,000\,000\, a^2 \log\big(a^2+1\big) + \\
& \quad 500\,000\, a^2 \log\big((a+1)^2+1\big) + 500\,000 \\
& \quad a^6 \log\big(a^2+1\big) + 1\,000\,000\, a^5 \log\big(a^2+1\big) - \\
& \quad 500\,000\, a^4 \log\big(a^2+1\big) - 2\,000\,000\, a^3 \log\big(\\
& \quad a^2+1\big) + 593\,430\,808\, a + 296\,715\,404\big)^2 - \\
& \quad 4 \big(-250\,000\, a^4 \log\big((a+1)^2+1\big) - 500\,000\, a^3 \\
& \quad \log\big((a+1)^2+1\big) + 250\,000\, a^2 \log\big(a^2+1\big) - \\
& \quad 250\,000\, a^2 \log\big((a+1)^2+1\big) + \\
& \quad 250\,000\, a^4 \log\big(a^2+1\big) + 500\,000\, a^3 \\
& \quad \log\big(a^2+1\big) - 148\,357\,702\, a - 74\,178\,851\big) \\
& \quad \big(-250\,000\, a^8 \log\big((a+1)^2+1\big) - \\
& \quad 1\,500\,000\, a^7 \log\big((a+1)^2+1\big) - \\
& \quad 2\,500\,000\, a^6 \log\big((a+1)^2+1\big) + 500\,000\, a^5 + \\
& \quad 1\,250\,000\, a^4 + 2\,750\,000\, a^4 \log\big((a+1)^2+1\big) + \\
& \quad 1\,000\,000\, a^3 + 1\,500\,000\, a^3 \log\big((a+1)^2+1\big) + \\
& \quad 250\,000\, a^2 + 1\,000\,000\, a^2 \log\big(a^2+1\big) + \\
& \quad 250\,000\, a^8 \log\big(a^2+1\big) + 500\,000\, a^7 \\
& \quad \log\big(a^2+1\big) - 1\,000\,000\, a^6 \log\big(a^2+1\big) - \\
& \quad 2\,500\,000\, a^5 \log\big(a^2+1\big) - 250\,000\, a^4 \\
& \quad \log\big(a^2+1\big) + 2\,000\,000\, a^3 \log\big(a^2+1\big) - \\
& \quad 593\,430\,808\, a - 296\,715\,404\big)\big) - \\
& 593\,430\,808\, a - 296\,715\,404\Big)\Big) \Big/ \\
& \Big(-250\,000\, a^4 \log\big((a+1)^2+1\big) - 500\,000\, a^3 \\
& \quad \log\big((a+1)^2+1\big) + \\
& \quad 250\,000\, a^2 \log\big(a^2+1\big) - \\
& \quad 250\,000\, a^2 \log\big((a+1)^2+1\big) + \\
& \quad 250\,000\, a^4 \log\big(a^2+1\big) + \\
& \quad 500\,000\, a^3 \log\big(a^2+1\big) - \\
& \quad 148\,357\,702\, a - \\
& \quad 74\,178\,851\Big)\Big) - 3\Big)
\end{aligned}
$$

$$b \approx 0.5 \left(\sqrt{\left(9 - \left(2 \left(500000\, a^{6} \log\left((a+1)^{2}+1\right) + 2000000\, a^{5} \log\left((a+1)^{2}+1\right) + 2000000\, a^{4} \log\left((a+1)^{2}+1\right) + 1000000\, a^{2} \log\left(a^{2}+1\right) - 500000\, a^{2} \log\left((a+1)^{2}+1\right) - 500000\, a^{6} \log\left(a^{2}+1\right) - 1000000\, a^{5} \log\left(a^{2}+1\right) + 500000\, a^{4} \log\left(a^{2}+1\right) + 2000000\, a^{3} \log\left(a^{2}+1\right) + \sqrt{\left(\left(-500000\, a^{6} \log\left((a+1)^{2}+1\right) - 2000000\, a^{5} \log\left((a+1)^{2}+1\right) - 2000000\, a^{4} \log\left((a+1)^{2}+1\right) - 1000000\, a^{2} \log\left(a^{2}+1\right) + 500000\, a^{2} \log\left((a+1)^{2}+1\right) + 500000\, a^{6} \log\left(a^{2}+1\right) + 1000000\, a^{5} \log\left(a^{2}+1\right) - 500000\, a^{4} \log\left(a^{2}+1\right) - 2000000\, a^{3} \log\left(a^{2}+1\right) + 593430808\, a + 296715404\right)^{2} - 4 \left(-250000\, a^{4} \log\left((a+1)^{2}+1\right) - 500000\, a^{3} \log\left((a+1)^{2}+1\right) + 250000\, a^{2} \log\left(a^{2}+1\right) - 250000\, a^{2} \log\left((a+1)^{2}+1\right) + 250000\, a^{4} \log\left(a^{2}+1\right) + 500000\, a^{3} \log\left(a^{2}+1\right) - 148357702\, a - 74178851\right) \left(-250000\, a^{8} \log\left((a+1)^{2}+1\right) - 1500000\, a^{7} \log\left((a+1)^{2}+1\right) - 2500000\, a^{6} \log\left((a+1)^{2}+1\right) + 500000\, a^{5} + 1250000\, a^{4} + 2750000\, a^{4} \log\left((a+1)^{2}+1\right) + 1000000\, a^{3} + 1500000\, a^{3} \log\left((a+1)^{2}+1\right) + 250000\, a^{2} + 1000000\, a^{2} \log\left(a^{2}+1\right) + 250000\, a^{8} \log\left(a^{2}+1\right) + 500000\, a^{7} \log\left(a^{2}+1\right) - 1000000\, a^{6} \log\left(a^{2}+1\right) - 2500000\, a^{5} \log\left(a^{2}+1\right) - 250000\, a^{4} \log\left(a^{2}+1\right) + 2000000\, a^{3} \log\left(a^{2}+1\right) - 593430808\, a - 296715404\right)\right)} - 593430808\, a - 296715404\right)\right) \Big/ \left(-250000\, a^{4} \log\left((a+1)^{2}+1\right) - 500000\, a^{3} \log\left((a+1)^{2}+1\right) + 250000\, a^{2} \log\left(a^{2}+1\right) - 250000\, a^{2} \log\left((a+1)^{2}+1\right) + 250000\, a^{4} \log\left(a^{2}+1\right) + 500000\, a^{3} \log\left(a^{2}+1\right) - 148357702\, a - 74178851\right)\right)} - 3 \right)$$

for b = 10, we obtain :

(a^2 (a+1)^2 ((a-10-1)(a+10+2)((a-10-2) (a+10+1) log(a^2+1)-(a-5) (a+5+3) log((a+1)^2+1)) + (2a+1)))/(2 (2a+1) (5+1)^2 (5+2)^2) = 148.357702

Input interpretation

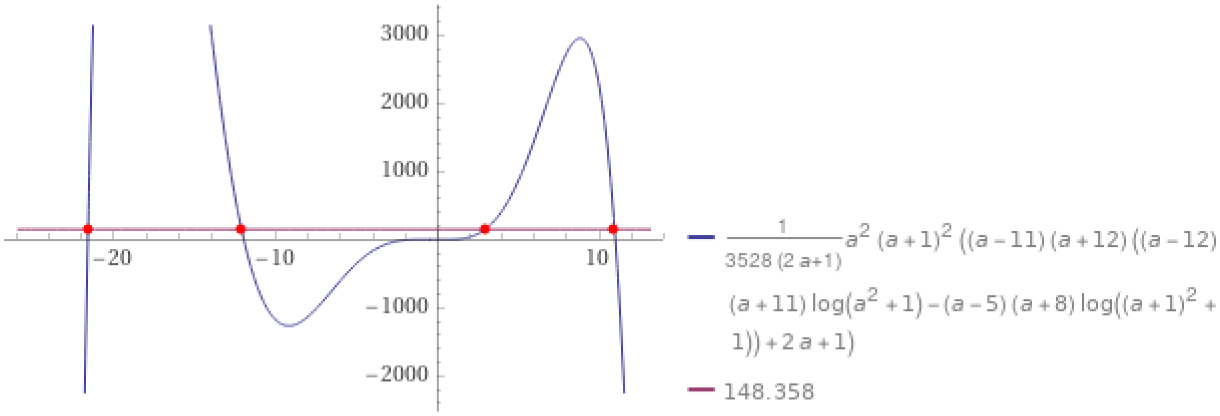
$$\left(a^2 (a+1)^2 \left((a-10-1)(a+10+2)\left((a-10-2)(a+10+1)\log\left(a^2+1\right) - (a-5)(a+5+3)\log\left((a+1)^2+1\right)\right) + (2a+1)\right)\right) / \left(2(2a+1)(5+1)^2(5+2)^2\right) = 148.357702$$

log(x) is the natural logarithm

Result

$$\frac{1}{3528(2a+1)} a^2 (a+1)^2 \left((a-11)(a+12)\left((a-12)(a+11)\log\left(a^2+1\right) - (a-5)(a+8)\log\left((a+1)^2+1\right)\right) + 2a+1\right) = 148.358$$

Plot



Solutions

$a = -21.5994$

$a = -12.1356$

$a = -0.499842$

$a = 2.93925$

$a = 10.9561$

Numerical solution

$a \approx 2.93924957302642\ldots$

For a = 2.93925, we obtain :

(2.93925^2 (2.93925+1)^2 ((2.93925-10-1)(2.93925+10+2)((2.93925-10-2)(2.93925+10+1) log(2.93925^2+1)-(2.93925-5) (2.93925+5+3) log((2.93925+1)^2+1)) + (2*2.93925+1)))/(2 (2*2.93925+1)36*49)

Input interpretation

$$\left(2.93925^2 (2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right) / (2(2\times 2.93925+1)\times 36\times 49)$$

log(x) is the natural logarithm

Result

148.358…

148.358….

The study of this function provides the following representations:

Alternative representations

$$\left(2.93925^2 \left((2.93925 + 1)^2 \left((2.93925 - 10 - 1)(2.93925 + 10 + 2) \left((2.93925 - 10 - 2)(2.93925 + 10 + 1)\log\left(2.93925^2 + 1\right) - (2.93925 - 5)(2.93925 + 5 + 3)\log\left((2.93925 + 1)^2 + 1\right)\right) + (2 \times 2.93925 + 1)\right)\right)\right) / (2(2 \times 2.93925 + 1)\, 36 \times 49) = \frac{1}{24\,267.3}\left(6.8785 - 120.422\left(-126.3\log(a)\log_a\left(1 + 2.93925^2\right) + 22.5431\log(a)\log_a\left(1 + 3.93925^2\right)\right)\right) 2.93925^2 \times 3.93925^2$$

$$\left(2.93925^2 \left((2.93925 + 1)^2 \left((2.93925 - 10 - 1)(2.93925 + 10 + 2) \left((2.93925 - 10 - 2)(2.93925 + 10 + 1)\log\left(2.93925^2 + 1\right) - (2.93925 - 5)(2.93925 + 5 + 3)\log\left((2.93925 + 1)^2 + 1\right)\right) + (2 \times 2.93925 + 1)\right)\right)\right) / (2(2 \times 2.93925 + 1)\, 36 \times 49) = \frac{1}{24\,267.3}\left(6.8785 - 120.422\left(-126.3\log_e\left(1 + 2.93925^2\right) + 22.5431\log_e\left(1 + 3.93925^2\right)\right)\right) 2.93925^2 \times 3.93925^2$$

$$\left(2.93925^2 \left((2.93925 + 1)^2 \left((2.93925 - 10 - 1)(2.93925 + 10 + 2) \left((2.93925 - 10 - 2)(2.93925 + 10 + 1)\log\left(2.93925^2 + 1\right) - (2.93925 - 5)(2.93925 + 5 + 3)\log\left((2.93925 + 1)^2 + 1\right)\right) + (2 \times 2.93925 + 1)\right)\right)\right) / (2(2 \times 2.93925 + 1)\, 36 \times 49) = \frac{1}{24\,267.3}\left(6.8785 - 120.422\left(126.3\,\mathrm{Li}_1\left(-2.93925^2\right) - 22.5431\,\mathrm{Li}_1\left(-3.93925^2\right)\right)\right) 2.93925^2 \times 3.93925^2$$

Series representations

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = 0.0379989 + 84.0206\log(8.63919) - 14.9967 \log(15.5177) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(14.9967\, e^{-2.74198\, k} - 84.0206\, e^{-2.15631\, k}\right)}{k}$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = 0.0379989 + 168.041\, i\, \pi \left\lfloor \frac{\arg(9.63919-x)}{2\pi} \right\rfloor - 29.9933\, i\, \pi \left\lfloor \frac{\arg(16.5177-x)}{2\pi} \right\rfloor + 69.0239 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-84.0206\,(9.63919-x)^k + 14.9967\,(16.5177-x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\,36\times 49) =$$

$$0.0379989 + 84.0206 \left\lfloor \frac{\arg(9.63919-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 14.9967 \left\lfloor \frac{\arg(16.5177-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 69.0239 \log(z_0) + 84.0206 \left\lfloor \frac{\arg(9.63919-z_0)}{2\pi} \right\rfloor \log(z_0) - 14.9967 \left\lfloor \frac{\arg(16.5177-z_0)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-84.0206\,(9.63919-z_0)^k + 14.9967\,(16.5177-z_0)^k\right) z_0^{-k}}{k}$$

Integral representation

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\,36\times 49) = 0.0379989 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{7.49834\, e^{-4.89829 s}\left(e^{2.15631 s} - 5.60261\, e^{2.74198 s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{i\pi\,\Gamma(1-s)}\, ds \quad \text{for } -1<\gamma<0$$

Now, for a = 64 and b = 128 ,

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a+\frac{1}{2}\right) \Gamma(b+1) \Gamma(b-a+1)}{\Gamma(a) \Gamma\left(b+\frac{1}{2}\right) \Gamma\left(b-a+\frac{1}{2}\right)}$$

we obtain:

(sqrt(π) Γ(64 + 1/2) Γ(128 + 1) Γ(-64 + 128 + 1))/(2 Γ(64) Γ(128 + 1/2) Γ(-64 + 128 + 1/2))

Input

$$\frac{\sqrt{\pi}\ \Gamma\left(64+\frac{1}{2}\right) \Gamma(128+1) \Gamma(-64+128+1)}{2\ \Gamma(64) \Gamma\left(128+\frac{1}{2}\right) \Gamma\left(-64+128+\frac{1}{2}\right)}$$

$\Gamma(x)$ is the gamma function

Exact result

1 852 673 427 797 059 126 777 135 760 139 006 525 652 319 754 650 249 024 631 321 344 126 610 074 238 976 / 2 884 329 411 724 603 169 044 874 178 931 143 443 870 105 850 987 581 016 304 218 283 632 259 375 395

Decimal approximation

642.32379986352025789577314705862646447370857549025692089819461318 …

642.32379986….

The study of this function provides the following representations:

Alternative representations

$$\frac{\sqrt{\pi}\ \left(\Gamma\left(64+\frac{1}{2}\right) \Gamma(128+1) \Gamma(-64+128+1)\right)}{2\ \Gamma(64) \Gamma\left(128+\frac{1}{2}\right) \Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{\frac{127}{2}! \times 64! \times 128!\ \sqrt{\pi}}{2 \times 63! \times \frac{127}{2}! \times \frac{255}{2}!}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{\Gamma\left(\frac{129}{2},0\right)\Gamma(65,0)\,\Gamma(129,0)\sqrt{\pi}}{2\,\Gamma(64,0)\,\Gamma\left(\frac{129}{2},0\right)\Gamma\left(\frac{257}{2},0\right)}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{(1)_{\frac{127}{2}}\,(1)_{64}\,(1)_{128}\sqrt{\pi}}{2\,(1)_{63}\,(1)_{\frac{127}{2}}\,(1)_{\frac{255}{2}}}$$

Series representations

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\Gamma(65)\,\Gamma(129)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\,\Gamma(64)\,\Gamma\left(\frac{257}{2}\right)}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{\Gamma(65)\,\Gamma(129)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(\pi-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}{2\,\Gamma(64)\,\Gamma\left(\frac{257}{2}\right)}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{\sqrt{-1+\pi}\,\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{(-1+\pi)^{-k_1}\binom{\frac{1}{2}}{k_1}(65-z_0)^{k_2}(129-z_0)^{k_3}\,\Gamma^{(k_2)}(z_0)\,\Gamma^{(k_3)}(z_0)}{k_2!\,k_3!}}{2\left(\sum_{k=0}^{\infty}\frac{(64-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{257}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

Integral representations

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \int_0^1\int_0^1 \log^{64}\left(\frac{1}{t_1}\right)\log^{128}\left(\frac{1}{t_2}\right)dt_2\,dt_1$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{1}{2}\exp\left(\int_0^1 \frac{-3-3\sqrt{x}+2x^{64}+2x^{129/2}+2x^{257/2}}{2\left(1+\sqrt{x}\right)\log(x)}\,dx\right)\sqrt{\pi}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(64+\frac{1}{2}\right)\Gamma(128+1)\Gamma(-64+128+1)\right)}{2\,\Gamma(64)\,\Gamma\left(128+\frac{1}{2}\right)\Gamma\left(-64+128+\frac{1}{2}\right)} = \frac{1}{2}\exp\left(-\frac{3\gamma}{2}+\int_0^1 \frac{x^{64}-x^{65}+x^{257/2}-x^{129}-\log\left(x^{64}\right)+\log\left(x^{65}\right)-\log\left(x^{257/2}\right)+\log\left(x^{129}\right)}{\log(x)-x\log(x)}\,dx\right)\sqrt{\pi}$$

$\log(x)$ is the natural logarithm

γ is the Euler-Mascheroni constant

From:

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx = \left(a^2(a+1)^2\left(\left(a^4-a^2\left(2b^2+6b+5\right)+\left(b^2+3b+2\right)^2\right)\log\left(a^2+x^2\right)-\left(a^4+4a^3+a^2\left(-2b^2-6b+1\right)-2a\left(2b^2+6b+3\right)+b\left(b^3+6b^2+11b+6\right)\right)\log\left(a^2+2a+x^2+1\right)+(2a+1)x^2\right)\right)\Big/\left(2(2a+1)\left(b^2+3b+2\right)^2\right) + \text{constant}$$

$$\left(a^2 (a+1)^2 \left((a-b-1)(a+b+2)\left((a-b-2)(a+b+1)\log\left(a^2+x^2\right)-(a-b)(a+b+3)\log\left((a+1)^2+x^2\right)\right)+(2a+1)x^2\right)\right)/\left(2(2a+1)(b+1)^2(b+2)^2\right) + \text{constant}$$

We consider:

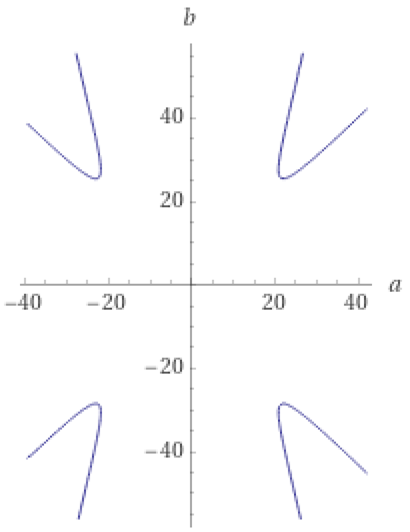
(a^2 (a + 1)^2 ((a - b - 1) (a + b + 2) ((a - b - 2) (a + b + 1) log(a^2 + 1) - (a - b) (a + b + 3) log((a + 1)^2 + 1)) + (2 a + 1)))/(2 (2 a + 1) (b + 1)^2 (b + 2)^2) = 642.323799

Input interpretation

$$\left(a^2 (a+1)^2 \left((a-b-1)(a+b+2)\left((a-b-2)(a+b+1)\log\left(a^2+1\right)-(a-b)(a+b+3)\log\left((a+1)^2+1\right)\right)+(2a+1)\right)\right)/\left(2(2a+1)(b+1)^2(b+2)^2\right) = 642.323799$$

$\log(x)$ is the natural logarithm

Implicit plot



Solutions for the variable b

$$b \approx 0.5\Big(-\sqrt{\Big(9-\Big(2\Big(1000000\,a^6 \log\big((a+1)^2+1\big)+4000000\,a^5 \log\big((a+1)^2+1\big)+4000000\,a^4 \log\big((a+1)^2+1\big)+2000000\,a^2 \log\big(a^2+1\big)-1000000\,a^2 \log\big((a+1)^2+1\big)-1000000\,a^6 \log\big(a^2+1\big)-2000000\,a^5 \log\big(a^2+1\big)+1000000\,a^4 \log\big(a^2+1\big)+4000000\,a^3 \log\big(a^2+1\big)-\sqrt{\Big(\Big(-1000000\,a^6 \log\big((a+1)^2+1\big)-4000000\,a^5 \log\big((a+1)^2+1\big)-4000000\,a^4 \log\big((a+1)^2+1\big)-2000000\,a^2 \log\big(a^2+1\big)+1000000\,a^2 \log\big((a+1)^2+1\big)+1000000\,a^6 \log\big(a^2+1\big)+2000000\,a^5 \log\big(a^2+1\big)-1000000\,a^4 \log\big(a^2+1\big)-4000000\,a^3 \log\big(a^2+1\big)+5138590392\,a+2569295196\Big)^2-4\Big(-500000\,a^4 \log\big((a+1)^2+1\big)-1000000\,a^3 \log\big((a+1)^2+1\big)+500000\,a^2 \log\big(a^2+1\big)-500000\,a^2 \log\big((a+1)^2+1\big)+500000\,a^4 \log\big(a^2+1\big)+1000000\,a^3 \log\big(a^2+1\big)-1284647598\,a-642323799\Big)\Big(-500000\,a^8 \log\big((a+1)^2+1\big)-3000000\,a^7 \log\big((a+1)^2+1\big)-5000000\,a^6 \log\big((a+1)^2+1\big)+1000000\,a^5+2500000\,a^4+5500000\,a^4 \log\big((a+1)^2+1\big)+2000000\,a^3+3000000\,a^3 \log\big((a+1)^2+1\big)+500000\,a^2+2000000\,a^2 \log\big(a^2+1\big)+500000\,a^8 \log\big(a^2+1\big)+1000000\,a^7 \log\big(a^2+1\big)-2000000\,a^6 \log\big(a^2+1\big)-5000000\,a^5 \log\big(a^2+1\big)-500000\,a^4 \log\big(a^2+1\big)+4000000\,a^3 \log\big(a^2+1\big)-5138590392\,a-2569295196\Big)\Big)}-5138590392\,a-2569295196\Big)\Big)\Big/\Big(-500000\,a^4 \log\big((a+1)^2+1\big)-1000000\,a^3 \log\big((a+1)^2+1\big)+500000\,a^2 \log\big(a^2+1\big)-500000\,a^2 \log\big((a+1)^2+1\big)+500000\,a^4 \log\big(a^2+1\big)+1000000\,a^3 \log\big(a^2+1\big)-1284647598\,a-642323799\Big)\Big)}-3\Big)$$

$$b \approx 0.5\Big(\sqrt{}\Big(9 - \Big(2\Big(1000000\, a^6 \log\big((a+1)^2+1\big) + 4000000\, a^5 \log\big((a+1)^2+1\big) + 4000000\, a^4 \log\big((a+1)^2+1\big) + 2000000\, a^2 \log\big(a^2+1\big) - 1000000\, a^2 \log\big((a+1)^2+1\big) - 1000000\, a^6 \log\big(a^2+1\big) - 2000000\, a^5 \log\big(a^2+1\big) + 1000000\, a^4 \log\big(a^2+1\big) + 4000000\, a^3 \log\big(a^2+1\big) - \sqrt{}\Big(\Big(-1000000\, a^6 \log\big((a+1)^2+1\big) - 4000000\, a^5 \log\big((a+1)^2+1\big) - 4000000\, a^4 \log\big((a+1)^2+1\big) - 2000000\, a^2 \log\big(a^2+1\big) + 1000000\, a^2 \log\big((a+1)^2+1\big) + 1000000\, a^6 \log\big(a^2+1\big) + 2000000\, a^5 \log\big(a^2+1\big) - 1000000\, a^4 \log\big(a^2+1\big) - 4000000\, a^3 \log\big(a^2+1\big) + 5138590392\, a + 2569295196\Big)^2 - 4 \Big(-500000\, a^4 \log\big((a+1)^2+1\big) - 1000000\, a^3 \log\big((a+1)^2+1\big) + 500000\, a^2 \log\big(a^2+1\big) - 500000\, a^2 \log\big((a+1)^2+1\big) + 500000\, a^4 \log\big(a^2+1\big) + 1000000\, a^3 \log\big(a^2+1\big) - 1284647598\, a - 642323799\Big) \Big(-500000\, a^8 \log\big((a+1)^2+1\big) - 3000000\, a^7 \log\big((a+1)^2+1\big) - 5000000\, a^6 \log\big((a+1)^2+1\big) + 1000000\, a^5 + 2500000\, a^4 + 5500000\, a^4 \log\big((a+1)^2+1\big) + 2000000\, a^3 + 3000000\, a^3 \log\big((a+1)^2+1\big) + 500000\, a^2 + 2000000\, a^2 \log\big(a^2+1\big) + 500000\, a^8 \log\big(a^2+1\big) + 1000000\, a^7 \log\big(a^2+1\big) - 2000000\, a^6 \log\big(a^2+1\big) - 5000000\, a^5 \log\big(a^2+1\big) - 500000\, a^4 \log\big(a^2+1\big) + 4000000\, a^3 \log\big(a^2+1\big) - 5138590392\, a - 2569295196\Big)\Big) - 5138590392\, a - 2569295196\Big)\Big) \Big/ \Big(-500000\, a^4 \log\big((a+1)^2+1\big) - 1000000\, a^3 \log\big((a+1)^2+1\big) + 500000\, a^2 \log\big(a^2+1\big) - 500000\, a^2 \log\big((a+1)^2+1\big) + 500000\, a^4 \log\big(a^2+1\big) + 1000000\, a^3 \log\big(a^2+1\big) - 1284647598\, a - 642323799\Big)\Big) - 3\Big)$$

$$
\begin{aligned}
b \approx 0.5\Big(-\sqrt{}\Big(9 - \Big(2\Big(&1000000\,a^6 \log\big((a+1)^2+1\big) + 4000000\,a^5 \log\big((a+1)^2+1\big) + \\
&4000000\,a^4 \log\big((a+1)^2+1\big) + \\
&2000000\,a^2 \log\big(a^2+1\big) - 1000000\,a^2 \log\big((a+1)^2+1\big) - \\
&1000000\,a^6 \log\big(a^2+1\big) - 2000000\,a^5 \log\big(a^2+1\big) + \\
&1000000\,a^4 \log\big(a^2+1\big) + 4000000\,a^3 \log\big(a^2+1\big) + \\
&\sqrt{}\big(\big(-1000000\,a^6 \log\big((a+1)^2+1\big) - 4000000\,a^5 \log\big(\\
&\quad (a+1)^2+1\big) - 4000000\,a^4 \log\big((a+1)^2 + \\
&\quad 1\big) - 2000000\,a^2 \log\big(a^2+1\big) + 1000000\,a^2 \\
&\quad \log\big((a+1)^2+1\big) + 1000000\,a^6 \log\big(a^2 + \\
&\quad 1\big) + 2000000\,a^5 \log\big(a^2+1\big) - 1000000\,a^4 \\
&\quad \log\big(a^2+1\big) - 4000000\,a^3 \log\big(a^2+1\big) + \\
&\quad 5138590392\,a + 2569295196\big)^2 - \\
&\quad 4\,\big(-500000\,a^4 \log\big((a+1)^2+1\big) - 1000000\,a^3 \\
&\quad \log\big((a+1)^2+1\big) + 500000\,a^2 \log\big(a^2+1\big) - \\
&\quad 500000\,a^2 \log\big((a+1)^2+1\big) + 500000 \\
&\quad a^4 \log\big(a^2+1\big) + 1000000\,a^3 \log\big(a^2+1\big) - \\
&\quad 1284647598\,a - 642323799\big) \\
&\quad \big(-500000\,a^8 \log\big((a+1)^2+1\big) - \\
&\quad 3000000\,a^7 \log\big((a+1)^2+1\big) - \\
&\quad 5000000\,a^6 \log\big((a+1)^2+1\big) + 1000000\,a^5 + \\
&\quad 2500000\,a^4 + 5500000\,a^4 \log\big((a+1)^2+1\big) + \\
&\quad 2000000\,a^3 + 3000000\,a^3 \log\big((a+1)^2+1\big) + \\
&\quad 500000\,a^2 + 2000000\,a^2 \log\big(a^2+1\big) + \\
&\quad 500000\,a^8 \log\big(a^2+1\big) + 1000000\,a^7 \\
&\quad \log\big(a^2+1\big) - 2000000\,a^6 \log\big(a^2+1\big) - \\
&\quad 5000000\,a^5 \log\big(a^2+1\big) - 500000\,a^4 \\
&\quad \log\big(a^2+1\big) + 4000000\,a^3 \log\big(a^2+1\big) - \\
&\quad 5138590392\,a - 2569295196\big)\big) - \\
&5138590392\,a - 2569295196\Big)\Big) \Big/ \\
\Big(-500000\,a^4 &\log\big((a+1)^2+1\big) - 1000000\,a^3 \\
&\log\big((a+1)^2+1\big) + \\
&500000\,a^2 \log\big(a^2+1\big) - \\
&500000\,a^2 \log\big((a+1)^2+1\big) + \\
&500000\,a^4 \log\big(a^2+1\big) + \\
&1000000\,a^3 \log\big(a^2+1\big) - \\
&1284647598\,a - \\
&642323799\Big)\Big) - 3\Big)
\end{aligned}
$$

$$
\begin{aligned}
b \approx 0.5 \Bigg(\sqrt{} \Bigg(9 - \Bigg(2 \Bigg(& 1\,000\,000\,a^6 \log\left((a+1)^2+1\right) + 4\,000\,000\,a^5 \log\left((a+1)^2+1\right) + \\
& 4\,000\,000\,a^4 \log\left((a+1)^2+1\right) + \\
& 2\,000\,000\,a^2 \log\left(a^2+1\right) - 1\,000\,000\,a^2 \log\left((a+1)^2+1\right) - \\
& 1\,000\,000\,a^6 \log\left(a^2+1\right) - 2\,000\,000\,a^5 \log\left(a^2+1\right) + \\
& 1\,000\,000\,a^4 \log\left(a^2+1\right) + 4\,000\,000\,a^3 \log\left(a^2+1\right) + \\
& \sqrt{} \Big(\big(-1\,000\,000\,a^6 \log\left((a+1)^2+1\right) - 4\,000\,000\,a^5 \log\big(\\
& \quad (a+1)^2+1\big) - 4\,000\,000\,a^4 \log\big((a+1)^2 + \\
& \quad 1\big) - 2\,000\,000\,a^2 \log\left(a^2+1\right) + 1\,000\,000\,a^2 \\
& \quad \log\left((a+1)^2+1\right) + 1\,000\,000\,a^6 \log\big(a^2 + \\
& \quad 1\big) + 2\,000\,000\,a^5 \log\left(a^2+1\right) - 1\,000\,000\,a^4 \\
& \quad \log\left(a^2+1\right) - 4\,000\,000\,a^3 \log\left(a^2+1\right) + \\
& \quad 5\,138\,590\,392\,a + 2\,569\,295\,196\big)^2 - 4 \\
& \quad \big(-500\,000\,a^4 \log\left((a+1)^2+1\right) - 1\,000\,000 \\
& \quad a^3 \log\left((a+1)^2+1\right) + 500\,000\,a^2 \log\left(a^2+1\right) - \\
& \quad 500\,000\,a^2 \log\left((a+1)^2+1\right) + 500\,000 \\
& \quad a^4 \log\left(a^2+1\right) + 1\,000\,000\,a^3 \log\left(a^2+1\right) - \\
& \quad 1\,284\,647\,598\,a - 642\,323\,799\big) \\
& \quad \big(-500\,000\,a^8 \log\left((a+1)^2+1\right) - \\
& \quad 3\,000\,000\,a^7 \log\left((a+1)^2+1\right) - \\
& \quad 5\,000\,000\,a^6 \log\left((a+1)^2+1\right) + 1\,000\,000\,a^5 + \\
& \quad 2\,500\,000\,a^4 + 5\,500\,000\,a^4 \log\left((a+1)^2+1\right) + \\
& \quad 2\,000\,000\,a^3 + 3\,000\,000\,a^3 \log\left((a+1)^2+1\right) + \\
& \quad 500\,000\,a^2 + 2\,000\,000\,a^2 \log\left(a^2+1\right) + \\
& \quad 500\,000\,a^8 \log\left(a^2+1\right) + 1\,000\,000\,a^7 \\
& \quad \log\left(a^2+1\right) - 2\,000\,000\,a^6 \log\left(a^2+1\right) - \\
& \quad 5\,000\,000\,a^5 \log\left(a^2+1\right) - 500\,000\,a^4 \\
& \quad \log\left(a^2+1\right) + 4\,000\,000\,a^3 \log\left(a^2+1\right) - \\
& \quad 5\,138\,590\,392\,a - 2\,569\,295\,196\big)\Big) - \\
& 5\,138\,590\,392\,a - 2\,569\,295\,196\Bigg)\Bigg) \Big/ \\
& \big(-500\,000\,a^4 \log\left((a+1)^2+1\right) - \\
& 1\,000\,000 \\
& a^3 \\
& \log\left((a+1)^2+1\right) + \\
& 500\,000\,a^2 \log\left(a^2+1\right) - \\
& 500\,000\,a^2 \\
& \log\left((a+1)^2+1\right) + \\
& 500\,000\,a^4 \log\left(a^2+1\right) + \\
& 1\,000\,000\,a^3 \log\left(a^2+1\right) - \\
& 1\,284\,647\,598\,a - \\
& 642\,323\,799\big)\Bigg) - 3\Bigg)
\end{aligned}
$$

for b = 40, we obtain :

(a^2 (a + 1)^2 ((a - 40 - 1) (a + 40 + 2) ((a - 40 - 2) (a + 40 + 1) log(a^2 + 1) - (a - 40) (a + 40 + 3) log((a + 1)^2 + 1)) + (2 a + 1)))/(2 (2 a + 1) (40 + 1)^2 (40 + 2)^2) = 642.323799

Input interpretation

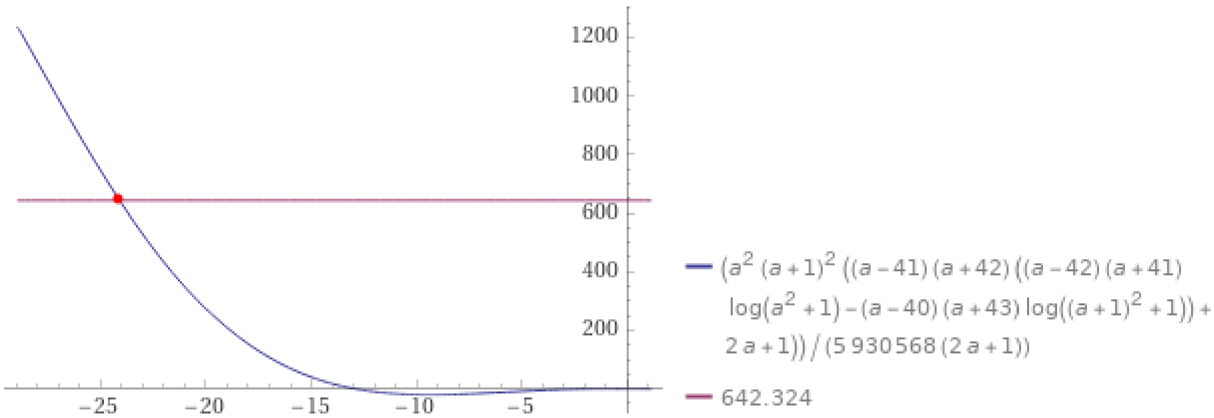
$$\left(a^2 (a+1)^2 \left((a-40-1)(a+40+2)\left((a-40-2)(a+40+1)\log\left(a^2+1\right) - (a-40)(a+40+3)\log\left((a+1)^2+1\right)\right) + (2a+1)\right)\right) / \left(2(2a+1)(40+1)^2(40+2)^2\right) = 642.323799$$

log(x) is the natural logarithm

Result

$$\frac{1}{5\,930\,568\,(2a+1)} a^2 (a+1)^2 \left((a-41)(a+42)\left((a-42)(a+41)\log\left(a^2+1\right) - (a-40)(a+43)\log\left((a+1)^2+1\right)\right) + 2a+1\right) = 642.324$$

Plot



Solutions

$a = -40.8018$

$a = -24.0923$

$a = 23.0923$

$a = 39.8018$

For a = 39.8018, we obtain :

(39.8018^2 (39.8018+1)^2 ((39.8018-40-1)(39.8018+40+2)((39.8018-40-2)(39.8018+40+1) log(39.8018^2+1)-(39.8018-40) (39.8018+40 + 3) log((39.8018+1)^2+1))+(2*39.8018+1)))/(2 (2*39.8018+1)(41)^2 (42)^2)

Input interpretation

$$\left(39.8018^2\,(39.8018+1)^2\left((39.8018-40-1)\,(39.8018+40+2)\left((39.8018-40-2)\,(39.8018+40+1)\log\left(39.8018^2+1\right)-(39.8018-40)\,(39.8018+40+3)\log\left((39.8018+1)^2+1\right)\right)+(2\times39.8018+1)\right)\right)/\left(2\left((2\times39.8018+1)\times41^2\right)\times42^2\right)$$

$\log(x)$ is the natural logarithm

Result

642.346671089816063066399849398202143794340906870699259604910788 53 …

642.346671089816…..

The study of this function provides the following representations:

Alternative representations

$$\left(39.8018^2\left((39.8018+1)^2\left((39.8018-40-1)\,(39.8018+40+2)\left((39.8018-40-2)\,(39.8018+40+1)\log\left(39.8018^2+1\right)-(39.8018-40)\,(39.8018+40+3)\log\left((39.8018+1)^2+1\right)\right)+(2\times39.8018+1)\right)\right)\right)/\left(2\left((2\times39.8018+1)\,41^2\right)42^2\right)=\frac{1}{161.207\times41^2\times42^2}\left(80.6036-98.0149\left(-177.619\log(a)\log_a\left(1+39.8018^2\right)+16.4113\log(a)\log_a\left(1+40.8018^2\right)\right)\right)39.8018^2\times40.8018^2$$

$$\left(39.8018^2\left((39.8018+1)^2\left((39.8018-40-1)(39.8018+40+2)\left((39.8018-40-2)(39.8018+40+1)\log\left(39.8018^2+1\right)-(39.8018-40)(39.8018+40+3)\log\left((39.8018+1)^2+1\right)\right)+(2\times 39.8018+1)\right)\right)\right)/\left(2\left((2\times 39.8018+1)\,41^2\right)42^2\right)=\frac{1}{161.207\times 41^2\times 42^2}\left(80.6036-98.0149\left(-177.619\log_e\left(1+39.8018^2\right)+16.4113\log_e\left(1+40.8018^2\right)\right)\right)39.8018^2\times 40.8018^2$$

$$\left(39.8018^2\left((39.8018+1)^2\left((39.8018-40-1)(39.8018+40+2)\left((39.8018-40-2)(39.8018+40+1)\log\left(39.8018^2+1\right)-(39.8018-40)(39.8018+40+3)\log\left((39.8018+1)^2+1\right)\right)+(2\times 39.8018+1)\right)\right)\right)/\left(2\left((2\times 39.8018+1)\,41^2\right)42^2\right)=\frac{1}{161.207\times 41^2\times 42^2}\left(80.6036-98.0149\left(177.619\,\mathrm{Li}_1\left(-39.8018^2\right)-16.4113\,\mathrm{Li}_1\left(-40.8018^2\right)\right)\right)39.8018^2\times 40.8018^2$$

Series representations

$$\left(39.8018^2\left((39.8018+1)^2\left((39.8018-40-1)(39.8018+40+2)\left((39.8018-40-2)(39.8018+40+1)\log\left(39.8018^2+1\right)-(39.8018-40)(39.8018+40+3)\log\left((39.8018+1)^2+1\right)\right)+(2\times 39.8018+1)\right)\right)\right)/\left(2\left((2\times 39.8018+1)\,41^2\right)42^2\right)=0.444701+96.0492\log(1584.18)-8.8746\log(1664.79)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(8.8746\,e^{-7.41745\,k}-96.0492\,e^{-7.36782\,k}\right)}{k}$$

$$\Big(39.8018^2 \big((39.8018+1)^2 \big((39.8018-40-1)(39.8018+40+2) \big((39.8018-40-2)(39.8018+40+1)\log\big(39.8018^2+1\big) - (39.8018-40)(39.8018+40+3) \log\big((39.8018+1)^2+1\big)\big) + (2\times 39.8018+1)\big)\big)\Big) \Big/ \Big(2\big((2\times 39.8018+1)\,41^2\big)\,42^2\Big) = 0.444701 + 192.098\, i\, \pi \left\lfloor \frac{\arg(1585.18-x)}{2\pi} \right\rfloor - 17.7492\, i\, \pi \left\lfloor \frac{\arg(1665.79-x)}{2\pi} \right\rfloor + 87.1746 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-96.0492\,(1585.18-x)^k + 8.8746\,(1665.79-x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\Big(39.8018^2 \big((39.8018+1)^2 \big((39.8018-40-1)(39.8018+40+2) \big((39.8018-40-2)(39.8018+40+1)\log\big(39.8018^2+1\big) - (39.8018-40)(39.8018+40+3) \log\big((39.8018+1)^2+1\big)\big) + (2\times 39.8018+1)\big)\big)\Big) \Big/ \Big(2\big((2\times 39.8018+1)\,41^2\big)\,42^2\Big) = 0.444701 + 96.0492 \left\lfloor \frac{\arg(1585.18-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 8.8746 \left\lfloor \frac{\arg(1665.79-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 87.1746 \log(z_0) + 96.0492 \left\lfloor \frac{\arg(1585.18-z_0)}{2\pi} \right\rfloor \log(z_0) - 8.8746 \left\lfloor \frac{\arg(1665.79-z_0)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-96.0492\,(1585.18-z_0)^k + 8.8746\,(1665.79-z_0)^k\right) z_0^{-k}}{k}$$

Integral representation

$$\big(39.8018^2 \big(((39.8018 + 1)^2 \big((39.8018 - 40 - 1)(39.8018 + 40 + 2) \big((39.8018 - 40 - 2)(39.8018 + 40 + 1)\log\big(39.8018^2 + 1\big) - (39.8018 - 40)(39.8018 + 40 + 3) \log\big((39.8018 + 1)^2 + 1\big)\big) + (2\times 39.8018 + 1)\big)\big)\big)\big) / \big(2\,\big((2\times 39.8018 + 1)\, 41^2\big)\, 42^2\big) = 0.444701 + \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} -\frac{4.4373\, e^{-14.7853\, s}\left(e^{7.36782\, s} - 10.8229\, e^{7.41745\, s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{i\,\pi\,\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

For a = 8 and b = 64 ,

from

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a + \frac{1}{2}\right)\Gamma(b+1)\,\Gamma(b-a+1)}{\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}$$

we obtain:

(sqrt(π) Γ(8 + 1/2) Γ(64 + 1) Γ(-8 + 64 + 1))/(2 Γ(8) Γ(64 + 1/2) Γ(-8 + 64 + 1/2))

Input

$$\frac{\sqrt{\pi}\ \Gamma\left(8 + \frac{1}{2}\right)\Gamma(64+1)\,\Gamma(-8+64+1)}{2\,\Gamma(8)\,\Gamma\left(64+\frac{1}{2}\right)\Gamma\left(-8+64+\frac{1}{2}\right)}$$

$\Gamma(x)$ is the gamma function

Exact result

13 479 973 333 575 319 897 333 507 543 509 815 336 818 572 211 270 286 240 551 805 124 608 / 90 861 297 665 263 806 397 852 504 259 184 867 012 180 701 150 408 708 366 012 722 575

Decimal approximation

148.35770212347189226490825070847834610348244898384466234402961177

…

148.357702123….

From:

$$\int \frac{\left(1+\frac{x^2}{(b+1)^2}\right)\left(1+\frac{x^2}{(b+2)^2}\right)x}{\left(1+\frac{x^2}{a^2}\right)\left(1+\frac{x^2}{(a+1)^2}\right)}\,dx = \Big(a^2 (a+1)^2 \big((a^4 - a^2 (2b^2 + 6b + 5) + (b^2 + 3b + 2)^2) \log(a^2 + x^2) - (a^4 + 4a^3 + a^2 (-2b^2 - 6b + 1) - 2a(2b^2 + 6b + 3) + b(b^3 + 6b^2 + 11b + 6)) \log(a^2 + 2a + x^2 + 1) + (2a+1)x^2\big)\Big) \Big/ \left(2(2a+1)(b^2 + 3b + 2)^2\right) + \text{constant}$$

$$\left(a^2 (a+1)^2 \left((a-b-1)(a+b+2)\left((a-b-2)(a+b+1)\log(a^2+x^2) - (a-b)(a+b+3)\log((a+1)^2+x^2)\right) + (2a+1)x^2\right)\right) \Big/ \left(2(2a+1)(b+1)^2(b+2)^2\right) + \text{constant}$$

(a^2 (a + 1)^2 ((a - b - 1) (a + b + 2) ((a - b - 2) (a + b + 1) log(a^2 + 1) - (a - b) (a + b + 3) log((a + 1)^2 + 1)) + (2 a + 1)))/(2 (2 a + 1) (b + 1)^2 (b + 2)^2) = 148.357702

For b = 10 and a = 2.93925, we obtain :

(2.93925^2 (2.93925+1)^2 ((2.93925-10-1)(2.93925+10+2)((2.93925-10-2) (2.93925+10+1) log(2.93925^2+1)-(2.93925-5) (2.93925+5+3) log((2.93925+1)^2+1)) + (2*2.93925+1)))/(2 (2*2.93925+1)36*49)

Input interpretation

$$\Big(2.93925^2 (2.93925+1)^2 \big((2.93925-10-1)(2.93925+10+2)\,((2.93925-10-2)(2.93925+10+1)\log(2.93925^2+1) - (2.93925-5)(2.93925+5+3)\log((2.93925+1)^2+1)) + (2\times 2.93925+1)\big)\Big) \Big/ \left(2(2\times 2.93925+1)\times 36\times 49\right)$$

log(x) is the natural logarithm

Result

148.358…

148.358….

The study of this function provides the following representations:

Alternative representations

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = \frac{1}{24267.3}\left(6.8785 - 120.422\left(-126.3\log(a)\log_a\left(1+2.93925^2\right) + 22.5431\log(a)\log_a\left(1+3.93925^2\right)\right)\right) 2.93925^2\times 3.93925^2$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = \frac{1}{24267.3}\left(6.8785 - 120.422\left(-126.3\log_e\left(1+2.93925^2\right) + 22.5431\log_e\left(1+3.93925^2\right)\right)\right) 2.93925^2\times 3.93925^2$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = \frac{1}{24267.3}\left(6.8785 - 120.422\left(126.3\,\mathrm{Li}_1\left(-2.93925^2\right) - 22.5431\,\mathrm{Li}_1\left(-3.93925^2\right)\right)\right) 2.93925^2\times 3.93925^2$$

Series representations

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = 0.0379989 + 84.0206\log(8.63919) - 14.9967\log(15.5177) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(14.9967\, e^{-2.74198\, k} - 84.0206\, e^{-2.15631\, k}\right)}{k}$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) / (2(2\times 2.93925+1)\, 36\times 49) = 0.0379989 + 168.041\, i\, \pi \left\lfloor \frac{\arg(9.63919-x)}{2\pi} \right\rfloor - 29.9933\, i\, \pi \left\lfloor \frac{\arg(16.5177-x)}{2\pi} \right\rfloor + 69.0239\log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-84.0206\,(9.63919-x)^k + 14.9967\,(16.5177-x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) \Big/ (2(2\times 2.93925+1)\,36\times 49) =$$

$$0.0379989 + 84.0206 \left\lfloor \frac{\arg(9.63919 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 14.9967 \left\lfloor \frac{\arg(16.5177 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 69.0239 \log(z_0) + 84.0206 \left\lfloor \frac{\arg(9.63919 - z_0)}{2\pi} \right\rfloor \log(z_0) - 14.9967 \left\lfloor \frac{\arg(16.5177 - z_0)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(-84.0206\,(9.63919 - z_0)^k + 14.9967\,(16.5177 - z_0)^k\right) z_0^{-k}}{k}$$

Integral representation

$$\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2) \left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right) - (2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right) + (2\times 2.93925+1)\right)\right)\right) \Big/ (2(2\times 2.93925+1)\,36\times 49) = 0.0379989 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{7.49834\, e^{-4.89829\, s} \left(e^{2.15631\, s} - 5.60261\, e^{2.74198\, s}\right) \Gamma(-s)^2\, \Gamma(1+s)}{i\,\pi\,\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

We obtain also:

233/(((2.93925^2 (2.93925+1)^2 ((2.93925-10-1)(2.93925+10+2)((2.93925-10-2)(2.93925+10+1) log(2.93925^2+1)-(2.93925-5) (2.93925+5+3) log((2.93925+1)^2+1)) + (2*2.93925+1)))/(2 (2*2.93925+1)36*49))-4)

Input interpretation

$$233/\left(\left(2.93925^2 (2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2)\left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)/(2(2\times 2.93925+1)\times 36\times 49)-4\right)$$

$\log(x)$ is the natural logarithm

Result

1.614045347919983251674706197108370472006412303951361285476086675 7...

1.6140453479…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

The study of this function provides the following representations:

Alternative representations

$$233/\left(\left(2.93925^2 \left((2.93925+1)^2 \left((2.93925-10-1)(2.93925+10+2)\left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)/(2(2\times 2.93925+1)\,36\times 49)-4\right) = \frac{233}{-4+\frac{\left(6.8785-120.422\left(-126.3\log_e\left(1+2.93925^2\right)+22.5431\log_e\left(1+3.93925^2\right)\right)\right)2.93925^2\times 3.93925^2}{24\,267.3}}$$

$$233/((2.93925^2((2.93925+1)^2((2.93925-10-1)(2.93925+10+2)((2.93925-10-2)(2.93925+10+1)\log(2.93925^2+1)-(2.93925-5)(2.93925+5+3)\log((2.93925+1)^2+1))+(2\times 2.93925+1))))/(2(2\times 2.93925+1)36\times 49)-4) = 233\Big/\Big(-4+\frac{1}{24\,267.3}\left(6.8785-120.422\left(-126.3\log(a)\log_a\left(1+2.93925^2\right)+22.5431\log(a)\log_a\left(1+3.93925^2\right)\right)\right)2.93925^2\times 3.93925^2\Big)$$

$$233/((2.93925^2((2.93925+1)^2((2.93925-10-1)(2.93925+10+2)((2.93925-10-2)(2.93925+10+1)\log(2.93925^2+1)-(2.93925-5)(2.93925+5+3)\log((2.93925+1)^2+1))+(2\times 2.93925+1))))/(2(2\times 2.93925+1)36\times 49)-4) = \frac{233}{-4+\frac{\left(6.8785-120.422\left(126.3\,\text{Li}_1\left(-2.93925^2\right)-22.5431\,\text{Li}_1\left(-3.93925^2\right)\right)\right)2.93925^2\times 3.93925^2}{24\,267.3}}$$

Series representations

$$233/((2.93925^2((2.93925+1)^2((2.93925-10-1)(2.93925+10+2)((2.93925-10-2)(2.93925+10+1)\log(2.93925^2+1)-(2.93925-5)(2.93925+5+3)\log((2.93925+1)^2+1))+(2\times 2.93925+1))))/(2(2\times 2.93925+1)36\times 49)-4) = 2.77313\Big/\left(-0.0471551+\log(8.63919)-0.178488\log(15.5177)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(0.178488\,e^{-2.74198\,k}-e^{-2.15631\,k}\right)}{k}\right)$$

$$233/\Big(\Big(2.93925^2\,\big((2.93925+1)^2\,\big((2.93925-10-1)\,(2.93925+10+2)\,\big((2.93925-10-2)\,(2.93925+10+1)\log\big(2.93925^2+1\big)-(2.93925-5)\,(2.93925+5+3)\log\big((2.93925+1)^2+1\big)\big)+(2\times 2.93925+1)\big)\big)\Big)\Big/\big(2\,(2\times 2.93925+1)\,36\times 49)-4\big)=1.38657\Big/$$

$$\left(-0.0235776+i\,\pi\left\lfloor\frac{\arg(9.63919-x)}{2\,\pi}\right\rfloor-0.178488\,i\,\pi\left\lfloor\frac{\arg(16.5177-x)}{2\,\pi}\right\rfloor+0.410756\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-0.5\,(9.63919-x)^k+0.0892441\,(16.5177-x)^k\right)x^{-k}}{k}\right)$$

for $x<0$

$$233/\Big(\Big(2.93925^2\,\big((2.93925+1)^2\,\big((2.93925-10-1)\,(2.93925+10+2)\,\big((2.93925-10-2)\,(2.93925+10+1)\log\big(2.93925^2+1\big)-(2.93925-5)\,(2.93925+5+3)\log\big((2.93925+1)^2+1\big)\big)+(2\times 2.93925+1)\big)\big)\Big)\Big/\big(2\,(2\times 2.93925+1)\,36\times 49)-4\big)=2.77313\Big/$$

$$\left(-0.0471551+\left\lfloor\frac{\arg(16.5177-z_0)}{2\,\pi}\right\rfloor\left(-0.178488\log\left(\frac{1}{z_0}\right)-0.178488\log(z_0)\right)+0.821512\log(z_0)+\left\lfloor\frac{\arg(9.63919-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-(9.63919-z_0)^k+0.178488\,(16.5177-z_0)^k\right)z_0^{-k}}{k}\right)$$

Integral representation

$$233/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-10-1)(2.93925+10+2)\left((2.93925-10-2)(2.93925+10+1)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)/(2(2\times 2.93925+1)\,36\times 49)-4\right)=$$

$$-\frac{58.8087\,i\,\pi}{i\,\pi+\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{1.89256\,e^{-4.89829\,s}\left(e^{2.15631\,s}-5.60261\,e^{2.74198\,s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s}$$

for $-1<\gamma<0$

We obtain also:

(236+3/2)/(((2.93925^2 (2.93925+1)^2 ((2.93925-11)(2.93925+12)((2.93925-10-2) (2.93925+11) log(2.93925^2+1)-(2.93925-5) (2.93925+5+3) log((2.93925+1)^2+1)) + (2*2.93925+1)))/(2 (2*2.93925+1)36*49))-4)

Input interpretation

$$\left(236+\frac{3}{2}\right)/\left(\left(2.93925^2\,(2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)/(2(2\times 2.93925+1)\times 36\times 49)-4\right)$$

log(x) is the natural logarithm

Result

1.6452178975579228423722863597134677558005275630405506665260540149...

$1.64521789755\ldots \approx \zeta(2)=\frac{\pi^2}{6}=1.644934\ldots$

The study of this function provides the following representations:

Alternative representations

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=\frac{475}{2\left(-4+\frac{(6.8785-120.422(-126.3\log_e(1+2.93925^2)+22.5431\log_e(1+3.93925^2)))\,2.93925^2\times 3.93925^2}{24\,267.3}\right)}$$

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=475\Big/\left(2\left(-4+\frac{1}{24\,267.3}\left(6.8785-120.422\left(-126.3\log(a)\log_a\left(1+2.93925^2\right)+22.5431\log(a)\log_a\left(1+3.93925^2\right)\right)\right)2.93925^2\times 3.93925^2\right)\right)$$

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=\frac{475}{2\left(-4+\frac{(6.8785-120.422(126.3\,\mathrm{Li}_1(-2.93925^2)-22.5431\,\mathrm{Li}_1(-3.93925^2)))\,2.93925^2\times 3.93925^2}{24\,267.3}\right)}$$

Series representations

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=$$

$$2.82669\Big/\left(-0.0471551+\log(8.63919)-0.178488\log(15.5177)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(0.178488\,e^{-2.74198\,k}-e^{-2.15631\,k}\right)}{k}\right)$$

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=$$

$$1.41334\Big/\left(-0.0235776+i\pi\left\lfloor\frac{\arg(9.63919-x)}{2\pi}\right\rfloor-0.178488\,i\pi\left\lfloor\frac{\arg(16.5177-x)}{2\pi}\right\rfloor+0.410756\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-0.5\,(9.63919-x)^k+0.0892441\,(16.5177-x)^k\right)x^{-k}}{k}\right)$$

for $x < 0$

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=$$

$$2.82669\Big/\left(-0.0471551+\left\lfloor\frac{\arg(16.5177-z_0)}{2\pi}\right\rfloor\left(-0.178488\log\left(\frac{1}{z_0}\right)-0.178488\log(z_0)\right)+0.821512\log(z_0)+\left\lfloor\frac{\arg(9.63919-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-(9.63919-z_0)^k+0.178488\,(16.5177-z_0)^k\right)z_0^{-k}}{k}\right)$$

Integral representation

$$\left(236+\frac{3}{2}\right)\Big/\left(\left(2.93925^2\left((2.93925+1)^2\left((2.93925-11)(2.93925+12)\left((2.93925-10-2)(2.93925+11)\log\left(2.93925^2+1\right)-(2.93925-5)(2.93925+5+3)\log\left((2.93925+1)^2+1\right)\right)+(2\times 2.93925+1)\right)\right)\right)\Big/(2(2\times 2.93925+1)\,36\times 49)-4\right)=$$

$$-\frac{59.9445\,i\,\pi}{i\,\pi+\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{1.89256\,e^{-4.89829\,s}\left(e^{2.15631\,s}-5.60261\,e^{2.74198\,s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for } -1<\gamma<0$$

We obtain also:

((236+3/2)/(((2.9392^2 (2.9392+1)^2 ((2.9392-11)(2.9392+12)((2.9392-12)(2.9392+11) log(2.9392^2+1)-(2.9392-5) (2.9392+8) log((2.9392+1)^2+1))+(2*2.9392+1)))/(2 (2*2.9392+1)36*49))-4))^15-21-e

Input interpretation

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\,(2.9392+1)^2\left((2.9392-11)\,(2.9392+12)\left((2.9392-12)\,(2.9392+11)\log\left(2.9392^2+1\right)-(2.9392-5)\,(2.9392+8)\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\Big/\left(2\,(2\times 2.9392+1)\times 36\times 49\right)-4\right)\right)^{15}-21-e$$

log(x) is the natural logarithm

Result

1729.16…

1729.16….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternative representations

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\left((2.9392+1)^2\left((2.9392-11)\,(2.9392+12)\left((2.9392-12)\,(2.9392+11)\log\left(2.9392^2+1\right)-(2.9392-5)\,(2.9392+8)\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\right)\Big/\left(2\,(2\times 2.9392+1)\,36\times 49\right)-4\right)\right)^{15}-21-e=-21-e+\left(475\Big/\left(2\left(-4+\frac{1}{24267.}\left(6.8784-120.422\left(-126.3\log_e\left(1+2.9392^2\right)+22.5435\log_e\left(1+3.9392^2\right)\right)\right)2.9392^2\times 3.9392^2\right)\right)\right)^{15}$$

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\,((2.9392+1)^2\,\left((2.9392-11)\,(2.9392+12)\,\left((2.9392-12)\,(2.9392+11)\,\log\left(2.9392^2+1\right)-(2.9392-5)\,(2.9392+8)\,\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\right)\Big/(2\,(2\times 2.9392+1)\,36\times 49)-4\right)\right)^{15}-21-e=$$

$$-21-e+\left(475\Big/\left(2\left(-4+\frac{1}{24\,267.}\left(6.8784-120.422\left(-126.3\log(a)\log_a\left(1+2.9392^2\right)+22.5435\log(a)\log_a\left(1+3.9392^2\right)\right)\right)2.9392^2\times 3.9392^2\right)\right)\right)^{15}$$

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\,((2.9392+1)^2\,\left((2.9392-11)\,(2.9392+12)\,\left((2.9392-12)\,(2.9392+11)\,\log\left(2.9392^2+1\right)-(2.9392-5)\,(2.9392+8)\,\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\right)\Big/(2\,(2\times 2.9392+1)\,36\times 49)-4\right)\right)^{15}-21-e=-21-e+$$

$$\left(\frac{475}{2\left(-4+\frac{(6.8784-120.422\,(126.3\,\mathrm{Li}_1(-2.9392^2)-22.5435\,\mathrm{Li}_1(-3.9392^2)))\,2.9392^2\times 3.9392^2}{24\,267.}\right)}\right)^{15}$$

Series representations

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\left((2.9392+1)^2\left((2.9392-11)(2.9392+12)\left((2.9392-12)(2.9392+11)\log\left(2.9392^2+1\right)-(2.9392-5)(2.9392+8)\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\right)\Big/\left(2(2\times 2.9392+1)\,36\times 49)-4\right)\right)^{15}-21-e=$$

$$-21-e+14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875\Big/\left(32\,768\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(\log(8.6389)-\sum_{k=1}^{\infty}\frac{(-0.115756)^k}{k}\right)+22.5435\left(\log(15.5173)-\sum_{k=1}^{\infty}\frac{(-0.0644442)^k}{k}\right)\right)\right)\right)^{15}\right)$$

$$\left(\left(236 + \frac{3}{2}\right) \Big/ \left(\left(2.9392^2 \left((2.9392 + 1)^2 \left((2.9392 - 11)(2.9392 + 12) \left((2.9392 - 12)(2.9392 + 11) \log\left(2.9392^2 + 1\right) - (2.9392 - 5)(2.9392 + 8) \log\left((2.9392 + 1)^2 + 1\right)\right) + (2 \times 2.9392 + 1)\right)\right)\right) \Big/ (2 (2 \times 2.9392 + 1)\, 36 \times 49) - 4\right)\right)^{15} - 21 - e =$$

$$-21 - e + 14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875 \Big/ \left(32\,768 \left(-4 + 0.00552406 \left(6.8784 - 120.422 \left(-126.3 \left(2 i \pi \left\lfloor \frac{\arg(9.6389 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (9.6389 - x)^k x^{-k}}{k}\right) + 22.5435 \left(2 i \pi \left\lfloor \frac{\arg(16.5173 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (16.5173 - x)^k x^{-k}}{k}\right)\right)\right)\right)^{15}\right)$$

for $x < 0$

$$\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\left((2.9392+1)^2\left((2.9392-11)(2.9392+12)\left((2.9392-12)(2.9392+11)\log\left(2.9392^2+1\right)-(2.9392-5)(2.9392+8)\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\right)\Big/\left(2(2\times 2.9392+1)\,36\times 49)-4\right)\right)^{15}-21-e=$$

$$-21-e+14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875\Big/\left(32\,768\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(\log(z_0)+\left\lfloor\frac{\arg(9.6389-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(9.6389-z_0)^k z_0^{-k}}{k}\right)+22.5435\left(\log(z_0)+\left\lfloor\frac{\arg(16.5173-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(16.5173-z_0)^k z_0^{-k}}{k}\right)\right)\right)\right)^{15}\right)$$

We obtain also:

(1/27(((236+3/2)/(((2.9392^2(3.9392)^2((2.9392-11)(2.9392+12)((2.9392-12)(13.9392)log(2.9392^2+1)-(2.9392-5)(10.9392)log((2.9392+1)^2+1))+(2*2.9392+1)))/(2(2*2.9392+1)36*49))-4))^15-21-π))^2-4+1/2

Input interpretation

$$\left(\frac{1}{27}\left(\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\times 3.9392^2\left((2.9392-11)(2.9392+12)\left((2.9392-12)\times 13.9392\log\left(2.9392^2+1\right)+(2.9392-5)\log\left((2.9392+1)^2+1\right)\times(-10.9392)\right)+(2\times 2.9392+1)\right)\right)\Big/\left(2(2\times 2.9392+1)(36\times 49)\right)-4\right)\right)^{15}-21-\pi\right)\right)^2-4+\frac{1}{2}$$

log(x) is the natural logarithm

Result

4096.02…

4096.02….≈ 4096 = 64^2

The study of this function provides the following representations:

Alternative representations

$$\left(\frac{1}{27}\left(\left(\left(236+\frac{3}{2}\right)\Big/\left((2.9392^2\times 3.9392^2\,((2.9392-11)\,(2.9392+12)\,((2.9392-12)\,13.9392\log\left(2.9392^2+1\right)-(2.9392-5)\,10.9392\log\left((2.9392+1)^2+1\right))+(2\times 2.9392+1)))\Big/(2\,(2\times 2.9392+1)\,(36\times 49))-4\right)\right)^{15}-21-\pi\right)\right)^2-4+\frac{1}{2}=-\frac{7}{2}+\left(\frac{1}{27}\left(-21-\pi+\left(475\Big/\left(2\left(-4+\frac{1}{24267.}\left(6.8784-120.422\left(-126.3\log_e\left(1+2.9392^2\right)+22.5435\log_e\left(1+3.9392^2\right)\right)\right)2.9392^2\times 3.9392^2\right)\right)\right)^{15}\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(236+\frac{3}{2}\right)\Big/\left((2.9392^2\times 3.9392^2\,((2.9392-11)\,(2.9392+12)\,((2.9392-12)\,13.9392\log\left(2.9392^2+1\right)-(2.9392-5)\,10.9392\log\left((2.9392+1)^2+1\right))+(2\times 2.9392+1)))\Big/(2\,(2\times 2.9392+1)\,(36\times 49))-4\right)\right)^{15}-21-\pi\right)\right)^2-4+\frac{1}{2}=-\frac{7}{2}+\left(\frac{1}{27}\left(-21-\pi+\left(475\Big/\left(2\left(-4+\frac{1}{24267.}\left(6.8784-120.422\left(-126.3\log(a)\log_a\left(1+2.9392^2\right)+22.5435\log(a)\log_a\left(1+3.9392^2\right)\right)\right)2.9392^2\times 3.9392^2\right)\right)\right)^{15}\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\times 3.9392^2\left((2.9392-11)\,(2.9392+12)\left((2.9392-12)\right.\right.\right.\right.\right.\right.$$

$$13.9392\log\left(2.9392^2+1\right)-(2.9392-5)$$

$$\left.10.9392\log\left((2.9392+1)^2+1\right)\right)+$$

$$\left.\left.(2\times 2.9392+1)\right)\right)\Big/\left(2\,(2\times 2.9392+1)\right.$$

$$\left.\left.\left.(36\times 49))-4\right)\right)^{15}-21-\pi\right)\Big)^2-4+\frac{1}{2}=$$

$$-\frac{7}{2}+\left(\frac{1}{27}\left(-21-\pi+\left(475\Big/\left(2\left(-4+\frac{1}{24\,267.}\left(6.8784-120.422\left(126.3\right.\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\mathrm{Li}_1\left(-2.9392^2\right)-22.5435\,\mathrm{Li}_1\left(-3.9392^2\right)\right)\right)$$

$$\left.\left.\left.\left.\left.\left.2.9392^2\times 3.9392^2\right)\right)\right)^{15}\right)\right)^2$$

Series representations

$$\left(\frac{1}{27}\left(\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\times 3.9392^2\left((2.9392-11)(2.9392+12)\left((2.9392-12)\,13.9392\log\left(2.9392^2+1\right)-(2.9392-5)\,10.9392\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\Big/\left(2(2\times 2.9392+1)(36\times 49)\right)-4\right)\right)^{15}-21-\pi\right)\right)^2-4+\frac{1}{2}=-\frac{469}{162}+\frac{14\pi}{243}+\frac{\pi^2}{729}+$$

$$199\,897\,926\,247\,620\,551\,792\,110\,523\,732\,099\,320\,281\,564\,713\,841\,504\,499\,214\,352\,108\,538\,150\,787\,353\,515\,625\Big/\left(782\,757\,789\,696\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(\log(8.6389)-\sum_{k=1}^{\infty}\frac{(-0.115756)^k}{k}\right)+22.5435\left(\log(15.5173)-\sum_{k=1}^{\infty}\frac{(-0.0644442)^k}{k}\right)\right)\right)\right)^{30}\right)-$$

$$98\,969\,684\,177\,193\,407\,055\,921\,852\,588\,653\,564\,453\,125\Big/\left(3\,981\,312\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(\log(8.6389)-\sum_{k=1}^{\infty}\frac{(-0.115756)^k}{k}\right)+22.5435\left(\log(15.5173)-\sum_{k=1}^{\infty}\frac{(-0.0644442)^k}{k}\right)\right)\right)\right)^{15}\right)-$$

$$(14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875\,\pi)\Big/\left(11\,943\,936\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(\log(8.6389)-\sum_{k=1}^{\infty}\frac{(-0.115756)^k}{k}\right)+22.5435\left(\log(15.5173)-\sum_{k=1}^{\infty}\frac{(-0.0644442)^k}{k}\right)\right)\right)\right)^{15}\right)$$

$$\left(\frac{1}{27}\left(\left(\left(\left(236+\frac{3}{2}\right)\Big/\left(\left(2.9392^2\times 3.9392^2\left((2.9392-11)(2.9392+12)\left((2.9392-12)\,13.9392\log\left(2.9392^2+1\right)-(2.9392-5)\,10.9392\log\left((2.9392+1)^2+1\right)\right)+(2\times 2.9392+1)\right)\right)\Big/\left(2\,(2\times 2.9392+1)\,(36\times 49)\right)-4\right)\right)^{15}-21-\pi\right)\right)^2-4+\frac{1}{2}=-\frac{469}{162}+\frac{14\,\pi}{243}+\frac{\pi^2}{729}+$$

$$199\,897\,926\,247\,620\,551\,792\,110\,523\,732\,099\,320\,281\,564\,713\,841\,504\,499\,214\ldots 352\,108\,538\,150\,787\,353\,515\,625\Big/\left(782\,757\,789\,696\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(2\,i\,\pi\left\lfloor\frac{\arg(9.6389-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-x)^k\,x^{-k}}{k}\right)+22.5435\left(2\,i\,\pi\left\lfloor\frac{\arg(16.5173-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-x)^k\,x^{-k}}{k}\right)\right)\right)\right)^{30}\right)-$$

$$98\,969\,684\,177\,193\,407\,055\,921\,852\,588\,653\,564\,453\,125\Big/\left(3\,981\,312\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(2\,i\,\pi\left\lfloor\frac{\arg(9.6389-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-x)^k\,x^{-k}}{k}\right)+22.5435\left(2\,i\,\pi\left\lfloor\frac{\arg(16.5173-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-x)^k\,x^{-k}}{k}\right)\right)\right)\right)^{15}\right)-$$

$$\left(14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875\,\pi\right)\Big/\left(11\,943\,936\left(-4+0.00552406\left(6.8784-120.422\left(-126.3\left(2\,i\,\pi\left\lfloor\frac{\arg(9.6389-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-x)^k\,x^{-k}}{k}\right)+22.5435\left(2\,i\,\pi\left\lfloor\frac{\arg(16.5173-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-x)^k\,x^{-k}}{k}\right)\right)\right)\right)^{15}\right)$$

for $x < 0$

$$\Bigg(\frac{1}{27}\Bigg(\Bigg(\Bigg(236+\frac{3}{2}\Bigg)\Big/\big((2.9392^2\times 3.9392^2\,((2.9392-11)\,(2.9392+12)$$

$$\big((2.9392-12)\,13.9392\log\big(2.9392^2+1\big)-$$

$$(2.9392-5)\,10.9392\log\big($$

$$(2.9392+1)^2+1\big)\big)+(2\times 2.9392+1)\big)\big)\Big/$$

$$(2\,(2\times 2.9392+1)\,(36\times 49))-4\big)\Bigg)^{15}-$$

$$21-\pi\Bigg)\Bigg)^2-4+\frac{1}{2}=-\frac{469}{162}+\frac{14\,\pi}{243}+\frac{\pi^2}{729}+$$

$$199\,897\,926\,247\,620\,551\,792\,110\,523\,732\,099\,320\,281\,564\,713\,841\,504\,499\,214\cdot$$

$$352\,108\,538\,150\,787\,353\,515\,625\Big/$$

$$\Bigg(782\,757\,789\,696\Bigg(-4+0.00552406\Bigg(6.8784-$$

$$120.422\Bigg(-126.3\Bigg(\log(z_0)+\left\lfloor\frac{\arg(9.6389-z_0)}{2\,\pi}\right\rfloor\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+$$

$$\log(z_0)\Bigg)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-z_0)^k\,z_0^{-k}}{k}\Bigg)+$$

$$22.5435\Bigg(\log(z_0)+\left\lfloor\frac{\arg(16.5173-z_0)}{2\,\pi}\right\rfloor$$

$$\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+\log(z_0)\Bigg)-$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-z_0)^k\,z_0^{-k}}{k}\Bigg)\Bigg)\Bigg)\Bigg)^{30}\Bigg)-$$

$$98\,969\,684\,177\,193\,407\,055\,921\,852\,588\,653\,564\,453\,125\Big/$$

$$\Bigg(3\,981\,312$$

$$\Bigg(-4+0.00552406\Bigg(6.8784-$$

$$120.422\Bigg(-126.3\Bigg(\log(z_0)+\left\lfloor\frac{\arg(9.6389-z_0)}{2\,\pi}\right\rfloor\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+$$

$$\log(z_0)\Bigg)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-z_0)^k\,z_0^{-k}}{k}\Bigg)+$$

$$22.5435\Bigg(\log(z_0)+\left\lfloor\frac{\arg(16.5173-z_0)}{2\,\pi}\right\rfloor$$

$$\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+\log(z_0)\Bigg)-$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-z_0)^k\,z_0^{-k}}{k}\Bigg)\Bigg)\Bigg)\Bigg)^{15}\Bigg)-$$

$$(14\,138\,526\,311\,027\,629\,579\,417\,407\,512\,664\,794\,921\,875$$

$$\pi)\Big/$$

$$\Bigg(11\,943\,936\Bigg(-4+0.00552406\Bigg(6.8784-120.422\Bigg(-126.3\Bigg(\log(z_0)+$$

$$\left\lfloor\frac{\arg(9.6389-z_0)}{2\,\pi}\right\rfloor\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+\log(z_0)\Bigg)-$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\,(9.6389-z_0)^k\,z_0^{-k}}{k}\Bigg)+22.5435$$

$$\Bigg(\log(z_0)+\left\lfloor\frac{\arg(16.5173-z_0)}{2\,\pi}\right\rfloor\Bigg(\log\Bigg(\frac{1}{z_0}\Bigg)+\log($$

$$z_0)\Bigg)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(16.5173-z_0)^k\,z_0^{-k}}{k}\Bigg)\Bigg)\Bigg)\Bigg)^{15}\Bigg)$$

Now, we analyze the following equation:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

We obtain:

(2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity

Input interpretation

$$\frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4\,k)!\,(1103 + 26\,390\,k)}{(k!)^4 \times 396^{4\,k}}$$

$n!$ is the factorial function

Result

$$\frac{1}{\pi} \approx 0.31831$$

0.31831

From the following expression:

$$24 = \frac{\pi\sqrt{142}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}.$$

we have:

(Pi*sqrt(142))/ln[sqrt(1/4*(10+11sqrt2))+sqrt(1/4*(10+7sqrt2))]

Input

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)}$$

Decimal approximation

24.000000000000000848609271479359429436295501181641940224711161612

…

≈ 24

The study of this function provides the following representations:

Alternate forms

$$\frac{2\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}$$

$$\frac{2\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}$$

$$\frac{\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}$$

Alternative representations

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=\frac{\pi\sqrt{142}}{\log_e\left(\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}\right)}$$

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=\frac{\pi\sqrt{142}}{\log(a)\log_a\left(\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}\right)}$$

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=-\frac{\pi\sqrt{142}}{\mathrm{Li}_1\left(1-\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}-\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}\right)}$$

Series representations

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=$$
$$\frac{\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\left(-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{2}{-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)^k}{k}}$$

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=$$
$$\frac{\sqrt{142}\,\pi}{\log\left(-1+\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{2}{-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)^k}{k}}$$

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=$$
$$-\left(\left(i\sqrt{142}\,\pi\right)\Big/\left(2\pi\left\lfloor\frac{\arg\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}-2x\right)}{2\pi}\right\rfloor-\right.\right.$$
$$\left.\left.i\left(\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}-2x\right)^k x^{-k}}{k}\right)\right)\right)$$
for $x<0$

Integral representations

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}=\frac{\sqrt{142}\,\pi}{\int_1^{\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}\frac{1}{t}\,dt}$$

$$\frac{\pi\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)} = \frac{2\,i\sqrt{142}\,\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(\frac{2}{-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)^s\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

Thence, inverting the previous expression

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}.$$

we obtain:

(((1/((2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity)))*sqrt(142))/ln[sqrt(1/4*(10+11sqrt2))+sqrt(1/4*(10+7sqrt2))]

Input interpretation

$$\frac{\frac{1}{\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103+26\,390\,k)}{(k!)^4\times 396^{4\,k}}}\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result

$$\frac{\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)} \approx 24$$

24

The study of this function provides the following representations:

Alternate forms

$$\frac{2\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}$$

$$\frac{2\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}$$

$$\frac{\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}$$

From which, we obtain:

72*(((1/((2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity)))*sqrt(142))/ln[sqrt(1/4*(10+11sqrt2))+sqrt(1/4*(10+7sqrt2))]+1

Input interpretation

$$72\times\frac{\frac{1}{\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103+26\,390\,k)}{(k!)^4\times 396^{4k}}}\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}+1$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result

$$1+\frac{72\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)}\approx 1729.$$

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternate forms

$$1+\frac{144\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}$$

$$1+\frac{144\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}$$

$$1+\frac{72\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}$$

(1/27((72*(((1/((2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity)))*sqrt(142))/ln[sqrt(1/4*(10+11sqrt2))+sqrt(1/4*(10+7sqrt2))])))^2

Input interpretation

$$\left(\frac{1}{27}\left(72\times\frac{\frac{1}{\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103+26390k)}{(k!)^4\times 396^{4k}}}\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}\right)\right)^2$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result

$$\frac{9088\,\pi^2}{9\log^2\left(\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)}\approx 4096$$

$4096 = 64^2$

The study of this function provides the following representations:

Alternate forms

$$\frac{36352\,\pi^2}{9\log^2\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}$$

$$\frac{36\,352\,\pi^2}{9\log^2\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}$$

$$(36352\,\pi^2)\Big/\left(9\left(-5\log(2)+2\log\left(\sqrt{2\left(10-7\sqrt{2}\right)}+2\sqrt{10-\sqrt{2}}+2^{3/4}\sqrt{7+5\sqrt{2}}+2\sqrt{10-i\sqrt{142}}+2\sqrt{10+i\sqrt{142}}\right)\right)^2\right)$$

And also:

(72*(((1/((2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity)))*sqrt(142))/ln[sqrt(1/4*(10+11sqrt2))+sqrt(1/4*(10+7sqrt2))]+1)^1/15

Input interpretation

$$\sqrt[15]{72\times\frac{\frac{1}{\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103+26\,390\,k)}{(k!)^4\times 396^{4k}}}\sqrt{142}}{\log\left(\sqrt{\frac{1}{4}\left(10+11\sqrt{2}\right)}+\sqrt{\frac{1}{4}\left(10+7\sqrt{2}\right)}\right)}+1}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result

$$\sqrt[15]{1+\frac{72\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\sqrt{10+7\sqrt{2}}+\frac{1}{2}\sqrt{10+11\sqrt{2}}\right)}}\approx 1.64382$$

$1.64382 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternate forms

$$\sqrt[15]{1+\frac{144\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}}$$

$$\sqrt[15]{1+\frac{144\sqrt{142}\,\pi}{\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}}$$

$$\sqrt[15]{1+\frac{72\sqrt{142}\,\pi}{\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}}$$

And we have also:

(36*(((1/((2sqrt2)/9801 sum ((4k)!(1103+26390k)) / ((k!)^4 396^(4k)), k=0..infinity)))))+5

Input interpretation

$$36\times\frac{1}{\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4\,k)!\,(1103+26\,390\,k)}{(k!)^4\times 396^{4\,k}}}+5$$

$n!$ is the factorial function

Result

$5+36\,\pi\approx 118.097$

118.097

result very near to the value of the following soliton mass:

From:

The total energy or the soliton mass for a single soliton becomes.

$$E = \int dx 2U(\phi) = \int dx \left(\frac{\lambda}{2}(\phi^2 - v^2)^2\right) = \mp \frac{2\lambda v}{\sqrt{2}m} \int_0^{\pm v} d\phi \left(\phi^2 - v^2\right)$$
$$= \mp \frac{2\lambda v}{\sqrt{2}m} \left(\mp \frac{2v^3}{3}\right) = \frac{2\sqrt{2}m^3}{3\lambda}$$

(2*sqrt2*125.35^3)/(3*125.35^2)

Input interpretation

$$\frac{2\sqrt{2} \times 125.35^3}{3 \times 125.35^2}$$

Result

118.18111336231164291152778771979043609913891305233362731513120343
...

118.18111336…..

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

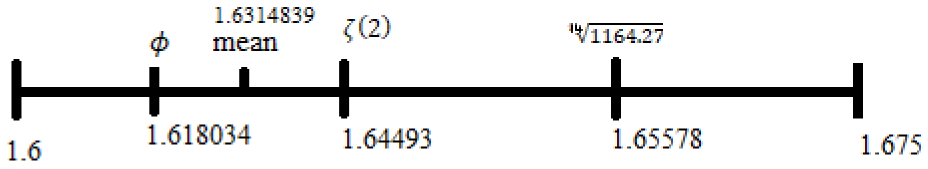
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

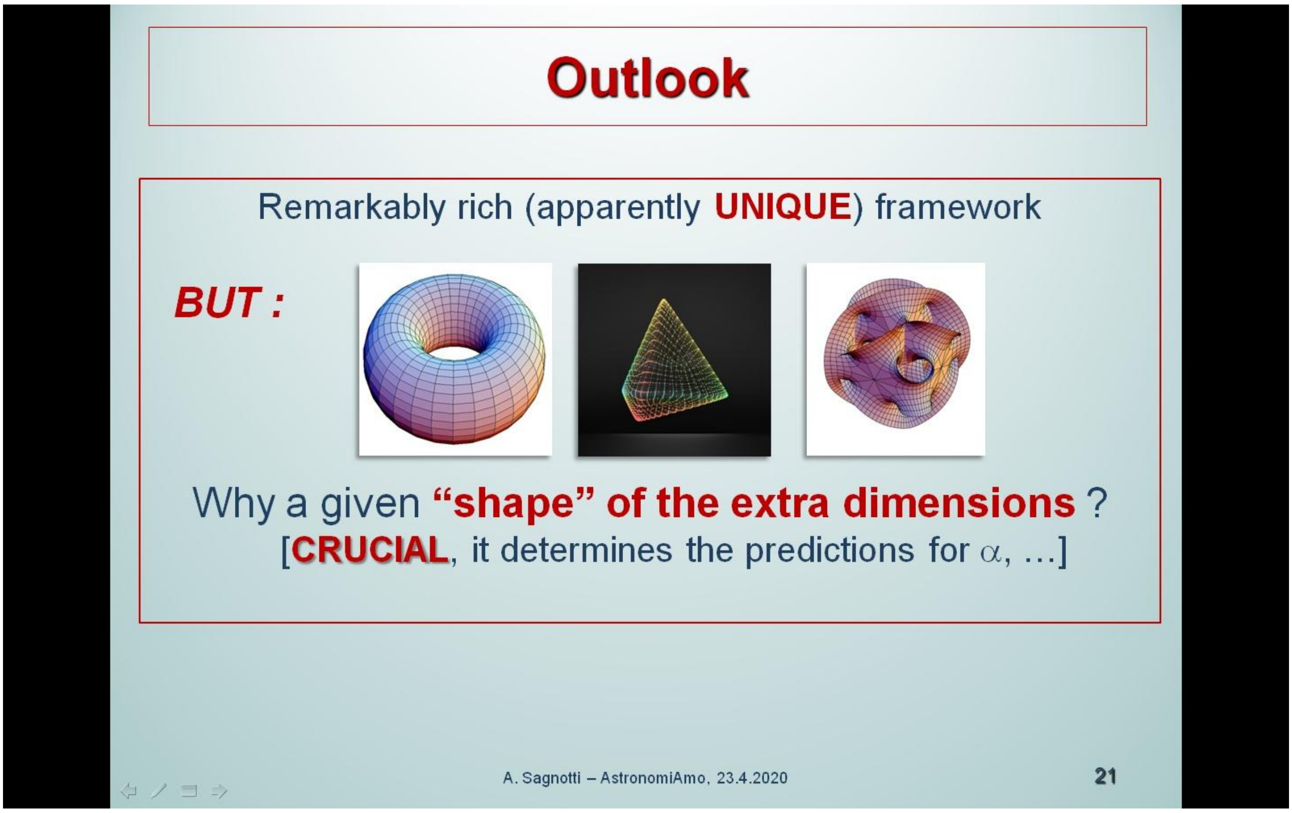
"Golden" Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to ζ(2) that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

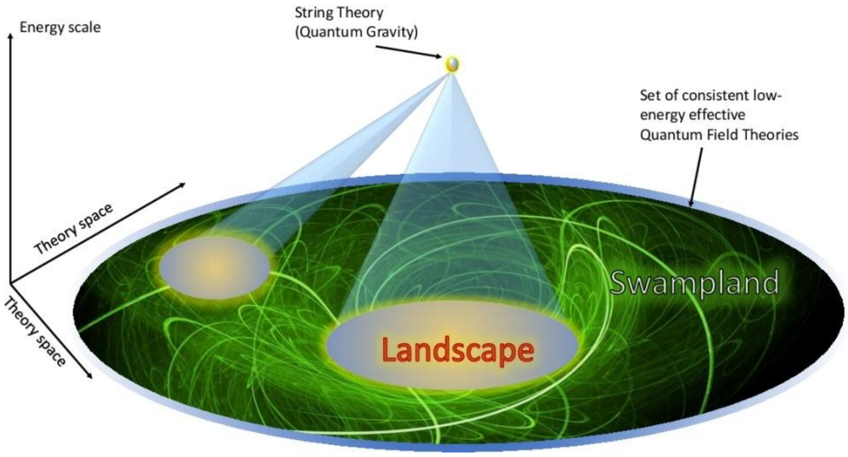


Fig. 1

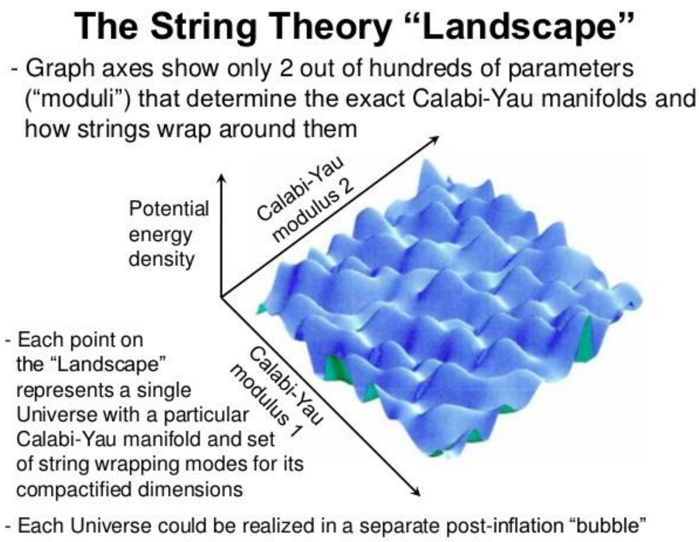


Fig. 2

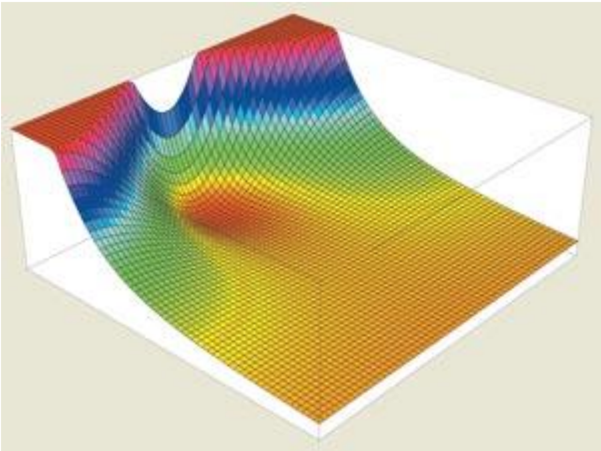


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

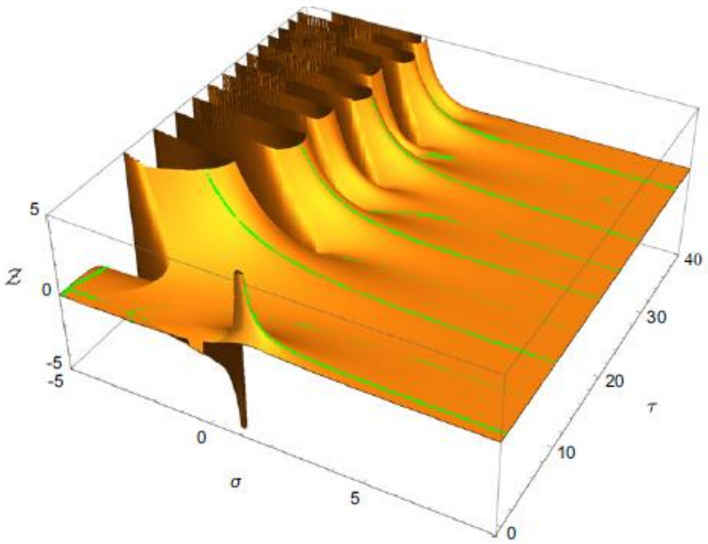


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8...

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4 i \pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\,\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.644934066848226436472415166646025189218949901206798437735558229 3
…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$
$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$64G_{37}^{24} = e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots,$$
$$64G_{37}^{-24} = 4096e^{-\pi\sqrt{37}} - \cdots,$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023
…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256 x^8 + 95744 x^7 - 3248750080 x^6 - 914210725504 x^5 + 15498355554921184 x^4 + 2911478392539914656 x^3 - 32941144911224677091680 x^2 - 3092528914069760354714456 x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\bigg/$$
$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\bigg/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023
...

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256 x^8 + 95744 x^7 - 3248750080 x^6 - 914210725504 x^5 + 15498355554921184 x^4 + 2911478392539914656 x^3 - 32941144911224677091680 x^2 - 3092528914069760354714456 x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$

$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$

$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$

$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

...

1.64381856858…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 15498355554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 15498355554921184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0 \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382$$ (real, principal root)

$$e^{(2i\pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 1.0999 + 1.2216\, i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634\, i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481\, i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right) 2}\right) + \phi} =$$

$$\frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right) \text{^} (1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}$$ for ($0 < \gamma < -\mathrm{Re}(a)$ and $|\arg(z)| < \pi$)

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-\,2\,C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\,e^{-\,2\,A} \,+\, \frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\,e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18))   we obtain:

Input:

$\exp\left(-\pi \sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

from which:

$\frac{1}{4096} e^{-6C+\phi}$ = 1.6272016… * 10^-6

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}\, e^{-6C+\phi} = \frac{1}{0.000244140625}\, e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

Result:

0.00666501785…

0.00666501785…

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

−5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^{5}\sqrt[4]{5^{3}}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.9904007327086440275509737557133014154607327961785555551684...

0.99040073.... result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity

E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\;. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\;. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\;.$$

$$\phi = \varphi - \frac{\sqrt{6}}{k},$$

$$a = \frac{b\gamma}{e} < 0, \qquad \gamma = \frac{k}{\sqrt{6}} < 0.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V = \frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\, b+1)^2 M^2\right\}=0 \text{ at } (b, M)=(-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3} M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226802245}{18480874}$

$$\min\left\{\frac{1}{3} M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\, M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M\neq 0)$$

we obtain

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

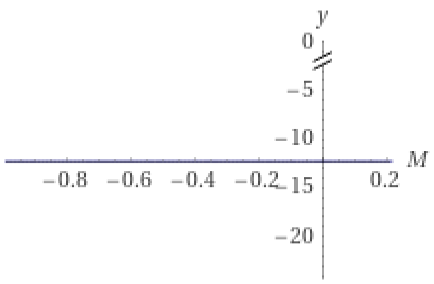
Input interpretation:

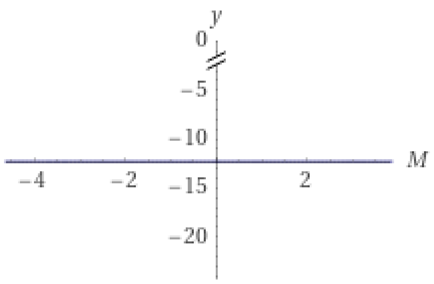
$$\frac{225.913\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4} - 0.054323\,M^2\right)}{M^2}$$

Plots:





Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2 - 1.21228\times 10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4} - 12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4} - 0.054323\,M^2\right)}{M^2}\right) = \frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times 10^{-7}\sqrt{M^4}}{M} - 12.2723\,M + \text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4} - 0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000} \text{ at } M = -1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4} - 0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000} \text{ at } M = -1$$

Limit:

$$\lim_{M\to\pm\infty} \frac{225.913\left(-0.054323\,M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{225.913\left(-0.054323\,M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} - -12.2723\right)dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}\,(0.0814845\, b + 1)^2\, M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

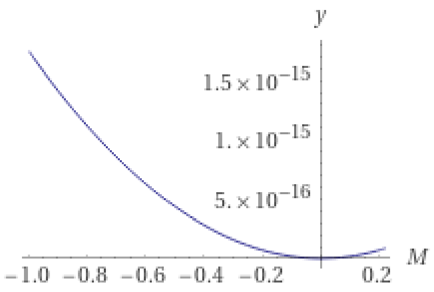
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

0

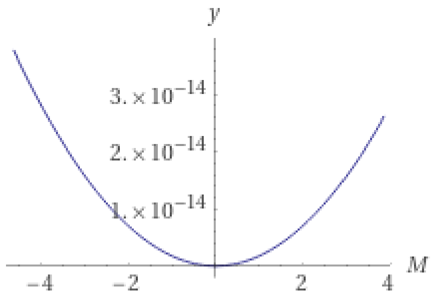
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$$O\left(M^{62194}\right)$$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{1}{3}\, M^2 \left(1 + \frac{18.4084\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2 - 1.75541\times 10^{-15}\, M^2\right) dM = 0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times \frac{225.913\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 0.2^2+6.58545\times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times 0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 3^2+6.58545\times 10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times 3^2$$

Result:

$1.579864841810872363256294820161116875\times 10^{-14}$

1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.912\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times2^2+6.58545\times10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times2^2$$

Result:

$7.0216215191594327255835325340494083333333333333333333333333\ldots\times10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right) \wedge (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_P = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_P = \frac{F_P}{q_P} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_P^E = \mathbf{E}_P l_P^2 = \phi_P l_P = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

$5.975498*10^{-8}$ V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

$1.042940*10^{27}$ V Lorentz-Heaviside valu

Relationship between Planck's Electric Flux and Planck's Electric Potential

$\mathbf{E_P * l_P}$ = $(1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306\times10^{61})\times5.729475}{10^{35}}$$

Result:

1042939771935000000000000000

Scientific notation:

$1.042939771935\times10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$\mathbf{E_P * l_P^2 / l_P}$ = $(5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

$$5.975498\times10^{-8}\times\frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

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