Quantum Cosmology: Cosmology linked to the Planck Scale

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November 11, 2021

Abstract

As we have recently shown, the Planck length can be found independently of G and $\hbar$, despite its common physical notion. This enabled us to make a series of cosmological predictions based on only two constants: the Planck length and the speed of light. The present paper explores further the link between the Planck scale and large-scale Universe structures. We look at both the Friedmann cosmology and the recently proposed Haug cosmology from this new perspective.

Keywords: Hubble constant, Hubble radius, universe equation, Friedmann universe, Haug universe, Planck length, Compton length, quantum cosmology.

1 Background

Max Planck [1, 2] assumed that there were three fundamental constants of Nature: G, $\hbar$ and c. Next he used the dimensional analysis to derive from them natural units of measurement, known today as the Planck length $l_p = \sqrt{\frac{G\hbar}{c^3}}$, the Planck time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, the Planck mass $m_p = \sqrt{\frac{\hbar}{Gc}}$, and the Planck temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_b^2}}$.

Already in 1984 Chahill [3] suggested that the gravitational constant can be expressed using the Planck mass as

$$G = \frac{\hbar c}{m_p^2} \tag{1}$$

which is simply the Planck mass formula solved for G. As pointed out by Cohen [4] in 1987, this seems to lead to a circular problem: one needs to know m_p to define G, but G has been already plugged into the definition of m_p. Therefore, it is of little or no use to express G in Planck units, a view held to this day. McCulloch [5] reminded us about it in 2016, looking at the same formula for G.

In 2016 Haug [6–8] suggested that the gravitational constant is a composite constant that can be obtained from the Planck length formula as

$$G = \frac{l_p^2 c^3}{\hbar} \tag{2}$$

This leads to a similar circular problem, this time involving G and l_p. However, in 2017, it was shown for the first time by Haug [31] how the Planck length can be found without knowing G (see in particular the appendix). In later works [9–12], Haug also demonstrated how to find the Planck length and Planck time with no knowledge of either G or $\hbar$ in a practical and feasible way. Similarly, he showed that one doesn't need G to find the Planck mass, but only c and $\hbar$.

In addition to this composite view of G represented by formula (2), we will consider that the mass of any size, small or large, can be expressed in kilograms as

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{3}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. This result is nothing more than the Compton wavelength formula solved for m [13]. Thus, we describe mass in terms of the Compton wavelength, rather than the other way around. Although it may seem trivial, looking at the relationship between the two quantities from this perspective was perhaps first proposed by Haug in 2016 [8, 14].

The above formula implies that a mass larger than the Planck mass has a reduced Compton wavelength shorter than the Planck length, which is impossible to determine according to the current understanding of quantum mechanics [32]. However, we will postulate that masses larger than the Planck mass are composite. Every composite mass consists of many elementary particles, even if it is smaller than the Planck mass, such as in the case of a proton. The reduced Compton frequencies of n elementary particles making up the composite mass can be aggregated according to the formula

$$\bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots \frac{1}{\bar{\lambda}_n}} \tag{4}$$

where $\bar{\lambda}$ now is the Compton wavelength of the composite mass. This can also be derived from the standard mass aggregation rule: $m = m_1 + m_2 + m_3 + \cdots + m_n$.

If we know the mass of an object in kilograms and the Planck constant, we can find the reduced Compton wavelength as

$$\bar{\lambda} = \frac{\hbar}{mc} \tag{5}$$

This formula can also be used for any mass size. However, we want to relay on as few constants as possible and we do not necessarily know the mass of large objects.

To find the Compton wavelength of any object without knowing its mass and the Planck constant, we look into the Compton scattering. In this process, photons are shoot at an electron, whose Compton wavelength is given by

$$\lambda_e = \frac{\lambda_2 - \lambda_1}{1 - \cos\theta} \tag{6}$$

where λ_1 and λ_2 are the wavelengths of the incident and scattered photon, and θ is the angle between the respective photon paths. We thus obtain the Compton wavelength of the electron without knowing its mass or the Planck constant. The Compton wavelength of a proton can be derived from the ratio of Compton wavelengths of the proton and the electron being equal to the ratio of their cyclotron frequencies:

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{\bar{\lambda}_P}{\bar{\lambda}_e} \approx \frac{1}{1836.15} \tag{7}$$

Hence, to obtain the proton Compton wavelength one can simply divide the electron Compton wavelength by 1836.15. The Compton wavelength of a proton attracted interest in the context of fundamental nuclear forces measured by Levitt back in 1958 [15] and again recently in theoretical calculations of the proton charge radius [16].

Once we know the Compton wavelength of a proton, we can find the Compton wavelength of larger macroscopic masses by counting the number of atoms of which they consist. Although tedious, the counting task can be performed for uniform masses of even a macroscopic size, see for example [17–21]. When one has established the Compton wavelength of a macroscopic object, thenone can measure the gravitational effect of such a mass to calculate the Compton wavelength from this larger object using the following formula:

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1} \tag{8}$$

where g_1, R_1 and g_2, R_2 are gravitational accelerations and radii of two masses, e.g. of the Earth and of a smaller object for which the Compton wavelength is known (we describe this procedure in Ref. [10] in more detail). Haug has also shown how one can extract the Compton wavelength of the Universe from cosmological redshift [22].

Both deriving G from the Planck unit formulas and mass from the Compton wavelength formula are trivial tasks. However, putting their outcomes together leads to new insightful observations. If we multiply them, we will notice that the Planck constant cancels out:

$$GM = \frac{l_p^2 c^3}{\hbar} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}} \tag{9}$$

Hence, the only constants we need to know to find the product of G and M are c and l_p (whilst we would need to know $\hbar$, c and l_p to find G and mass individually). At the same time, Haug's papers cited above show that the Planck length and the Compton wavelength can be found without knowing G or $\hbar$.

If we look at various observable gravity phenomena listed in Table 1, we can see that formulas related to them involve GM and not GMm. Similarly, the two body problem uses the gravity parameter $GM_1 + GM_2$ and not GMm. Hence, neither the Planck constant nor the gravity constant are needed for gravity predictions. All we need are l_p, c and, additionally, a variable deciding on the size of the gravitational object, namely its Compton wavelength. We naturally also need to know the distance to the object at which we want to make the predictions or test our model against observations.

Mass	$M = \frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$ (kg)
Non observable (contains GMm)	
Gravitational constant	$G, \left(G = \frac{l_p^2 c^3}{\hbar}\right)$
Gravity force	$F = G\frac{Mm}{R^2}$ $(kg \cdot m \cdot s^{-2})$
Observable predictions: (contains only GM)	
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2}{R^2}\frac{l_p^2}{\bar{\lambda}_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = cl_p\sqrt{\frac{1}{R\bar{\lambda}_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi\sqrt{\bar{\lambda}_M R^3}}{cl_p}$
Periodicity pendulum[a] (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi R\sqrt{\frac{L}{GM}} = \frac{2\pi R}{cl_p}\sqrt{L\bar{\lambda}_M}$
Frequency Newton spring	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}} = \frac{cl_p}{2\pi R}\sqrt{\frac{1}{\bar{\lambda}_M x}}$
Velocity ball Newton cradle[b]	$v_{out} = \sqrt{2\frac{GM}{R^2}H} = \frac{cl_p}{R}\sqrt{\frac{2H}{\bar{\lambda}_M}}$
Observable predictions (from GR): (contain only GM)	
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)}\frac{l_p^2}{\bar{\lambda}_M}$
Gravitational redshift	$z = \frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}} - 1 = \frac{\sqrt{1-\frac{2l_p^2}{R_1\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p^2}{R_2\bar{\lambda}_M}}} - 1$
Time dilation	$T_R = T_f\sqrt{1-\sqrt{\frac{2GM}{R}}^2/c^2} = T_f\sqrt{1-\frac{2l_p^2}{R\bar{\lambda}_M}}$
Deflection	$\delta = \frac{4GM}{c^2R} = \frac{4}{R}\frac{l_p^2}{\bar{\lambda}_M}$
Microlensing	$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{(d_S-d_L)}{d_Sd_L}} = 2l_p\sqrt{\frac{d_S-d_L}{\bar{\lambda}_M(d_Sd_L)}}$

Table 1: The table shows that formulas describing observable gravity phenomena contain GM and not GMm. Hence, treating G as a composite constant, the only actual constants we need to predict all these phenomena are l_p and c.

[a] The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for full circle, see [23].

[b] Where H is the height of the ball drop.

2 Quantum Cosmology linked to the Planck scale

Gravity theory is closely linked to cosmology. One of the most studied cosmological models, the Friedmann [24] model, was inspired by Einstein's [25] general relativity theory. In fact, in 1927 Lemaître [26] derived the solution of Einstein's equations for the case of an expanding universe unaware of Friedman's prior work. However, it has not been possible to find a link between cosmology and the Planck scale, despite considerable efforts, see [27] and references therein. In this section we will show how the Friedman equations can be rewritten in terms of Planck units. We will perform a similar operation on the new cosmological model proposed by Haug [28, 29] that takes into account the relativistic mass (which, abandoned by Einstein, it is not a part of his general relativity theory).

Table 2 presents the equations of the Friedmann model and the Haug model of the Universe written in the formalism of Planck units. Friedmann's model appears to be more complicated, even though we present its version for a critical universe, i.e. with the parameter k in the general Friedman solution set to zero, which only applies to a flat universe. In the Haug model, the k parameter cancels out in derivations when taking into account the relativistic mass [28]. The Haug model also gives a much better fit to the Planck scale [29] than general relativity theory for micro black holes.

The implications of the proposed Plack formalism for cosmological models go much further than just rewriting their equations to replace the gravity constant with Planck units. We can relatively easily find the Planck length, the Planck time, and the Compton wavelength of the total mass (and energy) in the observable universe with no knowledge of G [22]. Furthermore, we can replace the universal constants G, $\hbar$ and c with only c and l_p, both for observable gravity phenomena, as shown in Table 1, and for cosmological predictions in Table 2 and 3. The superstring theory has attempted to understand cosmology on the grounds on quantum physics acting on the atomic scale without the anticipated success, while the quantum gravity theory proposed by Haug [9, 30] offers a new understanding of the large-scale Universe structures and dynamics as originating from the physics at Planck scale. Our new approach to predicting cosmological phenomena using only two constants, the Planck length and the speed of light, demonstrates the potential of the theory to describe the Universe and thus unify the gravity theory with the quantum scale.

The proposed model of cosmos linking it directly to the Planck scale requires a physical interpretation. In our understanding, quantum gravity is hidden in Newton's gravity—not by assumption, but by construction and based on calibration. Most observable gravitational phenomena, if not all, are in our view indirect detections of the Planck scale physics, which is why we managed to extract the Planck length from most gravity phenomena and even cosmological redshift without knowing G or $\hbar$. This is naturally in stark contrast to the existing theory which distinguishes the gravity theory from the quantum scale.

	Freidmann Critical Universe	Haug Universe
Universe equation	$H_0^2 = \frac{8\pi G\rho_c}{3} = \frac{\bar{\lambda}_u^2 c^2}{4l_p^4}$	$H_0^2 = \frac{4\pi G\rho}{3} = \frac{\bar{\lambda}_u^2 c^2}{l_p^4}$
Universe kilogram mass denisty	$\rho_c = \frac{3H_0^2}{8\pi G} = \frac{3\hbar\bar{\lambda}_c^2}{32\pi c l_p^6}$	$\rho_u = \frac{3H_0^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_u^2}{4\pi c l_p^6}$
Universe energy Joule denisty	$\rho_c = \frac{3H_0^2c^2}{8\pi G} = \frac{3\hbar\bar{\lambda}_c^2 c}{32\pi l_p^6}$	$\rho_u = \frac{3H_0^2c^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_u^2 c}{4\pi l_p^6}$
Universe collision-time (mass) density		$\rho_u = \frac{3H_0^2}{4\pi c^3} = \frac{3t_p\bar{\lambda}_u^2}{4\pi l_p^5}$ see [9]
Hubble constant	$H_o = \frac{\lambda_c c}{2l_p^2}$	$H_o = \frac{\lambda_u c}{l_p^2}$
Hubble Radius	$R_H = \frac{c}{H_0} = \frac{2l_p^2}{\bar{\lambda}_c}$	$R_H = \frac{c}{H_0} = \frac{l_p^2}{\bar{\lambda}_u}$
Escape radius where $v_e = c$	$R_s = \frac{2l_p^2}{\bar{\lambda}_c}$	$R_h = \frac{l_p^2}{\bar{\lambda}_u}$
Hubble Circumference	$C_H = 2\pi\frac{c}{H_0} = \frac{4\pi l_p^2}{\bar{\lambda}_c}$	$C_H = 2\pi\frac{c}{H_0} = \frac{2\pi l_p^2}{\bar{\lambda}_u}$
Hubble volume	$V_H = \frac{4}{3}\pi R_H^3 = \frac{32\pi l_p^6}{3\bar{\lambda}_c^3}$	$V_H = \frac{4}{3}\pi R_H^3 = \frac{4\pi l_p^6}{3\bar{\lambda}_u^3}$
Hubble time	$T_H = \frac{R_H}{c} = \frac{1}{H_0} = \frac{2l_p^2}{\bar{\lambda}_c c} = 2t_p\frac{l_p}{\bar{\lambda}_c}$	$T_H = \frac{R_H}{c} = \frac{1}{H_0} = \frac{l_p^2}{\bar{\lambda}_u c} = t_p\frac{l_p}{\bar{\lambda}}$
Hubble frequency	$f_H = \frac{1}{T_H} = \frac{1}{2t_p}\frac{\bar{\lambda}_c}{l_p}$	$f_H = \frac{1}{T_H} = \frac{1}{t_p}\frac{\bar{\lambda}_u}{l_p}$
Escape radius time	$\frac{R_s}{c} = 2t_p\frac{l_p}{\bar{\lambda}_c} = T_H$	$\frac{R_h}{c} = t_p\frac{l_p}{\bar{\lambda}_c} = T_H$
Compton wavelength universe mass	$\bar{\lambda}_c = \frac{\hbar}{cM_c} = \frac{2l_p^2}{R_H}$	$\bar{\lambda}_u = \frac{\hbar}{cM_u} = \frac{l_p^2}{R_H}$
Cosmological redshift	$Z_c \approx \frac{dH_0}{c} = \frac{d\bar{\lambda}_c}{2l_p^2}$	$Z_c \approx \frac{dH_0}{c} = \frac{d\bar{\lambda}_u}{l_p^2}$
Planck length from Cosmological redshift	$l_p = \sqrt{\frac{d\bar{\lambda}_c}{2Z_c}}$	$l_p = \sqrt{\frac{d\bar{\lambda}_u}{Z_c}}$

Table 2: Some other ways to express the cosmological equations rooted in the Planck scale. However, for example making the cosmological observations linked to the Planck mass rather than the Planck length seems to just add complexity and it leads to one need one more constant, namely the Planck constant.

Table 3 shows more ways to express different aspects of cosmos in Planck formalism. When we use the Planck mass (in kilogram units) instead of the Planck length or Planck time, we additionally need to know the Planck constant. However, as we have demonstrated in Table 2, there is no need to use the Planck mass as we can predict these cosmological phenomena from the Planck length or the Planck time. Thus, we only need to know two constants, l_p and c, for cosmology predictions. Still, even the Planck mass can be found without knowledge of G.

The main difference between the Friedmann model and the Haug model of the Universe is that the latter takes into account the relativistic mass in its derivation. The Haug model gives simpler equations and predicts twice the mass (energy) density in the observable universe as the Friedmann model.

	Friedmann Critical Universe	Haug Universe
Universe equation	$H_0^2 = \frac{2c^2l_p^2}{\bar{\lambda}_c R_H^3}$	$H_0^2 = \frac{c^2l_p^2}{\bar{\lambda}_u R_H^3}$
Universe equation	$H_0^2 = \frac{\bar{\lambda}_c^2 c^2}{4l_p^4}$	$H_0^2 = \frac{\bar{\lambda}_u^2 c^2}{l_p^4}$
Hubble constant from t_p	$H_o = \frac{\bar{\lambda}_c}{2t_p^2 c}$	$H_o = \frac{\bar{\lambda}_u}{t_p^2 c}$
Hubble constant from m_p	$H_o = \frac{\bar{\lambda}_c m_p^2 c^3}{2\hbar^2}$	$H_o = \frac{\bar{\lambda}_u m_p^2 c^3}{\hbar^2}$
Hubble constant from l_p and t_p	$H_o = \frac{\bar{\lambda}_c}{2t_p l_p}$	$H_o = \frac{\bar{\lambda}_u}{t_p l_p}$
Hubble constant from m_p and t_p	$H_o = \frac{\bar{\lambda}_c m_p c}{2t_p\hbar}$	$H_o = \frac{\bar{\lambda}_u m_p c}{t_p\hbar}$
Radius universe from t_p	$R_H = \frac{c}{H_0} = \frac{2t_p^2c^2}{\bar{\lambda}_u}$	$R_H = \frac{c}{H_0} = \frac{t_p^2c^2}{\bar{\lambda}_u}$
Radius universe from m_p	$R_H = \frac{c}{H_0} = \frac{2\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$	$R_H = \frac{c}{H_0} = \frac{\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$
Radius universe from m_p and t_p	$R_H = \frac{c}{H_0} = \frac{2\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$	$R_H = \frac{c}{H_0} = \frac{t_p\hbar}{\bar{\lambda}_u m_p}$

Table 3: Cosmology written with its relation to the Planck scale, other ways to write it

3 Conclusion

We have presented how both the Friedmann model and the Haug cosmological model can be represented in the Planck formalism, namely in terms of the Planck length, the speed of light and the Compton wavelength of the mass in question, in this case the mass of the Universe. Importantly, we can find the Planck length and the Compton wavelength of the Universe with no knowledge of G or $\hbar$, and use them to predict a series of observable gravity phenomena. In this way, for the first time we are able to link the smallest, the Planck scale, with the largest, the cosmic scales of the Universe. We conclude that all gravitational phenomena, including cosmological redshift, are indirect manifestations of the Planck scale physics. This is in stark contrast to the current view of standard physics treating them as two separate domains and searching for new effects or formalism which could connect them. We believe that our simple but powerful theory deserves the consideration of the physics community.

Appendix, some derivations

The Friedmann universe

Just to demonstrate some derivations of the results given in the tables The Hubble constant in the Freedman universe can be written as

$$\begin{aligned} M_c &= \frac{c^3}{2GH_0} \\ \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c} &= \frac{c^3}{2\frac{l_p^2c^3}{\hbar}H_0} \\ H_0 &= \frac{c\bar{\lambda}_c}{2l_p^2} \end{aligned} \tag{10}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the critical mass in the Freedman universe. This means the (reduced) Compton wavelength of the critical mass in the Friedmann universe is given by

$$\begin{aligned} H_0^2 &= \frac{8\pi G\rho}{3} \\ \left(\frac{\bar{\lambda}_u c}{2l_p^2}\right)^2 &= \frac{2GM_c}{R_H^3} \\ \frac{\bar{\lambda}_u^2c^2}{4l_p^2} &= \frac{2l_p^2c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_c}\frac{1}{R_H^3} \\ \frac{\bar{\lambda}_c^3}{4l_p^4} &= \frac{2l_p^2}{R_H^3} \\ \bar{\lambda}_c^3 &= \frac{8l_p^6}{R_H^3} \\ \bar{\lambda}_c &= \left(\frac{8l_p^6}{R_H^3}\right)^{1/3} \\ \bar{\lambda}_c &= \frac{2l_p^2}{R_H} \end{aligned} \tag{11}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the universe, and since $R_H = \frac{c}{2H}$ we can also write this as

$$\bar{\lambda}_c = \frac{l_p^2H_0}{2c} \tag{12}$$

The Hubble radius

$$R_H = \frac{c}{H_0} = \frac{2l_p^2}{\bar{\lambda}_c} \tag{13}$$

Hubble time

$$T_H = \frac{1}{H_0} = \frac{2l_p^2}{c\bar{\lambda}_c} \tag{14}$$

Schwarzschild radius

$$\begin{aligned} R_s &= \frac{2GM_c}{c^2} \\ R_s &= \frac{2l_p^2}{\bar{\lambda}_c} \end{aligned} \tag{15}$$

Escape radius time (Schwarzschild time)

$$\begin{aligned} \frac{R_s}{c} &= \frac{2GM_c}{c^3} \\ \frac{R_s}{c} &= \frac{2l_p^2}{c\bar{\lambda}_c} \end{aligned} \tag{16}$$

The Haug universe

The Hubble constant in the Haug universe can be written as

$$H_0 = \frac{c\bar{\lambda}_u}{l_p^2} \tag{17}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the Haug universe. The mass in this universe is twice that of in the Friedmann universe. The reduced Compton wavelength of the mass in this universe we can express as

$$\begin{aligned}
H_0^2 &= \frac{8\pi G\rho}{3} \\
\left(\frac{\bar{\lambda}_u c}{l_p^2}\right)^2 &= \frac{GM_u}{R_H^3} \\
\frac{\bar{\lambda}_u^2 c^2}{l_p^2} &= \frac{l_p^2 c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_u}\frac{1}{R_H^3} \\
\frac{\bar{\lambda}_u^3}{l_p^4} &= \frac{l_p^2}{\bar{\lambda}_u R_H^3} \\
\bar{\lambda}_u^3 &= \frac{l_p^6}{R_H^3} \\
\bar{\lambda}_u &= \left(\frac{l_p^6}{R_H^3}\right)^{1/3} \\
\bar{\lambda}_u &= \frac{l_p^2}{R_H}
\end{aligned} \tag{18}$$

and since $R_H = \frac{c}{H_0}$ we can also re-write this as

$$\bar{\lambda}_u = \frac{l_p^2 H_0}{c} \tag{19}$$

The Hubble radius

$$R_H = \frac{c}{H_0} = \frac{l_p^2}{\bar{\lambda}_u} \tag{20}$$

Hubble time

$$T_H = \frac{1}{H_0} = \frac{l_p^2}{c\bar{\lambda}_u} \tag{21}$$

Haug radius (the radius where the escape velocity is c)

$$\begin{aligned}
R_h &= \frac{GM_u}{c^2} \\
R_h &= \frac{l_p^2}{\bar{\lambda}_c}
\end{aligned} \tag{22}$$

Escape radius time

$$\begin{aligned}
\frac{R_h}{c} &= \frac{GM_u}{c^3} \\
\frac{R_h}{c} &= \frac{l_p^2}{c\bar{\lambda}_u}
\end{aligned} \tag{23}$$

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