Quantum Entanglement is a White Noise?

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Abstract

We study the spin correlation (Quantum Entanglement) in a low energy proton-proton scattering. The nonrelativistic regime allows us to apply the first quantization framework. Finally, we will show that the Quantum Entanglement processes do not involve the transmission of information as the results of the measurements are in general 100% uncorrelated. In the framework of the Theory of signals, this corresponds to a *white noise*.

1 Low energy proton-proton scattering

Consider the low-energy proton-proton scattering process. In the eigenstate of the (total) orbital angular momentum with $L = 0$ by the Pauli exclusion principle, the system will be in the spin singlet state. To be more precise we denote by $\mathcal{H}_k^{(spin)}$ the Hilbert space of the spin states of a single proton ($k = 1, 2$). As is well known (being spin 1/2)

$$\mathcal{H}_k^{(spin)} = \mathbb{C}^2 \tag{1}$$

It follows that the Hilbert space of the spin states of the corresponding composite system is with obvious meaning of the symbols:

$$\mathcal{H}^{(spin)} = \mathcal{H}_1^{(spin)} \otimes \mathcal{H}_2^{(spin)} \tag{2}$$

If $\hat{\mathbf{S}}_1$ and $\hat{\mathbf{S}}_2$ are the self-adjoint operators that represent the spin observables of a single proton, we have the following eigenvalue equations [1]:

$$\begin{aligned} \hat{S}_k^2 \left|\mathbf{n}\pm\right\rangle_k &= \frac{3}{4}\hbar^2 \left|\mathbf{n}\pm\right\rangle_k \\ \hat{S}_k \left|\mathbf{n}\pm\right\rangle_k &= \pm\frac{\hbar}{2} \left|\mathbf{n}\pm\right\rangle_k \end{aligned} \tag{3}$$

where we oriented the spin analyzer along the versor axis n. The angular momentum of total spin is

$$\hat{\mathbf{S}} = \hat{\mathbf{S}}_1 \otimes \hat{1}_{\mathcal{H}_1^{(spin)}} + \hat{\mathbf{S}}_2 \otimes \hat{1}_{\mathcal{H}_2^{(spin)}} \tag{4}$$

where the unit operators of the respective Hilbert spaces appear. At this point it is essential to observe that we are using the first quantization frameworke, since being a low energy process we expect a nonrelativistic regime. We must therefore specify the inertial frame of reference with respect to which we observe the process. More precisely, we consider the case of two experimenters (Alice and Bob, fig. 1) who oriented the x axis of the aforementioned reference system, in the direction of scattering and with the origin O in the position where the interaction took place, and which are distant $d \gg 1$.

Subsequently, the two protons move away in the two opposite directions of the x axis. Note that this argument is somewhat imprecise, since Heisenberg's uncertainty principle prevents assigning a trajectory, so it is more correct to refer to the propagation of wave packets.

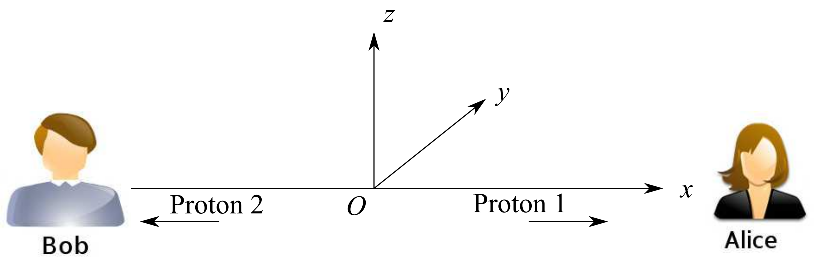

Figure 1: Experimental setup.

2 Entanglement and White Noise

From the angular momentum composition rules [1], the spin singlet state (total spin $S = 0$) is written:

$$|S=0\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{x}-;\mathbf{x}+\rangle - |\mathbf{x}+;\mathbf{x}-\rangle\right) \tag{5}$$

Where

$$|\mathbf{x}-;\mathbf{x}+\rangle = |\mathbf{x}-\rangle_1 |\mathbf{x}+\rangle_2$$
$$|\mathbf{x}+;\mathbf{x}-\rangle = |\mathbf{x}+\rangle_1 |\mathbf{x}-\rangle_2$$

In the second member appears the direct product of the eigenstates of the spin component along the x axis of a single proton.

Hypothesis:

1. Alice measures S_{1z} or S_{1x} or she doesn't measure.
2. Bob misures S_{2x}.

It is also clear that if Alice measures S_{1z} and finds for example spin up $|\mathbf{z}-\rangle_1$, Bob has a 1/2 probability of finding $|\mathbf{x}-\rangle_2$ or $|\mathbf{x}+\rangle_2$. It follows that the spin state of proton 2 is completely indeterminate, even following the measurement performed by A.

Notation 1 *Note that if unlike the assumptions, Bob measures the spin component along the z axis, he finds with certainty $|\mathbf{z}+\rangle_2$, if the result of Alice's measurement is $|\mathbf{z}-\rangle_1$. This conclusion is easily reached by writing the ket of the spin singlet in terms of the eigenket of the single components along the z axis.*

In the second option, that is, if Alice measures S_{1x}, from the (5) it follows that if A finds $|\mathbf{x}-\rangle_1$ then Bob will certainly find the eigenstate $|\mathbf{x}+\rangle_2$. Finally, if Alice does not make any measurements, the state of the spin component along the x axis of particle 2 is completely indeterminate.

In general, there is a 100% correlation between Alice's measurement and Bob's, if and only measurements of homonymous components of single particle spins are performed. Conversely, there is a completely random correlation in the case of measurements of heteronymous components. Also, if Alice doesn't measure, Bob's results are random.

It follows that Bob's results depend on what Alice *does*, noting that they may be light years away. In this framework, perform a measurement on a part of the system i.e. on one of the two particles, it is equivalent to making a measurement on the entire composite system. We will therefore say that the system is *entangled* i.e. non-separable.

Interpreting the effects of Alice's measurements on Bob's, in the case of heteronomous components, we conclude that this signal is actually a *white noise*, as the assumed values are 100% uncorrelated.

References

[1] Sakurai J.J., *Modern Quantum Mechanics* Pearson.