WHOLE NUMBERS IN SPECIFIED ARRAYS AND THEIR RELATIONSHIPS IN MULTI – DIMENSIONAL LOCALES

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Abstract :

The study details specified properties of whole numbers in conjunction with repetitive arrays and sequences. There prevails a common pattern for numbers when they exist in defined structures. The paper extends to the scope of progressions in regard to the specific number relationships and its reach in advanced mathematical studies. The properties of numbers enumerated have its scope in the field of recreational mathematical theories as well.

Finding 01 proves the relationships between numbers in specific patterns, with exceptions in effect. The exceptions hold true only in the mentioned patterns of numbers.

Findings 02 and 03 show the properties of numbers in Arithmetic Progression and the relationship of the numbers in specific arrangements with the Common Difference.

Findings 04 and 05 work on the value of Determinants of matrices of which numbers are in Arithmetic and Geometric Progressions and its application in regard to positioning of multiple points in varying dimensions.

Findings 06, 07 and 08 details the unique properties of numbers when applied in addition, subtraction and multiplication and the 'CONSTANCY' in the end results.

Findings 09 and 10 explains the relationships and properties of numbers with reference to the structures of regular polygons and the positioning of points in multiple dimensions.

FINDING 01 :

The difference between two consecutive numbers of the form a x [10a + b] b ;

where a = Infinity, ……, -2, -1, 0, 1, 2,……, Infinity

b = 0, 1, 2, …., 9 and

x = Infinity, ……, -2, -1, 0, 1, 2,……, Infinity

IS + or - [(10x) + 1]

The condition satisfies all cases, with an exception.

i.e. when b = 9 and the next consecutive number.

eg: The difference between 1242 and 1263 is 21, since x= 2; but the difference between 1389 and 2400 is not equal to 21.

PROOFS :

i) **a x [10a + b] b = 100a + 10 x. 10 a + 10 x . b + b = 100 a + 100 ax + 10 bx + b**

a x [10a + (b+1)] (b+1) = 100a + 10 x. 10 a + 10 x . b + 10 x + (b+1) = 100 a + 100 ax + 10 bx + 10 x + b + 1

Difference = + [10 x + 1]; where a = 1, x = 1 and b = 1 (a is +ve, x is +ve and b is +ve).

ii) **– a x [-10a + b] b = - 100a - 100ax + 10bx + b**

– a x [-10a + (b+1)] (b+1) = - 100a -100ax + 10bx + 10x + b + 1

Difference = + [10x + 1]; where a is –ve, x is +ve and b is +ve.

iii) $\underline{-a}$ $\underline{-x[-10a+b]}$ $\underline{b}$ = - 100a + 100ax - 10bx + b

$\underline{-a}$ $\underline{-x[-10a+(b+1)]}$ $(\underline{b+1})$ = - 100a + 100ax - 10bx – 10x + b + 1

Difference = - [(10x) + 1]; where a is –ve, x is -ve and b is +ve.

iv) $\underline{a}$ $\underline{-x[10a+b]}$ $\underline{b}$ = 100a - 100ax - 10bx + b

$\underline{a}$ $\underline{-x[10a+(b+1)]}$ $(\underline{b+1})$ = 100a - 100ax - 10bx – 10x + b + 1

Difference = - [(10x) + 1]; where a is +ve, x is -ve and b is +ve.

eg : The difference between (100, 101), (101,102); where a = 1, b = 0, 1, 2 and x = 0 is <u>01</u>.

The difference between (1100, 1111), (1111, 1122); where a = 1, b = 0, 1, 2 and x = 1 is <u>11</u>.

The difference between (1300, 1331), (1331, 1362), (1362, 1393) ; where a = 1, b = 0, 1, 2,3 and x = 3 is <u>31</u>.

The difference between (1400, 1441), (1441, 1482), (1482, 1523) ; where a = 1, b = 0, 1, 2,3 and x = 4 is <u>41</u>.

The difference between (1500, 1551), (1551, 1602), (1602, 1653) ; where a = 1, b = 0, 1, 2,3 and x = 5 is <u>51</u>.

The difference between (-2 -40 0, -2 -42 1) i.e. (2421 - 2400) ; where a = -2, b = 0, 1 and x = 2 is <u>+21</u>.

The difference between (-3 93 1, -3 96 2) i.e. (-3962 - -3931) ; where a = -3, b = 1, 2, and x = - 3 is <u>- 31</u>.

The difference between (-6 +330 6, -6 +335 7) i.e. (-63357 - -63306) ; where a = -6, b = 6, 7 and x = - 5 is <u>-51</u>.

The difference between (-2 88 2, -2 92 3) i.e. (-2923 - -2882) ; where a = -2, b = 2, 3 and x = -4 is <u>-41</u>.

When <u>x</u> [<u>10a + b</u>] and <u>x</u> [<u>10a + (b+1)</u>] differs in the number of digits, <u>x</u> [<u>10a + (b+1)</u>] is to be adjusted such that the number of digits equal. It should be done in such a way by adding the first digit of

<u>x</u> [<u>10a + (b+1)</u>] to the last digit of 'a' in <u>a</u> <u>x [10a + (b+1)]</u> <u>(b+1)</u>.

eg: i) Consider 2964 and 21005. In this case, 21005 is to be treated as 3005, thus the difference happens to be + 41. a = 2, b = 4, 5 and x = 4.

ii) For 21005 and 21046 also, the difference is + 41. In this case, no change is to be made.

iii) Consider 1962 and 11043. In this case, 11043 is to be treated as 2043, thus the difference happens to be + 81. a = 1, b = 2, 3 and x = 8.

iv) Consider 99900 and 910011. In this case, 910011 is to be treated as 100011, thus the difference happens to be + 111. a = 9, b = 0, 1 and x = 11.

<u>FINDING</u> <u>02</u> :

For a set of three numbers in Arithmetic Progression, the difference between the square of the middle number and the product of the end numbers is the square of the Common Difference.

i.e. If a,b,c are in A.P, with a Common Difference of 'd', then b^2 – ac = d^2

eg: For 12,13,14 ; (13^2 – 12*14) = (+1)^2

For 14,13,12 ; (13^2 – 14*12) = (-1)^2

For 15,17,19 ; (17^2 – 15*19) = (+2)^2

For 19,17,15 ; (17^2 – 19*15) = (-2)^2

For 06,10,14; (10^2 – 06*14) = (+4)^2

For 98,106,114; (106^2 – 98*114) = (+8)^2

Proof :- In the mentioned A.P of 3 numbers, b^2 – ac = (a+d)^2 – a(a+2d) = a^2+2ad+d^2-a^2-2ad = d^2.

FINDING 03 :

For a set of four numbers in Arithmetic Progression, the difference between the products of the middle numbers and the end numbers is 2(d^2).

i.e. If p,q,r,s are in A.P, with a Common Difference of ‘d’, then qr-ps = 2(d^2).

eg: For 2,4,6,8 ; (4*6 – 2*8) = 2(2^2)

For 8,6,4,2 ; (6*4 – 8*2) = 2(-2^2)

For 5,7,9,11 ; (7*9 – 5*11) = 2(2^2)

For 11,9,7,5 ; (9*7 –11*5) = 2(-2^2)

For 25,50,75,100 ; (50*75 – 25*100) = 2(25^2)

For 10,60,110,160 ; (60*110 – 10*160) = 2(50^2)

For 02,14,26,38 ; (14*26 – 02*38) = 2(12^2)

For 63,54,45,36 ; (54*45 – 63*36) = 2(-9^2)

For 49,42,35,28 ; $(42*35 - 49*28) = 2(-7^2)$

Proof :- In the mentioned A.P of 4 numbers, $qr-ps = (p+d)(p+2d) - p(p+3d) = p^2 + 3pd + 2d^2 - p^2 - 3pd = 2(d^2)$.

<u>FINDING 04 :</u>

Determinant value of numbers in Arithmetic Progression in a $n * n$ Determinant = 0, for $n > 2$.

- **For a 3*3 Determinant, with numbers in Arithmetic Progression :**

Consider a 3*3 Determinant of which first term is 'a' and Common Difference 'd'.

$$\begin{vmatrix} a & a+d & a+2d \\ a+3d & a+4d & a+5d \\ a+6d & a+7d & a+8d \end{vmatrix} = a[(a+4d)(a+8d) - (a+5d)(a+7d)] - (a+d)[(a+3d)(a+8d) - (a+5d)(a+6d)] + (a+2d)[(a+3d)(a+7d) - (a+4d)(a+6d)]$$

$$= a[a^2 + 12ad + 32d^2 - a^2 - 12ad - 35d^2] - (a+d)[a^2 + 11ad + 24d^2 - a^2 - 10ad - 24d^2] + (a+2d)[a^2 + 10ad + 21d^2 - a^2 - 10ad - 24d^2]$$

$$= a[-3d^2] - (a+d)[-6d^2] + (a+2d)[-3d^2] = -3ad^2 + 6ad^2 + 6d^3 - 3ad^2 - 6d^3 = 0.$$

Consider a 4 * 4 Determinant with numbers in A.P; where first term is, say, (a-8d) and Common Difference 'd'.

$$\begin{vmatrix} a-8d & a-7d & a-6d & a-5d \\ a-4d & a-3d & a-2d & a-d \\ a & a+d & a+2d & a+3d \end{vmatrix} = \underset{(1)}{(a-8d)} \begin{vmatrix} a-3d & a-2d & a-d \\ a+d & a+2d & a+3d \\ a+5d & a+6d & a+7d \end{vmatrix} - \underset{(2)}{(a-7d)} \begin{vmatrix} a-4d & a-2d & a-d \\ a & a+2d & a+3d \\ a+4d & a+6d & a+7d \end{vmatrix} + \underset{(3)}{(a-6d)} \begin{vmatrix} a-4d & a-3d & a-d \\ a & a+d & a+3d \\ a+4d & a+5d & a+7d \end{vmatrix}$$

$$\underset{(4)}{(a-5d)} \begin{vmatrix} a-4d & a-3d & a-d \\ a & a+d & a+2d \\ a+4d & a+5d & a+6d \end{vmatrix}$$

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(1) => $(a - 8d)\ [\ (a - 3d) \begin{vmatrix} a + 2d & a + 3d \\ a + 6d & a + 7d \end{vmatrix} - (a - 2d) \begin{vmatrix} a + d & a + 3d \\ a + 5d & a + 7d \end{vmatrix} + (a - d) \begin{vmatrix} a + d & a + 3d \\ a + 5d & a + 6d \end{vmatrix}$

= (a – 8d) { (a – 3d) [(a+ 2d) (a + 7d) – (a + 3d) (a + 6d)] – (a – 2d) [(a+ d) (a + 7d) – (a + 3d) (a + 5d)] + (a – d) [(a+ d) (a + 6d) – (a + 2d) (a + 5d)] }

= (a – 8d) { [(a – 3d) (a^2 + 9ad + 14 * d^2 – a^2 – 9ad – 18 * d^2)] - [(a – 2d) (a^2 + 8ad + 7 * d^2 – a^2 – 8ad – 15 * d^2)] + [(a – d) (a^2 + 7ad + 6 * d^2 – a^2 – 7ad – 10 * d^2)] }

= (a – 8d) { [(a – 3d) (– 4 * d^2)] – [(a - 2d) (- 8 * d^2)] + [(a – d) (-4 * d^2)]

= (a – 8d) [-4 a*d^2 + 12 d^3 + 8 a*d^2 – 16 d ^3 – 4 a*d^2 + 4 d^3] = 0.

(2) => $(a - 7d)\ [\ (a - 4d) \begin{vmatrix} a + 2d & a + 3d \\ a + 6d & a + 7d \end{vmatrix} - (a - 2d) \begin{vmatrix} a & a + 3d \\ a + 4d & a + 7d \end{vmatrix} + (a - d) \begin{vmatrix} a & a + 2d \\ a + 4d & a + 6d \end{vmatrix}$

= (a – 7d) { (a – 4d) [(a+ 2d) (a + 7d) – (a + 3d) (a + 6d)] – (a – 2d) [a (a + 7d) – (a + 3d) (a + 4d)] + (a – d) [a (a + 6d) – (a + 2d) (a + 4d)] }

= (a – 7d) { [(a – 4d) (-4 * d^2)] - [(a – 2d) (-12 * d^2)] + [(a – d) (-8 * d^2)] }

= (a – 7d) [-4a * d^2 + 16 * d ^3 + 12a * d^2 – 24 * d ^3 – 8a * d ^2 + 8 * d^3]

= (a – 7d) * 0 = 0.

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(3) => $(a - 6d)\ [\ (a - 4d) \begin{vmatrix} a + d & a + 3d \\ a + 5d & a + 7d \end{vmatrix} - (a - 3d) \begin{vmatrix} a & a + 3d \\ a + 4d & a + 7d \end{vmatrix} + (a - d) \begin{vmatrix} a & a + d \\ a + 4d & a + 5d \end{vmatrix}$

= (a – 6d) { (a – 4d) [(a+ d) (a + 7d) – (a + 3d) (a + 5d)] – (a – 3d) [a (a + 7d) – (a + 3d) (a + 4d)] + (a – d) [a (a + 5d) – (a + d) (a + 4d)] }

= (a – 6d) { [(a – 4d) (-8 * d^2)] - [(a – 3d) (-12 * d^2)] + [(a – d) (-4 * d^2)] }

= (a – 6d) [-8a * d^2 + 32 * d ^3 + 12a * d^2 – 36 * d ^3 – 4a * d ^2 + 4 * d^3]

= (a – 6d) * 0 = 0.

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(4) => $(a - 5d)\ [\ (a - 4d) \begin{vmatrix} a + d & a + 2d \\ a + 5d & a + 6d \end{vmatrix} - (a - 3d) \begin{vmatrix} a & a + 2d \\ a + 4d & a + 6d \end{vmatrix} + (a - 2d) \begin{vmatrix} a & a + d \\ a + 4d & a + 5d \end{vmatrix}$

= (a – 5d) { (a – 4d) [(a+ d) (a + 6d) – (a + 2d) (a + 5d)] – (a – 3d) [a (a + 6d) – (a + 2d) (a + 4d)] + (a – 2d) [a (a + 5d) – (a + d) (a + 4d)] }

= (a – 5d) { [(a – 4d) (-4 * d^2)] - [(a – 3d) (-8 * d^2)] + [(a – 2d) (-4 * d^2)] }

= (a – 5d) [-4a * d^2 + 16 * d ^3 + 8a * d^2 – 24 * d ^3 – 4a * d ^2 + 8 * d^3]

= (a – 5d) * 0 = 0.

Therefore, (1) - (2) + (3) - (4) = 0

Thus, the Determinant value of numbers in Arithmetic Progression for n = 3 and n = 4 is 0.

Hence, the Determinant value of 5 * 5 matrix, which is a* [4 * 4 Determinant] – b * [4 * 4 Determinant] + c [4 * 4 Determinant] - ……. = 0; where a,b,c,…. are in A.P.

Also, the Determinant value of a 6 * 6 matrix, which is a* [5 * 5 Determinant] – b * [5 * 5 Determinant] + c [5 * 5 Determinant] - ……. = 0; where a,b,c,…. are in A.P.

Similarly, the Determinant value of a n * n matrix where n > 2 = 0.

Hence the Proof.

FINDING 05 :

Determinant value of a n * n matrix with a set of 'n' numbers in Geometric Progression = 0; where n >2.

Consider a 3 * 3

Determinant with numbers in G.P; where the first term is 'a' and Common Ratio is 'r'.

$$\begin{vmatrix} a & ar & ar^2 \\ ar^3 & ar^4 & ar^5 \\ ar^6 & ar^7 & ar^8 \end{vmatrix} = a\,(a^2 * r^{12} - a^2 * r^{12}) - ar\,(a^2 * r^{11} - a^2 * r^{11}) + a * r^2\,(a^2 * r^{10} - a^2 * r^{10}) = 0.$$

For a 4 * 4 Determinant, the value is A * [3 * 3 Determinant] – B * [3 * 3 Determinant] + C * [3 * 3 Determinant] - ……….= 0; where A,B,C etc. are in G.P.

Similarly, for a 5 * 5 Determinant, the value is A * [4 * 4 Determinant] – B * [4 * 4 Determinant] + C * [4 * 4 Determinant] - ……….= 0; where A,B,C etc. are in G.P.

Thus, the Determinant value of any n * n matrix; where n > 2 = 0.

Hence the Proof.

FINDING 06 :

The resultant sum of the digits of a product obtained by multiplying numbers in the form of sets, in multiples of 3 and the numbers being in Arithmetic Progression is a constant and = 9.

eg: 78 76 74 * 76 74 72 = 604517740128 (6+0+4+5+1+7+7+4+0+1+2+8 =45 ; 4+5 = 9)

125 095 065 * 095 065 035 = 11892166732552275; The resultant sum of the digits = 9

Also, 125 95 65 * 95 65 35 = 1204818007275; The resultant sum of the digits = 9

810 610 410 * 610 410 210 = 494804870596286100; The resultant sum of the digits = 9

110 095 080 * 095 080 065 = 10467847362580200; The resultant sum of the digits = 9

Also, 110 95 80 * 95 80 65 = 1063049762700; The resultant sum of the digits = 9

110 95 80 * 35 50 65 = 393973022700; The resultant sum of the digits = 9

110 95 80 * 35 50 065 = 38890851122700; The resultant sum of the digits = 9

110 095 080 * 35 50 65 = 39090909580200; The resultant sum of the digits = 9

110 095 080 * 035 050 065 = 3858839710180200; The resultant sum of the digits = 9

This holds true for any sets, which are multiples of 3 i.e. 6,9,12,…

Proof :

1. **For 3 Sets : Suppose the 3 sets of the first number, which are in A.P are a, a+d, a+2d, whose sum is 3a+3d and the second number are a-2d, a-d, a, whose sum is 3a – 3d.**

   Thus, the product of the two numbers will always be a multiple of 9 and hence the resultant sum of digits will be 9.

   Similarly,
2. **For 6 Sets : The Sum of Numbers in A.P = 6/2 [2a + (6 – 1) d] = 3 [2a + 5d]**
3. **For 9 Sets : The Sum of Numbers in A.P = 9/2 [2a + (9 - 1) d] = 9 [a + 4d]**

For any multiple of 3, the Sum of the numbers which are in A.P will be multiples of 3 and hence the product of such numbers will be a multiple of 9 and hence the resultant sum of digits will be 9.

Hence the Proof.

eg : 6 Sets : 24 26 28 30 32 34 * 20 22 24 26 28 30 = 490653297631893549922220; The resultant sum of the digits = 9

9 Sets : 78 76 74 72 70 68 66 64 62 * 56 54 52 50 48 46 44 42 40 = 445392647425216766599565659241078880; The resultant sum of the digits = 9

12 Sets: 55 54 53 52 51 50 49 48 47 46 45 44 * 575 576 577 578 579 580 581 582 583 584 585 586 =

31970603901007930177391420624227831402400826903219306846 2784; The resultant sum of the digits = 9

15 Sets: 88 90 92 94 96 98 100 102 104 106 108 110 112 114 116 * 102 100 098 096 094 092 090 088 086 084 082 080 078 076 074 =

90776477380721670623576525466395308201057850579359239620014861764727754850017260584; The resultant sum of the digits = 9

Also, 088 090 092 094 096 098 100 102 104 106 108 110 112 114 116 * 102 100 098 096 094 092 090 088 086 084 082 080 078 076 074 =

89940070441011742593524495466397247978548918864811077704732200148617647277548500172605841764727754850017260584; The resultant sum of the digits = 9

<u>FINDING</u> <u>07</u> :

The resultant sum of the digits of any number ‘n’ subtracted by any number formed by the permutations and combinations of the original number ‘n’, including the decimal combinations will be a constant and = 9.

Proof :

Consider a number,

$[10^n]a + [10^{(n-1)}b] + [10^{(n-2)}c] + \ldots\ldots$; where n = Infinity,……, -3, -2, -1, 0, 1, 2, 3,…., Infinity and a, b, c, any variable = 0,1,2,….,9

The difference between the original number and the number formed by the Permutations and Combinations of the number will always be a multiple of 9.

The original number is $[10^n]a + [10^{(n-1)}b] + [10^{(n-2)}c] + \ldots\ldots$ -----------(1)

The Permutations and Combinations of the number are :

$[10^n]b + [10^{(n-1)}a] + [10^{(n-2)}c] + \ldots\ldots$ -----------(2)

$[10^n]b + [10^{(n-1)}c] + [10^{(n-2)}a] + \ldots\ldots$ -----------(3)

$[10^n]a + [10^{(n-1)}c] + [10^{(n-2)}b] + \ldots\ldots$ -----------(4)

$[10^n]c + [10^{(n-1)}a] + [10^{(n-2)}b] + \ldots\ldots$ -----------(5)

$[10^n]c + [10^{(n-1)}b] + [10^{(n-2)}a] + \ldots\ldots$ -----------(6)

For any such combinations, the difference of every digit will be either 0 or be a multiple of 9 and hence the resultant sum of digits will be 9.

- **(1) - (2) => $a[10^n - 10^{(n-1)}] + b[10^{(n-1)} - 10^n] + c[10^{(n-2)} - 10^{(n-2)}] + \ldots\ldots\ldots$**
- **(2) - (3) => $a[10^n - 10^{(n-2)}] + b[10^{(n-1)} - 10^n] + c[10^{(n-2)} - 10^{(n-1)}] + \ldots\ldots\ldots$**

Similarly, for other results. Hence the proof.

eg : The resultant sum of the digits of (81285470865 – 56845721880) = (81285470865 – 84565270881) = (81285470865 – 65542807188) =

(81285470865 – All permutations and combinations of the original number).

The resultant sum of the digits of (2.31548936 – 6.31285439) = (2.31548936 – 8.61324593) = (2.31548936 – 4.23816953) =

(2.31548936 - All permutations and combinations of the original number).

<u>FINDING</u> <u>08</u> :

The resultant sum of the digits of a product obtained by multiplying any two numbers will be the resultant sum of the digits of the product obtained by multiplying the resultant sum of digits of those numbers.

eg : 782 * 638 = 498916

The resultant sum of 782 = 17; 1+7 = 8

The resultant sum of 638 = 17; 1+7 = 8

The resultant sum of 8 * 8 = 64; 6+4 = 10; 1+0 = 1

The resultant sum of the digits of the product of 782 and 638 i.e 498916 = 37; 3 +7 is also equal to 1

Consider two 2 digit numbers ab and cd.

The Product of the sum of the digits => (a+b) (c+d) = ac +ad +bc + bd and

The product of the numbers => (10a + b) (10c + d) = <u>ac</u> <u>ad+bc</u> <u>bd</u>

<u>Thus, the resultant sum of digits of the product of the sum of digits and the product of the numbers are the same.</u>

For two 3 digit numbers, <u>abc</u> and <u>def,</u>

The Product of the sum of the digits => (a+b+c) (d+e+f) = ad+ae+af+bd+be+bf+cd+ce+cf and

The product of the numbers => (100a + 10b + c) (100d +10e + f) = <u>ad</u> <u>ae+bd</u> <u>af+be+cd</u> <u>bf+ce</u> <u>cf</u>

For two 'n' digit numbers, <u>abcde.......</u> and <u>ABCDE......</u>,

(a+b+c+…..) (A+B+C+…..) = a A + a B +….+ b A+ b B+….+ c A + c B +…..

and {a * [10^n] + b *[10^(n-1)] + c * [10^(n-2)] + ………} * {A * [10^n] + B *[10^(n-1)] + C * [10^(n-2)] + ………}

= a A * [10 ^ 2n] + b B * [10 ^ (2n – 2)] + ………………….and the number formed i.e. the product of the two numbers will have the terms a A, a B, ….., b A, b B,….i.e. the terms in the product of the resultant sums of the numbers.

Hence the Proof.

<u>FINDING 09 :</u>

For a regular polygon, with vertices formed by the terms in Arithmetic or Geometric Progression:

i.e when x1,y1,x2,y2,x3,y3,…..are in A.P or G.P, where (x1,y1), (x2,y2), (x3,y3),…..are the vertices of a regular polygon; then,

$$\begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix} = \begin{vmatrix} x2 & y2 & 1 \\ x3 & y3 & 1 \\ x4 & y4 & 1 \end{vmatrix} = \begin{vmatrix} x1 & y1 & 1 \\ x4 & y4 & 1 \\ x5 & y5 & 1 \end{vmatrix} = \text{Any such permutations of the numbers in the vertices} =$$

$$\begin{vmatrix} x1 & x2 \\ y1 & y2 \end{vmatrix} + \begin{vmatrix} x2 & x3 \\ y2 & y3 \end{vmatrix} + \ldots\ldots\ldots\ldots + \begin{vmatrix} xn & x1 \\ yn & y1 \end{vmatrix}$$

; where 'n' is the number of sides of the polygon and the vertices plotted counter clock-wise around the Perimeter.

$|\quad\quad|$ denotes the Determinant Value.

<u>FINDING 10 :</u>

For a regular polygon, with vertices formed by the <u>Consecutive terms</u> of an Arithmetic or Geometric Progression:

i.e when x1,y1,x2,y2,x3,y3,.....are in A.P or G.P, where (x1,y1), (x2,y2), (x3,y3),.....are the vertices of a regular polygon; then,

$$\begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix} = \begin{vmatrix} x2 & y2 & 1 \\ x3 & y3 & 1 \\ x4 & y4 & 1 \end{vmatrix} = \begin{vmatrix} x1 & y1 & 1 \\ x4 & y4 & 1 \\ x5 & y5 & 1 \end{vmatrix} = \text{Any such permutations of the numbers in the vertices} =$$

$$\begin{vmatrix} x1 & x2 \\ y1 & y2 \end{vmatrix} + \begin{vmatrix} x2 & x3 \\ y2 & y3 \end{vmatrix} + \ldots\ldots\ldots\ldots + \begin{vmatrix} xn & x1 \\ yn & y1 \end{vmatrix} =$$

; where ‘n’ is the number of sides of the polygon and the vertices plotted counter clock-wise around the Perimeter

$$\begin{vmatrix} x1 \;\; y1 \;\; x2 \;\; y2 \;\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots xk \;\; yk \end{vmatrix}$$

; where ‘k’ is a Perfect Even square >= 16. i.e. For a polygon with number of sides = 8 and thereafter with number of sides increasing with a difference of 10,14,18,22,26,....... i.e. with a Common difference of 4.

i.e. The equation holds true for a polygon with number of sides = 8,18,32,50,72,98,128,162,....

$|\quad\quad|$ **denotes the Determinant Value.**

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