The ‘Quantum Game Show’: a very simple explanation of Bell’s Theorem in Quantum Mechanics

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Abstract: In this article give a very simple presentation of Bell’s inequality by comparing it to a “quantum game show”, followed by a simple description of Aspect’s 1985 experiment involving entangled photons which confirms the inequality. The entire article is non-technical and requires no mathematical background other than high school mathematics and an understanding of basic concepts in probability. The physics involved in Aspect’s experiment is also explained.
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1 A quantum game show

The following analogy will elucidate a fundamental difference between classical and quantum mechanics. The example involves a game show in which a single participant chooses two of three doors, each concealing one of two prizes (cash or car). Let us label our three doors as green (**G**), red (**R**), and blue (**B**). Figure 1 shows the setup.

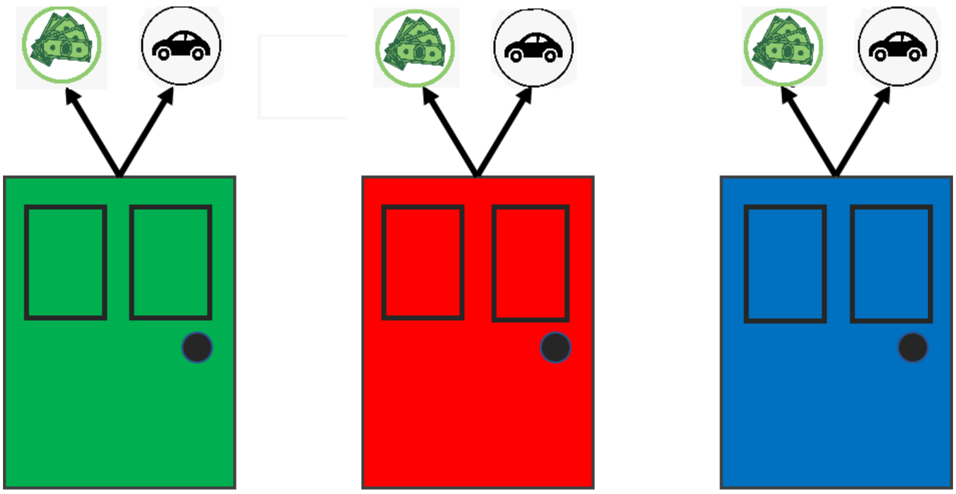

Figure 1: The doors, in order from left to right, are green (**G**), red (**R**), and blue (**B**). The arrows point to the two possible prizes (cash or car), one of which is behind each door.

The goal of this “Quantum Game Show” is to choose two doors with the *same* prize (similar to Wheel of Fortune’s “half-car” prize, where you only win the car if you get both halves). Dedicated fans of the show who’ve watched thousands of episodes have calculated the probability of winning by choosing adjacent doors is 85%, For example, since the green door **G** is next to the red door $\mathbf{R}$, it follows that that the probability that the green and red prizes are the same is 0.85. For short, we'll write this as $P(\mathbf{G} = \mathbf{R}) = 0.85$ Similarly, we also have $P(\mathbf{R} = \mathbf{B}) = 0.85$.

Now comes the big question. Knowing this information (and only this information), can we estimate the probability of winning by choosing the green and blue doors (i.e. $P(\mathbf{G} = \mathbf{B})$)?

Let's consider the possibilities. A little thought reveals that it must be true that at least two doors have the same prize at all times, and possibly all three doors have the same prize. To represent the different possibilities (and to brighten up the text), let's introduce some colorful notation. We'll indicate which door has a different prize from the other two by greying out its color and using lowercase. For example, **GR**b indicates the case where the green and red doors have the same prize, and the blue door has a different prize. Alternatively, **GRB** is the case where all three doors have the same prize. Table 1 summarizes all possible arrangements.

		G & **R** outcomes	
		$\mathbf{G} = \mathbf{R}$	$\mathbf{G} \neq \mathbf{R}$
R&**B**	$\mathbf{R} = \mathbf{B}$	**GRB**	g**RB**
outcomes	$\mathbf{R} \neq \mathbf{B}$	**GR**b	**G**r**B**

Table 1: Possible arrangements of matching prizes behind the three doors. For example, g**RB** represents the case when the red and blue doors have the same prize while the green door is different.

The four entries in the table represent all possible prize arrangements, and exactly one of these arrangements must be realized each time the game is played. It follows that the sum of the probabilities of the four outcomes must be 1:

$$P(\mathbf{GRB}) + P(\mathbf{GR}\mathrm{b}) + P(\mathrm{g}\mathbf{RB}) + P(\mathbf{G}\mathrm{r}\mathbf{B}) = 1 \tag{1}$$

The first column of Table 1 lists the two possible ways that the green prize can be the same as the red prize. As mentioned above, game fans have determined the probability of this happening is 0.85. We may then add the probabilities and obtain:

$$P(\mathbf{GRB}) + P(\mathbf{GR}\mathrm{b}) = P(\mathbf{G} = \mathbf{R}) = 0.85 \tag{2}$$

Also, the first row of Table 1 lists the two possible ways that the red prize can be the same as the blue prize. We know already this happens with probability 0.85. So by similar reasoning we have

$$P(\mathbf{GRB}) + P(\mathrm{g}\mathbf{RB}) = P(\mathbf{R} = \mathbf{B}) = 0.85. \tag{3}$$

Now for some algebra. Taking (2) + (3) − (1) and cancelling terms gives us:

$$P(\mathbf{GRB}) - P(\mathbf{G}\mathrm{r}\mathbf{B}) = 0.7,$$

and adding $2P(\mathbf{G}\mathrm{r}\mathbf{B})$ to both sides of the equation gives:

$$P(\mathbf{GRB}) + P(\mathbf{G}\mathrm{r}\mathbf{B}) = 0.7 + 2P(\mathbf{G}\mathrm{r}\mathbf{B}). \tag{4}$$

The left-hand side of (4) represents all possible arrangements where $\mathbf{G}$ is equal to $\mathbf{B}$ (i.e. $\mathbf{G} = \mathbf{B}$); while the right hand is 0.7 plus a non-negative number, which must be greater than or equal to 0.7. This leads us to our final result:

$$P(\mathbf{G} = \mathbf{B}) \geq 0.7. \tag{5}$$

In conclusion, the probability of having the same prize behind the green and blue doors must be at least 70%. So although we can't find the exact probability, at least we can get a lower estimate.

If we'd worked out the formula without putting in specific numbers, we'd have found that in general:

$$P(\mathbf{G} = \mathbf{B}) = P(\mathbf{G} = \mathbf{R}) + P(\mathbf{R} = \mathbf{B}) - 1 + 2P(\mathbf{G}\mathrm{r}\mathbf{B}), \tag{6}$$

or

$$P(\mathbf{G} = \mathbf{B}) \geq P(\mathbf{G} = \mathbf{R}) + P(\mathbf{R} = \mathbf{B}) - 1. \tag{7}$$

Equation (7) is one way of stating *Bell's inequality.*

There are no tricks in what we've done so far–it's all up-and-up mathematics, straight out of any probability textbook. But what's actually measured in the real-life quantum experiments doesn't agree, as we shall see shortly.

For an alternative presentation of Bell's inequality (without the game show, and with more physics) see Lorenzo Maccone's article [3].

2 Aspect's Experiment

A real-life "Quantum Game Show" experiment was conducted by French physicist Alain Aspect during the early 1980s [1]. In order to understand the experiment, we first need some background information about physics. The following sections introduce two key physical phenomena that play important roles in Aspect's experiment: polarization and photons.

Polarization of light waves

In physics, there are two basic wave types: longitudinal and transverse. Both types of waves have a direction of travel and direction of oscillation (the "back-and-forth" motion of the wave). The relation between these directions determines which type of wave it is. Longitudinal waves travel *parallel* to the direction of oscillation, whereas transverse waves travel *perpendicular* to the direction of oscillation. Traffic jams and sound are common examples of longitudinal waves, while light and ripples on the surface of water are examples of transverse waves.

Polarization is a property of transverse waves in which the direction of oscillation is consistent. For example, water waves are polarized because the oscillation is always in the up-and-down direction, and never from side to side. This is not true of light: for example the Sun (and other common light sources) emits *unpolarized* light waves, or *electromagnetic waves*, which we see in Figure 2. In this case, the direction of oscillation can be random, as long as it's always perpendicular to the direction of travel. Unpolarized light waves can become polarized by passing through a *polarizer* which fixes the oscillation in a consistent direction.

There is more than one way that a transverse wave can be polarized. Linear polarization is the simplest type of polarization. Suppose you are holding one end of a long rope and begin to move your hand directly up and down. The rope's consequential oscillating motion corresponds to a linearly polarized transverse wave. A mathematician would recognize the produced wave as a sinusoidal wave. If you were to move your hand continuously around in a circle, the rope's oscillation would then represent a circularly polarized transverse wave. Figure 3 demonstrates these two distinct types of polarized transverse waves (it's also possible to have elliptical polarization, but that's more complicated than we need to consider right now).

Light beams should actually not be considered as a single wave, but as a collection of many individual waves travelling in the same direction. Each individual wave carries a small amount of energy: the amount of energy is determined by frequency of the wave (the frequency of the wave also determines its color: for example, red light has a lower frequency (and less energy) than blue light). Each one of these individual waves is called a *photon.* Photons are often thought of as "particles" of light, and in some respects they behave like discrete objects. However, this conception is inaccurate and misleading, so we will avoid referring to photons in this way.

Each individual photon has a definite polarization, but a light beam may consist of many photons with different polarizations that happen to be traveling in the same direction. If you

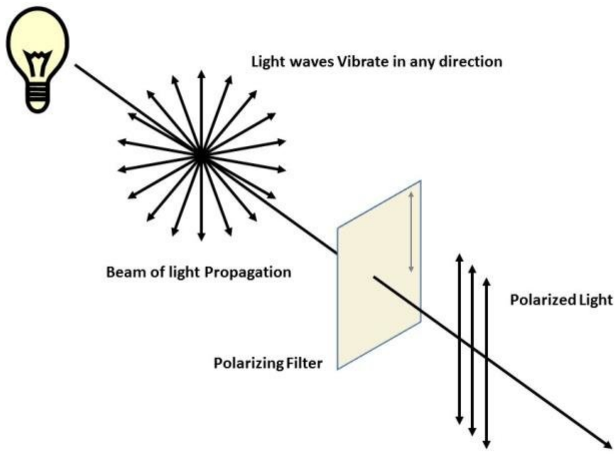

Figure 2: Light source producing unpolarized light which passes through a polarizer to produce a polarized light wave. Web Reference: `https://www.researchgate.net/publication/328100369_Polarised_infrared_light_enables_enhancement_of_histo-morphological_diagnosis_of_prostate_cancer`

send a single photon through a polarizer, it either makes it through or it doesn't. If the photon makes it through, then the outgoing photon has a polarization that is parallel to the polarizer's orientation. If the photon doesn't make it through, then the photon is reflected by the polarizer, and its outgoing polarization is perpendicular to the polarizer's orientation.

If a light beam consists of circularly polarized photons, then regardless of polarizer's orientation half of the photons will be measured as parallel and half will be measured as perpendicular. A key point here is that the measurement of a circularly polarized photon necessarily changes the polarization of the photon. Unlike ordinary 'classical' physics, in quantum physics it is impossible to make a measurement without changing the thing you're measuring: this is one of the biggest differences between classical and quantum physics.

The Experiment

We now have the physical background needed to understand Aspect's experiment, which is schematically represented in Figure 4. At the center of the experiment lies a source S which prepares two 'identical' circularly-polarized photons γ_1 and γ_2, travelling in opposite directions. These photons are 'identical' in the sense that their polarizations are the same according to any observer who measures them (in physics terminology, the two photons are said to be *entangled*).

For this experiment, the observers are linear polarizers placed on opposite sides of the source which measure the two photons' polarizations as they pass through. Let A be the polarizer that measures γ_1 and B be the polarizer that measures γ_2. There are two possible measurements A and B can give for their respective photons: parallel $(+)$ or perpendicular $(-)$. For example, if A reads $(+)$, then γ_1's polarization is parallel relative to the A's orientation.

Initially, A and B are oriented in the same direction and thus yield the same measurement, since γ_1 and γ_2 are entangled. In this case, the angular difference between A and B (denoted by θ) is zero. We thus have $P_{same}(\theta) = 1$ for $\theta = 0$.

In general, given the angle between A's and B's orientation is θ, quantum theory predicts that the probability that A and B have the same measurement is $P_{same}(\theta) = \cos^2\theta$.Since the photons either have the same or different polarizations, it follows that $P_{diff}(\theta) = 1 - \cos^2\theta = \sin^2\theta$. For a mathematical derivation of these results, see [4].

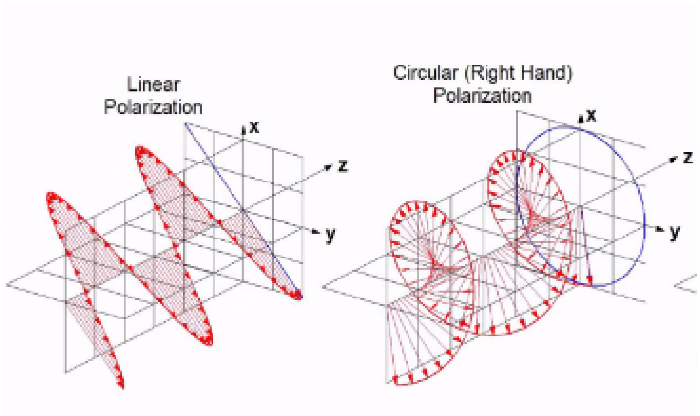

Figure 3: Linear and circular polarization of electromagnetic waves. The red arrow in each diagram shows the direction of the electric field in the electromagnetic wave (light wave). Taken from `https://www.emedicalprep.com/study-material/physics/wave-optics/polarization/`

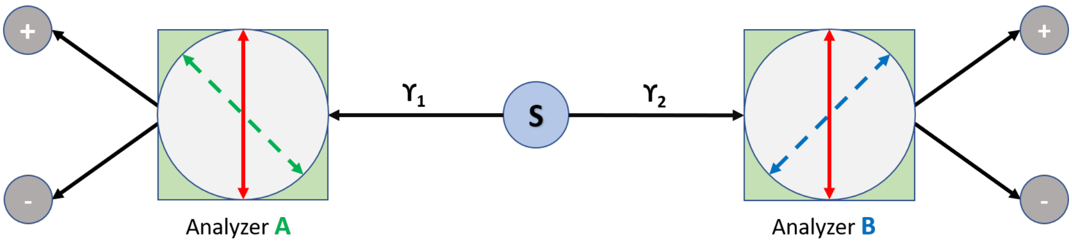

Figure 4: Schematic representation of Aspect's experiment, as described in [1]. The red line indicates the initial orientation of A and B. The green dashed line indicates A is oriented in the counter-clockwise direction while the blue dashed line indicates B is oriented in the clockwise direction. The angle between A and B is used to calculate the probability of the outcome.

For the purposes of this experiment, we use three different orientations for A and B (we'll be referring to Figure 4 above.)

(1) Polarizer A is rotated to align with green arrow while B remains on the red arrow, where the angle between the two is $\theta = 22.5°$.

(2) Polarizer A remains on the red arrow while B is rotated to line up with the blue arrow to give the angle $\theta = 22.5°$.

(3) Polarizers A and B are aligned with the green and blue arrows respectively, so that $\theta = 45°$.

In cases (1) and (2), we can easily see that the probability that A and B have the same measurement is $P_{same}(\theta = 22.5°) = \cos^2(22.5°) = .85$, rounded to the nearest hundredth. Using the quantum game show probabilities derived in Section 1 we obtain the following result:

$$\text{(According to game show)} \qquad P_{same}(45°) \geq .85 + .85 - 1 = 0.7.$$

Yet, when we use quantum mechanics to predict the angle of orientation between A and B, we get:

$$\text{(According to quantum theory)} \qquad P_{same}(45°) = \cos^2(45°) = 0.5$$

So who's right: quantum mechanics, or our intuition? Both can't be right! What Aspect showed is that quantum mechanics wins, and intuition fails. The probability turns out to be only 0.5.

The bottom line

What's going on here? It seems that we're missing something. Whenever something happens that we don't expect, that means that we've assumed something which is not actually the case. But what have we assumed? It seems that in the game-show example the only assumption we've made is that the laws of probability are valid. But in fact, we've also assumed something else, namely that the game-show host is unaware of the choice we are going to make, so the prizes are set regardless of what doors (measurements) we choose.

The violation of Bell's inequality demonstrates a similar hidden assumption in our analysis of Aspect's experiment. Table 1 assumes that there are four definite, distinct possibilities for the two prepared photons. In other words, the photons have "already decided" what they are going to do before the experimenter decides which measurement to make. By doing so, we assume the state they are in doesn't depend on which measurements are actually made. Mathematically, we describe this by saying that the photon states prior to measurement are *independent* of which measurements are subsequently made.

Based on our experience, this assumption is 'obviously' true. How could a measurement of a photon reach back into the past and change how the photon was created? But let us recall Sherlock Holmes' famous dictum: "When you have eliminated the impossible, whatever remains, however improbable, must be the truth." [2] A possible explanation is outlined in [8], and explored in more detail in ([6],[7]). A video series that presents the theory on a more popular level has been placed on YouTube at [5].

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