Collatz's Conjecture

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Conjecture: The following operation is applied on an arbitrary positive integer n

$$f(x) = \begin{cases} \frac{n}{2}, & if\ n = 0\ (mod\ 2) \\ 3n+1, & if\ n = 1\ (mod\ 2) \end{cases}$$

The Collatz's conjecture is: *This process will eventually reach the number 1, regardless of which positive integer is chosen initially.*

Abstract

3n+1 & n/2 we have multiplicative operation and inverse multiplicative operation. But in the case of odd element we have an extra additive operation involved. This makes any sort of analysis very difficult as it is not know how to combine additive and multiplicative operations together in a series of transformation. If we had known how to solve additive and multiplicative operations together in a series of transformations, primes would have been much easier to deal with.

In order to deal with this limitation;

- **We create a function that gives same results that the transformations would yield without really applying them**. The function shall be represented as $[r_b]$.
- We define the relationships between $[r_b]$ & n and various forms of $[r_b]$
- We analyze the conditions for failure of the conjecture and test them using $[r_b]$

Definition 0.1

- Transformation; application of 3n+1 or n/2 on any element.
- Transformed element; the result we get after transformation.

Notations

- [] : square brackets are used to represent sets. All the sets in the analysis are open sets.
- ≡ : Equivalence is used for operations which involve the defined transformations in the problem, i.e. 3n+1 & n/2. Example; 5 ≡ 16 ≡ 1. '≡' shall be used interchangeably with '=' as it is the value we get after the transformation. We need to be critical about =/≡ only when we are checking for special cases like loop.
- " * " notation is used for multiplication.

Let's take a universal set of all natural numbers: $[\mathrm{U}] = [1,2,3,4 ...]$. This consists of odd and even elements.

$$[\mathrm{U}] = [1,3,5,7, ...] \cup [2,4,6,8...]$$

We apply the defined transformation of n/2 for all the even elements i.e. 2,4,6,8... Upon repeated application the transformed set becomes: [1,3,5,7]

$$[\mathrm{U}] = [1,3,5,7, ...] \cup [1,3,5,7, ...] \Rightarrow [\mathrm{U}] = [1,3,5,7, ...]$$

Definition 0.2

r_0 is defined to be the serial number of all the elements of $[\mathrm{U}] = [1,3,5,7 ...]$

$$[\mathrm{r}_0] = [\mathrm{n}_{\mathrm{odd}}] = [\mathrm{U}]$$

We apply 3n+1 transformation to the universal set [U] and get all even numbers with varied divisibility of 2^b where $0 < b < \infty$. r_b: is the serial number set that is based upon the maximum divisibility of 2^b.

Example: all the elements divisible by 2, i.e 2^1 will lie in r_1, all the elements divisible by 4, i.e 2^2 will lie in r_2, all the elements divisible by 8 i.e 2^3 will lie in r_3 and so on.

We define the extended sets

r_1 Divisibility of all the elements of this set is 2^1

r_2 Divisibility of all the elements of this set is 2^2

r_3 Divisibility of all the elements of this set is 2^3 and so on

Definition 0.3

r_b is the extended set of r_{b-1} that is based upon the *Maximum* divisibility of 3n+1 transformed($\mathrm{r_0 = n_{odd}}$) as per the value of 2^b.

Rule for creating extended sets For r_b: when r=odd we keep on creating further sets for said element n till we reach the point r=even. This is same as dividing an element n by 2 till we get an odd number.

Example: element $n = 29$, its corresponding $\mathrm{r_1} = 15$ as $\mathrm{r_1} = \frac{\mathrm{n+1}}{2}$ ($\frac{n\pm1}{2}$ will be explained later) is odd, expand to serial set; $\mathrm{r_2} = \frac{\mathrm{r_1}-1}{2}$, so $\mathrm{r_2} = 7$, $\because \mathrm{r_b}$ is odd, expand the set further; $\mathrm{r_3} = \frac{\mathrm{r_2}+1}{2}$, so $\mathrm{r_3} = 4$, $\because$ $\mathrm{r_b}$ is even we stop. $b = 3$, this implies that the transformed element is divisible by $2^3 = 8$. So, for 29, $r_b = 4_3$.

n/2 till we get odd	**3n+1**	**N**	**b=1**	**b=2**	**b=3**	**b=4**
1	4	**1**	1			
5	10	**3**	2			
1	16	**5**	3	1		
11	22	**7**	4			
7	28	**9**	5	2		
17	34	**11**	6			
5	40	**13**	7	3	2	
23	46	**15**	8			
13	52	**17**	9	4		
29	58	**19**	10			
35	64	**21**	11	5	3	1
19	70	**23**	12			
11	76	**25**	13	6		
41	82	**27**	14			
11	88	**29**	15	7	4	
47	94	**31**	16			
25	100	**33**	17	8		
53	106	**35**	18			

Table 1.0: This table depicts how sets of r_b are created.

Definition 0.3 $[r_b^{even}]$ means that r =even for given b & $[r_b^{odd}]$ we mean that r =odd for given b

Analysis 0.2 Sets of r

$$[r_1] = [r_1^{even}] \cup [r_1^{odd}] = [r_1^{even}] \cup \left([r_2^{even}] \cup [r_2^{odd}]\right)$$
$$= [r_1^{even}] \cup ([r_2^{even}] \cup \left([r_3^{even}] \cup [r_3^{odd}]\right))$$
$$= [r_1^{even}] \cup ([r_2^{even}] \cup \left([r_3^{even}] \cup \left([r_4^{even}] \cup [r_4^{odd}]\right)\right)) \dots \text{ and so on}$$

$$[\mathbf{r_1}] = [\boldsymbol{r_1^{even}}] \cup [\boldsymbol{r_2^{even}}] \cup [\boldsymbol{r_3^{even}}] \cup [\boldsymbol{r_4^{even}}] \cup [\boldsymbol{r_5^{even}}] \cup \dots \boldsymbol{till\ we\ reach\ r_b^{even}}$$

Analysis 0.3 pattern based on (mod3)

We have an expression 3n+1 which is divisible by 2^b, that means $(3n+1) = L * 2^b$; L∈N.

$$3n + 1 \neq 3L$$

So the only possible values for n are 1 (mod3) and 2 (mod3).

Let $L_1 = 1\ (\text{mod}3)$ & $L_2 = 2\ (\text{mod}3)$

If L_1 is multiplied by odd b; we get

$$(3m+1) * 2^{\text{bodd}} = 3m * 2^{\text{bodd}} + 1 * 2^{\text{bodd}} = 0\ (mod3) +\ 2\ (mod3) =\ 2\ (mod3)$$

If L_1 is multiplied by even b; we get

$$(3m+1) *\ 2^{\text{beven}} = 3m * 2^{\text{beven}} + 1 * 2^{\text{beven}} = 0\ (mod3) +\ 1\ (mod3) =\ 1\ (mod3)$$

Same logic is applicable for L_2.

If L_2 is multiplied by odd b; we get

$$(3m+2) * 2^{\text{bodd}} = 3m * 2^{\text{bodd}} + 2 * 2^{\text{bodd}} = 3m * 2^{\text{bodd}} + 2^{\text{beven}} = 0\ (mod3) +\ 1\ (mod3)$$
$$=\ 1\ (mod3)$$

If L_2 is multiplied by even b; we get

$$(3m+2) * 2^{\text{beven}} = 3m * 2^{\text{beven}} + 2 * 2^{\text{beven}} = 3m * 2^{\text{beven}} + 2^{\text{bodd}} = 0\ (mod3) +\ 2\ (mod3)$$
$$=\ 2\ (mod3)$$

Definition 0.4

We create the master equation for extension of r based on Analysis 0.3.

The general expression for set expansion is: $r_b = \frac{r_{(b-1)} \pm 1}{2}$.

$$\mathbf{r_b} = \frac{\mathbf{r_{(b-1)} \pm 1}}{\mathbf{2}} = \begin{cases} \mathbf{r_b} = \dfrac{\mathbf{r_{(b-1)} + 1}}{\mathbf{2}} & \textbf{for b = odd 1 (mod3)} \\ \mathbf{r_b} = \dfrac{\mathbf{r_{(b-1)} - 1}}{\mathbf{2}} & \textbf{for b = even 2 (mod3)} \end{cases}$$

Analysis 0.4

We define sets of all the transformed elements according to divisibility according to powers of 2; $r_1 = \frac{r_0+1}{2} = \frac{n+1}{2}$, $r_2 = \frac{r_1-1}{2}$, $r_3 = \frac{r_2+1}{2}$, $r_4 = \frac{r_3-1}{2}$, $r_5 = \frac{r_4+1}{2}$, $r_6 = \frac{r_4-1}{2}$ $r_7 = \frac{r_6+1}{2}$ and so on.

$$[r_{1\to<\infty}\} = [\frac{n+1}{2}, \frac{n-1}{4}, \frac{n+3}{8}, \frac{n-5}{16}, \frac{n+11}{32}, \frac{n-21}{64}, \frac{n+43}{128}, \frac{n-85}{256}, \frac{n+171}{512}, \frac{n-341}{1024}, \frac{n+683}{2048}, \frac{n-1365}{4096}, \frac{n+2731}{8192} \dots]$$

Definition 0.5

$$n_{\text{bt(before transformation; applying 3n+1)}}$$
$$\equiv n_{\text{at(after transformation; applying 3n+1 and dividing it by max power of 2)}}$$

sum of r_b + r_{b-1}	n/2 till we get odd	3n+1	n	b=1	b=2	b=3	b=4
1	1	4	**1**	1			
5	5	10	**3**	2			
1	1	16	**5**	3	1		
11	11	22	**7**	4			
7	7	28	**9**	5	2		
17	17	34	**11**	6			
5	5	40	**13**	7	3	2	
23	23	46	**15**	8			
13	13	52	**17**	9	4		
29	29	58	**19**	10			
35	35	64	**21**	11	5	3	1
19	19	70	**23**	12			
11	11	76	**25**	13	6		
41	41	82	**27**	14			
11	11	88	**29**	15	7	4	
47	47	94	**31**	16			
25	25	100	**33**	17	8		
53	53	106	**35**	18			

In Table 1 we notice that column: "sum of r_b + r_{b-1}" & column "n/2 till we get odd" have the same values

Proposition 0.1

$$\mathbf{n_{at} = r_b + r_{b-1}}$$

Validity 0.1

Expansion of n_{at}according to definition for initial few values of

$$n_{at} = \begin{cases} for\ r_2 = \frac{n+1}{2} + \frac{n-1}{4} = \frac{2n+2+n-1}{4} = \frac{3n+1}{4} \\ for\ r_3 = \frac{n-1}{4} + \frac{n+3}{8} = \frac{2n-2+n+3}{8} = \frac{3n+1}{8} \\ for\ r_4 = \frac{n+3}{8} + \frac{n-5}{16} = \frac{2n+6+n-5}{16} = \frac{3n+1}{16} \\ for\ r_4 = \frac{n-5}{16} + \frac{n+11}{32} = \frac{2n+6+n-5}{32} = \frac{3n+1}{32} \\ and\ so\ on\ ... \end{cases}$$

Values for r_b & r_{b-1} have been taken from Analysis 0.5

Generalized from of the above expansion:

$$n_{at} = \frac{n \pm \tau}{2^b} + \frac{n \mp \sigma}{2^{b-1}} \text{ such that } \sigma, \tau \in W$$

$$n_{at} = \frac{n - \tau}{2^b} + \frac{n + \sigma}{2^{b-1}} = \frac{2n + n + 2\sigma - \tau}{2^b} = \frac{3n + (2\sigma - \tau)}{2^{(b+1)}} \Rightarrow 2\sigma - \tau = 1$$

$$\sigma = \frac{\tau + 1}{2} \; We\ get\ the\ defining\ eqution\ for\ odd\ b\ even\ r$$

Similarly,

$$n_{at} = \frac{n + \tau}{2^b} + \frac{n - \sigma}{2^{b-1}} \Rightarrow : \sigma = \frac{(\tau - 1)}{2} \; We\ get\ the\ defining\ equation\ for\ even\ b\ even\ r$$

Thus, we have all the values of n_{at} without having to apply 3n+1 and n/2 anymore. We can get value after transformation and it being divided till we get an odd entity by just analyzing the r of any number.

Corollary 0.1 from Proposition 0.1

$r_0 = n_{bt}$ by definition 0.2. For b=1; Substituting values in $n_{at} = r_b + r_{b-1}$

$$n_{at} = r_1 + r_0 = r_1 + n_{bt}$$

This is the condition for increase for all elements.

Exception 0.1

There exists a set of number/s that do not follow the relationship in proposition 0.1

Definition 0.4 implies $r_b = \frac{r_{(b-1)}+1}{2}$. When substituting the value $\tau = 1$, we get: $\sigma = 0$

The value for r_b ends up to be 1 which cannot be divided by 2 leaving us with odd number 1 and not allowing us to break it further into even sets. Such entities defined as

$$r_b + r_{b-1} = 4^x such\ that\ x \in N,$$

We get $n_{at} = 4^x$ which is even. n_{at} by definition should be odd. Thus, we have an exception.

Definition 0.6

Let us define $[2^y] = [2,4,8,16,32 \dots]$. y corresponds to value of b: r_b=1.

We apply 3n+1 and n/2 in reverse on$[2^y]$: we define operations to be; $\frac{n-1}{3}$ and 2*n y times.

Corollary 0.2

Taking the set $[2^y]$ and applying $\frac{n-1}{3}$ we get a set of valid natural number values for alternating elements of $[2^y]$. This can be validated by the fact that [2m]∩[3k] for every alternate element of [3k], where $m \& k \in N$

Analysis 0.5

$[2^y] \equiv [\frac{2-1}{3}, \frac{4-1}{3}, \frac{8-1}{3}, \frac{16-1}{3}, \dots]$; Ignoring invalid elements; $[2^y] \equiv$ [1,5, 21,85,341 ,1365].

Corollary 0.3

Every third element of this set is divisible by 3.

Validity 0.2

Corollary 0.3 implies that $2^{6y} - 1 = 9m \Rightarrow 9m + 1 = 2^{6y}$

We know $2^{6y} - 1 = 9m$ for y=1.

When we increase the value of y for 2^{6^y};

$$2^{6^2} = (9m + 1)^2 = ((9m)^2 + (9m) * 1 + 1 * (9m) + 1^2)$$

Generalized form;

$$2^{6^y} = ((9m)^y + \alpha_1(9m)^{(y-1)}1^2 + \alpha_2(9m)^{y-2}1^3 \dots \alpha_{y-1}(9m)^{y-(y-1)}1^{y-1} + 1^y)$$

α_n is the numerical constant we get after expanding 2^{6y} depending upon the value of y. $9m$ is common factor for the entities of expansion except the last term i.e. 1^y

$$2^{6y} = 9m((9m)^{y-1} + \alpha_1(9m)^{(y-2)}1^2 + \alpha_2(9m)^{y-3}1^3 \dots \alpha_{y-1}(9m)^{y-(y)}1^{y-1})) + 1^y$$

We compress the above expression:

$$2^{6y} = 9mk + 1$$

Where k is the result when we factor out 9m out of all the entities except the last one 1^y;

$$k = (9m)^{y-1} + \alpha_1(9m)^{(y-2)}1^2 + \alpha_2(9m)^{y-3}1^3 \dots \alpha_{y-1}(9m)^{y-(y)}1^{y-1})$$

So, every third element is divisible by 3.

$$[2^{6y}] = [64 \equiv \left(\frac{64-1}{3} = 21 = 9 * 7\right), 4096 \equiv \left(\frac{4096-1}{3} = 1365 = 9 * 455\right), 261144 \equiv \left(\frac{261144-1}{3} = 87381 = 9 * 29127\right) \dots]$$

Thus, we need not concern ourselves with this set defined by (St 6 & St 8) as $3n + 1 \neq 3m$

Analysis 0.6

Corollary 0.2 & corollary 0.3 imply that exception 0.1 is not a set of [3n+1]. For odd b: $\tau = 1$ & $\sigma = 0$. The element related to said condition is n=1 which we shall explore individually.

Analysis 1.0 **Testing for conditions for failure of conjecture.**

Condition 0.1 $\exists\, \boldsymbol{n'} \notin [\mathbf{r_b}]$

$[r_b]$ is incomplete and there exists some element n' that lie outside $[\mathrm{r_b}]$.

$$\mathrm{r_b} \in \mathrm{r_{b-1}} \in \mathrm{r_{b-2}} \in \mathrm{r_{b-3}} \in \mathrm{r_{b-4}} \in \cdots \mathrm{r_1}$$

$\mathrm{r_b}$ is extended set for $\mathrm{r_{b-1}}$ that is extended set for $\mathrm{r_{b-2}}$ that is extended set for $\mathrm{r_{b-3}}$ that is extended set for $\mathrm{r_{b-4}}$ and so on that is extended set for $\mathrm{r_1}$. $\mathrm{r_1}$ is just the serial number set for all elements of [U] thus is defined for all the elements of [U]. Every r has some b such that $0 < b <= \infty$.

$$\therefore\ \nexists\, \boldsymbol{n'} \notin [\mathbf{r_b}]$$

Condition 0.2 $\boldsymbol{n'} \equiv \infty$

Analysis 1.1

To grow to infinity, the transformations should continue for infinite steps; implying there exists least infinite numbers that explode to infinity.

Weak argument by definition all the points any element will land after being transformed are finitely defined by r_b. $n_{at} = r_b + r_{b-1}$; both the variables on right hand side are finitely represented and thus left hand side i.e. n_{at} also can be finitely represented. n cannot transform to infinity.

Strong argument

Defination1.1

For n=odd, upon application of 3n+1, the element grows. We define: Max growth factor =3n+1 ~3n, max growth factor is fixed.

For n = even, we apply $\frac{n}{2}$ once or more, said process is : $\frac{n}{2^b}$. We define: Reduction factor=2^b, reduction factor is variable.

Weighted avg reduction factor for range 2^1 to 2^y such that 1<b<y

$$= \frac{2^1 * 2^{y-1} + 2^2 * 2^{y-2} + 2^3 * 2^{y-3} + \cdots + 2^y * 2^0}{2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^y}$$

Weighted avg reduction factor for y=1 is 2, for y=2 is 2.67, for y=3 is 3.43, for y=4 is 4.27 and so on.

	Weighted avg reduction factor	2^b			Repetition count					
y=1	2.00	2.00		Y	2	4	8	16	32	64
y=2	2.67	4.00	2.00	y-1	1	2	4	8	16	32
y=3	3.43	8.00	4.00	y-2		1	2	4	8	16
y=4	4.27	16.00	8.00	y-3			1	2	4	8
y=5	5.16	32.00	16.00	y-4				1	2	4
y=6	6.10	64.00	32.00	y-5					1	2
y=7	7.06	128.00	64.00	y-6						1

and so on

Table of weighted avg reduction factor

Analysis 1.2 Larger the number larger is the reduction factor.

For y=1, Weighted avg reduction factor < max growth factor, so we are certain of growth.

For y=2, Weighted avg reduction factor < max growth factor, so we are certain of growth.

For y>2, Weighted avg reduction factor > max growth factor, so we are not certain of growth.

All the elements that lie between 2^0 & 2^1, will definitely increase. All the elements that lie between 2^1 & 2^2, will definitely increase. Elements for both the cases are known to be convergent to 1. Beyond that, for y>2, all the elements will not definitively increase. For y>2, we cannot be sure of definitive increase at for all elements as weighted avg reduction factor > max growth factor.

For elements such that y>2, they may increase or decrease. We can predict the increase or decrease depending upon the value of b. As per corollary 0.1, if b =1, the number is certain to grow at the rate of approx 3/2. For b>1, the number is certain to reduce which can be checked by value of b such that the net transformation is $\frac{n}{2^b}$. Analysis based upon behaviour of max growth factor and weighted avg growth factor suggests that element should eventually reduce. However, any analysis involving 'avg' cannot point out the statistical outliers aka the exception case scenarios.

To be sure that any transformation explodes to infinity, there has to be non negative growth at every step; in this case definitive increase at every step as increase by approx 3/2 is the only possible increment factor. For any element that would transform to infinity, the number may increase or decrease but at some point we have to be certain about its increase at every step of transformation.

Exception 0.2

The exception condition, where we can be sure of definitive increase; there exists an element that would definitively increase for y>2: b is always 1, resulting it to continuously explode to infinity.

Proposition 1.1

let's consider some n' which explodes to infinity with continuous increase. n' has r_b where b =1.

$$r_{1(at)} = r_{1(bt)} + \frac{r_{1(bt)}}{2} = \frac{3}{2} r_{1(bt)}$$

Validity1.1

Proposition 0.1 states $\mathrm{r}_{1(\mathrm{bt})} = n_{at} - \mathrm{n}_{\mathrm{bt}}$

$r_{1(at)} = \frac{n_{at}+1}{2} \Rightarrow n_{at} = 2r_{1(at)} - 1$

Similarly,

$$\mathrm{n}_{\mathrm{bt}} = 2r_{1(bt)} - 1$$

So,

$$\mathrm{r}_{1(\mathrm{bt})} - (2r_{1(at)} - 1) - (2r_{1(bt)} - 1) = 3\mathrm{r}_{1(\mathrm{bt})} = 2r_{1(at)}$$

$$\Rightarrow r_{1(at)} = \frac{3}{2} r_{1(bt)}$$

Corollary 1.1

We keep applying 3n+1 and dividing it by exactly 2 every time, implying r increasing by $\frac{3}{2}$. No matter the value of r_{at}, when we multiply it by $\frac{3}{2}$, we will eventually get an odd number resulting net transformation to be <1 as b>1.

Validity 1.2

Say, there exists some element: even r' such that $\frac{3^z}{2^z} r_1 = r'$

$$\frac{r'}{2} = \frac{3^z}{2^{(z-1)}} r_2$$

$$\frac{r'}{2^2} = \frac{3^z}{2^{(z-2)}} r_3$$

... and we keep repeating this till:

$$\frac{r'}{2^z} = 3^z r_z \; such \; that \; z \in N$$

$\frac{r\prime}{2^z}$ should be even for corollary 1.1 to be incorrect. On the right hand side of the equation, we have 3^z, the solution for this expression is always odd. So r_z has to be even for $\frac{r\prime}{2^z}$ to be even, r_z has been defined as odd definition (except for elements following exception 0.1). Apart from element '1' none of the elements in transformed [U] lie outside proposition 0.1

r_z being even is false. By contradiction we establish the validity of corollary 1.1.

Thus, there does not exist any element that will continuously explodes to infinity.

In fact, we can divide all the numbers into sets that define the number of steps any element would take to reach 1. (step counting; attached as addon).

$$\therefore \boldsymbol{n'} \not\equiv \infty$$

Condition 0.3 $\quad \exists \; \boldsymbol{a \; loop \; such \; that \; n_u = n_w}$

We have some number n_u such that

$$n_s \equiv n_u \equiv n_v \equiv n_w \; such \; that \; n_u = n_w$$

There may be single transformation or several transformations from n_u to n_w; the penultimate transformation before n_w yields n_v.

Definition 1.2 $\quad r_{b(s)}$ is value of r_b for element 's' & $r_{b(v)}$ is value of r_b for element 'v'

Proposition 0.1 states $\mathrm{n_{at}} = \mathrm{r_b} + \mathrm{r_{b-1}}$

$$\Rightarrow n_u = r_{b(s)} + r_{b-1(s)} \;\&\; n_w = r_{b(v)} + r_{b(v-1)}$$

$$n_u = n_w \Rightarrow r_{b(v)} + r_{b-1(v)} = r_{b(s)} + r_{b-1(s)}$$

Analysis 0.4 states $r_{b(v)} = \frac{r_{b-1(v)} \pm 1}{2} \Rightarrow 2r_{b(v)} = r_{b-1(v)} \pm 1$

$$n_u = n_w \Rightarrow 3r_{b(v)} \pm 1 = 3r_{b(s)} \pm 1$$

Analysis 1.3

After $3r_b$, we have +'1' or –'1' depending upon the value of b as explained in analysis 0.4. So, we have 2 cases; Case 1; when both have different signs before '1' such that $3r_{b(v)} = 3r_{b(s)} \pm 2$ & Case 2; when both have same sign before '1' such that $3r_{b(v)} = 3r_{b(s)}$.

Case1

$$3r_{b(v)} = 3r_{b(s)} \pm 2$$

There does not exist any element that satisfies the equation. So, we rule out this case.

Case 2

$$3r_{b(v)} = 3r_{b(s)} \Rightarrow r_{b(v)} = r_{b(s)}$$

Both $r_{b(s)}$&$r_{b(v)}$are uniquely defined for specific elements, thus $r_{b(s)} \neq r_{b(v)}$ for all elements that follow Proposition 0.1: $\mathrm{n_{at} = r_b + r_{b-1}}$.

The set [2^{6y}] (St 8) is not a solution to 3n+1, so we need not concern ourselves with it.

There lies one single element that does not follow Proposition 0.1: $\mathrm{n_{at} = r_b + r_{b-1}}$and is not an element of [2^{6y}]: Exception 0.1: when $\tau = 1$, we get $\sigma = 0$. The element corresponding to said condition is n=1, the only possible loop point in the universal set of numbers [U] that we have analyzed. For n=1, we get the same; n_{bt}=n_{at}= 1, satisfying the condition of case 2: 3$r_{b(v)}$ =3$r_{b(s)}$.

$$\boldsymbol{n_u = n_w \; only \; for \; n_u = 1}$$

Conclusion: $\forall \boldsymbol{n \equiv 1}$

We have checked for incompleteness of set $[r_b]$: $\exists \, n' \notin [\mathrm{r_b}]$

We have checked for any number exploding to infinity : $n' \equiv \infty$

We have checked for possibility of a loop point: $\exists \; a \; loop \; such \; that \; n_u = n_w$

Thus we conclude the only possible loop point is n=1 implying that the conjecture is true.