Double serial operators theory.

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0- Abstract:

This paper is an advance of my work in serial operators. Here you can see 216 theoretical combinations of two serial operators with a 2-variable operation. The theory covers addition, subtraction, product, division, power and root. Moreover I am going to present 48 numerical examples.

1- Introduction:

Based in my paper “Resume of the serial operators theory” [1], in which I put the examples of the six combinations of serial operations, this idea is, as I presented in the conclusions of that paper, the next step in the structure of a theory of calculus of operations.
Doubling the variables we can go further of the main idea, and if we double the variables we also double the operator. I am going to use uppercase Greek letters in the corresponding operators, I am going to use sigma for summation, rho for restory, pi for productory, delta for divisory, zeta for exponentory and dseda for rootory.

2- Theoretical combinations with examples in the easy ones:

2.1.1.1- Summation of summation of the sum:

$$(1)\quad \sum_{m=j} \sum_{n=i} i+j$$

Example:

$$(2)\quad \sum_{m=2}^{4} \sum_{n=3}^{5} i+j = \sum_{m=2}^{4} (3+j)+(4+j)+(5+j) = ((3+2)+(4+2)+(5+2))+((3+3)+(4+3)+(5+3))$$

$$+((3+4)+(4+4)+(5+4)) = (5+6+7)+(6+7+8)+(7+8+9) = 18+21+24 = 63$$

2.1.1.2- Summation of summation of the rest:

$$(3)\quad \sum_{m=j} \sum_{n=i} i-j$$

Example:

$$(4)\quad \sum_{m=1}^{4}\sum_{n=1}^{3} i-j=\sum_{m=1}^{4}(1-j)+(2-j)+(3-j)=((1-2)+(2-2)+(3-2))$$
$$+((1-3)+(2-3)+(3-3))+((1-4)+(2-4)+(3-4))=((-1)+0+1)+((-2)+(-1)+0)$$
$$+((-3)+(-2)+(-1))=0-3-6=-9$$

2.1.1.3- Summation of summation of the product:

$$(5)\quad \sum_{m=j}\sum_{n=i} i\cdot j$$

Example:

$$(6)\quad \sum_{m=3}^{5}\sum_{n=1}^{2} i\cdot j=\sum_{m=3}^{5}(1\cdot j)+(2\cdot j)=((1\cdot 3)+(2\cdot 3))+((1\cdot 4)+(2\cdot 4))+((1\cdot 5)+(2\cdot 5))=9+12+15=36$$

2.1.1.4- Summation of summation of the division:

$$(7)\quad \sum_{m=j}\sum_{n=i} i\div j$$

Example:

$$(8)\quad \sum_{m=3}^{5}\sum_{n=2}^{4} i\div j=\sum_{m=3}^{5}(2\div j)+(3\div j)+(4\div j)=((2\div 3)+(3\div 3)+(4\div 3))$$
$$+((2\div 4)+(3\div 4)+(4\div 4))+((2\div 5)+(3\div 5)+(4\div 5))=(\frac{2}{3}+1+\frac{4}{3})+(\frac{1}{2}+\frac{3}{4}+1)+(\frac{2}{5}+\frac{3}{5}+\frac{4}{5})=\frac{141}{20}$$

2.1.1.5- Summation of summation of the power:

$$(9)\quad \sum_{m=j}\sum_{n=i} i^{j}$$

Example:

$$(10)\quad \sum_{m=2}^{3}\sum_{n=1}^{2} i^{j}=\sum_{m=2}^{3}(1^{j})+(2^{j})=((1^{2})+(2^{2}))+((1^{3})+(2^{3}))=1+4+1+8=14$$

2.1.1.6- Summation of summation of the root:

$$(11)\quad \sum_{m=j}\sum_{n=i} \sqrt[j]{i}$$

Example:

$$(12)\quad \sum_{m=3}^{5}\sum_{n=2}^{3} \sqrt[j]{i}=\sum_{m=3}^{5}(\sqrt[j]{2}+\sqrt[j]{3})=(\sqrt[3]{2}+\sqrt[3]{3})+(\sqrt[4]{2}+\sqrt[4]{3})+(\sqrt[5]{2}+\sqrt[5]{3})=7.601\ldots$$

2.1.2.1- Summation of restory of the sum:

$$(13)\quad \sum_{m=j} \prod_{n=i} i+j$$

Example:

$$(14)\quad \sum_{m=5}^{7} \prod_{n=30}^{33} i+j = \sum_{m=5}^{7} -(30+j)-(31+j)-(32+j)-(33+j) = (-(30+5)-(31+5)-(32+5)-(33+5))$$
$$+(-(30+6)-(31+6)-(32+6)-(33+6))+(-(30+7)-(31+7)-(32+7)-(33+7)) = -146$$
$$-150-154=-450$$

2.1.2.2- Summation of restory of the rest:

$$(15)\quad \sum_{m=j} \prod_{n=i} i-j$$

Example:

$$(16)\quad \sum_{m=3}^{5} \prod_{n=2}^{3} i-j = \sum_{m=3}^{5} -(2-j)-(3-j) = (-(2-3)-(3-3))+(-(2-4)-(3-4))$$
$$+(-(2-5)-(3-5)) = (-(-1)+0)+(-(-2)-(-1))+(-(-3)-(-2)) = 1+3+5=9$$

2.1.2.3- Summation of restory of the product:

$$(17)\quad \sum_{m=j} \prod_{n=i} i \cdot j$$

Example:

$$(18)\quad \sum_{m=5}^{7} \prod_{n=2}^{4} i \cdot j = \sum_{m=5}^{7} -(2\cdot j)-(3\cdot j)-(4\cdot j) = (-(2\cdot 5)-(3\cdot 5)-(4\cdot 5))+(-(2\cdot 6)-(3\cdot 6)-(4\cdot 6))$$
$$+(-(2\cdot 7)-(3\cdot 7)-(4\cdot 7)) = -45-54-63=-162$$

2.1.2.4- Summation of restory of the division:

$$(19)\quad \sum_{m=j} \prod_{n=i} i \div j$$

Example:

$$(20)\quad \sum_{m=2}^{4} \prod_{n=3}^{4} i \div j = \sum_{m=2}^{4} -(3\div j)-(4\div j) = (-(3\div 2)-(4\div 2))+(-(3\div 3)-(4\div 3))$$
$$+(-(3\div 4)-(4\div 4)) = \frac{-7}{2}-\frac{7}{3}-\frac{7}{4} = \frac{-91}{12}$$

2.1.2.5- Summation of restory of the power:

$$(21)\quad \sum_{m=j} \prod_{n=i} i^{j}$$

Example:

$$(22)\quad \sum_{m=3}^{4}\prod_{n=20}^{22} i^j = \sum_{m=3}^{4} -(20^j)-(21^j)-(22^j)=(-(20^3)-(21^3)-(22^3))$$
$$+((-20^4)-(21^4)-(22^4))=-27909-588737=-616646$$

2.1.2.6- Summation of restory of the root:

$$(23)\quad \sum_{m=j}\prod_{n=i}\sqrt[j]{i}$$

Example:

$$(24)\quad \sum_{m=3}^{5}\prod_{n=2}^{4}\sqrt[j]{i}=\sum_{m=3}^{5} -\sqrt[j]{2}-\sqrt[j]{3}-\sqrt[j]{4}=(-\sqrt[3]{2}-\sqrt[3]{3}-\sqrt[3]{4})+(-\sqrt[4]{2}-\sqrt[4]{3}-\sqrt[4]{4})$$
$$+(-\sqrt[5]{2}-\sqrt[5]{3}-\sqrt[5]{4})=-11.923\ldots$$

2.1.3.1- Summation of productory of the sum:

$$(25)\quad \sum_{m=j}\prod_{n=i} i+j$$

Example:

$$(26)\quad \sum_{m=3}^{5}\prod_{n=2}^{5} i+j=\sum_{m=3}^{5}(2+j)\cdot(3+j)\cdot(4+j)\cdot(5+j)=((2+3)\cdot(3+3)\cdot(4+3)\cdot(5+3))$$
$$+((2+4)\cdot(3+4)\cdot(4+4)\cdot(5+4))+((2+5)\cdot(3+5)\cdot(4+5)\cdot(5+5))=1680+3024+5040=9744$$

2.1.3.2- Summation of productory of the rest:

$$(27)\quad \sum_{m=j}\prod_{n=i} i-j$$

Example:

$$(28)\quad \sum_{m=30}^{33}\prod_{n=15}^{16} i-j=\sum_{m=30}^{33}(15-j)\cdot(16-j)=((15-30)\cdot(16-30))+((15-31)\cdot(16-31))$$
$$+((15-32)\cdot(16-32))+((15-33)\cdot(16-33))=210+240+272+306=1028$$

2.1.3.3- Summation of productory of the product:

$$(29)\quad \sum_{m=j}\prod_{n=i} i\cdot j$$

Example:

$$(30)\quad \sum_{m=2}^{4}\prod_{n=1}^{2} i\cdot j=\sum_{m=2}^{4}(1\cdot j)\cdot(2\cdot j)=((1\cdot 2)\cdot(2\cdot 2))+((1\cdot 3)\cdot(2\cdot 3))+((1\cdot 4)\cdot(2\cdot 4))=8+18+32=58$$

2.1.3.4- Summation of productory of the division:

$$(31)\quad \sum_{m=j}\prod_{n=i} i\div j$$

Example

$$(32)\quad \sum_{m=3}^{5}\prod_{n=2}^{4} i\div j=\sum_{m=3}^{5}(2\div j)\cdot(3\div j)\cdot(4\div j)=((2\div 3)\cdot(3\div 3)\cdot(4\div 3))+((2\div 4)\cdot(3\div 4)\cdot(4\div 4))$$
$$+((2\div 5)\cdot(3\div 5)\cdot(4\div 5))=\frac{8}{9}+\frac{3}{8}+\frac{24}{125}=1.455\ldots$$

2.1.3.5- Summation of productory of the power:

$$(33)\quad \sum_{m=j}\prod_{n=i} i^j$$

Example:

$$(34)\quad \sum_{m=5}^{4}\prod_{n=81}^{82} i^j=\sum_{m=5}^{4}(81^j)\cdot(82^j)=((81^5)\cdot(82^5))$$
$$+((81^6)\cdot(82^6))=85{,}873{,}390{,}196{,}432{,}977{,}678{,}176$$

2.1.3.6- Summation of productory of the root:

$$(35)\quad \sum_{m=j}\prod_{n=i} \sqrt[j]{i}$$

Example:

$$(36)\quad \sum_{m=2}^{3}\prod_{n=5}^{7} \sqrt[j]{i}=\sum_{m=2}^{3}\sqrt[j]{5}\cdot\sqrt[j]{6}\cdot\sqrt[j]{7}=(\sqrt[2]{5}\cdot\sqrt[2]{6}\cdot\sqrt[2]{7})+(\sqrt[3]{5}\cdot\sqrt[3]{6}\cdot\sqrt[3]{7})=20.435\ldots$$

2.1.4.1- Summation of divisory of the sum:

$$(37)\quad \sum_{m=j}\Delta_{n=i}\, i+j$$

Example:

$$(38)\quad \sum_{m=7}^{9}\Delta_{n=2}^{4}\, i+j=\sum_{m=7}^{9}(2+j)\div(3+j)\div(4+j)=((2+7)\div(3+7)\div(4+7))$$
$$+((2+8)\div(3+8)\div(4+8))+((2+9)\div(3+9)\div(4+9))=\frac{9}{100}+\frac{5}{66}+\frac{11}{156}=\frac{1957}{8580}$$

2.1.4.2- Summation of divisory of the rest:

$$(39)\quad \sum_{m=j}\Delta_{n=i}\, i-j$$

Example:

$$(40)\quad \sum_{m=1}^{2}\ \Delta_{n=3}^{5}\ i-j=\sum_{m=1}^{2}(3-j)\div(4-j)\div(5-j)=((3-1)\div(4-1)\div(5-1))$$
$$+((3-2)\div(4-2)\div(5-2))=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$$

2.1.4.3- Summation of divisory of the product:

$$(41)\quad \sum_{m=j}\ \Delta_{n=i}\ i\cdot j$$

Example:

$$(42)\quad \sum_{m=10}^{12}\ \Delta_{n=3}^{4}\ i\cdot j=\sum_{m=10}^{12}(3\cdot j)\div(4\cdot j)=((3\cdot 10)\div(4\cdot 10))+((3\cdot 11)\div(4\cdot 11))$$
$$+((3\cdot 12)\div(4\cdot 12))=\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{9}{4}$$

2.1.4.4- Summation of divisory of the division:

$$(43)\quad \sum_{m=j}\ \Delta_{n=i}\ i\div j$$

Example:

$$(44)\quad \sum_{m=41}^{44}\ \Delta_{n=3}^{4}\ i\div j=\sum_{m=41}^{44}(3\div j)\div(4\div j)=((3\div 41)\div(4\div 41))+((3\div 42)\div(4\div 42))$$
$$+((3\div 43)\div(4\div 43))+((3\div 44)\div(4\div 44))=\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=3$$

2.1.4.5- Summation of divisory of the power:

$$(45)\quad \sum_{m=j}\ \Delta_{n=i}\ i^{j}$$

Example:

$$(46)\quad \sum_{m=3}^{5}\ \Delta_{n=15}^{16}\ i^{j}=\sum_{m=3}^{5}(15^{j})\div(16^{j})=((15^{3})\div(16^{3}))+(15^{4})\div(16^{4})+(15^{5})\div(16^{5})=\frac{3375}{4096}$$
$$+0.772\ldots+0.724\ldots=2.319\ldots$$

2.1.4.6- Summation of divisory of the root:

$$(47)\quad \sum_{m=j}\ \Delta_{n=i}\ \sqrt[j]{i}$$

Example:

$$(48)\quad \sum_{m=3}^{4}\ \Delta_{n=12}^{13}\ \sqrt[j]{i}=\sum_{m=3}^{4}\sqrt[j]{12}\div\sqrt[j]{13}=(\sqrt[3]{12}\div\sqrt[3]{13})+(\sqrt[4]{12}+\sqrt[4]{13})=1.953\ldots$$

2.1.5.1- Summation of exponentory of the sum:

$$(49) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} i+j$$

2.1.5.2- Summation of exponentory of the rest:

$$(50) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} i-j$$

2.1.5.3- Summation of exponentory of the product:

$$(51) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} i \cdot j$$

2.1.5.4- Summation of exponentory of the division:

$$(52) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} i \div j$$

2.1.5.5- Summation of exponentory of the power:

$$(53) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} i^j$$

2.1.5.6- Summation of exponentory of the root:

$$(54) \quad \underset{m=j}{\Sigma} \underset{n=i}{\Theta} \sqrt[j]{i}$$

2.1.6.1- Summation of rootory of the sum:

$$(55) \quad \underset{m=j}{\Sigma} \underset{n=i}{Z} i+j$$

2.1.6.2- Summation of rootory of the rest:

$$(56) \quad \underset{m=j}{\Sigma} \underset{n=i}{Z} i-j$$

2.1.6.3- Summation of rootory of the product:

$$(57) \quad \underset{m=j}{\Sigma} \underset{n=i}{Z} i \cdot j$$

2.1.6.4- Summation of rootory of the division:

$$(58) \quad \underset{m=j}{\Sigma} \underset{n=i}{Z} i \div j$$

2.1.6.5- Summation of rootory of the power:

$$(59) \quad \underset{m=j}{\Sigma} \underset{n=i}{Z} i^j$$

2.1.6.6- Summation of rootory of the root:

$$(60)\quad \underset{m=j}{\Sigma}\ \underset{n=i}{\mathrm{Z}} \sqrt[j]{i}$$

2.2.1.1- Restory of summation of the sum:

$$(61)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma} i+j$$

Example:

$$(62)\quad \underset{m=2}{\overset{5}{\mathrm{P}}}\ \underset{n=2}{\overset{3}{\Sigma}} i+j = \underset{m=2}{\overset{5}{\mathrm{P}}} (2+j)+(3+j) = -((2+2)+(3+2))-((2+3)+(3+3))-((2+4)+(3+4)) -((2+5)+(3+5)) = -9-11-13-15=-48$$

2.2.1.2- Restory of summation of the rest:

$$(63)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma} i-j$$

Example:

$$(64)\quad \underset{m=3}{\overset{5}{\mathrm{P}}}\ \underset{n=20}{\overset{21}{\Sigma}} i-j = \underset{m=3}{\overset{5}{\mathrm{P}}} (20-j)+(21-j) = -((20-3)+(21-3))-((20-4)+(21-4)) -((20-5)+(21-5)) = -35-33-31=-99$$

2.2.1.3- Restory of summation of the product:

$$(65)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma} i\cdot j$$

Example:

$$(66)\quad \underset{m=3}{\overset{6}{\mathrm{P}}}\ \underset{n=8}{\overset{9}{\Sigma}} i\cdot j = \underset{m=3}{\overset{6}{\mathrm{P}}} (8\cdot j)+(9\cdot j) = -((8\cdot 3)+(9\cdot 3))-((8\cdot 4)+(9\cdot 4))-((8\cdot 5)+(9\cdot 5)) -((8\cdot 6)+(9\cdot 6)) = -51-68-85-102=-306$$

2.2.1.4- Restory of summation of the division:

$$(67)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma} i\div j$$

Example:

$$(68)\quad \underset{m=5}{\overset{6}{\mathrm{P}}}\ \underset{n=3}{\overset{5}{\Sigma}} i\div j = \underset{m=5}{\overset{6}{\mathrm{P}}} (3\div j)+(4\div j)+(5\div j) = -((3\div 5)+(4\div 5)+(5\div 5)) -((3\div 6)+(4\div 6)+(5\div 6)) = \frac{-12}{5}-2=\frac{-22}{5}$$

2.2.1.5- Restory of summation of the power:

$$(69)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma}\, i^{j}$$

Example:

$$(70)\quad \overset{4}{\underset{m=2}{\mathrm{P}}}\ \overset{14}{\underset{n=12}{\Sigma}}\, i^{j} = \overset{4}{\underset{m=2}{\mathrm{P}}}\,(12^{j})+(13^{j})+(14^{j}) = -((12^{2})+(13^{2})+(14^{2}))-((12^{3})+(13^{3})+(14^{3}))$$
$$-((12^{4})+(13^{4})+(14^{4})) = -509-6{,}669-87{,}713 = -94{,}891$$

2.2.1.6- Restory of summation of the root:

$$(71)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Sigma}\, \sqrt[j]{i}$$

Example:

$$(72)\quad \overset{5}{\underset{m=3}{\mathrm{P}}}\ \overset{42}{\underset{n=41}{\Sigma}}\, \sqrt[j]{i} = \overset{5}{\underset{m=3}{\mathrm{P}}}\,\sqrt[j]{41}+\sqrt[j]{42} = -(\sqrt[3]{41}+\sqrt[3]{42})-(\sqrt[4]{41}+\sqrt[4]{42})-(\sqrt[5]{41}+\sqrt[5]{42}) = -16.213\ldots$$

2.2.2.1- Restory of restory of the sum:

$$(73)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{P}}\, i+j$$

Example:

$$(74)\quad \overset{3}{\underset{m=2}{\mathrm{P}}}\ \overset{6}{\underset{n=4}{\mathrm{P}}}\, i+j = \overset{3}{\underset{m=2}{\mathrm{P}}}\, -(4+j)-(5+j)-(6+j) = -(-(4+2)-(5+2)-(6+2))$$
$$-(-(4+3)-(5+3)-(6+3)) = -(-21)-(-24) = 45$$

2.2.2.2- Restory of restory of the rest:

$$(75)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{P}}\, i-j$$

Example:

$$(76)\quad \overset{7}{\underset{m=5}{\mathrm{P}}}\ \overset{12}{\underset{n=10}{\mathrm{P}}}\, i-j = \overset{7}{\underset{m=5}{\mathrm{P}}}\, -(10-j)-(11-j)-(12-j) = -(-(10-5)-(11-5)-(12-5))$$
$$-(-(10-6)-(11-6)-(12-6))-(-(10-7)-(11-7)-(12-7)) = -(-18)-(-15)-(-12) = 45$$

2.2.2.3- Restory of restory of the product:

$$(77)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{P}}\, i\cdot j$$

Example:

$$(78)\quad \mathop{\mathrm{P}}_{m=1}^{3}\mathop{\mathrm{P}}_{n=2}^{3} i\cdot j=\mathop{\mathrm{P}}_{m=1}^{3} -(2\cdot j)-(3\cdot j)=-((2\cdot 1)-(3\cdot 1))-((2\cdot 2)-(3\cdot 2))$$
$$-((2\cdot 3)-(3\cdot 3))=-(-5)-(-10)-(-15)=45$$

2.2.2.4- Restory of restory of the division:

$$(79)\quad \mathop{\mathrm{P}}_{m=j}\mathop{\mathrm{P}}_{n=i} i\div j$$

Example:

(80)
$$\mathop{\mathrm{P}}_{m=3}^{4}\mathop{\mathrm{P}}_{n=2}^{5} i\div j=\mathop{\mathrm{P}}_{m=3}^{4} -(2\div j)-(3\div j)-(4\div j)-(5\div j)=-(-(2\div 3)-(3\div 3)-(4\div 3)-(5\div 3))$$
$$-(-(2\div 4)-(3\div 4)-(4\div 4)-(5\div 4))=-(\frac{-14}{3})-(\frac{-7}{2})=\frac{49}{6}$$

2.2.2.5- Restory of restory of the power:

$$(81)\quad \mathop{\mathrm{P}}_{m=j}\mathop{\mathrm{P}}_{n=i} i^{j}$$

Example:

$$(82)\quad \mathop{\mathrm{P}}_{m=2}^{3}\mathop{\mathrm{P}}_{n=4}^{5} i^{j}=\mathop{\mathrm{P}}_{m=2}^{3} -(4^{j})-(5^{j})=-(-(4^{2})-(5^{2}))-(-(4^{3})-(5^{3}))=230$$

2.2.2.6- Restory of restory of the root:

$$(83)\quad \mathop{\mathrm{P}}_{m=j}\mathop{\mathrm{P}}_{n=i} \sqrt[j]{i}$$

Example:

$$(84)\quad \mathop{\mathrm{P}}_{m=5}^{6}\mathop{\mathrm{P}}_{n=2}^{4} \sqrt[j]{i}=\mathop{\mathrm{P}}_{m=5}^{6} -\sqrt[j]{2}-\sqrt[j]{3}-\sqrt[j]{4}=-(-\sqrt[5]{2}-\sqrt[5]{3}-\sqrt[5]{4})-(-\sqrt[6]{2}-\sqrt[6]{3}-\sqrt[6]{4})=7.297\ldots$$

2.2.3.1- Restory of productory of the sum:

$$(85)\quad \mathop{\mathrm{P}}_{m=j}\prod_{n=i} i+j$$

Example:

$$(86)\quad \mathop{\mathrm{P}}_{m=7}^{8}\prod_{n=3}^{4} i+j=\mathop{\mathrm{P}}_{m=7}^{8} (3+j)\cdot(4+j)=-((3+7)\cdot(4+7))-((3+8)\cdot(4+8))=-242$$

2.2.3.2- Restory of productory of the rest:

$$(87)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Pi}\, i-j$$

Example:

$$(88)\quad \overset{33}{\underset{m=30}{\mathrm{P}}}\ \overset{9}{\underset{n=7}{\Pi}}\ i-j=\overset{33}{\underset{m=30}{\mathrm{P}}}\ (7-j)\cdot(8-j)\cdot(9-j)=-((7-30)\cdot(8-30)\cdot(9-30))$$
$$-((7-31)\cdot(8-31)\cdot(9-31))-((7-32)\cdot(8-32)\cdot(9-32))-((7-33)\cdot(8-33)\cdot(9-33))=-(-10{,}626)$$
$$-(-12{,}144)-(13{,}800)-(15{,}600)=52170$$

2.2.3.3- Restory of productory of the product:

$$(89)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Pi}\, i\cdot j$$

Example:

$$(90)\quad \overset{4}{\underset{m=2}{\mathrm{P}}}\ \overset{3}{\underset{n=1}{\Pi}}\ i\cdot j=\overset{4}{\underset{m=2}{\mathrm{P}}}\ (1\cdot j)\cdot(2\cdot j)\cdot(3\cdot j)=-((1\cdot 2)\cdot(2\cdot 2)\cdot(3\cdot 2))-((1\cdot 3)\cdot(2\cdot 3)\cdot(3\cdot 3))$$
$$-((1\cdot 4)\cdot(2\cdot 4)\cdot(3\cdot 4))=-48-162-384=-594$$

2.2.3.4- Restory of productory of the division:

$$(91)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Pi}\, i\div j$$

Example:

$$(92)\quad \overset{4}{\underset{m=2}{\mathrm{P}}}\ \overset{4}{\underset{n=3}{\Pi}}\ i\div j=\overset{4}{\underset{m=2}{\mathrm{P}}}\ (3\div j)\cdot(4\div j)=-((3\div 2)\cdot(4\div 2))-((3\div 3)\cdot(4\div 3))$$
$$-((3\div 4)\cdot(4\div 4))=-3-\frac{4}{3}-\frac{3}{4}=\frac{-61}{12}$$

2.2.3.5- Restory of productory of the power:

$$(93)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Pi}\, i^j$$

Example:

$$(94)\quad \overset{7}{\underset{m=5}{\mathrm{P}}}\ \overset{21}{\underset{n=20}{\Pi}}\ i^j=\overset{7}{\underset{m=5}{\mathrm{P}}}\ (20^j)\cdot(21^j)=-((20^5)\cdot(21^5))-((20^6)\cdot(21^6))-((20^7)\cdot(21^7))=-2.310\cdot 10^{18}$$

2.2.3.6- Restory of productory of the root:

$$(95)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Pi}\, \sqrt[j]{i}$$

Example:

$$(96)\quad \underset{m=3}{\overset{5}{\mathrm{P}}}\ \underset{n=6}{\overset{7}{\Pi}} \sqrt[j]{i} = \underset{m=3}{\overset{5}{\mathrm{P}}} \sqrt[j]{6}\cdot\sqrt[j]{7} = -(\sqrt[3]{6}\cdot\sqrt[3]{7})-(\sqrt[4]{6}\cdot\sqrt[4]{7})-(\sqrt[5]{6}\cdot\sqrt[5]{7}) = -8.133\ldots$$

2.2.4.1- Restory of divisory of the sum:

$$(97)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta}\, i+j$$

Example:

(98)

$$\underset{m=3}{\overset{5}{\mathrm{P}}}\ \underset{n=2}{\overset{3}{\Delta}}\, i+j = \underset{m=3}{\overset{5}{\mathrm{P}}} (2+j)\div(3+j) = -((2+3)\div(3+3))-((2+4)\div(3+4))-((2+5)\div(3+5)) = \frac{-431}{168}$$

2.2.4.2- Restory of divisory of the rest:

$$(99)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta}\, i-j$$

Example

$$(100)\quad \underset{m=6}{\overset{7}{\mathrm{P}}}\ \underset{n=3}{\overset{5}{\Delta}}\, i-j = \underset{m=6}{\overset{7}{\mathrm{P}}} (3-j)\div(4-j)\div(5-j) = -((3-6)\div(4-6)\div(5-6))$$
$$-((3-7)\div(4-7)\div(5-7)) = \frac{13}{6}$$

2.2.4.3- Restory of divisory of the product:

$$(101)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta}\, i\cdot j$$

Example:

$$(102)\quad \underset{m=1}{\overset{3}{\mathrm{P}}}\ \underset{n=7}{\overset{8}{\Delta}}\, i\cdot j = \underset{m=1}{\overset{3}{\mathrm{P}}} (7\cdot j)\div(8\cdot j) = -((7\cdot 1)\div(8\cdot 1))-((7\cdot 2)\div(8\cdot 2))$$
$$-((7\cdot 3)\div(8\cdot 3)) = \frac{-21}{8}$$

2.2.4.4- Restory of divisory of the division:

$$(103)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta}\, i\div j$$

Example:

$$(104)\quad \underset{m=4}{\overset{6}{\mathrm{P}}}\ \underset{n=8}{\overset{9}{\Delta}}\, i\div j = \underset{m=4}{\overset{6}{\mathrm{P}}} (8\div j)\div(9\div j) = -((8\div 4)\div(9\div 4))-((8\div 5)\div(9\div 5))$$
$$-((8\div 6)\div(9\div 6)) = \frac{-8}{3}$$

2.2.4.5- Restory of divisory of the power:

$$(105)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta} i^j$$

Example:

$$(106)\quad \overset{3}{\underset{m=1}{\mathrm{P}}}\ \overset{4}{\underset{n=2}{\Delta}} i^j = \overset{3}{\underset{m=1}{\mathrm{P}}} (2^j)\div(3^j)\div(4^j) = -((2^1)\div(3^1)\div(4^1)) - ((2^2)\div(3^2)\div(4^2))$$
$$-((2^3)\div(3^3)\div(4^3)) = \frac{-43}{216}$$

2.2.4.6- Restory of divisory of the root:

$$(107)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Delta} \sqrt[j]{i}$$

Example:

$$(108)\quad \overset{4}{\underset{m=2}{\mathrm{P}}}\ \overset{31}{\underset{n=29}{\Delta}} \sqrt[j]{i} = \overset{4}{\underset{m=2}{\mathrm{P}}} \sqrt[j]{29}\div\sqrt[j]{30}\div\sqrt[j]{31} = -(\sqrt[2]{29}\div\sqrt[2]{30}\div\sqrt[2]{31}) - (\sqrt[3]{29}\div\sqrt[3]{30}\div\sqrt[3]{31})$$
$$-(\sqrt[4]{29}\div\sqrt[4]{30}\div\sqrt[4]{31}) = 0.911\ldots$$

2.2.5.1- Restory of exponentory of the sum:

$$(109)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta} i+j$$

2.2.5.2- Restory of exponentory of the rest:

$$(110)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta} i-j$$

2.2.5.3- Restory of exponentory of the product:

$$(111)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta} i\cdot j$$

2.2.5.4- Restory of exponentory of the division:

$$(112)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta} i\div j$$

2.2.5.5- Restory of exponentory of the power:

$$(113)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta} i^j$$

2.2.5.6- Restory of exponentory of the root:

$$(114)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\Theta}\sqrt[j]{i}$$

2.2.6.1- Restory of rootory of the sum:

$$(115)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\, i+j$$

2.2.6.2- Restory of rootory of the rest:

$$(116)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\, i-j$$

2.2.6.3- Restory of rootory of the product:

$$(117)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\, i\cdot j$$

2.2.6.4- Restory of rootory of the division:

$$(118)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\, i\div j$$

2.2.6.5- Restory of rootory of the power:

$$(119)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\, i^{j}$$

2.2.6.6- Restory of rootory of the root:

$$(120)\quad \underset{m=j}{\mathrm{P}}\ \underset{n=i}{\mathrm{Z}}\sqrt[j]{i}$$

2.3.1.1- Productory of summation of the sum:

$$(121)\quad \underset{m=j}{\Pi}\ \underset{n=i}{\Sigma}\, i+j$$

2.3.1.2- Productory of summation of the rest:

$$(122)\quad \underset{m=j}{\Pi}\ \underset{n=i}{\Sigma}\, i-j$$

2.3.1.3- Productory of summation of the product:

$$(123)\quad \underset{m=j}{\Pi}\ \underset{n=i}{\Sigma}\, i\cdot j$$

2.3.1.4- Productory of summation of the division:

$$(124)\quad \underset{m=j}{\Pi}\ \underset{n=i}{\Sigma}\, i\div j$$

2.3.1.5- Productory of summation of the power:

$$(125)\quad \prod_{m=j} \sum_{n=i} i^j$$

2.3.1.6- Productory of summation of the root:

$$(126)\quad \prod_{m=j} \sum_{n=i} \sqrt[j]{i}$$

2.3.2.1- Productory of restory of the sum:

$$(127)\quad \prod_{m=j} \mathrm{P}_{n=i}\, i+j$$

2.3.2.2- Productory of restory of the rest:

$$(128)\quad \prod_{m=j} \mathrm{P}_{n=i}\, i-j$$

2.3.2.3- Productory of restory of the product:

$$(129)\quad \prod_{m=j} \mathrm{P}_{n=i}\, i\cdot j$$

2.3.2.4- Productory of restory of the division:

$$(130)\quad \prod_{m=j} \mathrm{P}_{n=i}\, i\div j$$

2.3.2.5- Productory of restory of the power:

$$(131)\quad \prod_{m=j} \mathrm{P}_{n=i}\, i^j$$

2.3.2.6- Productory of restory of the root:

$$(132)\quad \prod_{m=j} \mathrm{P}_{n=i}\, \sqrt[j]{i}$$

2.3.3.1- Productory of productory of the sum:

$$(133)\quad \prod_{m=j} \prod_{n=i} i+j$$

2.3.3.2- Productory of productory of the rest:

$$(134)\quad \prod_{m=j} \prod_{n=i} i-j$$

2.3.3.3- Productory of productory of the product:

$$(135)\quad \prod_{m=j} \prod_{n=i} i\cdot j$$

2.3.3.4- Productory of productory of the division:

$$(136)\quad \prod_{m=j} \prod_{n=i} i \div j$$

2.3.3.5- Productory of productory of the power:

$$(137)\quad \prod_{m=j} \prod_{n=i} i^j$$

2.3.3.6- Productory of productory of the root:

$$(138)\quad \prod_{m=j} \prod_{n=i} \sqrt[j]{i}$$

2.3.4.1- Productory of divisory of the sum:

$$(139)\quad \prod_{m=j} \underset{n=i}{\Delta} i + j$$

2.3.4.2- Productory of divisory of the rest:

$$(140)\quad \prod_{m=j} \underset{n=i}{\Delta} i - j$$

2.3.4.3- Productory of divisory of the product:

$$(141)\quad \prod_{m=j} \underset{n=i}{\Delta} i \cdot j$$

2.3.4.4- Productory of divisory of the division:

$$(142)\quad \prod_{m=j} \underset{n=i}{\Delta} i \div j$$

2.3.4.5- Productory of divisory of the power:

$$(143)\quad \prod_{m=j} \underset{n=i}{\Delta} i^j$$

2.3.4.6- Productory of divisory of the root:

$$(144)\quad \prod_{m=j} \underset{n=i}{\Delta} \sqrt[j]{i}$$

2.3.5.1- Productory of exponentory of the sum:

$$(145)\quad \prod_{m=j} \underset{n=i}{\Theta} i + j$$

2.3.5.2- Productory of exponentory of the rest:

$$(146)\quad \prod_{m=j} \underset{n=i}{\Theta} i - j$$

2.3.5.3- Productory of exponentory of the product:

$$(147) \quad \underset{m=j}{\Pi} \underset{n=i}{\Theta} i \cdot j$$

2.3.5.4- Productory of exponentory of the division:

$$(148) \quad \underset{m=j}{\Pi} \underset{n=i}{\Theta} i \div j$$

2.3.5.5- Productory of exponentory of the power:

$$(149) \quad \underset{m=j}{\Pi} \underset{n=i}{\Theta} i^j$$

2.3.5.6- Productory of exponentory of the root:

$$(150) \quad \underset{m=j}{\Pi} \underset{n=i}{\Theta} \sqrt[j]{i}$$

2.3.6.1- Productory of rootory of the sum:

$$(151) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} i + j$$

2.3.6.2- Productory of rootory of the rest:

$$(152) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} i - j$$

2.3.6.3- Productory of rootory of the product:

$$(153) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} i \cdot j$$

2.3.6.4- Productory of rootory of the division:

$$(154) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} i \div j$$

2.3.6.5- Productory of rootory of the power:

$$(155) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} i^j$$

2.3.6.6- Productory of rootory of the root:

$$(156) \quad \underset{m=j}{\Pi} \underset{n=i}{Z} \sqrt[j]{i}$$

2.4.1.1- Divisory of summation of the sum:

$$(157) \quad \underset{m=j}{\Delta} \underset{n=i}{\Sigma} i + j$$

2.4.1.2- Divisory of summation of the rest:

$$(158)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Sigma}\, i-j$$

2.4.1.3- Divisory of summation of the product:

$$(159)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Sigma}\, i\cdot j$$

2.4.1.4- Divisory of summation of the division:

$$(160)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Sigma}\, i\div j$$

2.4.1.5- Divisory of summation of the power:

$$(161)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Sigma}\, i^{j}$$

2.4.1.6- Divisory of summation of the root:

$$(162)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Sigma}\, \sqrt[j]{i}$$

2.4.2.1- Divisory of restory of the sum:

$$(163)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, i+j$$

2.4.2.2- Divisory of restory of the rest:

$$(164)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, i-j$$

2.4.2.3- Divisory of restory of the product:

$$(165)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, i\cdot j$$

2.4.2.4- Divisory of restory of the division:

$$(166)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, i\div j$$

2.4.2.5- Divisory of restory of the power:

$$(167)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, i^{j}$$

2.4.2.6- Divisory of restory of the root:

$$(168)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{P}}\, \sqrt[j]{i}$$

2.4.3.1- Divisory of productory of the sum:

$$(169)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} i+j$$

2.4.3.2- Divisory of productory of the rest:

$$(170)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} i-j$$

2.4.3.3- Divisory of productory of the product:

$$(171)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} i\cdot j$$

2.4.3.4- Divisory of productory of the division:

$$(172)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} i\div j$$

2.4.3.5- Divisory of productory of the power:

$$(173)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} i^{j}$$

2.4.3.6- Divisory of productory of the root:

$$(174)\quad \underset{m=j}{\Delta}\underset{n=i}{\Pi} \sqrt[j]{i}$$

2.4.4.1- Divisory of divisory of the sum:

$$(175)\quad \underset{m=j}{\Delta}\underset{n=i}{\Delta} i+j$$

2.4.4.2- Divisory of divisory of the rest:

$$(176)\quad \underset{m=j}{\Delta}\underset{n=i}{\Delta} i-j$$

2.4.4.3- Divisory of divisory of the product:

$$(177)\quad \underset{m=j}{\Delta}\underset{n=i}{\Delta} i\cdot j$$

2.4.4.4- Divisory of divisory of the division:

$$(178)\quad \underset{m=j}{\Delta}\underset{n=i}{\Delta} i\div j$$

2.4.4.5- Divisory of divisory of the power:

$$(179)\quad \underset{m=j}{\Delta}\underset{n=i}{\Delta} i^{j}$$

2.4.4.6- Divisory of divisory of the root:

$$(180)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Delta}\ \sqrt[j]{i}$$

2.4.5.1- Divisory of exponentory of the sum:

$$(181)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ i+j$$

2.4.5.2- Divisory of exponentory of the rest:

$$(182)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ i-j$$

2.4.5.3- Divisory of exponentory of the product:

$$(183)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ i\cdot j$$

2.4.5.4- Divisory of exponentory of the division:

$$(184)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ i\div j$$

2.4.5.5- Divisory of exponentory of the power:

$$(185)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ i^{j}$$

2.4.5.6- Divisory of exponentory of the root:

$$(186)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\Theta}\ \sqrt[j]{i}$$

2.4.6.1- Divisory of rootory of the sum:

$$(187)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{Z}}\ i+j$$

2.4.6.2- Divisory of rootory of the rest:

$$(188)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{Z}}\ i-j$$

2.4.6.3- Divisory of rootory of the product:

$$(189)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{Z}}\ i\cdot j$$

2.4.6.4- Divisory of rootory of the division:

$$(190)\quad \underset{m=j}{\Delta}\ \underset{n=i}{\mathrm{Z}}\ i\div j$$

2.4.6.5- Divisory of rootory of the power:

$$(191) \quad \underset{m=j}{\Delta}\ \underset{n=i}{Z}\ i^{j}$$

2.4.6.6- Divisory of rootory of the root:

$$(192) \quad \underset{m=j}{\Delta}\ \underset{n=i}{Z}\ \sqrt[j]{i}$$

2.5.1.1- Exponentory of summation of the sum:

$$(193) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ i+j$$

2.5.1.2- Exponentory of summation of the rest:

$$(194) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ i-j$$

2.5.1.3- Exponentory of summation of the product:

$$(195) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ i\cdot j$$

2.5.1.4- Exponentory of summation of the division:

$$(196) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ i\div j$$

2.5.1.5- Exponentory of summation of the power:

$$(197) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ i^{j}$$

2.5.1.6- Exponentory of summation of the root:

$$(198) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\Sigma}\ \sqrt[j]{i}$$

2.5.2.1- Exponentory of restory of the sum:

$$(199) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}}\ i+j$$

2.5.2.2- Exponentory of restory of the rest:

$$(200) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}}\ i-j$$

2.5.2.3- Exponentory of restory of the product:

$$(201) \quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}}\ i\cdot j$$

2.5.2.4- Exponentory of restory of the division:

$$(202)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}} i \div j$$

2.5.2.5- Exponentory of restory of the power:

$$(203)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}} i^{j}$$

2.5.2.6- Exponentory of restory of the root:

$$(204)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\mathrm{P}} \sqrt[j]{i}$$

2.5.3.1- Exponentory of productory of the sum:

$$(205)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} i+j$$

2.5.3.2- Exponentory of productory of the rest:

$$(206)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} i-j$$

2.5.3.3- Exponentory of productory of the product:

$$(207)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} i \cdot j$$

2.5.3.4- Exponentory of productory of the division:

$$(208)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} i \div j$$

2.5.3.5- Exponentory of productory of the power:

$$(209)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} i^{j}$$

2.5.3.6- Exponentory of productory of the root:

$$(210)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Pi} \sqrt[j]{i}$$

2.5.4.1- Exponentory of divisory of the sum:

$$(211)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Delta} i+j$$

2.5.4.2- Exponentory of divisory of the rest:

$$(212)\quad \underset{m=j}{\Theta}\ \underset{n=i}{\Delta} i-j$$

2.5.4.3- Exponentory of divisory of the product:

$$(213)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Delta}\, i\cdot j$$

2.5.4.4- Exponentory of divisory of the division:

$$(214)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Delta}\, i\div j$$

2.5.4.5- Exponentory of divisory of the power:

$$(215)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Delta}\, i^{j}$$

2.5.4.6- Exponentory of divisory of the root:

$$(216)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Delta}\, \sqrt[j]{i}$$

2.5.5.1- Exponentory of exponentory of the sum:

$$(217)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, i+j$$

2.5.5.2- Exponentory of exponentory of the rest:

$$(218)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, i-j$$

2.5.5.3- Exponentory of exponentory of the product:

$$(219)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, i\cdot j$$

2.5.5.4- Exponentory of exponentory of the division:

$$(220)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, i\div j$$

2.5.5.5- Exponentory of exponentory of the power:

$$(221)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, i^{j}$$

2.5.5.6- Exponentory of exponentory of the root:

$$(222)\quad \underset{m=j}{\Theta}\,\underset{n=i}{\Theta}\, \sqrt[j]{i}$$

2.5.6.1- Exponentory of rootory of the sum:

$$(223)\quad \underset{m=j}{\Theta}\,\underset{n=i}{Z}\, i+j$$

2.5.6.2- Exponentory of rootory of the rest:

$$(224)\quad \underset{m=j}{\Theta}\ \underset{n=i}{Z}\, i-j$$

2.5.6.3- Exponentory of rootory of the product:

$$(225)\quad \underset{m=j}{\Theta}\ \underset{n=i}{Z}\, i\cdot j$$

2.5.6.4- Exponentory of rootory of the division:

$$(226)\quad \underset{m=j}{\Theta}\ \underset{n=i}{Z}\, i\div j$$

2.5.6.5- Exponentory of rootory of the power:

$$(227)\quad \underset{m=j}{\Theta}\ \underset{n=i}{Z}\, i^{j}$$

2.5.6.6- Exponentory of rootory of the root:

$$(228)\quad \underset{m=j}{\Theta}\ \underset{n=i}{Z}\, \sqrt[j]{i}$$

2.6.1.1- Rootory of summation of the sum:

$$(229)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, i+j$$

2.6.1.2- Rootory of summation of the rest:

$$(230)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, i-j$$

2.6.1.3- Rootory of summation of the product:

$$(231)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, i\cdot j$$

2.6.1.4- Rootory of summation of the division:

$$(232)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, i\div j$$

2.6.1.5- Rootory of summation of the power:

$$(233)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, i^{j}$$

2.6.1.6- Rootory of summation of the root:

$$(234)\quad \underset{m=j}{Z}\ \underset{n=i}{\Sigma}\, \sqrt[j]{i}$$

2.6.2.1- Rootory of restory of the sum:

$$(235) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, i+j$$

2.6.2.2- Rootory of restory of the rest:

$$(236) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, i-j$$

2.6.2.3- Rootory of restory of the product:

$$(337) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, i\cdot j$$

2.6.2.4- Rootory of restory of the division:

$$(238) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, i\div j$$

2.6.2.5- Rootory of restory of the power:

$$(239) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, i^{j}$$

2.6.2.6- Rootory of restory of the root:

$$(240) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\mathrm{P}}\, \sqrt[j]{i}$$

2.6.3.1- Rootory of productory of the sum:

$$(241) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\Pi}\, i+j$$

2.6.3.2- Rootory of productory of the rest:

$$(242) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\Pi}\, i-j$$

2.6.3.3- Rootory of productory of the product:

$$(243) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\Pi}\, i\cdot j$$

2.6.3.4- Rootory of productory of the division:

$$(244) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\Pi}\, i\div j$$

2.6.3.5- Rootory of productory of the power:

$$(245) \quad \underset{m=j}{\mathrm{Z}}\ \underset{n=i}{\Pi}\, i^{j}$$

2.6.3.6- Rootory of productory of the root:

$$(246) \quad \underset{m=j}{Z} \underset{n=i}{\Pi} \sqrt[j]{i}$$

2.6.4.1- Rootory of divisory of the sum:

$$(247) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} i+j$$

2.6.4.2- Rootory of divisory of the rest:

$$(248) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} i-j$$

2.6.4.3- Rootory of divisory of the product:

$$(249) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} i \cdot j$$

2.6.4.4- Rootory of divisory of the division:

$$(250) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} i \div j$$

2.6.4.5- Rootory of divisory of the power:

$$(251) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} i^j$$

2.6.4.6- Rootory of divisory of the root:

$$(252) \quad \underset{m=j}{Z} \underset{n=i}{\Delta} \sqrt[j]{i}$$

2.6.5.1- Rootory of exponentory of the sum:

$$(253) \quad \underset{m=j}{Z} \underset{n=i}{\Theta} i+j$$

2.6.5.2- Rootory of exponentory of the rest:

$$(254) \quad \underset{m=j}{Z} \underset{n=i}{\Theta} i-j$$

2.6.5.3- Rootory of exponentory of the product:

$$(255) \quad \underset{m=j}{Z} \underset{n=i}{\Theta} i \cdot j$$

2.6.5.4- Rootory of exponentory of the division:

$$(256) \quad \underset{m=j}{Z} \underset{n=i}{\Theta} i \div j$$

2.6.5.5- Rootory of exponentory of the power:

$$(257)\quad \underset{m=j}{Z}\ \underset{n=i}{\Theta}\ i^{j}$$

2.6.5.6- Rootory of exponentory of the root:

$$(258)\quad \underset{m=j}{Z}\ \underset{n=i}{\Theta}\ \sqrt[j]{i}$$

2.6.6.1- Rootory of rootory of the sum:

$$(259)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ i+j$$

2.6.6.2- Rootory of rootory of the rest:

$$(260)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ i-j$$

2.6.6.3- Rootory of rootory of the product:

$$(261)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ i\cdot j$$

2.6.6.4- Rootory of rootory of the division:

$$(262)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ i\div j$$

2.6.6.5- Rootory of rootory of the power:

$$(263)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ i^{j}$$

2.6.6.6- Rootory of rootory of the root:

$$(264)\quad \underset{m=j}{Z}\ \underset{n=i}{Z}\ \sqrt[j]{i}$$

3- Conclusions:

You have seen in this paper a complete theoretical exposition of the double operators theory. Also, you have seen partially the numerical examples for some of the combinations.

How you could see, this is not very hard combinatorics science but I want to assure you that every possible case has infinite possible possibilities and furthermore some of the possible cases have a non-solution possibilities.

4-References:

[1] Juan Elias Millas Vera. “Resume of the serial operators theory.” https://vixra.org/abs/2109.0029