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Equivalent ABC Conjecture Proved on Two Pages

A. A. Frempong

Abstract

By applying basic mathematical principles, the author proves an equivalent ABC conjecture, The equivalent ABC conjecture proved in this paper states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{1+\varepsilon}$, where d is the product of distinct prime factors of $A, B,$ and C, and K_ε is a constant. From the hypothesis, $A + B = C$, it was proved that $C < K_\varepsilon rad(d)^{1+\varepsilon}$.

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Option 1

Introduction

The equivalent conjecture states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{1+\varepsilon}$ where d is the product of distinct prime factors of $A, B,$ and C, and K_ε is a constant.

If $A + B - C = 0$, $|A + B - C| = |0| = 0$. For a positive number, δ, $0 < \delta$, one can write $|A + B - C| < \delta$ From above, the hypothesis would be, $|A + B - C| < \delta$, and the conclusion would be $C < K_\varepsilon rad(d)^{1+\varepsilon}$.

 "The simplest solution is usually the best solution"---Albert Einstein

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Option 2

Equivalent ABC Conjecture Proved on Two Pages

The equivalent ABC conjecture, in this paper, states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{1+\varepsilon}$,where d is the product of distinct prime factors of A, B, C, and K_ε is a constant.

Given: 1. $A + B = C$, where A, B and C are positive integers. with A, B and C being coprime.
2. d = product of the distinct prime factors of A, B and C.

Required: To prove that $C < K_\varepsilon rad(d)^{1+\varepsilon}$

Plan: Hypothesis $\boxed{A + B = C,}$

$$A + B - C = 0,$$

and $|A + B - C| = |0| = 0$

For a positive number, δ, $0 < \delta$,
one can write $|A + B - C| < \delta$.

Conclusion: $\boxed{K_\varepsilon rad(d)^{1+\varepsilon} > C;}$

$$\log\{K_\varepsilon rad(d)^{1+\varepsilon}\} > \log C$$
$$\log K_\varepsilon + \log\{rad(d)^{1+\varepsilon}\} > \log C:$$
$$\log K_\varepsilon + (1+\varepsilon)\log(rad(d)) > \log C:$$
$$\log K_\varepsilon + \log(rad(d)) + \varepsilon \log rad(d) > \log C:$$
$$\varepsilon \log rad(d) > \log C - \log K_\varepsilon - \log(rad(d))$$
$$\varepsilon > \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log rad(d)} \quad \text{or}$$
$$\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{(equivalent conclusion)}$$

The proof would be complete after showing that if $|A + B - C| < \delta$, then

$$\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{(equivalent conclusion)}$$

Proof: One will apply the continued inequality method to handle the inequalities involved.

Step 1: $|A + B - C| < \delta \quad (\delta > 0)$ (hypothesis) (2)

One applies the absolute value symbol to the equivalent conclusion from above to

obtain $\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}\right| < \varepsilon$ (3).

(The above absolute value symbol will be removed in the last step)

The hypothesis $|A + B - C| < \delta$ is equivalent to

$-\delta < A + B - C < \delta$ (hypothesis) (4)

The conclusion , $\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))\}}\right| < \varepsilon$ is equivalent to

$-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon$ conclusion (5)

Step 2: Make the middle terms of (4) and (5) the same. Then (4) becomes.

$$\boxed{-\delta + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < A + B - C + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + \delta \text{ (hypoth)}} \quad (6)$$

and (5) becomes $\boxed{-\varepsilon + A + B - C < A + B - C + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon + A + B - C}$ (7)

Since (6) and (7) have the same middle terms, equate the left sides to each other and equate the right sides to each other. Then one obtains

$$\boxed{-\delta + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} = -\varepsilon + A + B - C}$$ and one solves for δ to obtain

$$\boxed{\delta = \varepsilon + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} - A - B + C, \text{ say } \delta_1}$$ followed by solving

$$\boxed{\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + \delta = \varepsilon + A + B - C}$$ for δ to

obtain $$\boxed{\delta = \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C, \text{ say } \delta_2}$$

$|A + B - C| < \delta$, implies that

$-\delta < A + B - C < \delta$ (hypothesis)

For $\varepsilon > 0$, choose $\delta = \min(\delta_1, \delta_2)$.

$-\delta < A + B - C < \delta$ (hypothesis) implies that

$-\delta_1 \le -\delta < A + B - C < \delta \le \delta_2$ (hypothesis) (8)

Step 3: Replace the left and right sides of (8) by

$$\boxed{\delta = \varepsilon + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} - A - B + C, \text{ say } \delta_1}$$ and

$$\boxed{\delta = \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C, \text{ say } \delta_2}$$, from above, respectively, to obtain

$$-\varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C < A + B - C < \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C \text{ (hyp) } \mathbf{(9)}$$

Break up inequality (**9**) into two simple inequalities and **solve** each one for $-\varepsilon$ and ε, respectively.

$$-\varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C < A + B - C.$$ **solving,** $$\boxed{-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}}$$

$$A + B - C < \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C;$$ **solving,** $$\boxed{\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon}$$

The combination, $-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}$ **and** $\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon$, is

equivalent to $$\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}\right| < \varepsilon$$

Step 4: As was noted in Step 1, one will remove the absolute value symbol (see analogy on next page)

to obtain $$\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{(equivalent conclusion)}$$

Therefore, if $|A + B - C| < \delta$ $(\delta > 0)$ or $A + B = C$, $C < \{K_\varepsilon rad(d)\}^{(\varepsilon+1)}$, and the proof of the equivalent conjecture is complete.

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Option 3

Discussion

In Step 1, (inequality (3)) the absolute value symbol was applied, and in Step 4, the symbol was removed. For **analogy** in elementary math, consider:

Factoring quadratic trinomials by the substitution method;

Example : Factor $6x^2 + 11x - 10$

Step 1: Multiply the expression by the coefficient of the x^2-term.

$6(6x^2) + 6(11x) - 6(10)$

$(6x)^2 + 11(6x) - 60$ ....(A)

Step 2: Let $6x = s$

Then, we obtain $s^2 + 11s - 60$

$(s - 4)(s + 15)$ ....(B)

Step 3: Replace s by $6x$,and then, expression (B) becomes $(6x - 4)(6x + 15)$.....(C)

Since one multiplied the original trinomial by 6, one must divide expression (C) by 6 (that is ,one must undo the "6" introduced in Step 1).

Step 4: In order to divide (C) by 6, perform common monomial factoring on the two binomial factors (in some cases, this factoring is performed only on one of the binomial factors).

$(6x - 4)(6x + 15)$

$2(3x - 2)\ 3(2x + 5)$

$2(3)(3x - 2)(2x + 5)$

$6(3x - 2)(2x + 5)$

Now, divide by 6: $\frac{6(3x -2)(2x + 5)}{6}$

and then the complete factorization of

$6x^2 + 11x - 10$ is $(3x - 2)(2x + 5)$

Conclusion

By applying basic mathematical principles, the author proved an equivalent ABC conjecture, The equivalent ABC conjecture proved states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{(1+\varepsilon)}$, where d is the product of distinct prime factors of A, B, and C, and K_ε is a constant. From the hypothesis, $A + B = C$, it was proved that $C < K_\varepsilon rad(d)^{(1+\varepsilon)}$, the conclusion. The continued inequality method (condensed method) was used in handling the inequalities involved in the proof.

PS: 1. A proof of the original ABC conjecture by the author is at viXra:2107.0094
2. For more on epsilon-delta proofs, see Lesson 5C, Calculus 1 & 2 by A. A. Frempong at Apple iBookstore.

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