Adding boundary terms to Anderson localized Hamiltonians leads to unbounded growth of entanglement

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Abstract

It is well known that in Anderson localized systems, starting from a random product state the entanglement entropy remains bounded at all times. However, we show that adding a single boundary term to an otherwise Anderson localized Hamiltonian leads to unbounded growth of entanglement. Our results imply that Anderson localization is not a local property. One cannot conclude that a subsystem has Anderson localized behavior without looking at the whole system, as a term that is arbitrarily far from the subsystem can affect the dynamics of the subsystem in such a way that the features of Anderson localization are lost.

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1 Introduction

In the presence of quenched disorder, the phenomenon of localization can occur not only in single-particle systems, but also in interacting many-body systems. The former is known as Anderson localization (AL) [1], and the latter is called many-body localization (MBL) [2–7]. In the past decade, significant progress has been made towards understanding AL and especially MBL.

A characteristic feature that distinguishes MBL from AL lies in the dynamics of entanglement. Initialized in a random product state, the entanglement entropy remains bounded at all times in AL systems [8], but grows logarithmically with time in MBL systems [9–11]. The logarithmic growth of entanglement can be understood heuristically [12, 13] from a

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phenomenological model of MBL [14, 15]. Recently, it was rigorously proved that in MBL systems, the entanglement entropy obeys a volume law at long times [16].

Consider the random-field XXZ chain with open boundary conditions

$$H_{XXZ} = \sum_{j=1}^{N-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z) + \sum_{j=1}^{N} h_j \sigma_j^z, \tag{1}$$

where $\sigma_j^x, \sigma_j^y, \sigma_j^z$ are the Pauli matrices at site j, and h_j's are independent and identically distributed uniform random variables on the interval $[-h, h]$. For $\Delta = 0$, this model reduces to the random-field XX chain

$$H_{XX} = \sum_{j=1}^{N-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) + \sum_{j=1}^{N} h_j \sigma_j^z. \tag{2}$$

Using the Jordan–Wigner transformation, H_{XX} is equivalent to a model of free fermions hopping in a random potential. It is AL for any $h > 0$. The Δ term in Eq. (1) introduces interactions between fermions. H_{XXZ} is MBL for any $\Delta \neq 0$ and sufficiently large h [17–19].

In H_{XXZ}, the Δ term representing interactions between fermions is extensive in that it is the sum of $N-1$ local terms between adjacent qubits. Let

$$H_{XXb} = H_{XX} + \Delta \sigma_{N-1}^z \sigma_N^z = \sum_{j=1}^{N-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) + \sum_{j=1}^{N} h_j \sigma_j^z + \Delta \sigma_{N-1}^z \sigma_N^z. \tag{3}$$

Without the last term, H_{XXb} is AL. In this paper, we show that in the dynamics generated by H_{XXb}, the effect of this boundary term invades into the bulk: Starting from a random product state the entanglement entropy obeys a volume law at long times. For large h, the coefficient of the volume law is almost the same as that in the dynamics generated by H_{XXZ}.

Our results imply that AL is not a local property. One cannot conclude that a subsystem has AL behavior without looking at the whole system, as a term that is arbitrarily far from the subsystem can affect the dynamics of the subsystem in such a way that the features of AL are lost.

We briefly discuss related works. Khemani et al. [20] showed nonlocal response to local manipulations in localized systems. This work considers time-dependent Hamiltonians, and is thus different from ours. Vasseur et al. [21] studied the revival of a qubit coupled to one end of an AL system, but the coupling is chosen such that the whole system (including the additional qubit) is a model of free fermions. This is in contrast to H_{XXb}.

2 Results

Definition 1 (entanglement entropy)**.** The entanglement entropy of a bipartite pure state ρ_{AB} is defined as the von Neumann entropy

$$S(\rho_A) := -\operatorname{tr}(\rho_A \ln \rho_A) \tag{4}$$

of the reduced density matrix $\rho_A = \operatorname{tr}_B \rho_{AB}$.

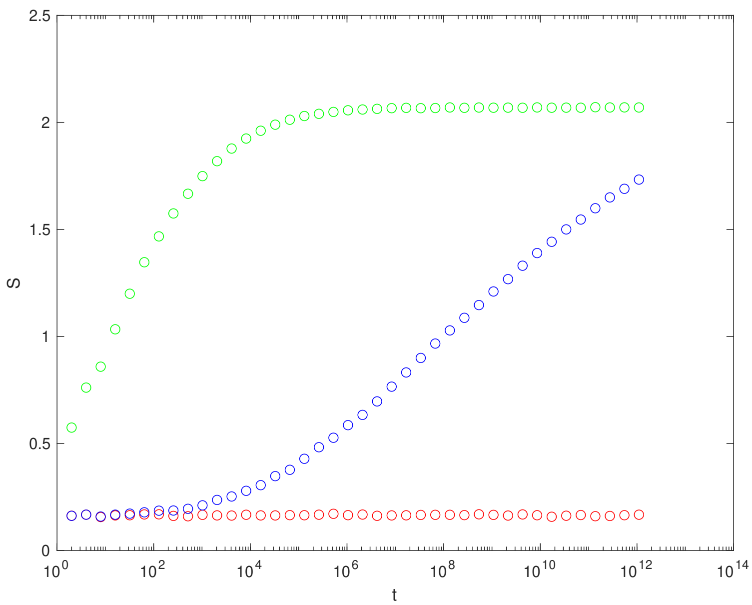

Figure 1: Dynamics of the half-chain entanglement entropy for H_{XXb} (blue), H_{XXZ} (green), and H_{XX} (red).

We initialize the system in a Haar-random product state.

Definition 2 (Haar-random product state)**.** In a system of N qubits, let $|\Psi\rangle = \bigotimes_{j=1}^{N} |\Psi_j\rangle$ be a Haar-random product state, where each $|\Psi_j\rangle$ is chosen independently and uniformly at random with respect to the Haar measure.

For our numerical results, we choose $h = 10$ and $\Delta = 1$, and average over 1000 disorder realizations. We choose $N = 10$ in Figure 1 and in the left panel of Figure 2.

Figure 1 shows the dynamics of the entanglement entropy between the left and right halves of the system for H_{XXb}, H_{XXZ}, and H_{XX}. We clearly see that the last term in Eq. (3) leads to slow entanglement growth.

Figure 2 shows that the entanglement entropy at long times obeys a volume law for H_{XXb} and H_{XXZ}, and the coefficient of the volume law is very close to $1/2$. This is consistent with the analytical prediction of Ref. [16], which assumes that the spectrum of the Hamiltonian has non-degenerate gaps.

Definition 3 (non-degenerate gap)**.** The spectrum $\{E_j\}$ of a Hamiltonian has non-degenerate gaps if the differences $\{E_j - E_k\}_{j \neq k}$ are all distinct, i.e., for any $j \neq k$,

$$E_j - E_k = E_{j'} - E_{k'} \implies (j = j') \text{ and } (k = k'). \tag{5}$$

Indeed, we have numerically verified that the spectra of both H_{XXb} and H_{XXZ} almost surely have non-degenerate gaps.

In the right panel of Figure 2, we observe a constant correction to the volume law. This is expected, for such corrections also exist in other contexts [22–27].

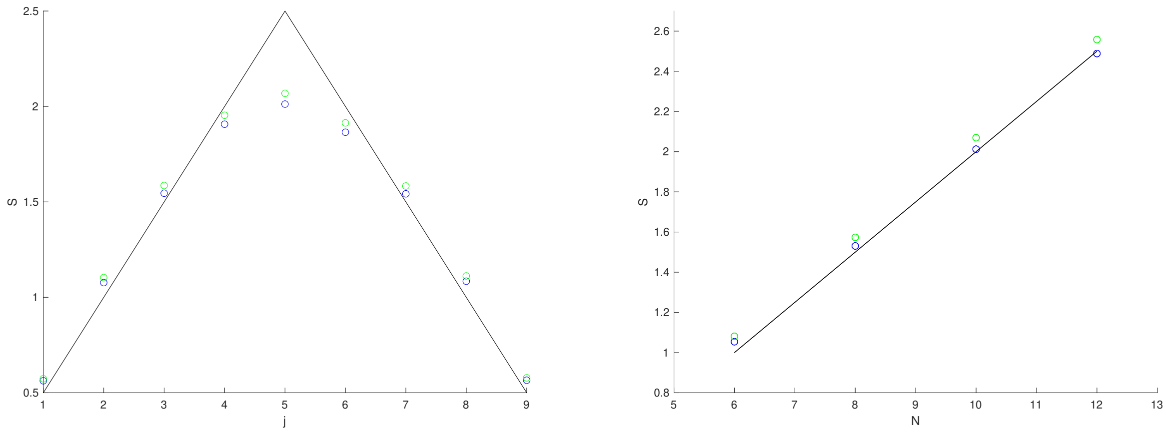

Figure 2: Left panel: The entanglement entropy between the first j and the last $N-j$ qubits at long times for H_{XXb} (blue) and H_{XXZ} (green). The black lines are $S = \min\{j, N-j\}/2$. Right panel: Finite-size scaling of the half-chain entanglement entropy at long times for H_{XXb} (blue) and H_{XXZ} (green). The black line is $S = N/4 - 1/2$.

3 Discussion

We have numerically shown that adding a single boundary term to an otherwise AL Hamiltonian leads to entanglement growth. Starting from a random product state the entanglement entropy obeys a volume law at long times, and the coefficient of the volume law is consistent with the analytical prediction of Ref. [16].

Here are some interesting problems that deserve further study.

- Can we prove that the spectrum of H_{XXb} almost surely has non-degenerate gaps? A positive answer to this question would allow us to rigorously prove some of the numerical results in this paper.
- Can we develop an analytical understanding of how the entanglement entropy grows with time for H_{XXb} by adapting the phenomenological model of MBL [14, 15]?
- How does H_{XXb} scramble local information as measured by the out-of-time-ordered correlator [28–34]?
- It was argued that MBL is less stable in two and higher spatial dimensions [35]. To what extent a single boundary term delocalizes an AL system in higher dimensions?

Acknowledgments

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