Is Gravity Quantum?

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ABSTRACT: We'll discuss several experiments aimed at proving gravity is a quantum, not classical, field theory. Some of the experiments are more "thought experiments" than practical; others are aimed toward being practically feasible; others have already been done. The net effect of our arguments, I think (even without performing any of the not-yet-done experiments), is to make it clear gravity must be quantized, in the sense gravitational fields are made of "gravitons" obeying Planck's energy-quantization condition E=hf.

1. Introduction

> **Dyson:** We can observe crowds and crowds of gravitons coherently because then they become a classical field. But there could be a classical field without having gravitons.
> **Q:** So your world view is completely comfortable if gravity is never integrated into quantum theory?
> **Dyson:** Right. It's certainly quite plausible. I don't say it's true, but there's no evidence against it as far as I can see.
> **...Q:** You have no discomfort if [quantum field theory and gravity] remain permanently, and in some final physics, different?
> **Dyson:** Yes, absolutely. I think that it would be a much more interesting universe if it has these two quite-disconnected parts. (2017 interview.)

This work was inspired by Freeman J. **Dyson**'s 2012 Poincare Prize Lecture. Dyson advocated the minority opinion that perhaps gravity (Einstein general relativity, a non-quantum theory) and quantum field theories (e.g. "standard model") did *not* need to be reconciled/unified. As arguably *the* founder of QED (quantum electrodynamics, the first successful quantum field theory), Dyson deserves some respect.

Dyson's first argument for his view (or perhaps it would better to call it a "criticism of the mainstream approach") went back to Bohr & Rosenfeld's 1933 demonstration, via purely classical arguments about measurability/not of electric and magnetic fields, that the photon "must" exist. Due to lack of negative masses to use for "compensation," the analogous arguments fail to demonstrate that gravitons must exist. Re that, it should be noted that Baym & Ozawa 2009 also considered – in a much more careful/detailed manner than Dyson – a related Bohr argument for the logical necessity of quantizing the electromagnetic field, again finding that it failed to generalize to gravitation (and the skeleton of their argument already had been raised by Rosenfeld at the 1957 Chapel Hill conference). Dyson then noted that the present version of LIGO cannot detect waves containing fewer than 10^{37} gravitons, and analysed several kinds of detectors:

1. Any LIGO-like detector based on measuring distance-changes, Dyson argued, is inherently incapable of single-graviton detection. (Actually, Dyson's anti-LIGO argument was already known long before Dyson 2012; endnotes 32 & 33 of Baym & Ozawa trace its history.)
2. Hot gravitons ionizing atoms in a detector. Dyson first considers converting Earth's entire mass to a detector of hot gravitons from the sun, finding that over the entire life of the sun (as the 80 MW hot-graviton source, see Gould 1985) there would be an expected total number of 4 detector atomic-ionization events. Since that is unsatisfactory, Dyson then ups the ante to an outrageous graviton detector the mass of our sun orbiting the brightest known hot-graviton sources – a hot white dwarf or neutron star – with 8-hour orbital period. These sources should radiate, respectively, about 1 and 10^6 terawatts worth of hot gravitons. Dyson then estimates respective detection rates of one per minute and

30000/second. Over the total lifetimes of these sources the total number of gravitons thus-detected would be 10^{13} and 10^{16}. Unfortunately Dyson finds there will be $\geq 10^{34}$ neutrino background confusor-events for each graviton absorption genuine-event. Even if every neutron star and white dwarf in our whole galaxy throughout its lifetime (probably about 10^{11} to 10^{12}) were harnessed to this task, we'd still be faced with $\leq 10^{28}$ genuine among $\geq 10^{61}$ confusor events.

3. The Gertsenshtein 1961 effect can convert gravitons passing through a large region filled with a high magnetic field (such as near pulsars), into photons (also vice versa), thus detecting them. Unfortunately the Euler-Heisenberg "effective lagrangian" shows that electromagnetism includes *nonlinear* QED effects, which, Dyson argues, are large enough to ruin this idea because they effectively alter the speed of photons (but not gravitons), tremendously reducing photon↔graviton interconversion rates. (Zeldovich 1973 made a similar complaint earlier.)

Given all that, Dyson conjectured that *any* detector capable of detecting 1 graviton either could not work, or must collapse into a black hole. I.e. gravitons, in some sufficiently strong sense, are *undetectable*, which both is an argument they do not exist, and/or that a non-quantum theory of gravity ought to be adequate.

But Dyson admitted that he found (2) to be the least impressive/convincing among his three analyses. Indeed, if some magic hypothetical neutrinoless graviton source were employed instead of a white dwarf / neutron star, then Dyson's analysis 2 would seem to show exactly the opposite of what he intended.

In the other direction, several people have tried to argue that gravity must be quantum, but all their arguments seem unsatisfactory.

R.P.Feynman at the Chapel Hill conference (Jan.1957) presented an argument something like the following: Consider a ^{250}Cm atom in vacuum. It spontaneously fissions, or not. (Halflife≈10^4 years. Another perhaps more-theoretically-interesting interesting variant would start not with ^{250}Cm but rather with an "atom" of "positronium" i.e. an electron and positron orbiting each other. This either self-annihilates into gamma rays, or survives; halflife≈179pSec if the spins antiparallel and 205nSec if aligned. Feynman in 1957 actually suggested a third variant where we begin with a nucleus in a superposition of spin-up and spin-down, then put it into a superposition of being in two locations by passing it through a Stern-Gerlach apparatus; for this version no radioactive decay is required.) If nobody observes it, it is in a superposition of both states. The ^{250}Cm atom, if still there, then gravitationally deflects something about 3 times as massive, for example a passing fullerene molecule (C_{60}, mass≈721 amu). That fullerene is now in a quantum superposition of two locations: deflected and not. If deflected, it in turn gravitationally deflects something about 3 times as massive, e.g. a Cr_{39} molecule. And so on: each actor, *if* deflected, gravitationally deflects the next, which has about 3× its mass. After 95 stages of this process, the final actor is the asteroid Ceres, whose mass is about 3^{95} times that of the original ^{250}Cm atom. If nobody observes any of this, then Ceres ends up in a quantum superposition of two far-apart locations. What gravity will *you* then experience from Ceres? Surely the only possible answer must involve a quantum description of gravity. As Feynman put it:

> Now, how do we analyze this experiment according to quantum mechanics? ... We have complex amplitudes [for the ball being in both positions]... Now, since the ball is big enough to produce a real gravitational field... we could use that gravitational field to move another ball, and amplify that, and [regard] the connection to the second ball as the *measuring equipment*. We would then have to analyze the channel provided by the gravitational field *itself* via the quantum mechanical amplitudes.

Feynman and/or his acolytes sometimes acted as though this made it a completely obvious settled fact that gravity had to be quantum, but I cannot accept his "proof." Suppose we instead postulate than quantum superpositions of masses in two locations simply *cannot happen* for masses larger than some threshold, say 10^6 amu. This postulate is unrefuted by any experiment so far! Then Feynman's argument would break down after only 7 stages. And in particular even you somehow built a giant ultracold refrigerated shield enclosing the whole experiment, and transported it all 10^8 light years away from every galaxy, then Ceres' location still would be "measured" to millimeter accuracy by the rest of the universe (for example via its interactions with the cosmic neutrino background) at least 10^{12} times per second, so no such quantum superposition could happen. So this kind of experiment is completely infeasible.

Feynman could counter that we do not need to go all 95 steps to Ceres. We could stop after only 70 stages since a mass 10^9 kg is readily detectable by commercial gravimeters. Indeed, presumably we could stop after only 43 stages, the last being a 136 milligram mass, because recent experiments have shown Newton's law seems to work for masses that size. But even for this, the total mass, distance and duration of quantum delocalization required seem well beyond anything humanity could ever achieve. Specifically, I **conjecture**: *Humanity will never be able to quantum superpose a ≥0.1 milligram mass in two positions ≥1 cm apart*. So, given that possibility, Feynman's argument is unconvincing. And indeed Feynman 1957 was aware of this objection:

> ...I would like to suggest that it is possible that quantum mechanics fails at large distances and for large objects. Now, mind you, I do not say that I think that quantum mechanics does fail at large distances, I only say that it is not inconsistent with what we know. If this failure of quantum mechanics is connected with gravity, we might speculatively expect this to happen for masses such that $GM^2 \approx \hbar c$, hence M near 10^{-5} grams, which corresponds to some 10^{18} particles... This would be a new [irreversibility] principle for masses>10^{-5} gram or whatever.

I personally believe there *is* a principle something like that (see Smith 2003) – albeit it is not incompatible with gravitons existing; it merely is incompatible with contending the ^{250}Cm→Ceres argument *proves* they exist.

Eppley & Hannah 1977 devised a thought-experiment using classical gravity waves to measure the position and momentum of a macroscopic body more accurately than the Heisenberg quantum uncertainty principle $\Delta p \Delta x \geq \hbar/2$ should allow. They claimed this showed "that the assumption that a classical gravitational field interacts with a quantum system leads to violations of either momentum conservation, the uncertainty principle, or yields transmission of signals faster than c. A similar argument holds for the electromagnetic field." But Eppley & Hannah's arguments were comprehensively destroyed by Mattingly 2006.

Another attempt was by **Page & Geilker** 1981. I find it entirely unconvincing, and even P&G themselves say it merely "supports (but does not prove) the hypothesis that a consistent theory of gravity coupled to quantized matter should also have the gravitational field quantized." The problem with arguments like theirs is that they insist on a very specific and rather obviously-ridiculous kind of sourcing for "semiclassical gravity" as their "only" alternative to quantum, then "rule that out." Dyson in his 2017 interview suggested that like "temperature" (a non-quantum statistical property of matter), gravity might be a nonquantum "statistical property of [large enough bulks of] matter." Such a hypothesis simply is not affected by Page & Geilker.

As long as we are here, I might as well mention the "**Casimir effect**" in which the vacuum between two parallel mirror-plates contains "negative pressure" and "negative energy" due to the quantum field theory of the photon. This has been experimentally confirmed by measuring the attraction it causes between the plates. Question: How does that region gravitate? At present, nobody knows. However, I would guess that it indeed produces "antigravity." Why? We know that nuclear binding energy certainly weighs. For example, if a gram of ^{4}He atoms are split into deuterium atoms, we gain 6.4 milligrams, which is easily detectable. I.e. helium's binding energy here weighs negative 6.4mg. Furthermore, any worries that perhaps different nuclear binding energies might somehow contribute more to "inertial mass" than "gravitational mass" (or vice versa – which would violate Einstein's "equivalence principle") were addressed by "Eötvös experiments" on many different substances, which have confirmed the equivalence principle to below 3 parts in 10^{11} (Roll, Krotkov, Dicke 1964). Also the "Nordtvedt effect" – the hypothesis that the gravitational binding energy holding the Moon together, should contribute differently to its gravitational versus its inertial mass – has been experimentally refuted, 5 orders of magnitude below the naive putative size of the effect, by lunar laser ranging and radar-transponding to Mercury spaceprobes. If we burn hydrogen in flourine – 1 gram of reactants total – the resulting liquid HF (after it cools) should weigh 1.7×10^{-10} grams less. If we burn carbon in oxygen, the resulting CO_2 (after it cools) should weigh 1 part in 10^{10} less. As far as I know few or no available balances are sensitive enough to detect these chemical energy losses, so nobody has ever confirmed these claims. (Incidentally, it has been estimated that national standard kilograms made of platinum-iridium alloy gained about 50 micrograms over 120 years, which is 4×10^{-10} fractional parts per year.) A gram of water vapor, if converted to ice, should weigh 3.3×10^{-11} grams less. A gram of argon gas, if solidified, should weigh about 2.3×10^{-12} grams less. This argon weight loss is measuring the energy of the Van der Waals forces binding argon atoms together in the solid. Now Van der Waals atom-attraction is essentially the same phenomenon as Casimir plate-attraction. So *presumably* Casimir forces really should "weigh" as a negative contribution to total mass.

A 1D-periodic structure consisting of 100nm-thick lithium films separated by 250nm vacuum gaps would have a ratio of the "negative mass" in the vacuum gaps, to the positive mass of the lithium metal, of about 5×10^{-16}.

Mattingly in a 1999 review **concludes** "*[any] real justification for quantizing gravity has yet to be articulated*" (and he reiterated this in his 2005 re-review). Indeed, given that all the inadequate justifications that *have* been suggested were already known to the participants at the 1957 Chapel Hill conference, arguably no progress whatever has been made (except for refining details) during 1957-2020. My purpose is to rectify that.

2. Blackbody radiation left over from the Big Bang

The Big Bang filled the universe with blackbody radiation at the temperature at which "decoupling" (of that kind of radiation from everything else) occurred, albeit today it all is redshifted by the factor by which the universe expanded between then and now. Decoupling occurred when the universe became "transparent" to that kind of radiation – which actually is not one precise moment, it occurs over some nonzero timespan. For photons, the measured present-day blackbody temperature ("cosmic microwave background," CMB) is 2.725 Kelvin. It is presumed that there is an analogous neutrino background (CNB) at about 1.95 Kelvin (which has energy density 69% of the photon background's). As of year 2021, the CNB has not yet been detected. An experiment called PTOLEMY has been suggested that might be able to detect the CNB by 2030, but I doubt it. If gravitons exist, then there should be an analogous thermal graviton background (CGB), presumably at ≤0.9 Kelvin, hence with energy density ≤2% of the photon energy density. The CGB should be far less directly-detectable even than the neutrino background. It could, however, be detected *indirectly* via its gravitational effect on cosmology, if all relevant aspects of cosmology (including the neutrino background, dark matter, and cosmological scale factor versus time curve) could be measured to 0.001 or better relative accuracy. That seems unlikely to happen anytime soon, but seems doable in principle.

But if gravitons do not exist, because the gravitational field is a classical field whose frequency-f modes are *not* subject to Planck's energy-quantization condition E=hf, then the spectral energy density of Big Bang relic-graviton blackbody radiation would obey the failed nonquantum classical "Rayleigh-Jeans" law that $\varrho_{RJ}(f)=8\pi c^{-3}k_B T f^2$. This law is, in fact, exactly valid for photons, *but* only asymptotically for low frequencies $f \ll k_B T/h$, or equivalently in the "classical limit" $h \to 0+$. For gravitational waves the same law would apply (given that weak-gravitation wave modes have only two possible polarization states, i.e. helicities ±2 for the graviton, cf. ch.10 of Weinberg 1972; this is the same as for electromagnetic waves, i.e. the photon has helicities ±1 only). The Rayleigh-Jeans law would predict *infinite* energy density in the CGB, obviously contradicting reality. Even assuming a magically-imposed high-frequency *cutoff* at the Planck frequency, which today has been redshifted by a factor of 10^{10}, we still would estimate the CGB's energy density today would exceed its (indirectly) observed value by over 80 orders of magnitude. You can postulate other values of a mysterious magically-caused frequency cutoff – for example the cutoff wavelength instead could be the diameter$\approx 1.8\times 10^{-15}$meter of a proton but redshifted today by factor 10^{10} – but this and most other reasonable assumptions yield vacuum energy densities vastly exceeding observational bounds.

Indeed, Planck's blackbody spectral density law is $\varrho_{Pl}(f)=8\pi c^{-3}k_B T f^3 h/[\exp(hf/[k_B T])-1]$, which is maximized when $f=f_{peak}\approx 2.82144 k_B T/h$. If we let Rat(f) denote the *ratio* $\mathrm{Rat}(f)=\int_{0<x<f} \varrho_{RJ}(x)dx \,/\, \int_{0<x<f} \varrho_{Pl}(x)dx$, then we find (writing the high-frequency cutoff f in units of $k_B T/h$)

f	**Rat(f)**	**f**	**Rat(f)**	**f**	**Rat(f)**
0.2	1.079	0.5	1.212	1.0	1.483
2.0	2.267	5.0	8.504	10	51.82
20	410.6	50	6416	100	51330

so that with any high-frequency-cutoff above $9.5 f_{peak}\approx 26.9 k_B T/h$, the ratio $\mathrm{Rat}(f_{cutoff})$ of Rayleigh-Jeans divided by Planck energy-density ought to exceed 1000, which should have been enough so that the CGB's gravitational effects ought already to have been noticed via their effects on the evolution of the early universe – but weren't. (E.g: the universe was "radiation dominated" for its first 47000 years, during which a'/a would have been quite different, altering the primordial synthesis of hydrogen, helium, and lithium isotopes during the first few minutes.)

This Rat(f)=1000 high-frequency cutoff today would be a frequency f obeying $hf/k_B \le 9$ Kelvin, i.e. $f \le 2\times 10^{11}$ Hz, i.e. all wavelengths **λ≤1.6mm** would be cut off. Such a cutoff would seem surprising in view of experiments by Kapner et al 2007, Lee et al 2020, and Westphal et al 2021. E.g. Kapner found "with 95% confidence that [Newton's gravitational] inverse-square law holds down to a length scale 56μm."

Conclusion: Already-done cosmological observations show that gravitons exist, and gravity is *not* unquantized classical fields, *unless* there presently is a high-frequency cutoff less than 2×10^{11} Hz for gravitational waves arising from heat from the Big Bang (this frequency f obeys $hf/k_B \approx 9$ Kelvin and corresponds to wavelength≈1.6 mm). Such a cutoff seems incompatible with already-done experiments probing Newton's law at short length and/or small mass scales.

First attack on that conclusion (and my response): The attacker worries that the universe never was a "blackbody," i.e. opaque, for gravitational radiation (cf. Smith 2003), which might vitiate our conclusion. To respond: Gould EQ 39 & 40 finds, based on a microscopic calculation, the luminosity of gravitational bremstrahlung radiation from hot plasma: it is proportional to $n^2 T^{3/2}$ per unit time per unit volume, where n is the number-density of electrons and nuclei in the plasma while T is its temperature. (This agrees with the less-generally-applicable calculation at the end of §10.4 in Weinberg 1972.) And because during the radiation-dominated era the universe (FLRW model; t>0 denotes time after the initial singularity) had length-scale proportional to $t^{1/2}$ and temperature proportional to $t^{-1/2}$ (Weinberg ch.15), we see that the total gravitational wave energy emitted by a chunk of plasma as it expands and cools after time t, including a then→now redshift factor in the radiated energy, would be at least proportional to $\int_{u>t} u^{1/2+3/2-3-3/4}\,du=\int_{u>t} u^{-7/4}\,du$, i.e. to $t^{-3/4}$ which goes to *infinity* when t→0+. (The reason I said "at least" is that when T gets hot enough actually n will grow due to additional particle-antiparticle pairs; this calculation was assuming particle numbers were conserved.) Hence the *expanding* universe backward *through time* as t→0+, indeed *must* have acted as a gravity-wave blackbody.

Also, an entirely different graviton-emission mechanism which works in a photon gas was examined by del Campo & Ford 1988. They do not need any charges and do not need to assume gravitons exist; all they need are fluctuations in photon energy-density. Their gravity-wave power radiated per unit time per unit volume is proportional to T^7. That suffices to yield our same conclusions, but even more strongly.

Second attack (by Boyer): Two papers (1969 & 2018; I'll focus almost entirely on the latter) by Timothy H. Boyer were brought to my attention. Boyer, a physics professor at City College of New York (who is not *Robert* H. Boyer, famed for the "Boyer Lindquist coordinatization" of the Kerr black hole, who was murdered in 1966), attacks the entire mainstream view about blackbody radiation. Specifically, the mainstream view, recounted in many physics textbooks, is: Max Planck in 1899-1900 realized that the spectrum of the "blackbody radiation" that shines out of holes in furnaces, was *not* explainable with classical physics (which incorrectly predicts the "Rayleigh-Jeans" unnormalizable spectrum). But it is explainable by also assuming the quantum condition that frequency-f modes of the electromagnetic field contain must contain energy $E=(n+½)hf$ where the $n\ge0$ are *integers*. This Nobel-prize winning discovery was the dawn of both "quantum mechanics" (QM) and "quantum field theory" (QFT).

Boyer disputes all that. As you can see from the title and abstract of his 1969 paper, he contends the "blackbody radiation spectrum" can be derived "*without* quantum assumptions" and hence the Rayleigh-Jeans law was *wrong*! The entire dawn of QM and QFT therefore presumably was *unjustified*! Boyer 2018 indeed specifically singles out certain textbooks as wrong (including the one they taught me QM from!) and asserts they should be redone because they are misleading students everywhere.

Boyer's claims would seem, on their face, to do major damage to the present paper. And certainly at least some of the things he says are insightful/valid. Nevertheless, his paper contains both false and misleading claims, which together suffice to refute his paper, and certainly refute its ability to damage the present paper.

Refutation of Boyer. Here is a false statement by Boyer 2018: "Electromagnetic radiation can be brought to thermal equilibrium only by the interaction of electromagnetic radiation with charged mechanical systems." (He simply *asserts* this, with no argument whatever to justify it.) Boyer then goes on to argue that this mechanical system necessarily has a special favored rest frame and thus *breaks relativistic invariance*. He emphasizes that is crucial; he then makes further arguments but all starting from this foundation. (At least one of those further arguments, about "smoothest possible interpolation," also seems illegitimate, which I'll refrain from explaining...)

Actually: a vacuum uniformly filled with some distribution of photons, but containing no charges or "mechanical systems" whatsoever, can, and generically will, reach thermal equilibrium via the small but well-understood QED phenomenon of "photon-photon scattering" (first discovered by Hans Euler & Bernhard Kockel in 1935; a recent review paper is Liang & Czarnecki 2012; effect experimentally observed by ATLAS in 2017 with 4.4σ confidence, then in 2018-2019 with 8.2σ). This process is nonlinear and causes electromagnetism to be slightly nonlinear (the "Euler-Heisenberg lagrangian") unlike the plain Maxwell equations which are linear. The simplest and most important version of this process inputs two photons and outputs two others (usually with altered frequencies and directions) having unaltered *total* momentum and energy. Also even without QED, photons also interact and scatter off each other *gravitationally*. I.e. gravity (Einstein general relativity) also causes electromagnetism to be slightly nonlinear – an even weaker, but entirely *non*quantum, effect.

> Although these effects are small (but nonzero!) at temperatures commonly encountered by humans, they become enormous at high temperatures. E.g, a 1 cubic meter ball of electromagnetic radiation at temperature 10^{10} Kelvin (mass≈8.4×10^7 kg) would radiate about 87 kWatts worth of gravitons (del Campo & Ford 1988) and also should exhibit very large photon-photon scattering, with photon mean free path<1 mm between scatterings. Electromagnetic radiation at temperature $T=0.1m_ec^2/k_B\approx6\times10^8$ Kelvin in vacuum should thermally equilibrate via the Euler-Kockel effect in less than 1 second.

Essentially *any* initial distribution of photons in our vacuum will in this way increase in entropy, tending asymptotically at large times toward the maximum-entropy distribution for a set of photons having with the same total energy and momentum as as the initial set. That maxent distribution is: the Planck distribution in the center-of-mass frame of the radiation. Notice this all happens with *no* charges anywhere, and in a 100% relativistically-invariant manner. Boyer was completely, and *foundationally*, wrong.

But *without* the Planck quantization assumption, no maxent spectral distibution *exists*, so the radiation field will keep unboundedly increasing its entropy forever because the nonlinearities gradually shift its energy into higher frequency modes forever. [E.g. the $x\to x^2$ nonlinear map causes "frequency doubling" effects due to the identity $2(\sin\theta)^2=1-\cos(2\theta)$.] This mathematical fact completely *refutes* Boyer's claim that any Planck-like spectral law can arise without Planck's quantal assumption. (I offered to correspond with Boyer via email about all this, but he ignored my offer. Another error in Boyer 2018's "classical derivation" of a Planckian law, is his false implicit assumption that a maxent distribution would exist.)

Remark re mathematical ugliness. Parsimony is desirable in mathematics: use the fewest assumptions possible to reach your conclusion. The *only* assumptions needed to derive the Planck blackbody spectrum are (i) Bose-Einstein statistics, i.e. quantization, (ii) Assume some mechanism – it does not matter what – to allow thermal equilibration, (iii) maximize entropy. If you instead find yourself (like Boyer) needing to postulate very specific equilibration mechanisms, involving e.g. Coulombic potentials, simple harmonic oscillators, demanding charge be discrete, etc, then you have lost the battle for mathematical beauty.

3. Hawking radiation

S.W.Hawking in 1974 discovered via semiclassical analysis of quantum fields in curved spacetime vacua (this nowadays is widely accepted and has been rederived in many ways by many authors) that black hole event horizons ought to *radiate* every possible form of radiation, like an ideal blackbody with temperature equal (for a mass=M Schwarzschild hole) to $T_{Hawk}=\hbar c^3/(8\pi G k_B M)$. Rather magically, the mathematics indicates black hole event horizons act to *convert* "virtual" vacuum modes into real radiated particles with an exactly-thermal distribution. The surface area of the event horizon for this hole is $16\pi G^2c^{-4}M^2$. The black hole thus loses mass/energy, causing its lifetime to be finite. Based on that, if the only form of radiation were photons, then the lifetime would equal $5120\pi G^2\hbar^{-1}c^{-4}M^3$. If *both* gravitons and photons are radiated, then *halve* this lifetime formula.

If gravitons exist, Hawking-type predictions would estimate they are radiated with same luminosity as photons, whereas with purely-classical gravity, there is *zero* gravitational Hawking radiation. **Warning:** these luminosity and lifetime formulas arose from the T_{Hawk} formula via a naive over-idealization of blackbody radiation, namely the Stefan-Boltzmann law that the power P radiated from an area=A temperature=T blackbody surface is $P=\sigma_{SB}T^4A$. That law actually only is valid if all radiated wavelengths are much shorter than the geometrical size of the source – which is false for Hawking radiation from black hole event horizons. Page 1976 (see also Gray & Visser 2018) overcomes that problem with more careful mode-dependent numerical computations. However, I warn you that Page in 1976 incorrectly thought neutrinos were massless, hence his calculations for neutrino radiation are only valid for hole temperatures $T \gg \max(m_\nu)c^2/k_B$. Here the greatest neutrino mass $\max(m_\nu)$ presumably lies in the interval $(0.02, 0.26)\text{eV}/c^2$, which would imply that a sufficient condition for Page's "massless" neutrino calculations to be valid is $T \gg 3020$ Kelvin, and a necessary condition is $T \gg 232$ Kelvin. For $T \ll \min(m_\nu)c^2/k_B$ the neutrino radiation should be negligibly small by comparison to the graviton (if any) and photon radiation. Probably $0.009\text{eV} \le \min(m_\nu)c^2 \le 0.09\text{eV}$, in which case neutrino radiation would be negligible when $T \ll 104$ Kelvin. Either way: If gravitons also are radiated, then the luminosity increases, and hole lifetime (and the "lifetime before T rises to $0.05\min(m_\nu)c^2/k_B$") correspondingly diminishes, versus the photons-only predictions.

A 1-sun mass black hole has temperature≈62nK and photon-only lifetime on the order of 2×10^{67} years. If somebody helpfully provided you with a black hole with Hawking temperature=10^9 Kelvin, it would have mass=1.227×10^{14} kg, luminosity=23661 watts, and remaining lifetime=4.923×10^{18} years based on the photon-only formula above – albeit you ought to be able to get enough accuracy after less than 10^{14} years, i.e. about 7000 times the present age of the universe, to decide the question of graviton existence.

> It is surprising Dyson missed this whole idea since it actually recapitulates the very birth of quantum physics – Max Planck's 1900 discovery that by postulating *quantization* of electromagnetic vibrations of frequency f into energy packets of energy E=hf – "photons" – the spectrum of blackbody radiation could be explained.

Conclusion: There is a very simple and entirely technologically feasible experiment (although admittedly it would take a very long time!) to decide whether gravitons exist: measure the lifetime of a black hole! If gravitons exist, it will be shorter, by a very-easily-detectable amount, than if they do not. [And knowing the M(t) mass-versus-time evaporation curve of a black hole also would reveal other interesting things, such as the entire particle spectrum.]

4. Entropy

This section will combine the ideas of both preceding sections to argue that anybody denying gravitons must deny the Second Law of thermodynamics.

> The law that entropy always increases [i.e. the Second Law] holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations – then so much the worse for Maxwell's equations. If it is found to be contradicted by observation – well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation. – Arthur S. Eddington, *The Nature of the Physical World* (1928) ch.4 (reprinted by Macmillan NY 1948, p.74).

A bound first stated independently by G.t'Hooft and L.Susskind in 1995 (building on a less-generally-applicable earlier bound by J.Bekenstein), and often called the "**holographic** principle" or "**Bekenstein**

bound," states that any physical system enclosed by a surface of area A (measured in Planck units), contains a number B of bits of information-entropy upper bounded by

$$B \le A/(4\ln 2).$$

This is 1.381×10^{69} bits per square meter and $1/(4\ln 2) \approx 0.3607$ bits per Planck area unit.

My own Derivation of Holographic Info Bound: Suppose A is (what general relativists call) a "trapped surface." By the Penrose-Hawking theorem in classical general relativity (Hawking & Ellis 1973 assuming certain "energy conditions"), everything enclosed by this surface is doomed eventually to fall into a black hole, and during this process,

1. the area A of the surface can only shrink,
2. The entropy B trapped within this surface, since it cannot escape, can only rise.

Assume the original surface is isolated far enough away from any external stuff that the collapse process cannot influence that external stuff. Since the holographic bound is known to be an *equality* for isolated black holes after long times, we conclude from the assumption that the total entropy of the universe cannot decrease, that the holographic bound must hold for such a surface. Now: we argue that the fact that the original surface is "trapped" and "isolated" is not perceivable by any local physical experiment. Any observer sitting either slightly within or slightly outside of the surface, "feels the same"; and there is "no way for him to know" that he is inside, outside, on, near, or far away from a trapped surface. (Also, incidentally, a surface which is not trapped, can be converted into one that is, by moving some suitable dense mass distribution inside it. This movement would not affect the local physics or entropy on or near the surface, and could only increase the entropy inside it.) Therefore, if the laws of physics are local, then the bound must hold for *all* surfaces, not merely ones which happen to be trapped and isolated. **Q.E.D.**

The assumptions that went into my above argument were (a) classical general relativity plus whatever "energy conditions" are required to make the Penrose-Hawking collapse theorem valid [the latter nowadays are quite weak, e.g. the "averaged null energy condition"] (b) Hawking-Bekenstein exact formula for entropy of a black hole [after a long-enough time has passed to get rid of all "transients"] with event-horizon area A, (c) entropy of universe cannot decrease, (d) physics exhibits spacetime locality. The bound is tight in the sense that a black hole event horizon meets it.

Actually we *shall not need* the precise formula for the Bekenstein bound. All we shall need is much weaker claims of form *if the holographic bound (or some weakened version of it) ever were violated, then the process of collapse into a black hole, would reduce entropy*. And if it were *enough*-violated the (longer) process of collapse followed by Hawking evaporation, also would reduce entropy. The Hawking radiation a black hole emits has *greater* entropy than the hole loses, which is the reason I needed the extra safety margin, implied by the word "enough," to force a Second Law violation. For example, if our black hole were contained in some finite size cubical box, say with sidelength $2N \gg 1$ Schwarzschild radii, with perfect-mirror walls, then the hole would shrink as it evaporated, hence always staying hotter than the radiation in the box, and hence would continue to radiate faster than it sucked in energy, until gone. The final state – a box filled with radiation – would then have entropy upper bounded by $O(3\log N)$ times the original black-hole entropy.

OK. Now observe that if gravitons exist, i.e. if gravitational wave-modes with frequency f have energies *quantized* in units of hf, then a graviton-gas, in order to have high entropy, i.e. in order to store a large number of bits of information-content, necessarily has high mass-energy. With gravitons the state of the metric is specified by *integer* mode-occupation numbers ("graviton counts") preventing any mode from storing too many bits. All that mass-energy causes collapse into a black hole if you try to store too much information. In contrast if gravitons do *not* exist, classical wave modes can store arbitrary noninteger real numbers. Even a single mode could store an unboundedly large amount of information. By storing information in metrical ripples of arbitrarily-short wavelengths using small enough amplitudes (now permitted since $E \neq hf$), for example if we stored an amount of energy of order $(1+f)^{-6}$ in each frequency=f mode [indeed, even if exactly *two* energy amounts, say $(1+f)^{-6}$ and 0, were permitted for each mode], then we clearly could store arbitrarily huge, indeed *infinite*, amounts of information in a region containing bounded mass-energy, thus violating the holographic upper bound by an arbitrarily-great factor. Indeed, if each mode of frequency $f>10$ were permitted to have only two energies 0 and $f^{-\ln f}$ (it does not matter for our present purposes what units we measure *f* in) then that would suffice to store an infinite amount of information in a finite-size, finite (and small) mass, box, *and* in such a way that the spacetime metric remained infinitely-differentiable! We could then toss that information-storing region into a black hole, thereby arbitrarily-hugely violating the Second Law.

Indeed, even there were some magic high-frequency cutoff preventing gravitational-wave modes of frequencies $f>F$ from existing, that *still* (regardless of the value of F!) would not hurt our argument, because by making the region containing our modes and information have large enough diameter, then by dropping it into a large-enough black hole, we still get a contradiction!

Conclusion: If gravity is a classical nonquantum field, then that arbitrarily-hugely contradicts the Second Law of thermodynamics; and the argument establishing this contradiction is not hurt even if we magically postulate an (arbitrary) high-frequency cutoff for gravitational waves.

Remark: Some readers might object to the idea above of a hypothetical large "box with perfect-mirror walls" surrounding our black hole. I only did that for simplicity/clarity. It is in no way essential to have any such box or mirror. The argument still works even if the black hole is Hawking-evaporating into unbounded space, because only a finite chunk of that space, of diameter $O(M^3)$ for a mass=M hole, will ever be reached by the Hawking radiation during the hole's lifetime. This multiplies the entropy bound by O(9logM) instead of O(3logN), which does not hurt the validity of my argument.

5. Attractive potential between two balls, from a general force(distance) law

Suppose that two volumes, each uniformly filled with matter say at mass densities ϱ_1 and ϱ_2, attract with the potential energy

$$E = -\iiint \iiint \Phi(R)\, dv_1\, dv_2$$

where $\Phi(R)$ is an arbitrary specified function of the Euclidean distance R between the two infinitesimal 3-dimensional volume-elements dv_1 and dv_2. For Newton's law of gravity, $\Phi(R)=G\varrho_1\varrho_2/R$, but this section shall mathematically investigate arbitrary $\Phi(R)$. Hamaker 1937 on pages 1059-1060 gave a nice method of evaluating these integrals when the two volumes are *balls* of radii A and B, with center separation C, where $0<A$, $0<B$, $A+B<C$. His method works in two stages: first we find the potential Ψ between a point and a ball (which is a spherically symmetric function of the point-location); then using Ψ instead of Φ we integrate over the second ball, which amounts to the same calculation. Each half-calculation is done by a 1-dimensional integration dR, i.e. partitioning the *ball* into infinitesimally-thin spherical-shell layers centered at the *point*. Hamaker for some unknown reason only described his method in the particular case $\Phi(R)=HR^{-6}$, but I point out it works for arbitrary $\Phi(R)$. The method yields closed forms for many $\Phi(R)$, in particular whenever $\Phi(R)$ is any *power law*. Also $\Phi(R)=R^n\exp(-kR)$ should be doable where n is any integer.
Hamaker's identity is:

$$E = -\pi^2 C^{-1} \int_{C-A<R<C+A} [A^2-(C-R)^2] \int_{R-B<x<R+B} [B^2-(R-x)^2]\, \Phi(x)\, x\, dx\, dR.$$

In the Newton-law case this yields $-E_1=16\pi^2A^3B^3\varrho_1\varrho_2G/(9C)=M_1M_2G/C$. Newton himself first proved this result, but I prefer Hamaker's derivation. Very simple formulas also arise for $\Phi(R)=R^p$ for each $p\in\{0,1,2\}$. For $\Phi(R)=HR^{-6}$, Hamaker found

$$-E_6 = (\pi^2H/12)\ [y/(x^2+xy+x) + y/(x^2+xy+x+y) + 2\ln([x^2+xy+x]/[x^2+xy+x+y])]$$

expressed in terms of the dimensionless variables $2x=S/B$ and $y=A/B$ where S is the *minimal* separation between the balls, as opposed to the *center*-separation $C=A+B+S$. Note that in the case of equal balls $A=B$ we have $y=1$, so this simplifies to

$$-E_{6eq} = (\pi^2H/12)\ [\ (x^2+2x)^{-1} + (x+1)^{-2} + 2\ln([x+1]^{-2}[x^2+2x])\ ]$$
$$= (\pi^2H/36)\ [z^{-6} + (3/2)z^{-8} + (9/5)z^{-10} + 2z^{-12} + O(z^{-14})]$$

where $z=x+1$, i.e. $2z=C/B$. And in the $y=\infty$ case of "a ball and a wall" ("wall" meaning halfspace) it instead simplifies to

$$-E_{6wall} = (\pi^2H/12)\ [x^{-1} + (x+1)^{-1} + 2\ln(x/[x+1])]$$
$$= (\pi^2H/36)\ [z^{-3} + (3/10)z^{-5} + (9/112)z^{-7} + (1/48)z^{-9} + O(z^{-11})].$$

where now $z=x+½$, i.e. $2z=(S+B)/B$.

I am interested in $\Phi(R)=KR^{-7}$. Casimir & Polder 1948 found from the quantum field theory of the vacuum that two atoms with polarizabilities P and Q would be attracted by such a potential ("Van der Waals force") with $K=23\hbar cPQ/(4\pi)$, for separations R exceeding perhaps 500 nm. Here the definition of "polarizability" is P=(induced dipole moment)/(applied electric field). The effect arises because the zeropoint electromagnetic field modes induce fluctuating polarizations in the two balls, which then attract via dipole-dipole force. The reason I said R>500nm is that Casimir & Polder's "retarded" Van der Waals effects dominate with separations R such that photons of wavelength≈R would have energies well below those required to ionize the atoms. The 13.6eV energy of ionization of hydrogen corresponds to wavelength 91 nm. The elements with the least and greatest ionization energies are cesium (3.89eV) and helium (24.6eV) corresponding to photon wavelengths 318 and 50.4nm respectively. If we consider photoelectric "work function" as probably more appropriate than "atomic ionization energy" then cesium is the metal with the least work function (2.1eV) while gold, platinum, iridium, osmium, and selenium have the greatest (5-6eV). In contrast, for separations below 10 nm, "unretarded" (aka "London regime") Van der Waals forces dominate, and for them the exponent is 6, not 7. (In the intermediate range 10<R<500nm it depends on the material.)

Let $C=S+A+B$ be the central, $L=C+A+B$ the maximal, and S the minimal separation. Then

$$30\pi^{-2}C\ E_7 = -\ln(\ (S+2A)(S+2B)L^{-1}S^{-1}\ ) + 4AB\ S^{-2}\ (S+2A)^{-2}\ (S+2B)^{-2}\ L^{-2} \times$$
$$(C^6-3B^2C^4-3A^2C^4+3B^4C^2-14A^2B^2C^2+3A^4C^2-B^6+A^2B^4+A^4B^2-A^6).$$

In the case of equal ball radii $A=B$ this becomes

$$E_{7eq} = 30^{-1}\ \pi^2\ K\ C^{-1}\ [\ \ln(SL/C^2) + 4B^2C^{-2}L^{-2}S^{-2}\ (C^4-6B^2C^2-8B^4)\ ]$$
$$= (-16/9)\pi^2\ K\ C^{-1}\ [x^{-6} + (42/5)x^{-8} + (1296/25)x^{-10} + O(x^{-12})].$$

where $x=C/B$. In the "ball and wall" limit $A\to\infty$ with $C=A+B+S$ with B and S held fixed, we get

$$-E_{7wall} = (2/15)\, K\, \pi^2\, B^3\, S^{-2}\, (S+2B)^{-2}.$$

In the limit $B \to 0+$ of a tiny ball of volume dv, this potential between a wall and ball becomes $(-\pi/10)KS^{-4}dv$, where S is the wall-ball separation. We may then integrate this over a halfspace to obtain the potential energy of interaction between two parallel halfspaces with gap S (per unit cross-sectional area)

$$E_{7wallwall} = (-\pi/30)\, K\, S^{-3}.$$

If we compare this expression with the potential per unit area $(-\pi/1440)hcS^{-3}$ arising from the attractive "Casimir force" between two parallel infinite ideal-metal plates separated by gap S, we see that the two are the same if and only if K=hc/48.

6. General Remarks about measuring the gravity of small mass-balls

Consider two identical radius=B balls of metal. Our purpose here is to work out the **crossover** point where Casimir-Polder Van der Waals potential drops below, or rises above, the Newtonian gravitational potential.

If we enquire when the Newtonian potential $(16/9)\pi^2 G B^6 \varrho^2/R$ equals the approximate Casimir-Polder potential $(16/9)\pi^2(hc/48)B^6R^{-7}$ (where here R denotes the inter-center separation), upon canceling common terms and taking the square root, we see that this equality is equivalent to

$$\varrho\, R^3 = (hc/G)^{1/2} / \sqrt{48} \approx 7.874 \text{ micrograms.}$$

The left hand side of this equation is the mass of a hypothetical cube of matter at the same density=ϱ as our balls, and with the sidelength of the cube equal to their center-separation R. If the "=" is replaced by "<" then Casimir-Polder dominates; if ">" Newton dominates. Interestingly, the radii B and masses $4\pi B^3\varrho/3$ of our balls are *irrelevant* for deciding this question. Also interestingly, note the natural appearance of the "Planck mass" $(\hbar c/G)^{1/2} \approx 21.8\mu g$ in this criterion.

7. Direct probe of quantum gravity via mass-interferometry

If we want to keep Casimir-Polder confusor-potentials less than a fraction f of Newton, then we want $\varrho R^3 > f^{-1/2} 7.874\mu g$. So if we want to experiment with *quantum* gravity, the most logical thing to do is to make our mass-balls be *single atoms* of, say, osmium. Given that the density of osmium metal is ϱ=22.59 gram/cm^3 (the greatest for any known uncompressed substance), we then find that the interatom separation should be $R > f^{-1/6} 70.4$ micrometers. (And for any separation this large, our approximate Casimir-Polder expression should be very accurate.)

So imagine an experiment where we send an osmium atom through a 2-slit interferometer, causing it to be in a superposition of two locations distance R apart at the same time, and then one of those locations interacts gravitationally with a second osmium atom, causing a quantum phase-shift of order $\Phi\Delta t/\hbar$, where Φ is the Newtonian potential and Δt is the duration of the interaction. If this phase shift has order 1, that will alter the interference pattern by a detectable amount. We detect it, thus performing the world's first quantum gravity experiment.

What would it take to make that actually work? First of all, to get our atom in a superposition of being in two locations $R \approx 100\mu m$ apart without the wavefunction collapsing, we need everything to be kept at **cold** enough temperature T so that the atom's thermal de Broglie wavelength $\lambda_{th}=[h\hbar/(mk_BT)]^{1/2}$ exceeds 100μm. Substituting in the mass of an osmium atom for m, we find this requires $T\le 3\times 10^{-12}$ Kelvin.

That is 2 to 6 orders of magnitude colder than the coldest temperatures yet reached by mankind, but might be feasible. Specifically, the coldest achievements I am aware of are

- for bulk solid matter: 1.5-2μK in a chunk of platinum, achieved at Bayreuth University in West Germany in about 1990. Their equipment can maintain temperatures below 20μK for 2 weeks.
- For "nuclear spin temperature" (which I regard as a fake temperature measure with essentially no real use) 100-300pK was achieved for the nuclei in a 2 gram piece of rhodium metal in Aalto University, Espoo Finland in 2001.
- A "Bose Einstein condensate" of 2500 atoms of ^{23}Na gas cooled to 450pK was produced at the MIT-Harvard Center for Ultracold Atoms, Cambridge USA in 2003.

There is propaganda alleging that cooling gases containing a small number (10^3-10^5) of atoms to 1pK for 20 seconds, should in future be possible with techniques like NASA's "cold atom lab" on the space station. I am unsure whether this kind of cooling really would be permissible for our purpose (and certainly 20 seconds would be very inadequate), but if it is then the experiment this section is considering might become feasible.

Second, the Newtonian potential between two osmium atoms 100μm apart is $\Phi=6.66\times 10^{-56}$ joules, which means to get a phase shift of order 1 radian, we need it to experience that potential for a duration $\Delta t\approx\hbar/\Phi\approx 5\times 10^{13}$ years, i.e. about 3600 times the current age of the universe!

That problem improves if instead of two osmium *atoms* attracting, we consider a 100μm-diameter osmium *ball* attracting a single osmium atom 100μm away from its surface, i.e. 150μm away from its center. This ball has mass=11.8μg and consists of $N=3.74\times 10^{16}$ osmium atoms. This reduces the required duration by a factor of N/1.5 to only $\Delta t\approx 17.6$ hours. This duration would happen with an atomic speed of order 200 microns per day (which is about the RMS speed osmium atoms would have in an ideal gas of temperature 3.5×10^{-10} Kelvin).

Conclusion: Directly probing quantum gravity via matter interferometry requires refrigeration to picoKelvin temperatures, a feat nobody presently knows how to do for bulk matter, but it might be achievable for *gases* with small numbers (10^3-10^5) of atoms. Unfortunately I have doubts that an experiment on gravity attracting ultracold delocalized gas atoms toward a much hotter osmium ball could be run for the necessary timespan without the gas becoming too warm. If it cannot, then we return to the problem of picoKelvin refrigeration of bulk matter.

But even if all this could be done, then this atom+ball experiment still would not necessarily convince anyone that gravity is quantum! The Schrödinger equation involves quantum matter (described by a "wave function" Ψ) interacting with a *classical* potential V. So the fact that that Ψ exhibits quantum (e.g. interference, tunneling) phenomena in no way demonstrates that V is non-classical. Atom-atom versions of the experiment might convince me (as Feynman would have put it, if the two atoms can via gravitation become "entangled", that indicates the gravitational field "channel" can transmit quantum information), but they seem infeasible. A "Bohr atom" made of two gravitationally-bound osmium atoms would have "Bohr radius" of order 1 light year, binding energy of order 10^{-56} eV, and characteristic periods of order 10^{34} years.

Speaking of inteference phenomena in gravitational potentials/fields, another experiment of that ilk – this one entirely doable – is to view photons from a faraway star that arrive via two paths. One path goes slightly to the left of our Sun, the other to the right ("gravitational lensing"). If we position our telescope just right, we can interfere these, which would be the largest-distance interference experiment ever done. The advantage of our Sun is: we can place our telescope on a spaceprobe and move it to just the right place. Specifically, the spaceprobe would need to be about 560 AU from the sun with the probe, sun, and source all collinear in that order. Note 560 AU is about 18 times the Sun-Neptune distance. Voyager 1, the furthest spaceprobe as of year 2021, will reach that distance roughly 170 years after launch. (It also would be possible in principle to use much larger gravitational lenses than our Sun, if the Earth, lensing mass, and photon source were luckily positioned just right. But this probably requires too much luck, i.e. will never happen because we can't move the Earth.) Again, if this were done then it would qualify as an "interesting quantum gravity experiment" but probably could not and would not show (or refute) the quantum nature of the gravitational field.

8. Conclusions

1. Dyson was completely wrong in any attempt to claim or conjecture that the existence of gravitons was undecidable, not even in principle, by experiment. It definitely is decidable.

2. We have arguments about gravitational waves from big-bang heat, suggesting that gravitons have *already* been shown, by observation, to exist.

3. Classical fields have too many degrees of freedom – an intuition we have leveraged to show that if gravity is not a quantum field, that enables arbitrarily-huge violations of the Second Law of thermodynamics.

4. At least one kind of direct probe of how quantum-delocalized mass gravitates looks experimentally possible if anybody ever can cool macrocopic amounts of matter to picoKelvin temperature; and perhaps cooling a small number of atoms in a gas could suffice. But this would be far colder than present world records; the durations needed before gathering enough data might be of order 1 year; and the experiment would not decide whether gravitons exist.

These all are interesting; and I personally find #3 extremely convincing that gravity must be a quantum field. I speculate #3 also would have convinced Dyson, but we'll probably never know because he died in Feb.2020.

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