Sedeonic Generalization of Hydrodynamic Model of Vortex Plasma

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Abstract

The noncommutative algebra of space-time sedeons is used for the generalization of the system of nonlinear self-consistent equations in hydrodynamic two-fluid model of vortex plasma. This system describes both longitudinal flows as well as the rotation and twisting of vortex tubes taking into account internal electric and magnetic fields generated by fluctuations of plasma parameters. As an illustration we apply the proposed equations for the description of sound waves in electron-ion and electron-positron plasmas.

1 Introduction

In the past few decades, significant progress has been achieved in the application of non-commutative algebras of hypercomplex numbers for the description of electromagnetic field in vacuum, a weak gravitational field in the model of the gravito-electromagnetism and the motion of a fluid in the framework of the Euler formalism. In particular, the simplest generalization of electrodynamics equations was developed on the basis of quaternions [1,2]. The structure of quaternions with four components (scalar and vector) corresponds to the relativistic four-vector concept that allows reformulating all relativistic mathematical expressions in terms of quaternionic algebra. However, the essential imperfection of the quaternionic algebra is that the quaternions do not include pseudoscalar and pseudovector components, which are transformed differently under spatial-temporal inversions. This problem can be overcome by increasing the dimensionality of hypercomplex numbers based on the Cayley-Dickson doubling procedure [3], which leads to eight-component octonions [4] and sixteen-component sedenions [5]. All these hypercomplex algebras are used for the description of electromagnetic field and in application to fluid dynamics [6-15]. However, a significant disadvantage of the Cayley-Dickson numbers is their nonassociativity, which partially complicates their physical applications, since one needs to fix a certain sequence of operators in each equation. Recently we proposed the alternative space-time algebra of sixteen-component sedeons, which is associative and takes into account the symmetry of physical values with respect to the space-time inversions [16]. In particular, the sedeonic approach was used for the generalization of equations describing the massive and massless fields and fluid dynamics [17-19].

The commonly used hydrodynamic description of plasma is based on a two-fluid model, which includes Euler and continuity equations separately for the electron and ion components, as well as Maxwell's equations for the electromagnetic field [20,21]. In the present time, much attention is paid to the description of fluid dynamics by vector fields including vector of local velocity of fluid and vector of vorticity, which satisfy the symmetric Maxwell-type equations [22-29]. In particular, similar approach has been recently used for the description of two-fluid model of plasma [30-34]. However, usually an additional equation for the vortex motion is obtained by taking the "curl" operator from the Euler equation, and, therefore, the resulting equation is not independent. Recently, we have developed an alternative approach based on droplet model of fluid described by Helmholtz [35], and obtained a closed system of Maxwell-type equations for vortex fluid taking into account the rotation and twisting of vortex tubes [19]. In the present paper we apply the sedeonic algebra for the generalization of self-consistent equations describing two-component vortex plasma.

2 Sedeonic equation for vortex fluid flow

To compactify the equations we use the algebra of 16-component sedeons [16,18], which are the tensor product of two 4-component algebras of Macfarlane quaternions [36]. In this algebra any vector can be presented as a linear combination of unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$:

$$\mathbf{A} = A_1\mathbf{a}_1 + A_2\mathbf{a}_2 + A_3\mathbf{a}_3, \tag{1}$$

with the following rules of multiplication and commutation

$$\mathbf{a}_n\mathbf{a}_m = \delta_{nm} + i\lambda_{nmk}\mathbf{a}_k, \tag{2}$$

where δ_{nm} is Kronecker delta, λ_{nmk} is Levi-Civita symbol ($n, m, k \in \{1, 2, 3\}$) and i is the imaginary unit ($i^2 = -1$). The main advantage of sedeonic algebra is the possibility to define Clifford product of vectors. For example, the Clifford product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is

$$\mathbf{AB} = \mathbf{A}\cdot\mathbf{B} + i\,\mathbf{A}\times\mathbf{B}, \tag{3}$$

that allows to write the equations in the very compact form. To compactify the operator parts of equations we use additional units $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ with the same rules of multiplication

$$\mathbf{e}_n\mathbf{e}_m = \delta_{nm} + i\lambda_{nmk}\mathbf{e}_k, \tag{4}$$

The basis $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ is associated with spatial rotation of vector values, while basis $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ connected with space-time inversion [18]. The unit vectors $\mathbf{a}_n$, commute with units $\mathbf{e}_m$

$$\mathbf{a}_n\mathbf{e}_m = \mathbf{e}_m\mathbf{a}_n. \tag{5}$$

Recently we have shown [19] that the free isentropic fluid is described by sedeonic equation for values corresponding to the longitudinal motion and rotational vortex flows

$$\left(i\mathbf{e}_1\frac{1}{s}\left(\frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla)\right) - \mathbf{e}_2\nabla\right)(i\mathbf{e}_1 u - i\mathbf{e}_2\xi + \mathbf{e}_2\mathbf{v} + \mathbf{e}_1\mathbf{w}) = 0. \tag{6}$$

This compact equation is equivalent to the following closed system of equations [19]:

$$\begin{aligned}
&\frac{1}{s}\left(\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)\right)\mathbf{v}+\nabla u+\nabla\times\mathbf{w}=0,\\
&\frac{1}{s}\left(\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)\right)u+\nabla\cdot\mathbf{v}=0,\\
&\frac{1}{s}\left(\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)\right)\mathbf{w}+\nabla\xi-\nabla\times\mathbf{v}=0,\\
&\frac{1}{s}\left(\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)\right)\xi+\nabla\cdot\mathbf{w}=0.
\end{aligned} \tag{7}$$

Here s is the speed of sound, $\mathbf{v}$ is the local velocity of the fluid. The value u is proportional to the enthalpy

$$\begin{aligned}
u&=\frac{1}{s}h,\\
dh&=\frac{s^2}{n_0}dn,
\end{aligned} \tag{8}$$

where h is the enthalpy per unit mass, n is the fluid particles concentration, n_0 is the equilibrium concentration of particles. The value $\mathbf{w}$ characterizes the rotation of the vortex tubes

$$\begin{aligned}
\mathbf{w}&=2s\mathbf{\Theta},\\
\mathbf{\Omega}&=\frac{d\mathbf{\Theta}}{dt},
\end{aligned} \tag{9}$$

where $\mathbf{\Theta}$ is the angle vector of vortex tube rotation, $\mathbf{\Omega}$ is the angle velocity of vortex tube rotation. The value ξ characterizes the twisting of vortex tube

$$|\xi|=s\beta, \tag{10}$$

where β is the twisting angle of vortex tube [19].

3 Equations of the two-fluid model of vortex plasma

In the commonly used hydrodynamic approach, plasma is represented as a mixture of two fluids in which the particles have different masses and charges. We will consider only low-frequency perturbations in electrically neutral, fully ionized, nonradiative plasma, which propagate in the form of sound waves. In this approximation, the internal electric and magnetic fields are generated only due to deviations of plasma parameters from their equilibrium values. In fact, these fields are quasi-static and move together with the plasma [37]. This representation is close to the concept of a frozen-in field applied in the description of Alfven waves [38–40] and London's model of superconductors [41,42]. Here on the base of sedeonic equation (6) we construct self-consisted two-fluid model of vortex plasma. The hydrodynamic equations for electron and ion fluids with internal electromagnetic field can be obtained using the following substitutions [30, 37] in the equation (6):

$$\begin{aligned}
&\mathbf{v}\Rightarrow\mathbf{v}_\alpha+a_\alpha\mathbf{A}_\alpha,\\
&u\Rightarrow u_\alpha+a_\alpha\varphi_\alpha,\\
&\mathbf{w}\Rightarrow\mathbf{w}_\alpha+a_\alpha\mathbf{M}_\alpha,\\
&\xi\Rightarrow\xi_\alpha+a_\alpha\phi_\alpha.
\end{aligned} \tag{11}$$

Here the index $\alpha \in \{e, i\}$, where e stands for electrons and i stands for ions, φ_α and $\mathbf{A}_\alpha$ are scalar and vector electric potentials, ϕ_α and $\mathbf{M}_\alpha$ are scalar and vector magnetic potentials of internal electromagnetic field. The parameter a_α is

$$a_\alpha = \frac{q_\alpha}{m_\alpha s_\alpha}, \tag{12}$$

where q_α is particle charge, m_α is particle mass, s_α is corresponding speed of sound. Let us introduce new operator

$$\widehat{\nabla}_\alpha = \left(i\mathbf{e}_1 \frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right) - \mathbf{e}_2 \nabla \right). \tag{13}$$

Then taking into account (11) and (13) we rewrite (6) as

$$\widehat{\nabla}_\alpha \{ i\mathbf{e}_1 (u_\alpha + a_\alpha \varphi_\alpha) - i\mathbf{e}_2 (\xi_\alpha + a_\alpha \phi_\alpha) + \mathbf{e}_2 (\mathbf{v}_\alpha + a_\alpha \mathbf{A}_\alpha) + \mathbf{e}_1 (\mathbf{w}_\alpha + a_\alpha \mathbf{M}_\alpha) \} = 0. \tag{14}$$

This sedeonic equation is equivalent to the following system:

$$\begin{aligned}
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)(\mathbf{v}_\alpha + a_\alpha \mathbf{A}_\alpha) + \nabla(u_\alpha + a_\alpha \varphi_\alpha) + \nabla \times (\mathbf{w}_\alpha + a_\alpha \mathbf{M}_\alpha) = 0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)(u_\alpha + a_\alpha \varphi_\alpha) + \nabla \cdot (\mathbf{v}_\alpha + a_\alpha \mathbf{A}_\alpha) = 0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)(\mathbf{w}_\alpha + a_\alpha \mathbf{M}_\alpha) + \nabla(\xi_\alpha + a_\alpha \phi_\alpha) - \nabla \times (\mathbf{v}_\alpha + a_\alpha \mathbf{A}_\alpha) = 0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)(\xi_\alpha + a_\alpha \phi_\alpha) + \nabla \cdot (\mathbf{w}_\alpha + a_\alpha \mathbf{M}_\alpha) = 0.
\end{aligned} \tag{15}$$

Assuming the following definitions for internal field strengths:

$$\begin{aligned}
\mathbf{E}_\alpha &= -\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{A}_\alpha - \nabla \varphi_\alpha - \nabla \times \mathbf{M}_\alpha,\\
\mathbf{B}_\alpha &= -\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{M}_\alpha - \nabla \phi_\alpha + \nabla \times \mathbf{A}_\alpha,
\end{aligned} \tag{16}$$

and taking the following gauge conditions:

$$\begin{aligned}
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\varphi_\alpha + \nabla \cdot \mathbf{A}_\alpha = 0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\phi_\alpha + \nabla \cdot \mathbf{M}_\alpha = 0,
\end{aligned} \tag{17}$$

for equation (14) we get

$$\left(i\mathbf{e}_1 \frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) \right) - \mathbf{e}_2 \nabla \right) (i\mathbf{e}_1 u - i\mathbf{e}_2 \xi + \mathbf{e}_2 \mathbf{v} + \mathbf{e}_1 \mathbf{w}) = -a_\alpha (\mathbf{e}_3 \mathbf{E}_\alpha - i \mathbf{B}_\alpha). \tag{18}$$

This sedeonic equation is equivalent to the following system:

$$\begin{aligned}
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)\mathbf{v}_\alpha+\nabla u_\alpha+\nabla{\times}\mathbf{w}_\alpha=a_\alpha\mathbf{E}_\alpha,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)u_\alpha+\nabla\cdot\mathbf{v}_\alpha=0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)\mathbf{w}_\alpha+\nabla\xi_\alpha-\nabla{\times}\mathbf{v}_\alpha=a_\alpha\mathbf{B}_\alpha,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)\xi_\alpha+\nabla\cdot\mathbf{w}_\alpha=0.
\end{aligned}\tag{19}$$

Besides, the system (19) becomes closed if we suppose that internal electromagnetic fields propagate with plasma disturbances and satisfy the following sedeonic equation:

$$\begin{aligned}
&\widehat{\nabla}_\alpha\widehat{\nabla}_\alpha\left(i\mathbf{e}_1\varphi_\alpha-i\mathbf{e}_2\phi_\alpha+\mathbf{e}_1\mathbf{A}_\alpha+\mathbf{e}_2\mathbf{M}_\alpha\right)=\\
&-i\mathbf{e}_1 4\pi e\left(n_i-n_e\right)-\mathbf{e}_2\frac{4\pi}{s_\alpha}e\left(n_i\mathbf{v}_i-n_e\mathbf{v}_e\right)+\\
&i\mathbf{e}_2 4\pi e\left(g_i-g_e\right)-\mathbf{e}_1\frac{4\pi}{s_\alpha}e\left(g_i\mathbf{w}_i-g_e\mathbf{w}_e\right).
\end{aligned}\tag{20}$$

Here e is the magnitude of electron charge, n_α is particle concentration, g_α is density of vortex tube twisting for different component of plasma. The plasma particle concentration n_α and value g_α are connected with values u_α and ξ_α as

$$\begin{aligned}
n_\alpha&=\frac{n_{0\alpha}}{s_\alpha}u_\alpha,\\
g_\alpha&=\frac{n_{0\alpha}}{s_\alpha}\xi_\alpha,
\end{aligned}\tag{21}$$

where $n_{0\alpha}$ is the equilibrium particle concentration.

Taking into account (16) and (17) the equation (20) is reduced to

$$\begin{aligned}
&\left(i\mathbf{e}_1\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha\cdot\nabla)\right)-\mathbf{e}_2\nabla\right)\left(\mathbf{e}_3\mathbf{E}_\alpha-i\mathbf{B}_\alpha\right)=\\
&-i\mathbf{e}_1 4\pi e\left(n_i-n_e\right)-\mathbf{e}_2\frac{4\pi}{s_\alpha}e\left(n_i\mathbf{v}_i-n_e\mathbf{v}_e\right)+\\
&i\mathbf{e}_2 4\pi e\left(g_i-g_e\right)-\mathbf{e}_1\frac{4\pi}{s_\alpha}e\left(g_i\mathbf{w}_i-g_e\mathbf{w}_e\right),
\end{aligned}\tag{22}$$

which is equivalent to the following system

$$\begin{aligned}
&\nabla\cdot\mathbf{E}_\alpha=4\pi e\left(n_i-n_e\right),\\
&\nabla\cdot\mathbf{B}_\alpha=4\pi e\left(g_i-g_e\right),\\
&\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)\mathbf{B}_\alpha+s_\alpha\nabla{\times}\mathbf{E}_\alpha=-4\pi e\left(n_i\mathbf{w}_i-n_e\mathbf{w}_e\right),\\
&\left(\frac{\partial}{\partial t}+(\mathbf{v}_\alpha{\cdot}\nabla)\right)\mathbf{E}_\alpha-s_\alpha\nabla{\times}\mathbf{B}_\alpha=-4\pi e\left(n_i\mathbf{v}_i-n_e\mathbf{v}_e\right),
\end{aligned}\tag{23}$$

The equations (19) and (23) form the self-consistent system describing the vortex plasma.

4 Linearized equations of vortex plasma

Further we will consider small oscillations in plasma, which propagate as the different types of sound waves. Neglecting the convective derivative in the system (19) and assuming $n_{0i} = n_{0e} = n_0$ we obtain the following linearaized hydrodynamic equations:

$$\begin{aligned} &\frac{1}{s_\alpha}\frac{\partial \mathbf{v}_\alpha}{\partial t} + \nabla u_\alpha + \nabla\times\mathbf{w}_\alpha = a_\alpha \mathbf{E}_\alpha, \\ &\frac{1}{s_\alpha}\frac{\partial u_\alpha}{\partial t} + \nabla\cdot\mathbf{v}_\alpha = 0, \\ &\frac{1}{s_\alpha}\frac{\partial \mathbf{w}_\alpha}{\partial t} + \nabla \xi_\alpha - \nabla\times\mathbf{v}_\alpha = a_\alpha \mathbf{B}_\alpha, \\ &\frac{1}{s_\alpha}\frac{\partial \xi_\alpha}{\partial t} + \nabla\cdot\mathbf{w}_\alpha = 0, \end{aligned} \tag{24}$$

and corresponding equations for internal electromagnetic field

$$\begin{aligned} &\nabla\cdot\mathbf{E}_\alpha = 4\pi e\left(n_i - n_e\right), \\ &\nabla\cdot\mathbf{B}_\alpha = 4\pi e\left(g_i - g_e\right), \\ &\frac{\partial \mathbf{B}_\alpha}{\partial t} + s_\alpha \nabla\times\mathbf{E}_\alpha = -4\pi e n_0\left(\mathbf{w}_i - \mathbf{w}_e\right), \\ &\frac{\partial \mathbf{E}_\alpha}{\partial t} - s_\alpha \nabla\times\mathbf{B}_\alpha = -4\pi e n_0\left(\mathbf{v}_i - \mathbf{v}_e\right). \end{aligned} \tag{25}$$

From the equations (24) and (25) we have the following relations:

$$\begin{aligned} &\frac{1}{2}\frac{\partial}{\partial t}\left(\mathbf{v}_\alpha^2 + \mathbf{w}_\alpha^2 + u_\alpha^2 + \xi_\alpha^2\right) + s_\alpha\left(\nabla\cdot\left\{u_\alpha\mathbf{v}_\alpha + \xi_\alpha\mathbf{w}_\alpha + \mathbf{v}_\alpha\times\mathbf{w}_\alpha\right\}\right) = \\ &a_\alpha s_\alpha\left\{\mathbf{v}_\alpha\cdot\mathbf{E}_\alpha + \mathbf{w}_\alpha\cdot\mathbf{B}_\alpha\right\}, \end{aligned} \tag{26}$$

$$\begin{aligned} &\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2\right) + \frac{s_\alpha}{4\pi}\left(\nabla\cdot\left[\mathbf{E}_\alpha\times\mathbf{B}_\alpha\right]\right) = \\ &e n_0\left\{\left(\mathbf{E}_\alpha\cdot\left(\mathbf{v}_i - \mathbf{v}_e\right)\right) + \left(\mathbf{B}_\alpha\cdot\left(\mathbf{w}_i - \mathbf{w}_e\right)\right)\right\}, \end{aligned} \tag{27}$$

which are the analogs of Poynting relation known in the theory of electromagnetic field. The value

$$W_m = \frac{1}{2}\left(\mathbf{v}_\alpha^2 + \mathbf{w}_\alpha^2 + u_\alpha^2 + \xi_\alpha^2\right) \tag{28}$$

is the mechanical energy per unit mass and value

$$\mathbf{P}_m = s_\alpha\left(u_\alpha\mathbf{v}_\alpha + \xi_\alpha\mathbf{w}_\alpha + \mathbf{v}_\alpha\times\mathbf{w}_\alpha\right) \tag{29}$$

is the density of mechanical energy flux. The value

$$W_{em} = \frac{1}{8\pi}\left(\mathbf{E}_\alpha^2 + \mathbf{B}_\alpha^2\right) \tag{30}$$

is the volume density of electromagnetic energy of internal fields, while the value

$$\mathbf{P}_{em} = \frac{s_\alpha}{4\pi}\left[\mathbf{E}_\alpha\times\mathbf{B}_\alpha\right] \tag{31}$$

is the density of electromagnetic energy flux.

5 Sound waves in vortex plasma

Let us consider small fluctuations of plasma near the equilibrium state,

$$\begin{aligned} n_\alpha &= n_{0\alpha} + \tilde{n}_\alpha, \\ \mathbf{v}_\alpha &= \tilde{\mathbf{v}}_\alpha, \\ g_\alpha &= \tilde{g}_\alpha, \\ \mathbf{w}_\alpha &= \tilde{\mathbf{w}}_\alpha, \\ n_{0i} &= n_{0e} = n_0, \end{aligned} \tag{32}$$

Then reformulating the system (22) in term of n_α and g_α values we have

$$\begin{aligned} &\frac{1}{s_\alpha}\frac{\partial \tilde{\mathbf{v}}_\alpha}{\partial t} + \frac{s_\alpha}{n_0}\nabla\tilde{n}_\alpha + \nabla\times\tilde{\mathbf{w}}_\alpha = a_\alpha\tilde{\mathbf{E}}_\alpha, \\ &\frac{1}{n_0}\frac{\partial \tilde{n}_\alpha}{\partial t} + \nabla\cdot\tilde{\mathbf{v}}_\alpha = 0, \\ &\frac{1}{s_\alpha}\frac{\partial \tilde{\mathbf{w}}_\alpha}{\partial t} + \frac{s_\alpha}{n_0}\nabla\tilde{g}_\alpha - \nabla\times\tilde{\mathbf{v}}_\alpha = a_\alpha\tilde{\mathbf{B}}_\alpha, \\ &\frac{1}{n_0}\frac{\partial \tilde{g}_\alpha}{\partial t} + \nabla\cdot\tilde{\mathbf{w}}_\alpha = 0, \end{aligned} \tag{33}$$

and

$$\begin{aligned} &\nabla\cdot\tilde{\mathbf{E}}_\alpha = 4\pi e\left(\tilde{n}_i - \tilde{n}_e\right), \\ &\nabla\cdot\tilde{\mathbf{B}}_\alpha = 4\pi e\left(\tilde{g}_i - \tilde{g}_e\right), \\ &\frac{\partial\tilde{\mathbf{B}}_\alpha}{\partial t} + s_\alpha\nabla\times\tilde{\mathbf{E}}_\alpha = -4\pi e n_0\left(\tilde{\mathbf{w}}_i - \tilde{\mathbf{w}}_e\right), \\ &\frac{\partial\tilde{\mathbf{E}}_\alpha}{\partial t} - s_\alpha\nabla\times\tilde{\mathbf{B}}_\alpha = -4\pi e n_0\left(\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_e\right). \end{aligned} \tag{34}$$

From the systems (33) and (34) we have the following wave equations for the electron and ion concentrations

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} - s_i^2\Delta + \omega_{ip}^2\right)\tilde{n}_i = \omega_{ip}^2\tilde{n}_e, \\ \left(\frac{\partial^2}{\partial t^2} - s_e^2\Delta + \omega_{ep}^2\right)\tilde{n}_e = \omega_{ep}^2\tilde{n}_i. \end{aligned} \tag{35}$$

Here Δ is Laplace operator, ω_{ip} is the ion plasma frequency and ω_{ep} is the electron plasma frequency:

$$\omega_{ip}^2 = \frac{4\pi n_0 e^2}{m_i}, \tag{36}$$

$$\omega_{ep}^2 = \frac{4\pi n_0 e^2}{m_e}. \tag{37}$$

The equations (35) can be separated as:

$$\left\{\left(\frac{\partial^2}{\partial t^2} - s_e^2\Delta + \omega_{ep}^2\right)\left(\frac{\partial^2}{\partial t^2} - s_i^2\Delta + \omega_{ip}^2\right) - \omega_{ip}^2\omega_{ep}^2\right\}\tilde{n}_i = 0, \tag{38}$$

$$\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{n}_e = 0. \tag{39}$$

The same equations we have for the rest plasma values

$$\begin{aligned} &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{v}}_i = 0, \\ &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{v}}_e = 0, \end{aligned} \tag{40}$$

$$\begin{aligned} &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{g}_i = 0, \\ &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{g}_e = 0, \end{aligned} \tag{41}$$

$$\begin{aligned} &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{w}}_i = 0, \\ &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{w}}_e = 0, \end{aligned} \tag{42}$$

$$\begin{aligned} &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{E}}_i = 0, \\ &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{E}}_e = 0, \end{aligned} \tag{43}$$

$$\begin{aligned} &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{B}}_i = 0, \\ &\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{\mathbf{B}}_e = 0. \end{aligned} \tag{44}$$

Let us define the generalized plasma parameter as

$$\tilde{P}_\alpha \in \left\{ \tilde{n}_\alpha, \tilde{g}_\alpha, \tilde{\mathbf{v}}_\alpha, \tilde{\mathbf{w}}_\alpha, \tilde{\mathbf{E}}_\alpha, \tilde{\mathbf{B}}_\alpha \right\}, \tag{45}$$

then the generalized sound wave equation can be written as

$$\left\{ \left(\frac{\partial^2}{\partial t^2} - s_e^2 \Delta + \omega_{ep}^2 \right) \left(\frac{\partial^2}{\partial t^2} - s_i^2 \Delta + \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{P}_\alpha = 0. \tag{46}$$

The sound waves described by equation (46) have the following dispersion relation

$$\left(\omega^2 - s_e^2 k^2 - \omega_{ep}^2 \right) \left(\omega^2 - s_i^2 k^2 - \omega_{ip}^2 \right) - \omega_{ip}^2 \omega_{ep}^2 = 0, \tag{47}$$

where ω is the frequency and $\mathbf{k}$ is the wave vector ($k = |\mathbf{k}|$). The schematic plots illustrating this dispersion relation are represented in Fig. 1.

If $k = 0$, then we have two roots of equation (47)

$$\begin{aligned} &\omega = 0, \\ &\omega = \omega_* = \sqrt{\omega_{ep}^2 + \omega_{ip}^2}. \end{aligned} \tag{48}$$

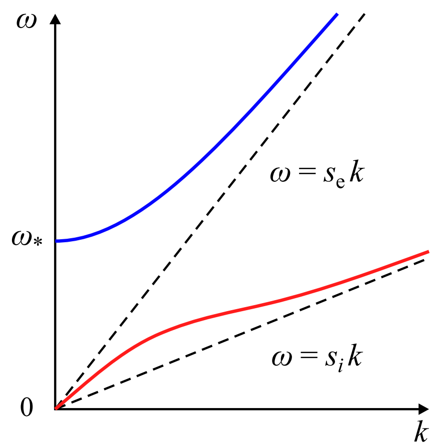

Figure 1: The schematic plots of dispersion curves for electron-ion sound waves [37].

If $k \to \infty$ we have two asymptotes

$$\omega = s_e k, \tag{49}$$

and

$$\omega = s_i k. \tag{50}$$

The upper curve in FIG. 1 corresponds to the electron sound, while the lower curve corresponds to the ion sound. The group velocity of ion sound in the long wave limit ($k \to 0$) is

$$v_{ig} = \frac{d\omega}{dk} = \sqrt{\frac{s_e^2\omega_{ip}^2 + s_i^2\omega_{ep}^2}{\omega_{ep}^2 + \omega_{ip}^2}}, \tag{51}$$

which is between $s_e > v_{ig} > s_i$.

6 Damping in vortex plasma

In rarefied plasma, the main damping mechanisms are electron-electron, electron-ion, and ion-ion collisions. Such damping of disturbances can be described by the following replacement of operators in all equations

$$\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) \Rightarrow \frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha. \tag{52}$$

Here the parameter ε_α is the characteristic collision frequency. In the case of dense plasma, it is also necessary to take into account the diffusion damping of plasma disturbances, which can be described by the following replacement of operators

$$\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) \Rightarrow \frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) - \mu_\alpha \triangle, \tag{53}$$

where μ_α is the coefficient of viscosity of plasma components. In general case both of these mechanisms must be taken into account, which leads to the following operator replacement:

$$\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) \Rightarrow \frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \triangle. \tag{54}$$

Thus instead of linearized equations (24) and (25) we have

$$
\begin{aligned}
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)\mathbf{v}_\alpha+\nabla u_\alpha+\nabla\times\mathbf{w}_\alpha=a_\alpha\mathbf{E}_\alpha,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)u_\alpha+\nabla\cdot\mathbf{v}_\alpha=0,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)\mathbf{w}_\alpha+\nabla\xi_\alpha-\nabla\times\mathbf{v}_\alpha=a_\alpha\mathbf{B}_\alpha,\\
&\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)\xi_\alpha+\nabla\cdot\mathbf{w}_\alpha=0,
\end{aligned}
\tag{55}
$$

and corresponding equations for internal electromagnetic field

$$
\begin{aligned}
&\nabla\cdot\mathbf{E}_\alpha=4\pi e\left(n_i-n_e\right),\\
&\nabla\cdot\mathbf{B}_\alpha=4\pi e\left(g_i-g_e\right),\\
&\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)\mathbf{B}_\alpha+s_\alpha\nabla\times\mathbf{E}_\alpha=-4\pi en_0\left(\mathbf{w}_i-\mathbf{w}_e\right),\\
&\left(\frac{\partial}{\partial t}+\varepsilon_\alpha-\mu_\alpha\triangle\right)\mathbf{E}_\alpha-s_\alpha\nabla\times\mathbf{B}_\alpha=-4\pi en_0\left(\mathbf{v}_i-\mathbf{v}_e\right).
\end{aligned}
\tag{56}
$$

Corresponding Poynting-type relations are written as

$$
\begin{aligned}
&\left(\frac{1}{2}\frac{\partial}{\partial t}+\varepsilon_\alpha\right)\left(\mathbf{v}_\alpha^2+\mathbf{w}_\alpha^2+u_\alpha^2+\xi_\alpha^2\right)+s_\alpha\left(\nabla\cdot\{u_\alpha\mathbf{v}_\alpha+\xi_\alpha\mathbf{w}_\alpha+\mathbf{v}_\alpha\times\mathbf{w}_\alpha\}\right)-\\
&\mu_\alpha\left(u_\alpha\Delta u_\alpha+\xi_\alpha\Delta\xi_\alpha+\mathbf{v}_\alpha\cdot\Delta\mathbf{v}_\alpha+\mathbf{w}_\alpha\cdot\Delta\mathbf{w}_\alpha\right)=a_\alpha s_\alpha\left\{\mathbf{v}_\alpha\cdot\mathbf{E}_\alpha+\mathbf{w}_\alpha\cdot\mathbf{B}_\alpha\right\},
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
&\left(\frac{1}{2}\frac{\partial}{\partial t}+\varepsilon_\alpha\right)\left(\mathbf{E}_\alpha^2+\mathbf{B}_\alpha^2\right)-\mu_\alpha\left\{(\mathbf{E}_\alpha\cdot\Delta\mathbf{E}_\alpha)+(\mathbf{B}_\alpha\cdot\Delta\mathbf{B}_\alpha)\right\}+\\
&s_\alpha\left(\nabla\cdot[\mathbf{E}_\alpha\times\mathbf{B}_\alpha]\right)=4\pi en_0\left\{\mathbf{E}_\alpha\cdot(\mathbf{v}_i-\mathbf{v}_e)+\mathbf{B}_\alpha\cdot(\mathbf{w}_i-\mathbf{w}_e)\right\}.
\end{aligned}
\tag{58}
$$

Let us denote

$$\widehat{\Box}_e=\left(\frac{\partial}{\partial t}+\varepsilon_e-\mu_e\Delta\right)^2-s_e^2\Delta, \tag{59}$$

$$\widehat{\Box}_i=\left(\frac{\partial}{\partial t}+\varepsilon_i-\mu_i\Delta\right)^2-s_i^2\Delta. \tag{60}$$

Then from equations (55) and (56) we obtain the following generalized sound wave equation

$$\left\{\left(\widehat{\Box}_e+\omega_{ep}^2\right)\left(\widehat{\Box}_i+\omega_{ip}^2\right)-\omega_{ip}^2\omega_{ep}^2\right\}\tilde{P}_\alpha=0. \tag{61}$$

All sound waves described by equation (61) have the following dispersion relation

$$\left(\left(i\omega+\varepsilon_e+\mu_e k^2\right)^2+s_e^2k^2+\omega_{ep}^2\right)\left(\left(i\omega+\varepsilon_i+\mu_i k^2\right)^2+s_i^2k^2+\omega_{ip}^2\right)-\omega_{ip}^2\omega_{ep}^2=0. \tag{62}$$

7 Electron-positron vortex plasma

In the case of electron-positron plasma the index α in equations (55) and (56) should be interpreted as $\alpha \in \{e, p\}$, where e is for electrons and p is for positrons. For electron-positron plasma we have the equality of speeds of sound, plasma frequencies and other parameters

$$\begin{aligned} s_e &= s_p = s_{ep}, \\ \omega_{ep} &= \omega_{pp} = \omega_{epp}, \\ \varepsilon_e &= \varepsilon_p = \varepsilon_{ep}, \\ \mu_e &= \mu_p = \mu_{ep}. \end{aligned} \tag{63}$$

This simplifies the system of equations (55) and (56). In the case of a cold, strongly interacting electron-positron plasma, the following approximation is valid:

$$\begin{aligned} \tilde{n}_e &= \tilde{n}_p, \\ \tilde{\mathbf{v}}_e &= \tilde{\mathbf{v}}_p, \\ \tilde{g}_e &= \tilde{g}_p, \\ \tilde{\mathbf{w}}_e &= \tilde{\mathbf{w}}_p, \\ n_{0e} &= n_{0p} = n_0. \end{aligned} \tag{64}$$

Such plasma does not generate electromagnetic fields and from equations (55) and (56) we obtain the usual wave equation for sound waves

$$\left(\left(\frac{\partial}{\partial t} + \varepsilon_{ep} - \mu_{ep}\Delta \right)^2 - s_{ep}^2 \Delta \right) \tilde{p}_{ep} = 0, \tag{65}$$

where

$$\tilde{p}_{ep} \in \{\tilde{n}_e, \tilde{g}_e, \tilde{\mathbf{v}}_e, \tilde{\mathbf{w}}_e, \tilde{n}_p, \tilde{g}_p, \tilde{\mathbf{v}}_p, \tilde{\mathbf{w}}_p\}. \tag{66}$$

The dispersion relation for the equation (65) is

$$\omega^2 - 2i\omega\left(\varepsilon_{ep} + \mu_{ep}k^2\right) - \left(\varepsilon_{ep}\mu_{ep} + s_{ep}^2\right)k^2 - \mu_{ep}^2 k^4 - \varepsilon_{ep}^2 = 0. \tag{67}$$

In the case of hot electron-positron plasma, relations (64) do not hold and from equations (55) and (56) we obtain the following generalized equation for sound waves:

$$\left\{ \left(\left(\frac{\partial}{\partial t} + \varepsilon_{ep} - \mu_{ep}\Delta \right)^2 - s_{ep}^2 \Delta + \omega_{epp}^2 \right)^2 - \omega_{epp}^4 \right\} \tilde{P}_{ep} = 0, \tag{68}$$

where

$$\tilde{P}_{ep} \in \left\{ \tilde{n}_e, \tilde{g}_e, \tilde{\mathbf{v}}_e, \tilde{\mathbf{w}}_e, \tilde{\mathbf{E}}_e, \tilde{\mathbf{B}}_e, \tilde{n}_p, \tilde{g}_p, \tilde{\mathbf{v}}_p, \tilde{\mathbf{w}}_p, \tilde{\mathbf{E}}_p, \tilde{\mathbf{B}}_p \right\}. \tag{69}$$

The dispersion relation for the equation (69) is

$$\left(\left(i\omega + \varepsilon_{ep} - \mu_{ep}k^2 \right)^2 + s_{ep}^2 k^2 + \omega_{epp}^2 \right)^2 - \omega_{epp}^4 = 0. \tag{70}$$

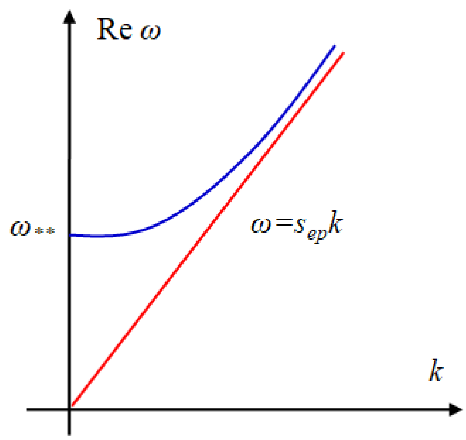

Figure 2: The schematic plots of dispersion curves for electron-positron sound waves.

This equation has two branches of dispersion characteristic

$$\omega = s_{ep}k + i\left(\varepsilon_{ep} + \mu_{ep}k^2\right) \tag{71}$$

and

$$\omega = \sqrt{s_{ep}^2 k^2 + 2\omega_{epp}^2} + i\left(\varepsilon_{ep} + \mu_{ep}k^2\right). \tag{72}$$

If $k = 0$, then we have two meaning of frequency

$$\omega = 0, \tag{73}$$

and

$$\omega_{**} = \sqrt{2}\,\omega_{epp}. \tag{74}$$

The damping parameter is

$$\delta = \varepsilon_{ep} + \mu_{ep}k^2. \tag{75}$$

The schematic plots illustrating dispersion relations (71) and (72) are represented in Fig. 2, where the lower curve corresponds to an ordinary sound wave with linear dispersion law and the upper curve corresponds to plasma sound wave with cutoff frequency ω_{**}.

8 Nonlinear equations with generalized Lorentz force

Let us denote the new operator

$$\hat{\mathbf{D}}_\alpha = i\mathbf{e}_1 \frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta\right) - \mathbf{e}_2 \nabla. \tag{76}$$

Then the generalized sedeonic equation for a two-liquid plasma model, which takes into account the Lorentz force can be represented in the following form:

$$\hat{\mathbf{D}}_\alpha \left(i\mathbf{e}_1 u - i\mathbf{e}_2 \xi + \mathbf{e}_2 \mathbf{v} + \mathbf{e}_1 \mathbf{w} \right) = -\alpha_\alpha \left(1 + \mathbf{e}_3 \frac{\mathbf{v}_\alpha}{s_\alpha} \right) \left(\mathbf{e}_3 \mathbf{E}_\alpha - i\mathbf{B}_\alpha \right). \tag{77}$$

The corresponding sedeonic equation for internal fields has the form

$$\begin{aligned} &\hat{\mathbf{D}}_\alpha \left(\mathbf{e}_3 \mathbf{E}_\alpha - i\mathbf{B}_\alpha \right) = -i\mathbf{e}_1 4\pi e \left(n_i - n_e \right) - \mathbf{e}_2 \frac{4\pi}{s_\alpha} e \left(n_i \mathbf{v}_i - n_e \mathbf{v}_e \right) + \\ &i\mathbf{e}_2 4\pi e \left(g_i - g_e \right) - \mathbf{e}_1 \frac{4\pi}{s_\alpha} e \left(n_i \mathbf{w}_i - n_e \mathbf{w}_e \right). \end{aligned} \tag{78}$$

Equations (77) and (78) are equivalent to the following system:

$$\begin{aligned} &\frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) \mathbf{v}_\alpha + \nabla u_\alpha + \nabla \times \mathbf{w}_\alpha = a_\alpha \mathbf{E}_\alpha + \frac{a_\alpha}{s_\alpha} \mathbf{v}_\alpha \times \mathbf{B}_\alpha, \\ &\frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) u_\alpha + (\nabla \cdot \mathbf{v}_\alpha) = \frac{a_\alpha}{s_\alpha} \left(\mathbf{v}_\alpha \cdot \mathbf{E}_\alpha \right), \\ &\frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) \mathbf{w}_\alpha + \nabla \xi_\alpha - \nabla \times \mathbf{v}_\alpha = a_\alpha \mathbf{B}_\alpha - \frac{a_\alpha}{s_\alpha} \mathbf{v}_\alpha \times \mathbf{E}_\alpha, \\ &\frac{1}{s_\alpha} \left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) \xi_\alpha + (\nabla \cdot \mathbf{w}_\alpha) = -\frac{a_\alpha}{s_\alpha} \left(\mathbf{v}_\alpha \cdot \mathbf{B}_\alpha \right), \end{aligned} \tag{79}$$

and

$$\begin{aligned} &\nabla \cdot \mathbf{E}_\alpha = 4\pi e \left(n_i - n_e \right), \\ &\nabla \cdot \mathbf{B}_\alpha = 4\pi e \left(g_i - g_e \right), \\ &\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) \mathbf{B}_\alpha + s_\alpha \nabla \times \mathbf{E}_\alpha = -4\pi e \left(n_i \mathbf{w}_i - n_e \mathbf{w}_e \right), \\ &\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) + \varepsilon_\alpha - \mu_\alpha \Delta \right) \mathbf{E}_\alpha - s_\alpha \nabla \times \mathbf{B}_\alpha = -4\pi e \left(n_i \mathbf{v}_i - n_e \mathbf{v}_e \right). \end{aligned} \tag{80}$$

Upon linearization, the equations (79) and (80) transform into equations (55) and (56).

The sedeonic equation for the plasma in an external electromagnetic field has the following form:

$$\begin{aligned} &\hat{\mathbf{D}}_\alpha \left(i\mathbf{e}_1 u - i\mathbf{e}_2 \xi + \mathbf{e}_2 \mathbf{v} + \mathbf{e}_1 \mathbf{w} \right) = \\ &-\alpha_\alpha \left(1 + \mathbf{e}_3 \frac{\mathbf{v}_\alpha}{s_\alpha} \right) \left(\mathbf{e}_3 \mathbf{E}_\alpha - i\mathbf{B}_\alpha \right) - \alpha_\alpha \left(1 + \mathbf{e}_3 \frac{\mathbf{v}_\alpha}{c} \right) \left(\mathbf{e}_3 \mathbf{E}_{ex} - i\mathbf{B}_{ex} \right), \end{aligned} \tag{81}$$

where $\mathbf{E}_{ex}$ and $\mathbf{H}_{ex}$ are electric and magnetic field strengths.

9 conclusion

The advantage of sedeons is that they form the associative noncommutative algebra taking into account properties of physical quantities in relation to space-time inversions and enabling correct calculations to be performed simultaneously with different physical values. We used this mathematical toll to formulate the system of self-consistent equations, which describes the vortex plasma within the framework of two-fluid hydrodynamic model. It is shown that the internal electric and magnetic fields generated by fluctuations of the mechanical parameters of the plasma can be taken into account separately for the electronic and ionic components. These fields satisfy modified Maxwell-like equations, which show that the fields are incorporated in plasma and propagate at the speed of sound. Linearized equations form a closed system that describes sound waves, in which values $\tilde{n}_\alpha$ and $\tilde{\mathbf{v}}_\alpha$ describe longitudinal expansion-compression waves and values $\tilde{g}_\alpha$ and $\tilde{\mathbf{w}}_\alpha$ describe vortex-twisting waves. We have shown that the system of hydrodynamic equations for electron-ion plasma motion and equations for internal electromagnetic field can be transformed in fourth-order wave equations, in which the spectrum of eigenwaves has two branches corresponding to the hybridization of electron and ion sound waves. In the case of electron-positron plasma the spectrum of eigenwaves has two branches corresponding to conventional sound waves with linear dispersion low and plasma waves having nonlinear dispersion low with cutoff frequency. Also we show that sedeonic equations can be easily generalized to take into account the damping in plasma. The proposed equations can be potentially applied for description of the ionization and recombination processes in multicomponent plasma.

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