Resume of the serial operators theory.

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September 2021

0- Abstract:

In this paper I want to explain with definitions and examples the 36 possible combinations of conjunction of a serial operator and operation inside it. I am going to remember to the reader the basic theory of the 6 basic serial operators (Summation or Sigma notation, Restory or Rho notation, Productory or Pi notation, Divisory or Delta notation, Exponentory or Theta notation and Divisory or Zeta notation), then I will do the 6 x 6 categories.

1- Introduction:

For this particular paper I am going to exclude interval (c) properties which we have seen before in my other papers, so the interval between one and other state of the function always will be c=1 as it have been normal in function calculus theories. In the other hand we will use Alfa and Omega notation for the equality in the notation but we are not going to see the generalization for all n of this notation, we will only see the first 3 operation cases in positive (A) and negative (Ω) serial operators. Also we introduce the change in the notation denoting f(x) as f(n) to create a correlation between the variable of the operator and the variable of the function inside it. Establishing c=1 we can define summation as:

$$(1.1)\quad \sum_{n=a}^{b} f(n) = \mathop{\mathrm{A}_1}_{n=a}^{b} f(n) = +f(a)+f(a+1)+f(a+2)+...+f(b-2)+f(b-1)+f(b)$$

Starting with the examples we can see a linear function example:

$$(1.1.1)\quad \sum_{n=3}^{6} n = 3+4+5+6 = 18$$

We define productory as:

$$(1.2)\quad \prod_{n=a}^{b} f(n) = \mathop{\mathrm{A}_2}_{n=a}^{b} f(n) = f(a)\cdot f(a+1)\cdot f(a+2)\cdot ...\cdot f(b-2)\cdot f(b-1)\cdot f(b)$$

We can see here a linear example too:

$$(1.2.1)\quad \prod_{n=5}^{8} n = 5\cdot 6\cdot 7\cdot 8 = 1{,}680$$

We define exponentory as:

$$\text{(1.3a)} \quad \underset{n=a}{\overset{b}{\Theta}} f(n) = \underset{n=a}{\overset{b}{\mathrm{A}_3}} f(n) = ((((((f(a))^{f(a+1)})^{f(a+2)})^{\dots})^{f(b-2)})^{f(b-1)})^{f(b)}$$

We can also define exponentory with Knuth's notation:

$$\text{(1.3b)} \quad \underset{n=a}{\overset{b}{\Theta}} f(n) = \underset{n=a}{\overset{b}{\mathrm{A}_3}} f(n) = f(a)\uparrow f(a+1)\uparrow f(a+2)\uparrow \dots \uparrow f(b-2)\uparrow f(b-1)\uparrow f(b)$$

Let's see now a linear example:

$$\text{(1.3.1)} \quad \underset{n=2}{\overset{5}{\Theta}} n = 2\uparrow 3\uparrow 4\uparrow 5 = 1{,}15\ldots\cdot 10^{18}$$

I define restory with the following notation:

$$\text{(1.4)} \quad \underset{n=a}{\overset{b}{\mathrm{P}}} f(n) = \underset{n=a}{\overset{b}{\Omega_1}} f(n) = -f(a)-f(a+1)-f(a+2)-\dots-f(b-2)-f(b-1)-f(b)$$

A numerical example with linear function can be:

$$\text{(1.4.1)} \quad \underset{n=4}{\overset{10}{\mathrm{P}}} n = -4-5-6-7-8-9-10 = -49$$

Also, I can define divisory as:

$$\text{(1.5)} \quad \underset{n=a}{\overset{b}{\Delta}} f(n) = \underset{n=a}{\overset{b}{\Omega_2}} f(n) = f(a)\div f(a+1)\div f(a+2)\div \dots \div f(b-2)\div f(b-1)\div f(b)$$

Lets see a numerical example:

$$\text{(1.5.1)} \quad \underset{n=7}{\overset{11}{\Delta}} n = 7\div 8\div 9\div 10\div 11 = \frac{7}{7920} = 8.83\ldots\cdot 10^{(-4)}$$

Finally, where ANS is the answer of the previous root, I can define rootory as:

$$\text{(1.6)} \quad \underset{n=a}{\overset{b}{\mathrm{Z}}} f(n) = \underset{n=a}{\overset{b}{\Omega_3}} f(n) = \sqrt[f(b)]{ANS}\,\sqrt[f(b-1)]{ANS}\,\sqrt[f(b-2)]{ANS}\,\ddot{\cdot}\,\sqrt{ANS}\,\sqrt[f(a+2)]{ANS}\,\sqrt[f(a+1)]{f(a)}$$

Lets see an example in this case too:

$$\text{(1.6.1)} \quad \underset{n=3}{\overset{6}{\mathrm{Z}}} n = \sqrt[6]{ANS}\,\sqrt[5]{ANS}\,\sqrt[4]{3} = \sqrt[6]{ANS}\,\sqrt[5]{1{,}316\ldots} = \sqrt[6]{1{,}056\ldots} = 1{,}009\ldots$$

2- Summation combinations:

It is relevant to say first, that in every cases k represent a constant for summation and other cases too.

2.1- Summation of sums:

Definition:

$$(2.1)\quad \sum_{n=a}^{b} n+k=+(a+k)+((a+1)+k)+((a+2)+k)+...+((b-2)+k)+((b-1)+k)+(b+k)$$

Example:

$$(2.1.1)\quad \sum_{n=2}^{5} n+3=+(2+3)+(3+3)+(4+3)+(5+3)=5+6+7+8=26$$

2.2- Summation of products:

Definition:

$$(2.2)\quad \sum_{n=a}^{b} n\cdot k=+(a\cdot k)+((a+1)\cdot k)+((a+2)\cdot k)+...+((b-2)\cdot k)+((b-1)\cdot k)+(b\cdot k)$$

Example:

$$(2.2.1)\quad \sum_{n=3}^{5} n\cdot 2=+(3\cdot 2)+(4\cdot 2)+(5\cdot 2)=6+8+10=24$$

2.3- Summation of exponents:

Definition:

$$(2.3)\quad \sum_{n=a}^{b} n^k=+a^k+(a+1)^k+(a+2)^k+...+(b-2)^k+(b-1)^k+b^k$$

Example:

$$(2.3.1)\quad \sum_{n=3}^{6} n^3=+3^3+4^3+5^3+6^3=9+64+125+216=414$$

2.4- Summation of rests:

Definition:

$$(2.4)\quad \sum_{n=a}^{b} n-k=+(a-k)+((a+1)-k)+((a+2)-k)+...+((b-2)-k)+((b-1)-k)+(b-k)$$

Example:

$$(2.4.1)\quad \sum_{n=3}^{8} n-2=+(3-2)+(4-2)+(5-2)+(6-2)+(7-2)+(8-2)=1+2+3+4+5+6=21$$

2.5- Summation of divisions:

Definition:

$$(2.5)\quad \sum_{n=a}^{b} n\div k=+(a\div k)+((a+1)\div k)+((a+2)\div k)+...+((b-2)\div k)+((b-1)\div k)+(b\div k)$$

Example:

$$(2.5.1)\quad \sum_{n=1}^{3} n\div 2=+(1\div 2)+(2\div 2)+(3\div 2)=\frac{6}{2}=3$$

2.6- Summation of roots:

Definition:

$$(2.6)\quad \sum_{n=a}^{b} \sqrt[k]{n}=+\sqrt[k]{a}+\sqrt[k]{a+1}+\sqrt[k]{a+2}+...+\sqrt[k]{b-2}+\sqrt[k]{b-1}+\sqrt[k]{b}$$

Example:

$$(2.6.1)\quad \sum_{n=5}^{8} \sqrt[5]{n}=+\sqrt[5]{5}+\sqrt[5]{6}+\sqrt[5]{7}+\sqrt[5]{8}=5{,}80\ldots$$

3- Productory combinations:

3.1-Productory of sums:

Definition:

$$(3.1)\quad \prod_{n=a}^{b} n+k=(a+k)\cdot((a+1)+k)\cdot((a+2)+k)\cdot\ldots\cdot((b-2)+k)\cdot((b-1)+k)\cdot(b+k)$$

Example:

$$(3.1.1)\quad \prod_{n=2}^{3} n+3=(2+3)\cdot(3+3)=5\cdot 6=30$$

3.2- Productory of products:

Definition:

$$(3.2)\quad \prod_{n=a}^{b} n\cdot k=(a\cdot k)\cdot((a+1)\cdot k)\cdot((a+2)\cdot k)\cdot\ldots\cdot((b-2)\cdot k)\cdot((b-1)\cdot k)\cdot(b\cdot k)$$

Example:

$$(3.2.1)\quad \prod_{n=1}^{4} n\cdot 3=(1\cdot 3)\cdot(2\cdot 3)\cdot(3\cdot 3)\cdot(4\cdot 3)=3\cdot 6\cdot 9\cdot 12=1{,}944$$

3.3- Productory of exponents:

Definition:

$$(3.3)\quad \prod_{n=a}^{b} n^k=a^k\cdot(a+1)^k\cdot(a+2)^k\cdot\ldots\cdot(b-2)^k\cdot(b-1)^k\cdot b^k$$

Example:

$$(3.3.1)\quad \prod_{n=6}^{8} n^3=6^3\cdot 7^3\cdot 8^3=216\cdot 343\cdot 512=37{,}933{,}056$$

3.4- Productory of rests:

Definition:

$$(3.4)\quad \prod_{n=a}^{b} n-k=(a-k)\cdot((a+1)-k)\cdot((a+2)-k)\cdot\ldots\cdot((b-2)-k)\cdot((b-1)-k)\cdot(b-k)$$

Examples:

$$(3.4.1)\quad \prod_{n=3}^{8} n-5=(3-5)\cdot(4-5)\cdot(5-5)\cdot(6-5)\cdot(7-5)\cdot(8-5)=(-2)\cdot(-1)\cdot 0\cdot 1\cdot 2\cdot 3=0$$

$$(3.4.2)\quad \prod_{n=4}^{7} n-3=(4-3)\cdot(5-3)\cdot(6-3)\cdot(7-3)=1\cdot 2\cdot 3\cdot 4=24$$

3.5- Productory of divisions:

Definition:

$$(3.5)\quad \prod_{n=a}^{b} n\div k=(a\div k)\cdot((a+1)\div k)\cdot((a+2)\div k)\cdot\ldots\cdot((b-2)\div k)\cdot((b-1)\div k)\cdot(b\div k)$$

Example:

$$(3.5.1)\quad \prod_{n=2}^{5} n\div 5=(2/5)\cdot(3/5)\cdot(4/5)\cdot(5/5)=24/125=0.192$$

3.6- Productory of roots:

Definition:

$$(3.6)\qquad \prod_{n=a}^{b} \sqrt[k]{n}=\sqrt[k]{a}\cdot\sqrt[k]{a+1}\cdot\sqrt[k]{a+2}\cdot...\cdot\sqrt[k]{b-2}\cdot\sqrt[k]{b-1}\cdot\sqrt[k]{b}$$

Example:

$$(3.6.1)\qquad \prod_{n=3}^{6} \sqrt[7]{n}=\sqrt[7]{3}\cdot\sqrt[7]{4}\cdot\sqrt[7]{5}\cdot\sqrt[7]{6}=2,31\ldots$$

4- Expontentory combinations:

For the examples we are going to use the Knuth up arrow notation.

4.1- Exponentory of sums:

Definitions:

$$(4.1a)\qquad \underset{n=a}{\overset{b}{\Theta}}\, n+k=((((((a+k)^{((a+1)+k)})^{((a+2)+k)})^{\cdots})^{((b-2)+k)})^{((b-1)+k)})^{(b+k)}$$

$$(4.1b)\qquad \underset{n=a}{\overset{b}{\Theta}}\, n+k=(a+k)\uparrow((a+1)+k)\uparrow((a+2)+k)\uparrow...\uparrow((b-2)+k)\uparrow((b-1)+k)\uparrow(b+k)$$

Example:

$$(4.1.1)\qquad \underset{n=3}{\overset{5}{\Theta}}\, n+2=(3+2)\uparrow(4+2)\uparrow(5+2)=5\uparrow 6\uparrow 7=2.27...\cdot 10^{29}$$

4.2- Exponentory of products:

Definitions:

$$(4.2a)\qquad \underset{n=a}{\overset{b}{\Theta}}\, n\cdot k=((((((a\cdot k)^{((a+1)\cdot k)})^{((a+2)\cdot k)})^{\cdots})^{((b-2)\cdot k)})^{((b-1)\cdot k)})^{(b\cdot k)}$$

$$(4.2b)\qquad \underset{n=a}{\overset{b}{\Theta}}\, n\cdot k=(a\cdot k)\uparrow((a+1)\cdot k)\uparrow((a+2)\cdot k)\uparrow...\uparrow((b-2)\cdot k)\uparrow((b-1)\cdot k)\uparrow(b\cdot k)$$

Example:

$$(4.2.1)\qquad \underset{n=1}{\overset{3}{\Theta}}\, n\cdot 3=(1\cdot 3)\uparrow(2\cdot 3)\uparrow(3\cdot 3)=3\uparrow 6\uparrow 9=5.81...\cdot 10^{25}$$

4.3- Exponentory of exponents:

Definitions:

$$(4.3a)\quad \underset{n=a}{\overset{b}{\Theta}} n^k = ((((((a^k)^{((a+1)^k)})^{((a+2)^k)})^{\cdots})^{((b-2)^k)})^{((b-1)^k)})^{(b^k)}$$

$$(4.3b)\quad \underset{n=a}{\overset{b}{\Theta}} n^k = (a^k)\uparrow((a+1)^k)\uparrow((a+2)^k)\uparrow\ldots\uparrow((b-2)^k)\uparrow((b-1)^k)\uparrow(b^k)$$

Example:

$$(4.3.1)\quad \underset{n=3}{\overset{5}{\Theta}} n^2 = (3^2)\uparrow(4^2)\uparrow(5^2) = 9\uparrow16\uparrow25 = 4{,}97\ldots\cdot10^{381}$$

4.4- Exponentory of rests:

Definitions:

$$(4.4a)\quad \underset{n=a}{\overset{b}{\Theta}} n-k = ((((((a-k)^{((a+1)-k)})^{((a+2)-k)})^{\cdots})^{((b-2)-k)})^{((b-1)-k)})^{(b-k)}$$

$$(4.4b)\quad \underset{n=a}{\overset{b}{\Theta}} n-k = (a-k)\uparrow((a+1)-k)\uparrow((a+2)-k)\uparrow\ldots\uparrow((b-2)-k)\uparrow((b-1)-k)\uparrow(b-k)$$

Example:

$$(4.4.1)\quad \underset{n=3}{\overset{4}{\Theta}} n-1 = (3-1)\uparrow(4-1) = 2\uparrow3 = 8$$

4.5- Exponentory of divisions:

Definitions:

$$(4.5a)\quad \underset{n=a}{\overset{b}{\Theta}} n\div k = ((((((a\div k)^{((a+1)\div k)})^{((a+2)\div k)})^{\cdots})^{((b-2)\div k)})^{((b-1)\div k)})^{(b\div k)}$$

$$(4.5b)\quad \underset{n=a}{\overset{b}{\Theta}} n\div k = (a\div k)\uparrow((a+1)\div k)\uparrow((a+2)\div k)\uparrow\ldots\uparrow((b-2)\div k)\uparrow((b-1)\div k)\uparrow(b\div k)$$

Example:

$$(4.5.1)\quad \underset{n=3}{\overset{6}{\Theta}} n\div5 = (3\div5)\uparrow(4\div5)\uparrow(5\div5)\uparrow(6\div5) = 0{,}66\ldots$$

4.6- Exponentory of roots:

Definitions:

$$\text{(4.6a)} \quad \underset{n=a}{\overset{b}{\Theta}} \sqrt[k]{n} = ((((((\sqrt[k]{a})^{\sqrt[k]{a+1}})^{\sqrt[k]{a+2}})^{\cdots})^{\sqrt[k]{b-2}})^{\sqrt[k]{b-1}})^{\sqrt[k]{b}}$$

$$\text{(4.6b)} \quad \underset{n=a}{\overset{b}{\Theta}} \sqrt[k]{n} = (\sqrt[k]{a}) \uparrow (\sqrt[k]{a+1}) \uparrow (\sqrt[k]{a+2}) \uparrow \ldots \uparrow (\sqrt[k]{b-2}) \uparrow (\sqrt[k]{b-1}) \uparrow (\sqrt[k]{b})$$

Example:

$$\text{(4.6.1)} \quad \underset{n=2}{\overset{5}{\Theta}} \sqrt[3]{n} = (\sqrt[3]{2}) \uparrow (\sqrt[3]{3}) \uparrow (\sqrt[3]{4}) \uparrow (\sqrt[3]{5}) = 2.47\ldots$$

5- Restory combinations:

5.1- Restory of sums:

Definition:

$$\text{(5.1)} \quad \underset{n=a}{\overset{b}{\mathrm{P}}} n+k = -(a+k)-((a+1)+k)-((a+2)+k)-\ldots-((b-2)+k)-((b-1)+k)-(b+k)$$

Example:

$$\text{(5.1.1)} \quad \underset{n=3}{\overset{6}{\mathrm{P}}} n+5 = -(3+5)-(4+5)-(5+5)-(6+5) = -8-9-10-11 = -38$$

5.2- Restory of products:

Definition:

$$\text{(5.2)} \quad \underset{n=a}{\overset{b}{\mathrm{P}}} n\cdot k = -(a\cdot k)-((a+1)\cdot k)-((a+2)\cdot k)-\ldots-((b-2)\cdot k)-((b-1)\cdot k)-(b\cdot k)$$

Example:

$$\text{(5.2.1)} \quad \underset{n=1}{\overset{4}{\mathrm{P}}} n\cdot 3 = -(1\cdot 3)-(2\cdot 3)-(3\cdot 3)-(4\cdot 3) = -3-6-9-12 = -30$$

5.3- Restory of exponents:

Definition:

$$\text{(5.3)} \quad \underset{n=a}{\overset{b}{\mathrm{P}}} n^k = -(a^k)-((a+1)^k)-((a+2)^k)-\ldots-((b-2)^k)-((b-1)^k)-(b^k)$$

Example:

(5.3.1)

$$\mathop{\mathrm{P}}_{n=3}^{8} n^4 = -(3^4)-(4^4)-(5^4)-(6^4)-(7^4)-(8^4) = -81-256-625-1296-2401-4096 = -8755$$

5.4- Restory of rests:

Definition:

$$(5.4) \quad \mathop{\mathrm{P}}_{n=a}^{b} n-k = -(a-k)-((a+1)-k)-((a+2)-k)-\ldots-((b-2)-k)-((b-1)-k)-(b-k)$$

Example:

(5.4.1)

$$\mathop{\mathrm{P}}_{n=5}^{8} n-13 = -(5-13)-(6-13)-(7-13)-(8-13) = -(-8)-(-7)-(-6)-(-5) = 8+7+6+5 = 26$$

5.5- Restory of divisions:

Definition:

$$(5.5) \quad \mathop{\mathrm{P}}_{n=a}^{b} n \div k = -(a \div k)-((a+1)\div k)-((a+2)\div k)-\ldots-((b-2)\div k)-((b-1)\div k)-(b \div k)$$

Example:

$$(5.5.1) \quad \mathop{\mathrm{P}}_{n=3}^{5} n \div 20 = -(3 \div 20)-(4 \div 20)-(5 \div 20) = -3/5 = -0.6$$

5.6- Restory of roots:

Definition:

$$(5.6) \quad \mathop{\mathrm{P}}_{n=a}^{b} \sqrt[k]{n} = -\sqrt[k]{a}-\sqrt[k]{a+1}-\sqrt[k]{a+2}-\ldots-\sqrt[k]{b-2}-\sqrt[k]{b-1}-\sqrt[k]{b}$$

Example:

$$(5.6.1) \quad \mathop{\mathrm{P}}_{n=2}^{5} \sqrt[3]{n} = -\sqrt[3]{2}-\sqrt[3]{3}-\sqrt[3]{4}-\sqrt[3]{5} = -5.99\ldots$$

6- Divisory combinations:

6.1- Divisory of sums:

Definition:

(6.1) $$\underset{n=a}{\overset{b}{\Delta}} n+k=(a+k)\div((a+1)+k)\div((a+2)+k)\div\ldots\div((b-2)+k)\div((b-1)+k)\div(b+k)$$

Example:

(6.1.1) $$\underset{n=3}{\overset{6}{\Delta}} n+8=(3+8)\div(4+8)\div(5+8)\div(6+8)=11\div12\div13\div14=11/2{,}184=5.03\ldots\cdot10^{(-3)}$$

6.2- Divisory of products:

Definition:

(6.2) $$\underset{n=a}{\overset{b}{\Delta}} n\cdot k=(a\cdot k)\div((a+1)\cdot k)\div((a+2)\cdot k)\div\ldots\div((b-2)\cdot k)\div((b-1)\cdot k)\div(b\cdot k)$$

Example:

(6.2.1)

$$\underset{n=3}{\overset{7}{\Delta}} n\cdot3=(3\cdot3)\div(4\cdot3)\div(5\cdot3)\div(6\cdot3)\div(7\cdot3)=9\div12\div15\div18\div21=1/7{,}560=1{,}32\ldots\cdot10^{(-4)}$$

6.3- Divisory of exponents:

Definition:

(6.3) $$\underset{n=a}{\overset{b}{\Delta}} n^k=(a^k)\div((a+1)^k)\div((a+2)^k)\div\ldots\div((b-2)^k)\div((b-1)^k)\div(b^k)$$

Example:

(6.3.1) $$\underset{n=1}{\overset{4}{\Delta}} n^3=1^3\div2^3\div3^3\div4^3=1\div8\div27\div64=1/13{,}824=7.23\ldots\cdot10^{(-5)}$$

6.4- Divisory of rests:

Definition:

(6.4) $$\underset{n=a}{\overset{b}{\Delta}} n-k=(a-k)\div((a+1)-k)\div((a+2)-k)\div\ldots\div((b-2)-k)\div((b-1)-k)\div(b-k)$$

Example:

(6.4.1)

$$\underset{n=4}{\overset{8}{\Delta}} n-2=(4-2)\div(5-2)\div(6-2)\div(7-2)\div(8-2)=2\div3\div4\div5\div6=1/180=5.55...\cdot10^{(-3)}$$

6.5- Divisory of divisions:

Definition:

$$(6.5)\quad \underset{n=a}{\overset{b}{\Delta}} n\div k=(a\div k)\div((a+1)\div k)\div((a+2)\div k)\div...\div((b-2)\div k)\div((b-1)\div k)\div(b\div k)$$

Example:

$$(6.5.1)\quad \underset{n=2}{\overset{5}{\Delta}} n\div5=(2\div5)\div(3\div5)\div(4\div5)\div(5\div5)=5/6=0.83....$$

6.6- Divisory of roots:

Definition:

$$(6.6)\quad \underset{n=a}{\overset{b}{\Delta}} \sqrt[k]{n}=\sqrt[k]{a}\div\sqrt[k]{a+1}\div\sqrt[k]{a+2}\div...\div\sqrt[k]{b-2}\div\sqrt[k]{b-1}\div\sqrt[k]{b}$$

Example:

$$(6.6.1)\quad \underset{n=8}{\overset{10}{\Delta}} \sqrt[5]{n}=\sqrt[5]{8}\div\sqrt[5]{9}\div\sqrt[5]{10}=0.61...$$

7- Rootory combinations:

7.1- Rootory of sums:

Definition:

$$(7.1)\quad \underset{n=a}{\overset{b}{Z}} n+k=\sqrt[(b+k)]{ANS}\sqrt[((b-1)+k)]{ANS}\sqrt[((b-2)+k)]{ANS}\because\sqrt{ANS}\sqrt[((a+2)+k)]{ANS}\sqrt[((a+1)+k)]{(a+k)}$$

Example:

$$(7.1.1)\quad \underset{n=5}{\overset{7}{Z}} n+3=\sqrt[7+3]{ANS}\sqrt[6+3]{5+3}=\sqrt[10]{ANS}\sqrt[9]{8}=1.023...$$

7.2- Rootory of products:

Definition:

$$(7.2)\quad \underset{n=a}{\overset{b}{Z}}\; n\cdot k = \sqrt[(b\cdot k)]{ANS}\,\sqrt[((b-1)\cdot k)]{ANS}\,\sqrt[((b-2)\cdot k)]{ANS}\,\sqrt[\cdots]{ANS}\,\sqrt[((a+2)\cdot k)]{ANS}\,\sqrt[((a+1)\cdot k)]{(a\cdot k)}$$

Example:

$$(7.2.1)\quad \underset{n=3}{\overset{6}{Z}}\; n\cdot 5 = \sqrt[6\cdot 5]{ANS}\,\sqrt[5\cdot 5]{ANS}\,\sqrt[4\cdot 5]{3\cdot 5} = \sqrt[30]{ANS}\,\sqrt[25]{ANS}\,\sqrt[20]{15} = 1.00018\ldots$$

7.3- Rootory of exponents:

Definition:

$$(7.3)\quad \underset{n=a}{\overset{b}{Z}}\; n^k = \sqrt[b^k]{ANS}\,\sqrt[(b-1)^k]{ANS}\,\sqrt[(b-2)^k]{ANS}\,\sqrt[\cdots]{ANS}\,\sqrt[(a+2)^k]{ANS}\,\sqrt[(a+1)^k]{a^k}$$

Example:

$$(7.3.1)\quad \underset{n=2}{\overset{4}{Z}}\; n^6 = \sqrt[4^6]{ANS}\,\sqrt[3^6]{2^6} = \sqrt[4{,}096]{ANS}\,\sqrt[729]{64} = 1.0000013\ldots$$

7.4- Rootory of rests:

Definition:

$$(7.4)\quad \underset{n=a}{\overset{b}{Z}}\; n-k = \sqrt[(b-k)]{ANS}\,\sqrt[((b-1)-k)]{ANS}\,\sqrt[((b-2)-k)]{ANS}\,\sqrt[\cdots]{ANS}\,\sqrt[((a+2)-k)]{ANS}\,\sqrt[((a+1)-k)]{(a-k)}$$

Example:

$$(7.4.1)\quad \underset{n=5}{\overset{8}{Z}}\; n-3 = \sqrt[8-3]{ANS}\,\sqrt[7-3]{ANS}\,\sqrt[6-3]{5-3} = \sqrt[5]{ANS}\,\sqrt[4]{ANS}\,\sqrt[3]{2} = 1.0116\ldots$$

7.5- Rootory of divisions:

Definition:

$$(7.5)\quad \underset{n=a}{\overset{b}{Z}}\; n\div k = \sqrt[(b\div k)]{ANS}\,\sqrt[((b-1)\div k)]{ANS}\,\sqrt[((b-2)\div k)]{ANS}\,\sqrt[\cdots]{ANS}\,\sqrt[((a+2)\div k)]{ANS}\,\sqrt[((a+1)\div k)]{(a\div k)}$$

Example:

$$(7.5.1)\quad \underset{n=2}{\overset{5}{Z}}\; n\div k = \sqrt[5\div 2]{ANS}\,\sqrt[4\div 2]{3\div 2} = \sqrt[2.5]{ANS}\,\sqrt[2]{1.5} = 1.084\ldots$$

7.6- Rootory of roots:

Definition:

$$(7.6)\qquad \underset{n=a}{\overset{b}{Z}} \sqrt[k]{n} = \sqrt[\sqrt[k]{b}]{ANS}\,\sqrt[\sqrt[k]{b-1}]{ANS}\,\sqrt[\sqrt[k]{b-2}]{ANS}\,\sqrt[\sqrt[k]{\ldots}]{ANS}\,\sqrt[\sqrt[k]{a+2}]{ANS}\,\sqrt[\sqrt[k]{a+1}]{\sqrt[k]{a}}$$

Example:

$$(7.6.1)\qquad \underset{n=2}{\overset{6}{Z}} \sqrt[3]{n} = \sqrt[\sqrt[3]{6}]{ANS}\,\sqrt[\sqrt[3]{5}]{ANS}\,\sqrt[\sqrt[3]{4}]{ANS}\,\sqrt[\sqrt[3]{3}]{\sqrt[3]{2}} = 1.033\ldots$$

8- Conclusions:

This is an extension of the theory in a global way, although the theory can be more large if we include a 2-variable notation instead of a 1-variable and a constant form. For this moment I think the theory is enough developed, but maybe in the future I will able to do a more extensive serial operators theory.

9- Dedication:

To my brother Honorio, he is a real support many days.