

# A Possible Logical Constraint on the Validity of General Relativity for Strong Gravitational Fields

Wolfgang Oehm

Bonn, Germany

This article is distributed under the Creative Commons by-nc-nd Attribution License.  
Copyright © 2021 Hikari Ltd.

## Abstract

The axiomatic foundation of Einstein's theory of General Relativity is discussed based on an epistemological point of view, yielding a possible logical restriction on the range of validity regarding the strength of gravitational fields. To be precise, the validity of the geodesic equations of motion derived from the Einstein Equivalence Principle is examined in view of Einstein's thought experiment, where an observer situated in a closed elevator cannot distinguish between acceleration and gravitation.

**Keywords:** General Relativity, Einstein Equivalence Principle

## 1 Introduction

Further to encouraging discussions following [10], the author decided for this publication with extended conviction and more details, especially regarding Section 2 (in [10] part of chapter 1). The postulated logical constraint on the validity of General Relativity for strong gravitational fields originates in the axiomatic derivation of the geodesic equations of motions based on the Einstein Equivalence Principle. As a consequence, at least the standard model of the Big Bang is being questioned, because the  $t = 0$  singularity implies the presence of unlimited strong gravitational forces at the beginning of the universe.

The Cosmic Microwave Background (CMB) is *the* essential observation for the establishment of the Big Bang theory in the frame of the cosmological standard model. However, in [11, 12] a new alternative interpretation of the

CMB as radiation by intergalactic dust was established, besides the commonly accepted interpretation of the CMB as a remnant of the Big Bang. Therefore, the questioning of the Big Bang theory for logical reasons does not contradict the observational presence of the CMB.

Furthermore, it is very interesting to note that on the other side, for extremely weak gravitational forces, recently obtained results [7] also question the Einstein Equivalence Principle within the scope of Modified Newtonian Dynamics (MOND, launched by M. Milgrom in 1983 [9]).

## 2 Observational tests

Regarding experimental and observational tests, General Relativity is quantitatively confirmed for weak gravitational fields only, mostly within the Solar system. Observations of black holes and gravitational waves are essentially of phenomenological character without a precise quantitative examination so far. Representative incidences further to the Solar system internal classical tests of General Relativity are:

- (i) PSR B1913+16: A binary pulsar where two neutron stars, each of them with a mass of  $1.4 M_{\odot}$ , are orbiting with a period of 7.75 hours [14]. The derived semi-major axis of about  $2 \cdot 10^6$  km is six orders of magnitudes larger than the Schwarzschild radius of the objects of about 4 km.
- (ii) S2 orbiting Sgr A\*: The star S2 of  $15 M_{\odot}$  is orbiting the massive black hole in the galactic centre with  $4 \cdot 10^6 M_{\odot}$  with a period of 16 years, and the pericentral distance is  $1.8 \cdot 10^{10}$  km [3]. The pericentral distance exceeds the Schwarzschild radius of the massive black hole of  $1.2 \cdot 10^7$  km by three orders of magnitude.
- (iii) GW150914 (first detection of gravitational waves) [1] and GW170817 (in combination with X-Ray observations) [2].
- (iv) Massive black holes in the centre of galaxies [3, 4].
- (v) Big Bang.

For the assessment of the strength of gravitational forces between objects, the comparison of distances between the objects and their Schwarzschild radii obviously is evident. Thus we are dealing with comparably weak gravitational forces in cases (i) and (ii). Cases (iii) and (iv) lack of a precise knowledge of the relevant physical entities. Case (v) lacks of a direct observation of any physical entity prior to the formation of the CMB (according to the standard model more than 300,000 years after the Big Bang).

### 3 Axiomatic foundation of General Relativity

As there are various forms of the equivalence principle circulating, this is the form suggested by [15]:

Weak Equivalence Principle (WEP): *“If an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its mass<sup>1</sup>, internal structure and composition.”*

Einstein Equivalence Principle (EEP): *“(i) WEP is valid, (ii) the outcome of any local non-gravitational test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local non-gravitational test experiment is independent of where and when in the universe it is performed.”*

Here the local non-gravitational test experiment is represented by the local Lorentz frame which is the freely falling reference frame in which the laws of Special Relativity are valid, and the term *local* indicates arbitrarily small spatial extensions.

The world-line of a force-free test particle in a Lorentz frame with coordinates  $\tilde{x}$  and proper time  $\tau$  is given by

$$\frac{d^2 \tilde{x}^a}{d\tau^2} = 0 . \quad (1)$$

The transformation to the reference frame of an observer with coordinates  $x$

$$\frac{d\tilde{x}^a}{d\tau} = \frac{\partial \tilde{x}^a}{\partial x^b} \frac{dx^b}{d\tau} \quad (2)$$

yields

$$\ddot{x}^a + \Gamma_{bc}^a \dot{x}^b \dot{x}^c = 0 , \quad (3)$$

with  $a, b, c = 0, 1, 2, 3$ . The derivative to the proper time  $\tau$  is denoted by the dot, and  $\Gamma_{bc}^a$  is the affine (metric) connection. Furthermore, the metric  $g_{ab}$  of the observer’s reference frame relates to the Minkowski metric  $\eta_{bc}$  via

$$g_{ab} = \frac{\partial \tilde{x}^c}{\partial x^a} \frac{\partial \tilde{x}^d}{\partial x^b} \eta_{cd} . \quad (4)$$

---

<sup>1</sup>The term “mass” added by the author for the sake of clarity. In [15] it is clear from the context.

This straightforward derivation is adopted from [13]. Of course, the transformation of the coordinates is not known from the outset and is associated with the solution of the field equations for the metric tensor  $g_{ab}$ . However, by means of this derivation the essential key aspect of the transition from Special Relativity to General Relativity becomes evident as it is directly related to the EEP.

## 4 Einstein's elevator

The assumption of a local Lorentz frame is based on Einstein's thought experiment, where an observer situated in a closed elevator is not able to distinguish between gravitational forces, and inertial forces due to acceleration. Since gravitational fields are always inhomogeneous, it is clear that the observer would be able to distinguish a gravitational field from an homogeneous acceleration field if he was able to measure with sufficient accuracy, depending on the size of the elevator and the inhomogeneity of the gravitational field (Fig. 1).

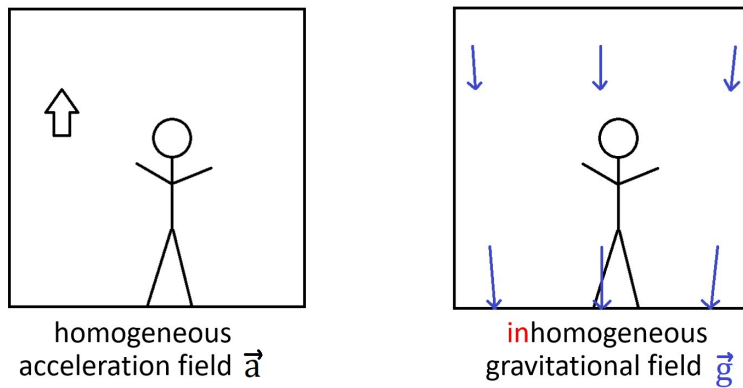


Figure 1: Einstein's elevator.

Commonly accepted is the way out to reduce the size of the elevator to infinitesimal small values, resulting in the local frame with an approximate metric

$$\eta_{ab} + o(\tilde{x}^2) . \quad (5)$$

## 5 *A priori* vs. *a posteriori* relation between theory and accuracy of measurements

Thus the axiomatic mathematical formulation of General Relativity apparently is directly related to the issue of the measurement capabilities of observers. If the observers were able to measure with sufficient accuracy then they would not consider their elevator as inertial (Lorentz) frame, and in principle this stays true down to infinitesimal extensions.

This suggests that the variety of axiomatic formulations in physics be distinguished between those with an *a posteriori* only relation to measurement capabilities and those with an additional *a priori* relation to measurement capabilities in the following sense:

- For an *a posteriori* formulation, only the predictions derived from a mathematical axiom are compared to observations and experimental results. Clearly have the capabilities of measurement an impact on this comparison but the formulation of the mathematical axiom itself is independent of the question of measurement accuracy.
- As already explained, General Relativity represents an *a priori* formulation, because this formulation “stands and falls” with measurement capabilities. This is an additional feature to the *a posteriori* comparison of mathematical predictions to measurement results, which of course also applies here.

The transition from Newtonian Dynamics to Quantum Mechanics is an example for the *a posteriori* case. Bell’s inequality [6] and its experimental examination [8, 5] prove that Quantum Mechanics is not a theory of hidden parameters which are not observed due to missing measurement accuracy.

## 6 Locality, a different view

As mentioned in Section 2, there are no quantitative experiments for strong gravitational fields available. Furthermore, referring to Section 5, the essential issue is the fact that the *a priori* relation to the measurement capabilities of an real existing observer on Earth is completely separated from the objects under investigation (black holes, Big Bang). With pithy words: What is the concern of cosmological objects regarding the measurement capabilities of an observer on Earth? Apparently (iii) of EEP manifests a vague extrapolation to situations which are incompatible to our local situation (Solar system).

In that regard the *a priori* aspect of EEP establishes a fundamental distinction from further *a posteriori* axiomatic formulations of basic physical concepts applied to cosmology.

## 7 Conclusion

Based on a given size of the “elevator” (apparatus) and a certain measurement accuracy, it is clear that there is a limiting ceiling value for the strength of the gravitational field in that sense that beyond this ceiling value the observer would be able to distinguish between acceleration and gravitation. As a logical consequence, this implies an intrinsic restriction on the validity range of General Relativity regarding the strength of gravitational forces. However, due to the locality aspect of Section 6 and the *a priori* feature of EEP in combination with the arbitrariness of a given measurement accuracy, an explicit quantification appears to be impossible.

Nevertheless that there is no quantified value for a limiting ceiling value for the strength of gravitational forces at our disposal, we have the following consequence for the theory of the Big Bang: The singularity at  $t = 0$  with unlimited strong gravitational forces implies that the application of (the current status of) General Relativity to this situation may not be justified.

Regarding the direct observation of massive black holes in galactic centres, however, the situation is principally different. The phenomenological existence of event horizons is not being questioned by the outlined arguments.

Future theoretical work possibly reveals structures beyond General Relativity coping with either situation, the questioning of EEP for weak gravitational forces [7], and the questioning of EEP for strong gravitational forces as discussed in this publication.

**Acknowledgements.** The author would like to thank *M. Křížek*, *P. Kroupa*, *M. Olschowy*, and *V. Vavryčuk* for encouraging discussions and communications.

## References

- [1] B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration): Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.*, **116** (2016), 061102.
- [2] B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration): GW170817: Observation of gravitational waves from a binary neutron star inspiral, *Phys. Rev. Lett.*, **119** (2017), 161101.
- [3] R. Abuter et al. (Gravity Collaboration), Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic centre massive black hole, *Astron. & Astrophys.*, **636** (2020), L5.  
<https://doi.org/10.1051/0004-6361/202037813>

- [4] K. Akiyama et al. (Event Horizon Telescope Collaboration), First M87 Event Horizon Telescope results. I. The shadow of the supermassive black hole, *Astrophys. J. Lett.*, **875** (2019), L1.
- [5] A. Aspect, Bell's inequality test: More ideal than ever, *Nature*, **398** (1999), 189–190. <https://doi.org/10.1038/18296>
- [6] J.S. Bell, On the Einstein Podolsky Rosen paradox, *Physics 1*, **3** (1964), 195–200. <https://doi.org/10.1103/physicsphysiquefizika.1.195>
- [7] K.H. Chae et al., Testing the Strong Equivalence Principle: Detection of the external field effect in rotationally supported galaxies, *Astrophys. J.*, **904** (2020), 51. <https://doi.org/10.3847/1538-4357/abbb96>
- [8] S.J. Freedman and J.F. Clauser, Experimental test of local hidden-variable theories, *Phys. Rev. Lett.*, **28** (1972), 938–941. <https://doi.org/10.1103/physrevlett.28.938>
- [9] M. Milgrom, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, *Astrophys. J.*, **270** (1983), 365–370. <https://doi.org/10.1086/161130>
- [10] W. Oehm, A possible constraint on the validity of General Relativity for strong gravitational fields? *Proceedings of the International Conference on Cosmology on Small Scales 2020*, M. Křížek and Y.V. Dumin, eds, Czech Academy of Sciences, Prague, 2020.
- [11] V. Vavryčuk, Universe opacity and EBL, *Mon. Not. R. Astron. Soc.*, **465** (2017), 1532–1542. <https://doi.org/10.1093/mnras/stw2825>
- [12] V. Vavryčuk, Universe opacity and CMB, *Mon. Not. R. Astron. Soc.*, **478** (2018), 283–301. <https://doi.org/10.1093/mnras/sty974>
- [13] S. Weinberg, *Gravitation and Cosmology*, pp. 70–72, Wiley, New York, 1972.
- [14] J.M. Weisberg, D.J. Nice and J.H. Taylor, Timing measurements of the relativistic binary pulsar PSR B1913+16, *Astrophys. J.*, **722** (2010), 1030–1034. <https://doi.org/10.1088/0004-637x/722/2/1030>
- [15] C.M. Will, *Theory and Experiment in Gravitational Physics*, p. 22, Cambridge University Press, Cambridge, 1993.

**Received: January 5, 2021; Published: January 30, 2021**