Differential Coefficients at Corners and Division by Zero Calculus

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Abstract: For a C_1 function $y = f(x)$ except for an isolated point $x = a$ having $f'(a-0)$ and $f'(a+0)$, we shall introduce its natural differential coefficient at the singular point $x = a$. Surprisingly enough, the differential coefficient is given by the division by zero calculus and it will give the gradient of the natural tangential line of the function $y = f(x)$ at the point $x = a$.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, differential coefficient, normal solutions of ordinary differential equations, Uchida's hyper exponential functions, isolated singular point, $1/0 = 0/0 = z/0 = \tan\frac{\pi}{2} = 0$, $y = |x|$.

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1 Introduction and the result

For a C_1 function $y = f(x)$ except for an isolated point $x = a$ having $f'(a-0)$ and $f'(a+0)$, we shall introduce its natural differential coefficient at the singular point $x = a$. Surprisingly enough, the differential coefficient is given by the division by zero calculus and it will give the gradient of the natural tangential line of the function $y = f(x)$ at the point $x = a$.

We obtain the very pleasant proposition

Proposition. *At the point $x = a$, we introduce the definition*

$$f'(a) = \frac{1}{2}\left(f'(a-0) + f'(a+0)\right).$$

Then, $f'(a)$ has the sense of the gradient of the natural tangential line at the point $x = a$ of the function $y = f(x)$ and it is given by the division by zero calculus at the point in the sense that: For the function

$$F(x) = \frac{f'(a-0) + f'(a+0)}{2}\left(|x-a| + (x-a)\right) - f'(a-0)|x-a|,$$

$$F'(a) = f'(a)$$

in the sense of the division by zero calculus.

2 Interpretation and the proof

For the background of Proposition, we shall state the typical examples.

First, for the function $y = |x|$ we obtain in [9]:

From the expression

$$y := \exp\left(\int_1^x \frac{dt}{t}\right) = \exp\left(\log|x|\right) = |x|,$$

$$y' = |x|\frac{1}{x} = \frac{1}{x}y$$

and

$$y'(0) = 0$$

by the division by zero calculus. For the division by zero calculus, see the papers in the references. Then, we see that Proposition is right for this concrete case, perfectly

Next, for some general case of the function

$$y = \frac{1}{m}\left(|x| + x\right),$$

we have

$$y' = \frac{1}{m}\left(\frac{|x|}{x} + 1\right)$$

and

$$y' = \frac{1}{x}y.$$

Then, we obtain

$$y'(0) = \frac{1}{m}.$$

Note that for this concrete case, Proposition is right, completely.

We thus obtain Proposition, directly.

3 Open problems and remarks

For Proposition, how will be a general version for the case of high dimensional surfaces?

Does there exist some good representations and some applications of $F'(a)$?

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