Dirac Equation in Cosmological Inertial Frame

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ABSTRACT

Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame.

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1.Introduction

Dirac equation is in special relativity theory,[8]

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mcI)\psi = 0 \quad ,$$

I is 4×4 unit matrix ,

$$\gamma^{0} = \begin{pmatrix} I' & 0 \\ 0 & -I' \end{pmatrix},\ \gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{i} & 0 \end{pmatrix},\ \sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

I' is 2×2 unit matrix, σ^{i} is Pauli's matrix. (1)

2. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,[2]

Wave function Type A:

$$r \to r\sqrt{\Omega(t_0)} \quad , \quad t \to \frac{t}{\sqrt{\Omega(t_0)}} \ ,$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

$$(i\hbar\sqrt{\Omega(t_0)}\gamma^{0}\partial_{0} + i\hbar\frac{1}{\sqrt{\Omega(t_0)}}\gamma^{i}\partial_{i} - mcI)\phi = 0 \tag{2}$$

If $\overline{\partial}_{\mu}$ is

$$\overline{\partial}_{\mu} = (\sqrt{\Omega(t_0)}\partial_{0}, \frac{1}{\sqrt{\Omega(t_0)}}\partial_{i}) \tag{3}$$

Dirac equation is in cosmological inertial frame,

$$(i\hbar\gamma^{\mu}\overline{\partial}_{\mu} - mcI)\phi = 0 \tag{4}$$

Eq(4) multiply $i\hbar\gamma^{\upsilon}\overline{\partial}_{\upsilon}$, hence

$$(-\hbar^{2}(\gamma^{\mu}\overline{\partial}_{\mu})(\gamma^{\upsilon}\overline{\partial}_{\upsilon}) - i\hbar(\gamma^{\upsilon}\overline{\partial}_{\upsilon})mcI)\phi = 0 \tag{5}$$

In this time,

$$i\hbar\gamma^{\upsilon}\overline{\partial}_{\upsilon}\phi = mcI\phi \tag{6}$$

Hence, Eq(5) is

$$(-\hbar^{2}\gamma^{\mu}\gamma^{\upsilon}\overline{\partial}_{\mu}\overline{\partial}_{\upsilon} - m^{2}c^{2}I)\phi = 0 \tag{7}$$

In this time, matrix γ^{μ} is

$$\frac{1}{2}(\gamma^{\mu}\gamma^{\upsilon}+\gamma^{\upsilon}\gamma^{\mu})=\frac{1}{2}\{\gamma^{\mu},\gamma^{\upsilon}\}=\eta^{\mu\upsilon}I \tag{8}$$

Therefore,[1],[3]

$$\frac{1}{2}(\gamma^{\mu}\gamma^{\upsilon}+\gamma^{\upsilon}\gamma^{\mu})\overline{\partial}_{\mu}\overline{\partial}_{\upsilon}\phi+\frac{m^2c^2}{\hbar^2}I\phi$$

$$=(\eta^{\mu\upsilon}\overline{\partial}_{\mu}\overline{\partial}_{\upsilon}+\frac{m^2c^2}{\hbar^2})I\phi=0 \tag{9}$$

Eq(9) is the matrix equation of Klein-Gordon.

Dirac spinor ϕ is $\phi=(\phi_1,\phi_2,\phi_3,\phi_4)$. ϕ 's hermitian conjugate $\phi^{+}=(\phi^{*}{}_1,\phi^{*}{}_2,\phi^{*}{}_3,\phi^{*}{}_4)$.

Hence, ϕ 's adjoint spinor $\overline{\phi}$ is

$$\overline{\phi}=\phi^{+}\gamma^{0},\ \ \overline{\phi}(i\gamma^{\mu}\overleftarrow{\overline{\partial}}_{\mu}+mcI)=0 \tag{10}$$

Hence, positive probability density j^0 is

$$j^0=\overline{\phi}\gamma^0\phi=\phi^{+}\phi=|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2 \tag{11}$$

3. Conclusion

We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude.

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