Definition and Applications of Anti-factorial.

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August 2021

0- Abstract:

In this paper I want to show a new concept, the anti-factorial. This is the inverse operator of the factorial. I introduce a full (and necessary) new notation for this concept. The main idea is to develop an operator (notated by n¡) that is able of do the inverse form of an expanded number n to a contracted number k and if you do the factorial of k you will end up back at n, that is k!=n.

1- Introduction:

Firstly we are going to introduce some concepts and notations. Factorial of n is the function defined on the set of non-negative integers with value at n equal to the product of the natural numbers from 1 to n. [1] Or what is the same, product from n to 1. In formula:

$$(1)\quad n!=1\cdot 2\cdot \ldots \cdot (n-2)\cdot (n-1)\cdot n=n\cdot (n-1)\cdot (n-2)\cdot \ldots \cdot 2\cdot 1$$

Now we are going to see a review of my first paper “New nomenclature in operators” [2]. In that article I submitted four new type of serial operators (restory, divisory, exponentory and rootory), but for this topic we only need the help of one of them, the divisory. To have a broad vision of the tools that we need lets to a review of the first two in comparative with the classic operators:

- Summation or Sigma notation: Is the serial operator which sums from a number a to a number b a determinate function f(n).

$$(2)\quad \sum_{n=a}^{b} f(n)=+f(a)+f(a+1)+f(a+2)+\ldots+f(b-2)+f(b-1)+f(b)$$

If the function are represented just for n, we have simply:

$$(3)\quad \sum_{n=a}^{b} n=+a+(a+1)+(a+2)+\ldots+(b-2)+(b-1)+b$$

- Restory or Rho notation: Is the serial operator which subtracts from a number a to a number b a determinate function f(n).

$$(4)\quad \underset{n=a}{\overset{b}{\mathrm{P}}} f(n)=-f(a)-f(a+1)-f(a+2)-\ldots-f(b-2)-f(b-1)-f(b)$$

In this case also, we can have the most simple theoretical example:

$$(5) \quad \underset{n=a}{\overset{b}{\mathrm{P}}} n=-a-(a+1)-(a+2)-...-(b-2)-(b-1)-b$$

- Product or Pi notation: Is the serial operator which multiplies from a number a to a number b in a determinate function f(n). If we want, we can see this operator as the evolution of the primitive factorial operator. Pi notation permits us to realize the ordered serial products between two different numbers or functions.

$$(5) \quad \prod_{n=a}^{b} f(n)=f(a)\cdot f(a+1)\cdot f(a+2)\cdot ...\cdot f(b-2)\cdot f(b-1)\cdot f(b)$$

We will see too which is the easiest form of this serial operator:

$$(6) \quad \prod_{n=a}^{b} n=a\cdot(a+1)\cdot(a+2)\cdot ...\cdot(b-2)\cdot(b-1)\cdot b$$

Now we are going to compare it to the factorial:

$$(7) \quad n!=\prod_{m=1}^{n} m=1\cdot 2\cdot ...\cdot(n-2)\cdot(n-1)\cdot n$$

Next step, is just define the inverse function of product, which is obviously division.

- Divisory of Delta notation: Is the serial operator which divides from a number a to a number b in a determinate function f(n).

$$(8) \quad \underset{n=a}{\overset{b}{\Delta}} f(n)=f(a)\div f(a+1)\div f(a+2)\div ...\div f(b-2)\div f(b-1)\div f(b)$$

As we did in the previous operators, we are going to see the basic form of this serial operator with a function n:

$$(9) \quad \underset{n=a}{\overset{b}{\Delta}} n=a\div(a+1)\div(a+2)\div ...\div(b-2)\div(b-1)\div b$$

2- Anti-factorial definition:

The anti-factorial (n¡) is defined as the necessary operation which implies an ordered serial divisions by the numbers [1,2 ..., (k-1), k] of a number n. An anti-factorial will be successful if only if (A) the operations end in the number 1 independently of the number of steps and (B) $k!=n$. In formula:

$$(10) \quad n¡=\underset{m=1}{\overset{k}{\Delta}} n\div m=((((n\div 1)\div 2)\div 3)\div ...\div(k-1))\div k=1$$

Where:
“n”: the number (constant) we want to analyze.
“m”: The variable of positive integers = [1,2, …,(k-1),k]
“k”: the number of the resulting factorial.

In the other hand if the result of any step of the ordered serial divisions we have a non-integer number, we can assure that n is not the result of any factorial number.

3- Examples:

Two cases with successful anti-factorial:

$$(11) \quad 6¡ = \underset{m=1}{\overset{k}{\Delta}} 6 \div m = (((6 \div 1) \div 2) \div 3) = ((6 \div 2) \div 3) = 3 \div 3 = 1 \quad => \quad k! = 3! = 6$$

$$(12) \quad 120¡ = \underset{m=1}{\overset{k}{\Delta}} 120 \div m = ((((120 \div 1) \div 2) \div 3) \div 4) \div 5 = (((120 \div 2) \div 3) \div 4) \div 5 \quad =$$
$$= \quad ((60 \div 3) \div 4) \div 5 = (20 \div 4) \div 5 = 5 \div 5 = 1 \quad => \quad k! = 5! = 120$$

Now two cases where an exact result is not possible:

$$(13) \quad 13¡ = \underset{m=1}{\overset{k}{\Delta}} 13 \div m = (13 \div 1) \div 2 = 13 \div 2 = 6.5 \quad => \quad 6.5 \notin \mathbb{N} \Rightarrow \nexists k$$

$$(14) \quad 140¡ = \underset{m=1}{\overset{k}{\Delta}} 140 \div m = (((140 \div 1) \div 2) \div 3 = (140 \div 2) \div 3) = 70 \div 3 = 23.\hat{3} \quad => \quad 23.\hat{3} \notin \mathbb{N} \Rightarrow \nexists k$$

4- Conclusions:

As we have seen, this is an application of the divisory operator, a powerful mathematical tool. It may have more applications of course, but for the moment this paper confirms the usefulness of the new serial operators.

5- References:

[1] https://encyclopediaofmath.org/wiki/Factorial

[2] Millas Vera, Juan Elias (2020) https://vixra.org/abs/2010.0077