# Deflection of light by kink mass

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The relation of an angle of deflection-mass is given where we replace mass with the mass of kink (anti-kink). The mass of kink (anti-kink), in turn, can be replaced with topological charge and winding number. Because mass is related with the refractive index, the angle of deflection can be formulated in relation with the decomposed form of refractive index.

## I. KINK, TOPOLOGICAL CHARGE AND WINDING NUMBER

A kink is a topological soliton in one-dimensional space, $\phi(x)$[1]. Its energy density, at any given location, does not vanish with time in the long time limit. By definition, the kink is a map[1]

$$\phi : Z_2 \rightarrow Z_2 \tag{1}$$

where $Z_2$ denotes the group of integer with size or modulo 2. In general, $Z_p$ is group of integer with size or modulo $p$. The elements of $Z_p$ are $0, \pm 1, .., \pm(p-1)$. So, the subscript 2 in $Z_2$ of eq.(1) indicates the modulo of the group of integer where the elements or members are 0 and $\pm 1$[2–4].

In the sine-Gordon model[5], the topological charge is given by[6]

$$Q = \frac{\phi_{(x\;=\;+\infty)} - \phi_{(x\;=\;-\infty)}}{2\pi} \tag{2}$$

Refer to Skyrme[7], the field configuration with boundary conditions for the sine-Gordon model[8] is given by

$$\phi(x \;=\; +\infty) - \phi(x \;=\; -\infty) = 2N\pi \tag{3}$$

where $N$ is the winding number.

If we accommodate the idea of Skyrme in eq.(3), by substituting eq.(3) into (2), we obtain

$$Q = \frac{2N\pi}{2\pi} = N \tag{4}$$

It means that the topological charge, $Q$, is equal to the winding number[9], $N$.

Topological charge is related to total energy[10]. The total energy or mass[11] of the kink (anti-kink)[12] in the sine-Gordon model is[6]

$$M = 8|Q| \tag{5}$$

In harmony with the Skyrme's idea[7], if we treat the topological charge, $Q$, is equal to the winding number, $N$, , then eq.(5) becomes

$$M = 8|N| \tag{6}$$

Here, we should remember that the mass of kink (anti-kink), eq.(5) or (6), is the mass which is formulated in the curved space.

## II. THE REFRACTIVE INDEX AND KINK MASS

There exist the relationship between the refractive index and the mass in curved space, as below[13]

$$n(r) = \left\{1 - \frac{2G}{c^2 r}\; M\right\}^{-1} \tag{7}$$

or

$$M = X\left(1 - n^{-1}\right) \tag{8}$$

where $X = c^2 r/2G$.

By substituting eqs.(5), (6), into (7) we obtain the relationship between the refractive index and the mass of the kink (anti-kink) in the sine-Gordon model

$$n(r) = \left\{1 - \frac{2G}{c^2 r}\; (8|Q|)\right\}^{-1} = \left\{1 - \frac{2G}{c^2 r}\; (8|N|)\right\}^{-1} \tag{9}$$

Eq.(9) show that the refractive index can be formulated related to the topological charge and the winding number.

How about the linear refractive index formulation related to the topological charge and the winding number in the sine-Gordon model? In case of the linear optics for sine-Gordon model, we treat the refractive index as the second rank tensor, and because $G$, $c$, $r$, $m$, $\lambda$ are scalars, so the topological charge and the winding number are the second rank tensors. Then, eqs.(9) become[13]

$$n_{\mu\nu} = \left\{1 - \frac{2G}{c^2 r}\; (8|Q^{\mu\nu}|)\right\}^{-1} = \left\{1 - \frac{2G}{c^2 r}\; (8|N^{\mu\nu}|)\right\}^{-1} \tag{10}$$

It means that the mass of kink (anti-kink) in the sine-Gordon model for the linear optics can be related with the topological charge and winding number as

$$M^{\mu\nu} = 8|Q^{\mu\nu}| = 8|N^{\mu\nu}| \tag{11}$$

We see from eqs.(8), (9), (10), (11), then the refractive index-mass of kink (anti-kink) relation can be written explicitly as

$$M^{\mu\nu} = X\left(1 - n_{\mu\nu}^{-1}\right) \tag{12}$$

$$|Q^{\mu\nu}| = \frac{X}{8}\left(1 - n_{\mu\nu}^{-1}\right) \tag{13}$$

$$|N^{\mu\nu}| = \frac{X}{8}\left(1 - n_{\mu\nu}^{-1}\right) \quad (14)$$

where[14]

$$\left|\partial_\nu \left\{\frac{c}{\omega} \arccos\left(A_\mu^{U(1)}\hat{m}^{U(1)} - \frac{1}{g}\hat{m}^{U(1)} \times \partial_\mu \hat{m}^{U(1)}\right) a_\mu^{-1} + ct\right\}\right| = n_{\mu\nu} \quad (15)$$

$n_{\mu\nu}$ is the second rank tensor of refractive index, $A_\mu^{U(1)}$ is *the unrestricted electric (scalar) potential* of $U(1)$ group and $\hat{m}^{U(1)}$ is *the restricted magnetic (vector) potential* of $U(1)$ group.

We see from eqs.(12)-(15), there exist relation between *the unrestricted electric (scalar) potential* of $U(1)$ group, $A_\mu^{U(1)}$, *the restricted magnetic (vector) potential* of $U(1)$ group, $\hat{m}^{U(1)}$, with the mass, $M^{\mu\nu}$, the topological charge, $Q^{\mu\nu}$, and the winding number, $N^{\mu\nu}$, of kink (anti-kink).

## III. DEFLECTION OF LIGHT BY THE KINK MASS

Gravitational lensing is a direct consequence of general relativity. If light passes near an object of massive mass, $M$, at an impact parameter, $D$, (i.e. its shortest distance to the object), the curvature of space-time (due to such the object of the massive mass) will cause light to be deflected by an angle of deflection, $\Phi$, as below[15]

:

$$\Phi = \frac{4G}{c^2 D} M \quad (16)$$

It means that the angle of deflection, $\Phi$, by which light is deflected depends on the impact parameter, $D$, and the massive mass of the object, $M$[16]. The linear mass of the kink (anti-kink) can be expressed in the sine-Gordon model as in the eq.(11).

How about the angle of deflection in case of the linear optics? By substituting eqs.(11) into eq.(16), we obtain the angle of deflection by the linear mass of the kink (anti-kink) in the sine-Gordon model as follows

$$\Phi^{\mu\nu} = VM^{\mu\nu} \quad (17)$$

$$\Phi^{\mu\nu} = V(8|Q^{\mu\nu}|) \quad (18)$$

$$\Phi^{\mu\nu} = V(8|N^{\mu\nu}|) \quad (19)$$

where $V = 4G/c^2D$. Here, $\Phi^{\mu\nu}$ is the second rank tensor (because we treat $G$, $c$, $D$, $m$, $\lambda$ as scalars). It is the consequence of the second rank tensor of the topological charge. *Eqs.(18),(19) show that the angle of the light deflection can be linked to the topological charge and the winding number respectively as the mass of kink (anti-kink).*

Substituting eqs.(12)-(14) into eqs.(17)-(19), we obtain the angle of deflection-the refractive index relation for kink (anti-kink) as below

$$\Phi^{\mu\nu} = \frac{2r}{D}\left(1 - n_{\mu\nu}^{-1}\right) \quad (20)$$

where $n_{\mu\nu}$ is given in eq.(15). We see from eq.(20), because the refractive index is decomposed, it has consequence that the angle of deflection is also decomposed.

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[1] Tanmay Vachaspati, *Kinks and Domain Walls: An Introduction to Classical and Quantum Solitons*, Cambridge University Press, 2015.

[2] Geoff Smith, *Integers modulo N* https://people.bath.ac.uk/masgcs/book1/amplifications/g.pdf

[3] *Group of integers*, https://groupprops.subwiki.org/wiki/Group_of_integers.

[4] Sze-Tsen Hu, *Homotopy Theory*, Academic Press, 1965.

[5] If we compare the sine-Gordon and $\phi^4$ models, the formulation of the topological charge for both models looked different. Does it mean that the formulation of the topological charge in the $\phi^4$ model is more general than the sine-Gordon (e.g. if we take $m = \pi$)? Here, $m$ is a arbitrary normalisation parameter for the topological charge. For the $\phi^4$ model, it is convenient to fix it to 1 so that the topological charge is in units of 1 $(0, \ +1, \ -1)$. For the sine-Gordon model, we can replace $2\pi$ by $m$ so that the topological charge will be in units of $\pi$ (if our fields go from 0 to $\pi$) (Wojtek Zakrzewski, *Private communication.*)

[6] Lidia Stocker, *Kinks*, ETH Zurich, 2018.

[7] T.H.R. Skyrme, *The Origins of Skyrmions*, International Journal of Modern Physics A, Vol. 3, No.12, 2745-2751, 1988.

[8] Actually, Skyrme uses $\alpha(x)$ as a notation for describing a single angle-type field variable instead of $\phi(x)$.

[9] In order to be classically stable, soliton should have energy with special lower bound. The bound usually involves the topological index: $\varepsilon_{\phi \in Q_n} \geq C|N|$, where $C$ is a constant and $N$ is topological index (A.P. Balachandran, G. Marmo, B.S. Skagerstam, A. Stern, *Classical Topology and Quantum States*, World Scientific, 1991, p.103). *$N$ is topological index which is similar with the winding number* (A.P. Balachandran, *Private communication*).

[10] Tanmay Vachaspati, *Private communication.*

[11] Here we use natural unit $c = 1$, so $E = Mc^2$ gives $E = M$.

[12] In this case, the kink (anti-kink) is a solution of the sine-Gordon equation.

[13] Miftachul Hadi, *A refractive index of a kink in curved space* https://osf.io/preprints/inarxiv/x2qtw, 2019.

[14] Miftachul Hadi, *Magnetic symmetry of geometrical optics*, https://vixra.org/abs/2104.0188, 2021 and all references therein.

[15] B. Ryden, *Introduction to Cosmology*, Cambridge University Press, 2017.

[16] Michael Richmond, *Gravitational Lensing Theory* http://spiff.rit.edu/classes/phys240/lectures/grav_lens/grav_lens.html