Refractive index and mass in curved space

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The refractive index-mass relation in curved space is derived using the decomposed form of refractive index. The curvature-mass relation is also showed.

I. GEOMETRICAL OPTICS

The ray propagation equation of a steady monochromatic wave where the frequency is a constant can be derived from Fermat's principle[1]. It gives[1]

$$\frac{1}{R} = \vec{N} \cdot \frac{\vec{\nabla} n}{n} \tag{1}$$

where R is the radius of curvature, $\vec{N}$ is the unit vector along the principal normal, n is the refractive index, a function of coordinates. Eq.(1) shows that the rays are therefore *bent* in the direction of increasing refractive index[1].

The curvature of space in one dimension, $1/R$, in eq.(1) can be generalized for higher dimension. In case of the dimension is more than two, the curvature can be expressed as the Riemann-Christoffel curvature tensor. It is a generalization of the Gauss' curvature of one and two dimensions[2]. The generalised form of eq.(1) is[3,4]

$$R_{\mu\nu\rho\sigma} = g N_\sigma \; \partial_\rho \ln n_{\mu\nu} \tag{2}$$

where $R_{\mu\nu\rho\sigma}$ is the Riemann-Christoffel curvature tensor.

In our previous work on the magnetic symmetry of the geometrical optics, we obtained the result as below[4]

$$\Big|\partial_\nu \left\{ \frac{c}{\omega} \arccos \left(A_\mu^{U(1)} \hat{m}^{U(1)} - \frac{1}{g} \hat{m}^{U(1)} \times \partial_\mu \hat{m}^{U(1)} \right) a_\mu^{-1} + ct \right\} \Big| = n_{\mu\nu} \tag{3}$$

where $n_{\mu\nu}$ is the second rank tensor of refractive index, $A_\mu^{U(1)}$ is *the unrestricted electric (scalar) potential* of $U(1)$ group and $\hat{m}^{U(1)}$ is *the restricted magnetic (vector) potential* of $U(1)$ group.

Eq.(3) shows that the refractive index is decomposed into the unrestricted electric (scalar) potential part and the restricted magnetic (vector) potential part. This decomposition is a consequence of the magnetic symmetry existence for the gauge potential in the geometrical optics[4].

II. THE REFRACTIVE INDEX AND A MASS IN CURVED SPACE

Let us consider the Schwarzschild metric[5,6] and assume that the space is isotropic and spherically symmetric. Then, the line element is[5]

$$\begin{aligned} ds^2 &= g_{00}(r) \; c^2 \; dt^2 - g_{rr}(r) \; dr^2 \\ &= \left(1 - \frac{2GM}{c^2 r}\right) c^2 \; dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 \end{aligned} \tag{4}$$

where r is the spatial coordinate, M is a mass of an object in curved space, G is the gravitational constant.

The world line corresponding to the propagation of light is defined as *null geodesic* as follows

$$ds^2 = 0 \tag{5}$$

Substitute this eq.(5) into (4), we obtain

$$\left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \frac{dr}{dt} = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} c \tag{6}$$

If we substitute $dr/dt = v$ into (6) and rearrange the terms, then we obtain

$$\left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} = \frac{c}{v} \tag{7}$$

where

$$\frac{c}{v} = n(r) \tag{8}$$

So, we have the space dependent refractive index, $n(r)$, related to the mass of an object, M, as below[5,7]

$$n(r) = \left(1 - \frac{2G}{c^2 r} \; M\right)^{-1} \tag{9}$$

How to formulate the space dependent linear the second rank tensor of refractive index related to the mass of an object as expressed in eq.(9)? In order to answer this question, we need to understand the quantities G, c in eq.(9). The simple understanding of G is coming from the Einstein field equation as follows[8–10]

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} \; T_{\mu\nu} \tag{10}$$

We are informed from eq.(10) that the gravitational constant, G, is a scalar (because the speed of light, c, is a scalar).

In the case of linear optics, we take the space dependent refractive index as the second rank tensor[7]. Because of the gravitational constant, G, the speed of light, c, the spatial coordinate (distance), r, are scalars, then eq.(9) can be written as

$$n_{\mu\nu} = \left(1 - \frac{2G}{c^2 r}\, M^{\mu\nu}\right)^{-1} \tag{11}$$

where $M^{\mu\nu}$ is the second rank tensor of mass[11,12].

The second rank tensor of mass can be expressed as

$$M^{\mu\nu} = X\left(1 - n_{\mu\nu}^{-1}\right) \tag{12}$$

where $X = c^2 r/2G$ and $n_{\mu\nu}$ is given in eq.(3).

Substituting eq.(11) into eq.(2), we obtain the curvature and mass relation as below

$$R_{\mu\nu\rho\sigma} = gN_\sigma\ \partial_\rho \ln\left(1 - \frac{2G}{c^2 r}\, M^{\mu\nu}\right)^{-1} \tag{13}$$

where $M^{\mu\nu}$ is given in eq.(12).

III. ACKNOWLEDGMENT

Thank to Professor Yongmin Cho for introducing me the magnetic symmetry. Thank to Juwita Armilia for continuous support, patience and much love. To beloved Aliya Syauqina Hadi for great hope. To Ibunda and Ayah. May Allah bless them with Jannatul Firdaus.

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