Using the methodology of the relativity theory in describing the orbital motion of Mercury

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Abstract

The theory of relativity, the subject of which is the comparison of the results of observations (calculations) carried out in different reference frames, is used in the description of the planets movement in connection with the following aspects. (1) Astronomers observe the motions of the planets in the geocentric coordinate system and then recalculate the elements of motion for the heliocentric system. (2) The motion of the planets relative to the Sun is composite and includes motion in the orbital plane and rotation of the orbital plane (precession). (3) Observers on Earth record the position of the planet that it occupied earlier. The time interval depends on the distance between the Earth and the planet and it changes depending on the change in this distance. These aspects are considered in the article on the example of the Mercury.

This article discusses the use of the methodology of the relativity theory in the study and description of the motion of Mercury. The same approaches are fair for any planet in the solar system, but Mercury stands out among them with the special attention that was given to it in connection with the work of Urbain Le Verrier and Albert Einstein.

In 1859, the French astronomer Urbain Le Verrier reported to the French Academy of Sciences [1] that he had found a discrepancy between the calculated and observed shift in the longitude of Mercury's perihelion over a hundred years, amounting to 38″, and that this discrepancy is difficult to explain by anything other than the presence of a certain celestial body revolving around the Sun inside the orbit of Mercury. The theory of the motion of Mercury developed by U. Le Verrier was published by the Paris Bureau of Longitudes in 1845 [2]. In 1859, he published an updated version of his theory [3], which gives the calculated value of the secular perihelion displacement equal to 527″, which differs from the value found from the results of astronomical observations - 565″. Subsequently, the observed value was refined by Simon Newcomb and amounted to 570″ [4].

Since earlier Le Verrier, based on the analysis of Uranus's revolution around the Sun, predicted the existence of Neptune [5], his hypothesis was not disregarded, but long-term searches for a new planet were unsuccessful [4].

This circumstance became a new reason for attempts to modify Newton's equation of universal gravitation, which also turned out to be unsuccessful [4]. A. Einstein made his contribution to the solution of this problem, who tried to combine the theory of relativity and the theory of gravitation using matrix transformations. In 1915 he published an article in which he argued that the theory of relativity qualitatively and quantitatively explains Le Verrier 's discovered secular rotation of the orbit of Mercury, which is about 45″ per century [6]. Mathematical transformations, the meaning of which is not entirely clear to a person who does not know tensor calculus, led him to the equation:

$$\varepsilon = 2\pi^3 \frac{a^2}{T^2c^2(1-e^2)}, \text{ in which:} \tag{1}$$

ε – perihelion displacement,
T – orbital period,
a – semi-major axis,
e – orbital eccentricity,
c – speed of light.

In conclusion, Einstein writes: the calculation gives the planet Mercurius a perihelion rotation of 43″ per century, while astronomers point to 45±5″ as an inexplicable difference between observations and Newton's theory. In 1920, A. Einstein published a separate brochure devoted to the special and general theory of relativity [7], in which a somewhat different from the original formula is given:

$$\varepsilon = 24\pi^3 \frac{a^2}{T^2c^2(1-e^2)}, \tag{2}$$

in which ε – the deviation of the perihelion displacement angle from the value found according to Newton's theory.

It is not clear how, using any of the two formulas, can obtain the value ε = 43″, especially since the dimension of the result is an angle to the third degree.

In 2019, the work of N.I. Amel’kin was published [8], which claims that he calculated the influence of the planets of the solar system on the Mercury orbit precession in the framework of the limited problem of three bodies: Sun-planet-Mercury. It is shown that the average displacement of the perihelion of the orbit of Mercury, calculated in the framework of the plane limited circular problem, is 556.5 arc seconds per century and coincides with the observed one (570″) with a relative accuracy of 2.5%. It is also shown that in the observed displacement of the Mercury perihelion , in addition to the average, there are oscillatory components with a total amplitude of up to 20″ and periods from several years to several tens of years.

Thus, there is no need to search for other factors influencing the results of astronomical observations. Nevertheless, a number of general questions remain.

U. Le Verrier claimed that his calculations were made in accordance with Newton's law of universal gravitation and Kepler's laws of motion for planets. Firstly, it should be noted that Kepler's laws relate to plane elliptical motion proper and do not affect such an aspect of them as perihelion displacement. Secondly, it should be emphasized that the displacement of the perihelion longitude is the result of two processes: the actual displacement of the perihelion (the argument of perihelion) in the orbital plane and the precession of the angular momentum of the celestial body, which is expressed in the rotation of the nodes line (displacement of the longitude of the ascending node) (see fig. 1).

The precession of angular momentum is associated with the influence of other celestial bodies on the motion of a given celestial body. It can be assumed that it has the same nature as the gyroscope precession, but so far this effect has not found a theoretical description based on Newton's law even for the motion of the Moon around the Earth. In order to calculate the secular change in the angular momentum, it is not enough to find an analytical relationship between the position of celestial bodies and their influence on the angular momentum. It is necessary to carry out the integration over 100 years, taking into account the movement of all participants in the considered system. And if Le Verrier succeeded such an integration with an error of 10% rel., then this is an excellent result. As for the shift of the perihelion argument, this effect is even more complicated. The ellipse along which the body moves in the central field was not drawn by someone and, in the general case, it may not be closed even in a system of two bodies. The influence of third bodies added to this. Therefore, Le Verrier, claiming that he gave a description of the change in the orbital elements of Mercury on the basis of the fundamental laws of physics, exaggerated a little. Today, a century and a half after the publication of his work, the displacement of the planets perihelion longitudes, as well as the change in other orbital parameters, are described using stochastic models, which are an expansion in a series in time. The longitude of the Mercury perihelion in the epoch J2000 can be found by the equation [9]:

$$\begin{aligned} L = 77^\circ.45611904 + 5719''.11590t - 4''.83016t^2 - 0''.02464t^3 - \\ - 0''.00016t^4 + 0''.00004t^5, \end{aligned} \tag{3}$$

In which t is the time measured in thousands of years from J2000 (JD 2451545.0).

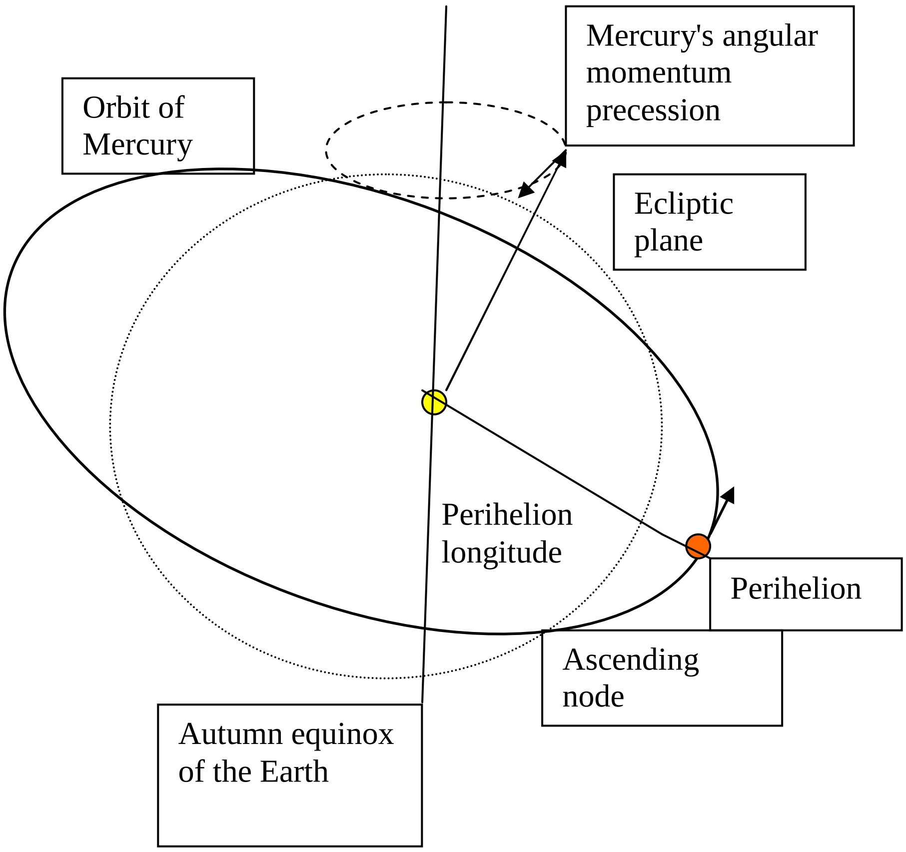

Figure 1. Orbit of Mercury.

According to this equation, the secular displacement of the longitude of the Mercury perihelion is 572″. Ascending node precession calculated using a similar equation for 100 years:

$$\Omega = 48°.33089304 - 4515''.21727t - 31''.79892t^2 - 0''.71933t^3 + 0''.01242t^4 \quad (4)$$

equals -452″. Hence, the displacement of the perihelion argument is 1023 ″ per hundred years.

But back to the work of A. Einstein. Another remark is that Einstein combined the theory of relativity and the theory of gravity. The subject of the theory of relativity is the comparison of the results of measurements (calculations) performed in different frames. The selection of a suitable frame of reference allows us to reveal the patterns that are hidden from us in other frames, an example of which is the discovery of the Coriolis effect. But the unification of the theory of relativity and the theory of gravity is fundamentally wrong.

While the geocentric model of the motion of celestial bodies was used in astronomy, the concept of "perihelion" did not exist. Astronomers observe the movements of the planets in the sky from the Earth. Then they recalculate the observation results from the geocentric coordinate system to the heliocentric one, which, by the way, introduces its own error.

If we use the terminology of theoretical mechanics, then the motion of the planets along the celestial vault will be relative, and their motion relative to the Sun is absolute. As we can see from Fig. 1, the motion relative to the Sun can also be divided into two types: the motion of the body in the orbital plane and the rotation of the orbital plane.

The next circumstance is related to the relativity of the observation results. Observing the planet from the Earth, we fix its position, which it occupied at the moment of time, which differs from the observation time by the value:

$\Delta t = \Delta l / c$, in which Δl – is the distance between the planet and the Earth. (5)

Those, when translating the coordinates of Mercury from geocentric to heliocentric, we must take into account that the scale of the Mercury time differs from the scale of the earth's time by the value Δt, which is a function of Δl.

Equation (5) is valid when the object and the observer are motionless relative to each other, but if they are moving, the problem becomes more complicated. Suppose we have two identical clocks, which move relative to each other with a speed v. Two impulses were sent from the moving clock through the time interval $\Delta t'$ towards the stationary clock. The time interval between pulses Δt, which will be fixed by a stationary clock, will be:

$$\Delta t = t_2 - t_1 . \quad (6)$$

Since the time of arrival of the first pulse $t_1 = \Delta l / c$, and the time of arrival of the second pulse $t_2 = \Delta t' + (\Delta l + v\Delta t')/c$, then:

$$\Delta t = t_2 - t_1 = \Delta t' + (\Delta l + v\Delta t')/c - \Delta l/c = \Delta t'(1 + v/c). \quad (7)$$

In this way, the time between events, information about which comes with a limited speed from the receding object, will be perceived by us as greater than it was on the object itself. For an approaching object, the perceived time interval will be shorter.

Let's try, now, to illustrate this on some real object. For example, let's observe the motion of Mercury around the Sun (see Fig. 2). On November 28, 1964, the orbits of the Earth and Mercury had the following parameters (see Table 1).

At 11:00:00, the distance between the planets was $1.583 \cdot 10^{11}$ m, so the light traveled this path in 528.15 s. This means that the position of Mercury, recorded on Earth at 11:00:00, corresponded to the position of Mercury, which it occupied 528.15 s earlier. By 12:00:00 the distance between the planets was reduced to $1.580 \cdot 10^{11}$ m and the time spent by light to cover the distance between the planets decreased by 1.07 s. This means that the Mercury hour recorded on the Earth has become less than the Earth hour and the real Mercury hour by 1.07 s. During this time (1.07 s), Mercury is displaced relative to the Sun by 0.18 arc seconds, which gives an error (if this effect is not taken into account) in the description of the orbit of Mercury $3 \cdot 10^{-4}$.

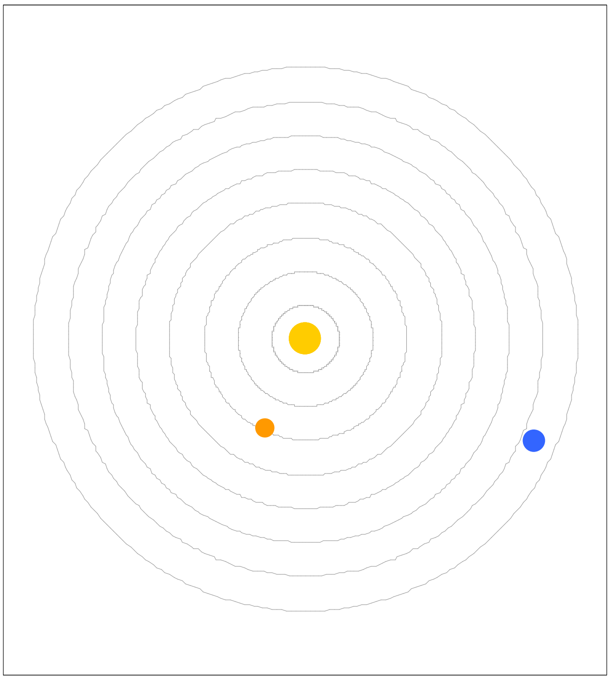

Figure 2. Mutual arrangement of the Earth, Sun and Mercury 11/28/1964.

Thus, by recalculating from a geocentric to a heliocentric coordinate system, it is necessary to take into account the mutual movement of the planets.

Table 1.

Parameters of the Earth and Mercury orbits on 11/28/1964[1]

Time	Longitude, grad	Latitude, grad	Radius, $m \cdot 10^{11}$	Angular velocity, radian/s	Radial velocity, m/s
Earth					
11:00:00	66.264402	0.000003	1.4776	0.017681	-282
12:00:00	66.306687	0.000003	1.4756	0.017681	-282
Mercury					
11:00:00	336.546356	-6.6406	0.57618	0.071031	-9891
12:00:00	336.716066	-6.6343	0.57582	0.071116	-9896

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