Bell's Spaceship Paradox from the Perspective of Length Expansion

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Many researchers argue that strings break when discussing Bell's spaceship paradox. This is described under the premise that the length contraction is correct. However, I studied the spaceship parable from the point of view of length expansion, not length contraction. I examined whether there were logical contradictions among observers, and I also examined the relationship between the basic rule of relativity and causality.

I. Introduction

Bell's Paradox makes us think a lot about the concept of length, the existence of a coordinate system, and causality. If we look deeply into this paradox, we may need to revise the concepts we know. If we cannot solve this problem using existing theories and rational thinking, we may need to modify some concepts. Dewan and Beran differentiated the concept of length, and Maudlin interpreted the coordinate system differently [1, 2]. And many researchers argue that the string should be broken as the result of this thought experiment. If this is correct, then causality should be abandoned. This problem goes beyond a simple thought experiment and challenges important concepts that have been built up by mankind. However, there is a way to preserve the theory of relativity and causality and to not damage the concepts of both length and coordinate systems. It is to interpret this paradox as length expansion, not length contraction.

II. Opinions on the Spaceship Paradox

Dewan and Beran, Bell, Petkov, Maudlin, Franklin, and other researchers have studied this paradox [3, 4, 5]. Two identical spaceships A and B on board with observers A and B are stationary. A thin string is hung between the two spaceships A and B. Discussing what this string would look like if the two spaceships were moving at relativistic speeds is central to Bell's spaceship paradox. If the spaceship moves at a relativistic speed, the spaceship will shrink under the influence of the Lorentz contraction, and the string will break. The distance between the two spaceships was initially assumed to be constant, but if they move at a relativistic speed, the distance between them increases even more. Petkov and Franklin derived it, and it is shown in the space-time diagram as follows (1) ~ (3) [4, 5]. Point E is the left end of the string and point F is the right end of the string. Points P and Q are the corresponding points in the moving system corresponding to point E and F.

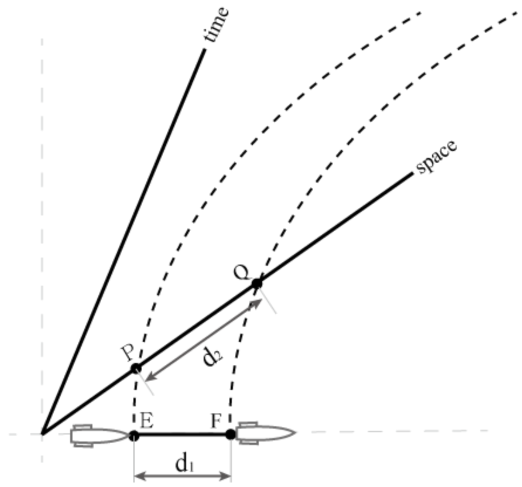

Fig. 1. Space expansion

$$x_P = \gamma(x_E - vt) \tag{1}$$

$$x_Q = \gamma(x_F + d - vt) \tag{2}$$

The distance between the two spaceships in the moving system is given by

$$d_2 = x_Q - x_P = \gamma d_1 \qquad \therefore d_2 = \gamma d_1 \tag{3}$$

If this is written in general symbols, it is as follows (L_o is the proper length, L is the observed length, γ is the Lorentz factor)

$$\therefore\ L = \gamma L_o \tag{4}$$

In Figure 1, the distance between points E and F is the proper distance, and the distance between points P and Q is the observed distance to the other party moving at relativistic speed. When moving at relativistic speed, time and length are expressed in oblique coordinates and the distance between two points on the spatial axis $\overline{PQ}$ is naturally longer than the proper distance $\overline{EF}$. This is natural when referring to the rules of the Minkowski space-time diagram. Dewan and Beran, who understood this from the beginning, divided the length of the spaceship and the distance between the two spaceships as follows [1].

(a) *The distance between two ends of a connected rod,*

(b) *The distance between two objects which are not connected but each of which independently and simultaneously moves with the same velocity for an inertial frame.*

Summing up the arguments of several researchers, when a relativistic speed change occurs, the length of the rigid body contracts, and the length of the two points in space rather expands. The space expansion is an unfamiliar conception and there is no name for it yet. Let's temporarily call this '*space expansion*'. Now let's add one spaceship. Spaceship C (observer C) is next to spaceships A and B. Spaceships A and B are at rest, and spaceship C is moving fast.[5]

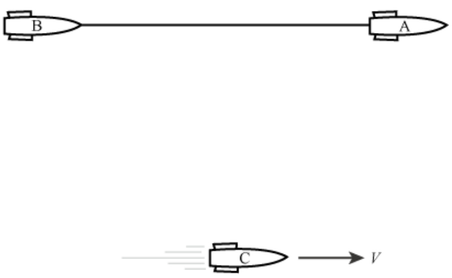

Fig 2. Destruction of causality

Then something strange happens. From the perspective of spaceship C, of course, it would be judged that the string between spaceship A and B was broken, but observers A and B did not. They (A, B) were standing still, but the string was broken. There is no cause, but there is an effect, which is a violation of causality. Maudlin would not have been unaware of these results. So, I think he created the concept of a slightly unusual coordinate system. Maudlin distinguished two types of length contraction. He argued that *coordinate-based Lorentz-FitzGerald contraction* and *physical Lorentz-FitzGerald contraction* are different [2]. This avoids the destruction of causality. These are the overall outlines of the bell's spaceship paradox being discussed so far. It is a very bizarre and difficult paradox. The important problems to solve this paradox are as follows.

1. *Two physical phenomena occur for one object*

2. *Relativity or causality, a matter of choice*

3. *Whether the observers agree on the braking of the string*

First, the fact that two physical phenomena occur for one object causes a very serious problem. Dewan and Beran divided the length into two, and we accept that Lorentz contraction is applied to general objects, and space expansion occurs in the distance between two points.

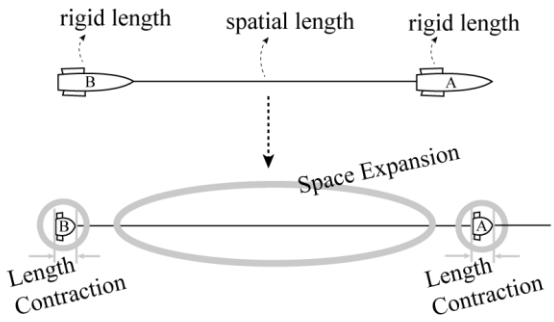

Fig. 3. Two physical effects on one object

The two spaceships and the string are bound together, so they are effectively one object. Then, different physical phenomena such as Lorentz contraction and space expansion are applied to a single object. This is strange. If we apply a slightly stricter standard to matter, all objects in the universe are connected by space. We are aware that the interior of atoms and molecules is mostly space. Then, why does Lorentz contraction occur without space expansion for atoms or molecules? We should be able to answer the question about this. There are currently no standards for this. Second, most researchers conclude that the string breaks, which results in ignoring causality. Looking back at Figure 2, they were just stationary, but observer C moves, so the string between observers A and B is broken. This is nonsense. It is a strange conclusion that cannot be accepted. Third, all observers must observe the same result. Whether the string is broken or not, only one of the two is true. Probabilistic interpretations or interpretations of multiple worlds do not exist in relativity.

III. Length Contraction Theory and Length Expansion Theory

It is not easy to logically solve these three difficult problems. However, this does not mean that there is no solution. If we discard the length contraction theory and choose the length expansion theory, this problem is solved. As we all know, length contraction is not a theory based purely on relativity. There are many problems in the process of origin and theory formation. Importantly, when length contraction is involved, inconsistencies occur most of the time. The way to solve this problem is to find the correct relativistic length and reinterpret the problem with it.

Recently, there has been a lot of discussion about the correct length for relativistic judgment. The problem of length contraction has been pointed out by many people. Strel'tsov pointed out the problem of length contraction by taking the concept of radar length [6], and Kwak insisted that the correct relativistic length is not length contraction, but the opposite length expansion [7]. Buenker insisted that length expansion, not length contraction, was found in GPS [8]. And Sato argued that if the length contraction was correct, GPS would not work [9]. In addition, Ashby said that they found the effect of time dilation in GPS, but he passed over the effect of length contraction. I think because he couldn't find any length contraction effect in GPS [10]. Some argue for partial length expansion [11]. Given the opinions of these various authors, it is reasonable to suspect that there is a problem with the relativistic length as we know it. First, we can simply prove the length expansion from the time dilation.

$$t = \gamma t_o \qquad \textit{time dilation} \tag{5}$$

$$ct = \gamma c t_o \qquad \textit{multiply both sides by } c \tag{6}$$

$$l = \gamma l_o \qquad \textit{length expansion} \tag{7}$$

The length expansion can also be proved simply by the constancy of the speed of light. Since the speed of light must always be constant regardless of the light source and the observer, the speed of light is always constant c in both a stationary system and a moving system. Therefore, the following relationship holds.

$$c = \frac{l}{t} = \frac{\gamma l_o}{\gamma t_o} = \frac{l_o}{t_o} = c \tag{8}$$

If so, the relationship below is also correct.

$$\therefore\ l = \gamma l_o \tag{9}$$

If the constancy of the speed of light is correct, the length expansion must be correct. If the length contraction is correct, the constancy of the speed of light is broken.

$$c = \frac{l}{t} = \frac{(1/\gamma) l_o}{\gamma t_o} = \frac{1}{\gamma^2}\frac{l_o}{t_o} = \frac{1}{\gamma^2} c \neq c \tag{10}$$

In fact, the expression $l = \gamma l_o$ which is the space expansion derived by Petkov and Franklin, is not an accidental expression, but a length expansion expression that I claim to be correct [4, 5]. Length is not divided into two concepts. Dividing the length of a rigid body and the length of space is not reasonable considering the properties of atoms and molecules. The interior of a rigid body is almost empty. This is very natural if we look at the properties of atoms and molecules. Now, let us see how the spaceship paradox is solved by introducing the length expansion theory.

IV. Solving the Paradox

The relativistic effect is a phenomenon that occurs when observing fast-moving systems. Therefore, all fast-moving objects need to apply the length expansion effect. If the length expansion theory is correct, the length of the spaceship increases, and the string increases at the same rate as the length of the spaceship increases. Therefore, the string is not under tension. Since the string is not under tension, of course, the string will not break. This can be expressed in the space-time diagram as follows.

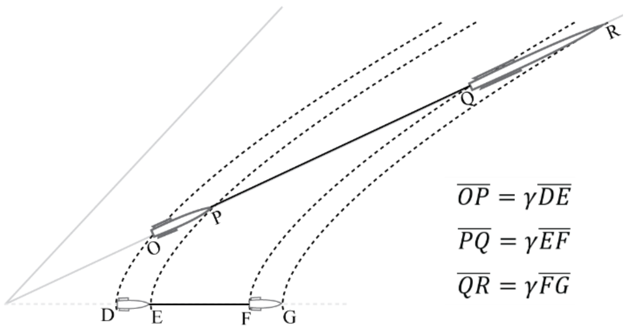

Fig. 4. Length expansion without tension on the string

Now let us look at the opposite case. There are two stationary spaceships A and B, with an observer C moving rapidly against them. To observer C, it will appear that spaceships A and B are moving. As stated above, spaceships A, B, and the string are stretched at the same rate, so the string is not in tension. Therefore, the string between the two spaceships A and B is not broken. What happens when observers A and B observe themselves? Since they stood still, of course, the string did not break. In any case, the string does not break. Since the observed results of A, B, and C are consistent, the causality is also not affected. The figure below compares when length contraction is applied and when length expansion is applied to the spaceship paradox.

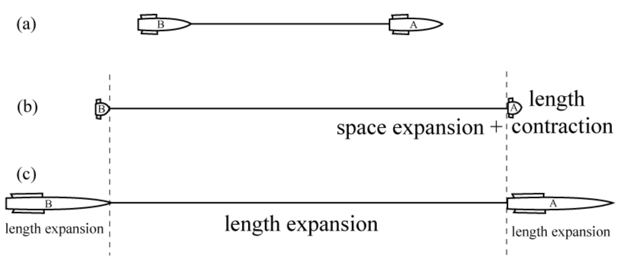

Fig. 5. Differences in the interpretation of length contraction and length expansion for the spaceship paradox.
(a) Non-relativistic interpretation
(b) Length contraction:
two physical phenomena appear in one object
(c) Length expansion:
one physical phenomenon occurs in one object,
No object is affected by tension.

This will solve all the problems we were worried about. Everyone's observations are consistent with each other. There is no need to divide the concept of length into two, and there is no need to interpret the physical phenomenon of the coordinate system differently. And there is no case to hurt causality. There is no logical contradiction or paradox. Now, the paradox is neatly solved. Therefore, the correct length of relativity should be length expansion, not length contraction.

VI. Conclusion

If the length contraction theory is correct, then we have to choose between the theory of relativity and causality. However, the theory of relativity is correct, and the causality is also correct. However, the length contraction theory is not correct. If we interpret this paradox with the length expansion instead of the length contraction, we can keep the theory of relativity and the causality. The spaceship paradox doesn't even happen in the first place.

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