Phase transition in Econophysics

Domenico Oricchio

August 6, 2021

Abstract

Thermodynamics applied to economy: phase and critical exponent. Liquid and solid economic phase with critical points.

It is possible to write the collective behavior of a large number of economic agents (customers, shareholders, small banks, etc.) who interact with each other according to physical laws that describe the interaction in the short and long range.

A statistical description of the agents (portfolio average, etc) allows to describe the dynamics of the system (like a thermodynamic system).

I am thinking that the temporal dynamics of an economic system could be modeled by an economics potential[1]:

$$\Phi(t, p, q)$$

and, that there is a economic dynamic[2]:

$$0 = a_1 \partial_t \Phi + a_2 \partial_p \Phi + a_3 \partial_q \Phi + a_4 \partial_{qt} \Phi + \cdots$$

when the system is in equilibrium

$$0 = a_2 \partial_p \Phi + a_3 \partial_q \Phi + \cdots$$

so that the economic system has an equilibrium point, phase transition and critical exponent (like in the Landau's phase transition).

These equilibrium point are solution of the dynamics, when the system

[1] each extensive quantity

[2] if $\Phi(t, q, p)$ is an analytic function in the variables then exist the differential equation

does not oscillate[3] , or when the initial condition[4] is $0 = \partial_t \Phi = \partial_{qt} \Phi = \cdots$

When the extensive quantity are near a critical point, then the equilibrium points change a lot with small perturbations, and so the trajectories change a lot with small pertubations: the economic system is unstable in the temporal evolution, and the economic system is near a critical point.

There are critical points if there is a linear approximation of the economics potential[5] in an order parameter and some intensive variable[6].

A possible differential equation[7] for a critical point $\Phi = \beta(T - T_c)^\gamma$ is $0 = \gamma \Phi \partial_{TT} \Phi - (\gamma - 1) \partial_T \Phi \partial_T \Phi$, that is $0 = \Phi\, \Phi_{TT} - \alpha\, \Phi_T\, \Phi_T$ where $\alpha = \frac{\gamma - 1}{\gamma}$; if there is a Lagrangian for the economic system, then it is possible to write an analogy with other real system (in chemistry, biology and physics).

Perhaps it might be possible to model thermodynamic systems that vary over time: thermodynamic oscillations or irreversible transformations.

If the Lagrangian[8] is

$$\mathcal{L} = \Phi^\alpha \partial_T \Phi \partial_T \Phi$$

the Euler-Lagrange equation is

$$0 = \alpha \Phi^{\alpha - 1} \partial_T \Phi \partial_T \Phi - \partial_T (2 \Phi^\alpha \partial_T \Phi) = -\alpha \Phi^{\alpha - 1} \partial_T \Phi \partial_T \Phi - 2 \Phi^\alpha \partial_{TT} \Phi$$

or

$$0 = \frac{\alpha}{2} \partial_T \Phi \partial_T \Phi + \Phi \partial_{TT} \Phi$$

there is another solution

$$\Phi^{\alpha - 1} = 0$$

so that a phase transition happen when the Lagrangian can be approximated[9] with the Lagrangian $\mathcal{L} = \Phi^\alpha \partial_T \Phi \partial_T \Phi$

---

[3]due to dispersion

[4]a constant economic potential

[5]with a change in the interaction between economic agents

[6]Statistical physics - Landau Lifshitz

[7]zeroing the products of the derivatives with the suitable polynomial

[8]this is a mass variable Lagrangian: $\mathcal{L} = \frac{1}{2} m(\Phi^\alpha) \partial_T \Phi \partial_T \Phi$

[9]in an interval