Radii of Electron, Proton and Neutron

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

In our previous papers, we once gave formulas and value of the classical electron radius (r_e=2.81794032658(43) fm) and the proton charge radius. In this paper, we give more reasonable forms of them for the proton charge radius. The values of the proton charge radius should be different according to the different measurement methods, so we give three values of it, i.e., $r_{p/H}$=0.8330977868 fm, $r_{p/H\text{-}\mu}$=0.8419605292 fm and $r_{p/e}$=0.8311047299 fm corresponding to the three measurement methods. In addition, we also give formula and value of the neutron charge radius (r_n=0.3312876729 fm).

Keywords: radius; electron; proton; neutron.

1. Introduction

In our previous papers[1-10], we gave formulas of the fine structure constant and their applications or relevant developments. Some typical formulas and definitions of the fine-structure constant are as follows.

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0}, \quad \alpha_2 = \frac{2\pi r_e}{\lambda_e}, \quad \alpha_c = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{v_e}{c}$$

$$\alpha_1 = \frac{36}{7\cdot(2\pi)_{Chen-112}} \frac{1}{112+\dfrac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13\cdot(2\pi)_{Chen-278}}{100} \frac{1}{112-\dfrac{1}{64\cdot 3\cdot 29}} = 1/137.035999111818$$

$$2\pi - e \text{ formula: } (2\pi)_{Chen\text{-}k} = (\frac{e}{e^{\gamma_{c-k}}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{\frac{2\pi a_0}{\lambda_e} \frac{\lambda_e}{2\pi r_e}} = \sqrt{\frac{a_0}{r_e}}$$
$$= \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$
$$= \sqrt{137.035999037435 \times 137.035999111818} = 137.035999074626$$

With these definitions and formulas, we calculated the classical electron radius r_e as follows[1].

$$\frac{a_0}{r_e} = \frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = 112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)}) = 18788.865042381$$
$$r_e = \alpha_c^2 a_0 = \alpha_1 \alpha_2 a_0 = \frac{5.29177210903(80) \times 10^{-11} m}{18788.865042381} = 2.81794032658(43)\ fm$$

The above calculated r_e=2.81794032658(43) was more precise (with a more effective digit) than current CODATA recommended value r_e=2.8179403262(13) fm.

In the same paper[1], we also proposed the similar formulas for the proton charge radius r_p as follows.

$$\frac{a_0}{r_p} = \frac{1}{\alpha_{p/c}^2} = \frac{1}{\alpha_{p/1} \alpha_{p/2}} = 225 \cdot (282 + \frac{1}{3} - \frac{1}{12 \cdot 47} + \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47}) = 63524.60147736$$
$$= 247 \cdot (257 + \frac{1}{5} - \frac{1}{5 \cdot 13} + \frac{1}{30 \cdot (28 \cdot (2 \cdot 100 - 1) + 1) + \frac{8}{45}})$$
$$= (252 + \frac{1}{24} - \frac{1}{2 \cdot 17 \cdot 37} + \frac{1}{11 \cdot 13 \cdot 19 \cdot (2 \cdot 11 \cdot 19 + 1) + \frac{11}{20}})^2 = 252.040872632515^2$$
$$r_p = \alpha_{p/c}^2 a_0 = \alpha_{p/1} \alpha_{p/2} a_0 = \frac{5.29177210903(80) \times 10^{-11}\ m}{63524.60147736} = 0.833027202999(13)\ fm$$

$\alpha_{p/c} \approx \alpha_{p/1} \approx \alpha_{p/2} \approx 252.04$, α_p could be called the second fine-structure constant.
2019/12/19-23

However, the above formulas foe r_p should not be satisfyingly correct and need some corrections. In this paper, we try to find more correct formulas and more precise values of r_p. In addition, we also give a formula and a value of the neutron charge radius.

2. The Proton-radius Puzzle

Scientists developed two approaches to measure the proton charge radius, one is hydrogen spectroscopy (Lamb shift measurement), the other is electron-proton scattering (p/e method). And the first approach was divided into two sub-branches which are ordinary-hydrogen spectroscopy (p/H method) and muonic hydrogen spectroscopy (p/H-μ method). There were always discrepancies among the results of the measurements with these three methods as shown in the following table and figure (**Table 1** and **Fig. 1**). This strange phenomenon is called the proton-radius puzzle.

Table 1. Values for the proton radius measured by the three methods

Year	Method	r_p	Value	Reference
2010	Electronic-proton scattering	$r_{p/e}$	0.879(8)	11
2010	Muonic-hydrogen spectroscopy	$r_{p/H\text{-}\mu}$	0.84184(67)	12
2013	Muonic-hydrogen spectroscopy	$r_{p/H\text{-}\mu}$	0.84087(39)	13
2017	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.8335(95)	14
2018	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.877(13)	15
2019	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.833(10)	16
2019	Electronic-proton scattering	$r_{p/e}$	0.831(14)	17

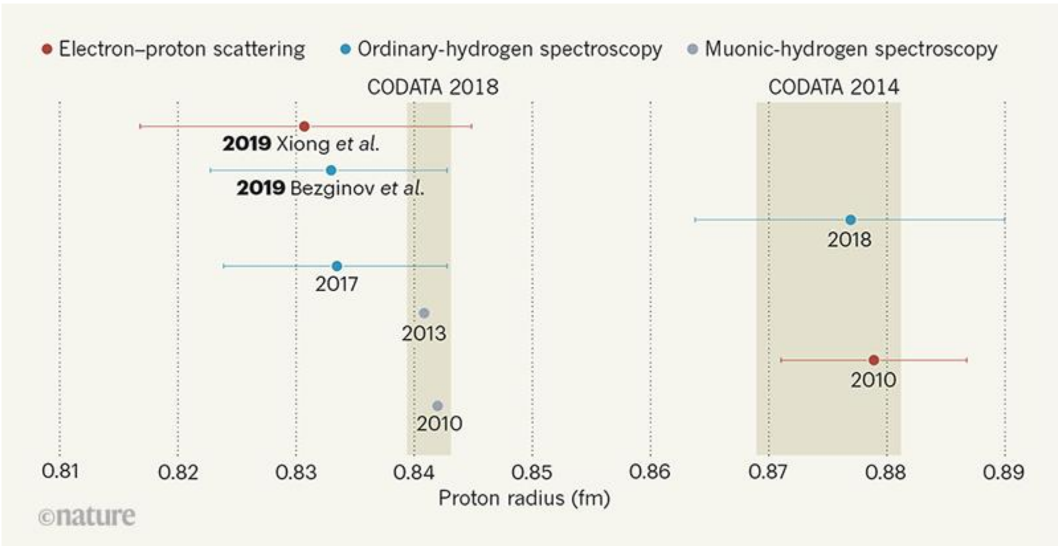

Figure 1. Values for the proton radius (figure from Nature[18]).

According to our points of view, these discrepancies are natural and should be exist reasonably. Proton should be just like an elastic football which should give different feelings (radii) to an adult or a child, by hand and by foot. So we suppose there should be three kinds of the proton charge radius which are denoted as $r_{p/H}$, $r_{p/H\text{-}\mu}$ and $r_{p/e}$.

3. Formulas of the Proton Charge Radius

In our previous formulas for the classical electron radius, we employed the fine-structure constant α as follows.

$$r_e = \alpha_c^2 a_0 = \alpha_1 \alpha_2 a_0 = \frac{5.29177210903(80)\times 10^{-11} m}{18788.865042381} = 2.81794032658(43)\ fm$$

$$\alpha_1 = \cfrac{1}{56+81+\cfrac{1}{28-\cfrac{13\cdot(2\cdot 56\cdot 11-1)}{3\cdot 5\cdot(2\cdot 56\cdot 43+1)}}} = 1/137.035999037435$$

$$\alpha_2 = \cfrac{1}{56+81+\cfrac{1}{28-\cfrac{2\cdot(16\cdot 27-1)}{3\cdot(16\cdot 81+1)}}} = 1/137.035999111818$$

$$\alpha_c = \cfrac{1}{56+81+\cfrac{1}{28-\cfrac{5\cdot(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}} = \cfrac{1}{56+81+\cfrac{1}{27+\cfrac{4\cdot(4\cdot(2\cdot 27\cdot 29+1)+1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}}$$

$$= 1/137.035999074626$$

the key element and its isotopes: ${}^{136,137,138}_{56}Ba_{80,81,82}$ (Note: $136 = 8\cdot 17,\ 138 = 6\cdot 23$)

It is supposed that the situation should be similar for the proton charge radius, and the critical point is to find the key element and its isotopes. And hence we construct the following formulas of the proton charge radius to give reasonable and precise values.

$$r_{p/H} = \frac{a_o}{(252+\delta)^2} = \frac{52917.72109\ fm}{\left(99+153+\cfrac{1}{33+\cfrac{2}{17}}\right)^2} = 0.8330977868\ fm$$

$$r_{p/H-\mu} = r_{p/H}\times\left(1+\frac{1}{2\cdot 47}\right) = \frac{a_o}{\left(251-\cfrac{1}{3}+\cfrac{1}{29}-\cfrac{1}{2\cdot 13\cdot 41+\cfrac{4}{31}}\right)^2} = 0.8419605292\ fm$$

$$r_{p/e} = r_{p/H}\times\left(1-\frac{1}{2\cdot 11\cdot 19}\right) = \frac{a_o}{\left(2\cdot 126+\cfrac{1}{3}-\cfrac{1}{7\cdot(126+1)+\cfrac{2\cdot 7}{41}}\right)^2} = 0.8311047299\ fm$$

the key element and its isotope: ${}^{252}_{99}Es^{*}_{153}$ (Note: $153 = 9\cdot 17$，$252 = 2\cdot 126 = 9\cdot 28$)

$$\frac{r_{p/H}}{r_e} = \frac{2\cdot 17}{5\cdot 23} - \cfrac{1}{2\cdot 3\cdot 7\cdot 19\cdot 109+\cfrac{107}{278}} = 0.295640677332428$$

$$\frac{r_{p/H}}{r_e} = \left(\cfrac{56+81+\cfrac{1}{28-\cfrac{5\cdot(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}}{99+153+\cfrac{1}{33+\cfrac{2}{17}}}\right)^2 = 0.29564067732429$$

$$\frac{r_{p/H}}{r_e} == \frac{112\times(168-\frac{1}{3}+\frac{1}{12\cdot47}-\frac{1}{14\cdot112\cdot(2\cdot173+1)}}{(99+153+\frac{1}{33+\frac{2}{17}})^2} = 0.29564067732429$$

$$\frac{r_e}{r_{p/H}} = 3+\frac{1}{2}-\frac{1}{8}+\frac{1}{7\cdot19}-\frac{1}{27\cdot13\cdot83-\frac{23}{30}} = 3.3824844707536$$

$2021/7/30-31$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{60,61,62}_{28}Ni_{32,33,34}$ ${}^{57,58}_{26}Fe_{31,32}$

${}^{61,62}_{28}Ni_{33,34}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{76,80,82}_{34}Se_{42,46,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{99,100,101,102}_{44}Ru_{55,56,57,58}$

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{124,125,126}_{52}Te_{72,73,74}$ ${}^{133}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$

${}^{140}_{58}Ce_{82}$ ${}^{3\cdot47}_{59}\mathrm{Pr}_{82}$ ${}^{142,143,144}_{60}Nd_{82,83,84}$ ${}^{149,150,152}_{62}Sm_{87,88,90}$ ${}^{153}_{63}Eu_{90}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$

${}^{169}_{69}Tm_{100}$ ${}^{4\cdot47}_{76}Os_{112}$ ${}^{206,207,208}_{82}Pt_{124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{251}_{98}Cf^{*}_{153}$

${}^{252}_{99}Es^{*}_{153}$ ${}^{2\cdot157}_{126}Ch^{ie}_{4\cdot47}$ ${}^{3\cdot112}_{133}Ch^{ie}_{203}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{384}_{153}Ch^{ie}_{7\cdot33}$ ${}^{418}_{166}Ch^{ie}_{252}$

These formulas and values would not be absolutely correct, but it seems that they are reasonable and should be relatively precise.

4. Formulas of the Neutron Charge Radius

With the same method, we construct the following formulas of the neutron charge radius and give reasonable and precise value of it.

$$r_n = \frac{a_o}{(400-\delta)^2} = \frac{52917.72109\ fm}{(157+243-\frac{1}{3})^2} = 0.3312876729\ fm$$

$$r_n^{\ 2} = 0.1097515222\ fm^2$$

the key element and its isotope: ${}^{400}_{157}Ch^{ie}_{243}$

${}^{157}_{64}Gd_{93}$ ${}^{200}_{80}Hg_{120}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{2\cdot157}_{126}Ch^{ie}_{4\cdot47}$ ${}^{400}_{157}Ch^{ie}_{243}$

$2021/7/31$

$$\frac{r_n}{r_e} = \frac{1}{17}-\frac{1}{2\cdot9\cdot23\cdot29-\frac{13}{2\cdot17}} = 0.1175637645$$

$$\frac{r_e}{r_n} = 9-\frac{1}{2}+\frac{1}{2\cdot83}-\frac{1}{81\cdot7\cdot(4\cdot243-1)} = 8.506022280$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{58}_{26}Fe_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80}_{34}Se_{46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{104}_{46}Pd_{58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{140,142}_{58}Ce_{82,84}$ ${}^{166,168,170}_{68}Er_{98,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{225}_{87}Fr^{*}_{138}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{2\cdot157}_{126}Ch^{ie}_{188}$ ${}^{344,346,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{418}_{166}Ch^{ie}_{252}$

$2021/8/3$

$$\frac{r_n}{r_{p/H}} = \left(\frac{99+153+\cfrac{1}{33+\cfrac{2}{17}}}{(157+243-\frac{1}{3})}\right)^2 = 0.39765760767979$$

$$\frac{r_n}{r_{p/H}} = \frac{4\cdot 17}{9\cdot 19} - \frac{1}{2\cdot 31\cdot(2\cdot 81\cdot 31+1)\ or\ 2\cdot 31\cdot(32\cdot 157-1)+\frac{11}{8\cdot 7}} = 0.39765760767979$$

$$\frac{r_{p/H}}{r_n} = \frac{5}{2} + \frac{1}{4\cdot 17} + \frac{1}{11^3\cdot 37-\frac{17}{59}} = 2.5147261882772$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{56,57}_{26}Fe_{30,31}$ ${}^{62}_{28}Ni_{34}$ ${}^{64,67,68}_{30}Zn_{34,37,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99,100,101,102}_{44}Ru_{55,56,57,58}$

${}^{12\cdot 59}_{50}Sn_{68}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{147,148}_{62}Sm_{85,86}$ ${}^{157}_{64}Gd_{93}$ ${}^{168}_{68}Er_{100}$ ${}^{9\cdot 19}_{70}Yb_{101}$ ${}^{201,204}_{80}Hg_{121,124}$

${}^{203,205}_{81}Tl_{122,124}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{302}_{121}Ch^{ie}_{181}$ ${}^{310}_{124}Ch^{ie}_{186}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{188}$ ${}^{400}_{157}Ch^{ie}_{243}$

2021/8/3

A recent paper[19] explained: "Despite the neutron zero-net electric charge, the asymmetric distribution of the positively- (up) and negatively-charged (down) quarks, a result of the complex quark-gluon dynamics, lead to a negative value for its squared charge radius, r_n^2." And it gave a relatively precise result of $r_n^2 = -0.110(8)\ fm^2$. By referring to this result, we give the above formulas and value for the neutron charge radius, and they seem to be still reasonable and much more precise.

5. Different Difficulties to Determine the Radii of Electron, Proton and Neutron

We noticed that the difficulties to determine the radii of electron, proton and neutron are quite different by experimental measurements or theoretical calculations. But why? Here we try to propose an explanation.

In our previous paper[3], we developed Mass Model of Elementary Particles. The main picture of the model is shown as follows (**Fig. 2**).

According to the model's stipulations, electron is a Yang particle, we can assume it should be hard, round and smooth like a steel ball, so its radius could be determined easily and precisely. However, a proton contains two up quarks (u) which are Yin and one down quark (d) which is Yang, so a proton is a net Yin particle, and we can assume it should be soft, not very round, elastic and rough like a football, so it is difficult to determine its radius and the determined values should depend on the measurement

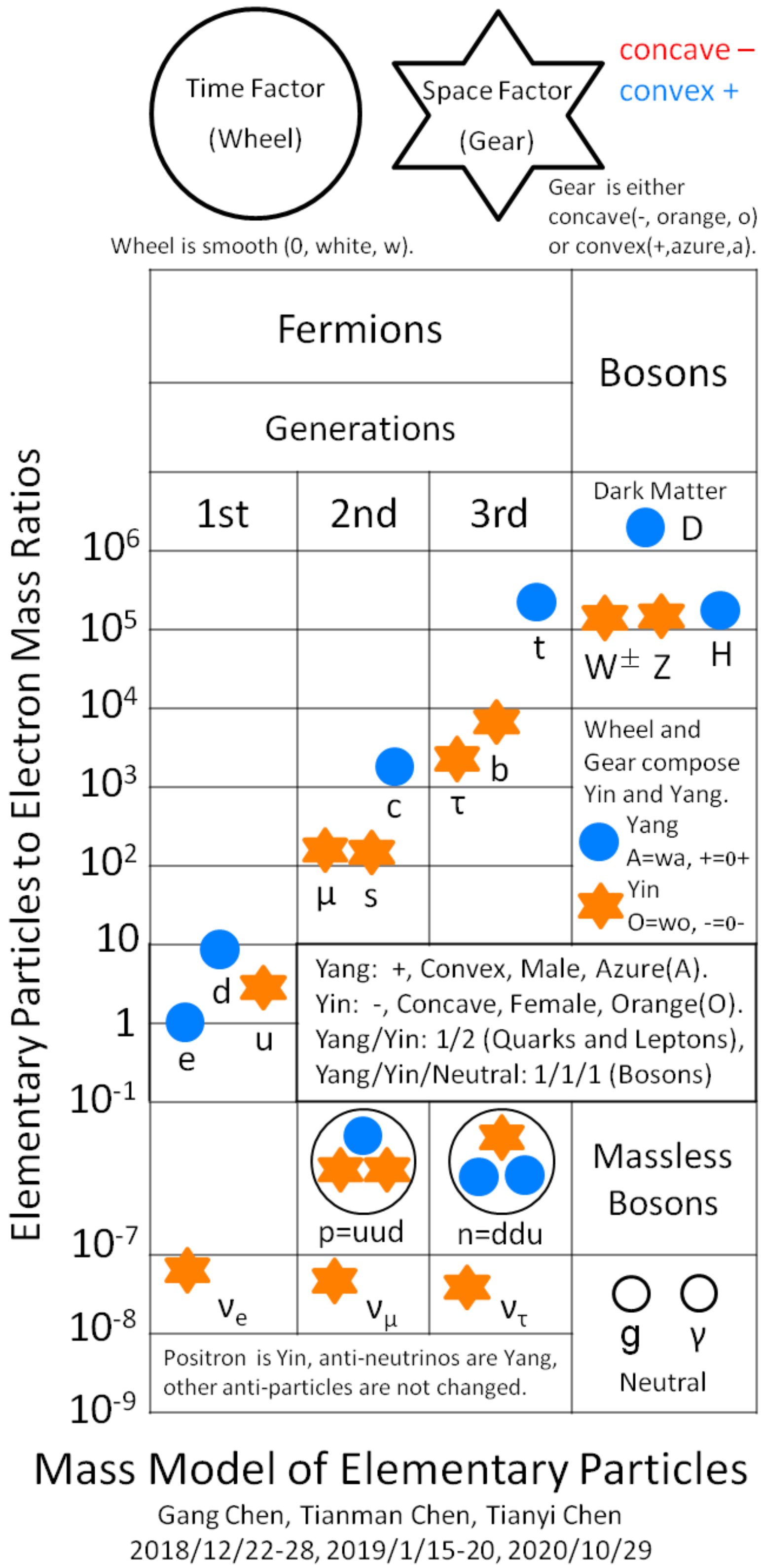

Figure 2.

methods. A neutron contains two down quarks (d) which are Yang and one up quark (u) which is Yin, so a neutron is a net Yang particle, it should be not very difficult to determine its radius, however, it is neutral in net electric charge, so it is still difficult to determine its electric charge radius which should be resulted from asymmetric distribution of its component quarks and gluons. According to this explanation, we could also predict that it should be relatively easy to determine the radii of Yang particles such as Higgs boson (H) and the particle of dark matter (D), and it should be relatively difficult to determine the radii of Yin particles such as muon, tauon, W boson, Z boson and the neutrino particles.

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Appendix I: Research History

Section	Page	Date	Location
Abstract	1	2021/7/30-8/1	Chengdu
1	1-2	2021/7/30-31	
2	2-3	2021/7/31-8/1	
3	3-5	2021/7/30-8/1	
4	5	2021/7/31-8/3	
5	5-7	2021/8/2-3	
Preparing this paper	1-9	2021/7/30-8/3	Chengdu
Note: Date was recorded according to Beijing Time.			