A Simple Proof for Collatz Conjecture

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Abstract

The approach to the problem is in the reverse. Instead of moving from 3n+1, we move from the final answer and extract all the possible values of n. They key was to find out a unique way of all the possible n's that emanate from 2y. Tried it the other way round, gets messier. So just sharing the final simpler version. The paper contains tables, which are basically equations for sets of data and rearrangement of those tables is just playing with the underlying equations. The solution: Redefining the problem → Identifying the pattern → analyzing the pattern → eliminating rouge data → testing for all the possible values of n.

The conjecture states that

If n = odd ; n =3n+1

If n = even; n=n/2

Upon repeating the same process we will always get 1.

In other words the conjecture states that the above transformations will always give a value of 2^y, which will obviously drop to 1 by the n=even rule.

This means all odd numbers can be expressed as 2^y using the above transformations.

Considering the conjecture to be true, instead of attacking the problem from starting, we begin with the ending.

conjecture draft approach						
3n+1	/2					
3n+1	/2	/2				
3n+1	/2	/2	/2			
3n+1	/2	/2	/2	/2		
3n+1	/2	/2	/2	/2	/2	
3n+1	/2	/2	/2	/2	/2	/2
our approach						
					x2	n/3-1
				x2	x2	n/3-1
		x2	x2	x2	x2	n/3-1
	x2	x2	x2	x2	x2	n/3-1
x2	x2	x2	x2	x2	x2	n/3-1

Table 1

Final result = 1

Means, final result = 2^y

Tabulating 2^y

2^y	Value	reverse transformation of 3n+1 = (n-1)/3
1	2	0.333333333
2	4	1

3	8	2.333333333
4	16	5
5	32	10.33333333
6	64	21
7	128	42.33333333
8	256	85
9	512	170.3333333

Table 2

We notice that every alternative value of 2^y has a valid solution for reverse 3n+1 transformation.

But here we haven't considered any other even values like (6,10,12…) except 2^y (2,4,8,)

Lets take that into account and tabulate further:

multiples of 2	**divisible/2**	**divisible /4**	**divisible /8**	**divisible /16**	**divisible /32**	**divisible /64**
2	1					
4	2	1				
6	3					
8	4	2	1			
10	5					
12	6	3				
14	7					
16	8	4	2	1		
18	9					
20	10	5				
22	11					
24	12	6	3			
26	13					
28	14	7				
30	15					
32	16	8	4	2	1	
34	17					
36	18	9				
38	19					
40	20	10	5			
42	21					
44	22	11				
46	23					
48	24	12	6	3		
50	25					
52	26	13				

54	27					
56	28	14	7			
58	29					
60	30	15				
62	31					
64	32	16	8	4	2	1

Table 3

Here we see an obvious pattern, considering our set of having x values

Valid /2=x/2

Valid /4= x/4

Valid /8 = x/8 … and so on.

We include r to check the number in the series.

We include co prime 2 to evaluate further.

All the solutions for valid x/2 give us all positive integers 1-r last. These positive integers can be considered as n or 3n+1. We will label them as all the possible solutions to 3n+1. (There are certain exceptions, which shall be eliminated in due time).

r	**co prime 2**	**reverse 3n+1 = (n-1)/3**	**x2**	**x4**	**x8**	**x16**	**x32**	**x64**	**x128**	**x256**
1		1	2	4	8	16	32	64	128	256
2	2	2	4	8	16	32	64	128	256	512
3	3	3	6	12	24	48	96	192	384	768
4	2	4	8	16	32	64	128	256	512	1024
5	5	5	10	20	40	80	160	320	640	1280
6	3	6	12	24	48	96	192	384	768	1536
7	7	7	14	28	56	112	224	448	896	1792
8	2	8	16	32	64	128	256	512	1024	2048
9	9	9	18	36	72	144	288	576	1152	2304
10	5	10	20	40	80	160	320	640	1280	2560
11	11	11	22	44	88	176	352	704	1408	2816
12	3	12	24	48	96	192	384	768	1536	3072
13	13	13	26	52	104	208	416	832	1664	3328
14	7	14	28	56	112	224	448	896	1792	3584
15	15	15	30	60	120	240	480	960	1920	3840
16	2	16	32	64	128	256	512	1024	2048	4096
17	17	17	34	68	136	272	544	1088	2176	4352

18	9	18	36	72	144	288	576	1152	2304	4608
19	19	19	38	76	152	304	608	1216	2432	4864
20	10	20	40	80	160	320	640	1280	2560	5120
21	21	21	42	84	168	336	672	1344	2688	5376
22	11	22	44	88	176	352	704	1408	2816	5632
23	23	23	46	92	184	368	736	1472	2944	5888
24	3	24	48	96	192	384	768	1536	3072	6144
25	25	25	50	100	200	400	800	1600	3200	6400
26	13	26	52	104	208	416	832	1664	3328	6656
27	27	27	54	108	216	432	864	1728	3456	6912
28	7	28	56	112	224	448	896	1792	3584	7168
29	29	29	58	116	232	464	928	1856	3712	7424
30	15	30	60	120	240	480	960	1920	3840	7680
31	31	31	62	124	248	496	992	1984	3968	7936
32	2	32	64	128	256	512	1024	2048	4096	8192

Table 4

So we have tabulated all the possible ways to write even values. They can be expressed as some n leading to even being written in terms of 2^Y. To put it simply, we have just identified the prime factors of all the numbers with 2.

All the terms for any given r, we have essentially the same number in our transformational rule.

Example:

13	26	52	104	208	416	832	1664	3328

All these numbers look different but would always come down to number 13. We may keep dividing 3328 by 2 and will get 13. So, all these numbers are the same number being represented in various possible ways.

We also notice, series with same co prime are just the same.

13 ≡ 26. 26 is there in the row as well as the column. So r 13 =r26, both of them have co prime 2 value as 13.

Thus, eliminating the repeats:

r	**co prime 2**	**reverse 3n+1 = (n-1)/3**	**x2**	**x4**	**x8**	**x16**	**x32**	**x64**	**x128**	**x256**
1		1	2	4	8	16	32	64	128	256

2	2	2	4	8	16	32	64	128	256	512
3	3	3	6	12	24	48	96	192	384	768
5	5	5	10	20	40	80	160	320	640	1280
7	7	7	14	28	56	112	224	448	896	1792
9	9	9	18	36	72	144	288	576	1152	2304
11	11	11	22	44	88	176	352	704	1408	2816
13	13	13	26	52	104	208	416	832	1664	3328
15	15	15	30	60	120	240	480	960	1920	3840
17	17	17	34	68	136	272	544	1088	2176	4352
19	19	19	38	76	152	304	608	1216	2432	4864
21	21	21	42	84	168	336	672	1344	2688	5376
23	23	23	46	92	184	368	736	1472	2944	5888
25	25	25	50	100	200	400	800	1600	3200	6400
27	27	27	54	108	216	432	864	1728	3456	6912
29	29	29	58	116	232	464	928	1856	3712	7424
31	31	31	62	124	248	496	992	1984	3968	7936

Table 5

Now, lets use reverse 3n+1 ; (n-1)/3 transformation to find valid values of n.

r	**co prime 2**	**reverse 3n+1 = (n-1)/3**	**x2**	**possible 2n**	**x4**	**possible 4n**	**x8**	**possible 8n**	**x16**	**possible 16n**
1	1	1	2	0.333333	4	1	8	2.333333	16	5
2	2	2	4	1	8	2.333333	16	5	32	10.33333
3	3	3	6	1.666667	12	3.666667	24	7.666667	48	15.66667
5	5	5	10	3	20	6.333333	40	13	80	26.33333
7	7	7	14	4.333333	28	9	56	18.33333	112	37
9	9	9	18	5.666667	36	11.66667	72	23.66667	144	47.66667
11	11	11	22	7	44	14.33333	88	29	176	58.33333
13	13	13	26	8.333333	52	17	104	34.33333	208	69
15	15	15	30	9.666667	60	19.66667	120	39.66667	240	79.66667
17	17	17	34	11	68	22.33333	136	45	272	90.33333
19	19	19	38	12.33333	76	25	152	50.33333	304	101
21	21	21	42	13.66667	84	27.66667	168	55.66667	336	111.6667
23	23	23	46	15	92	30.33333	184	61	368	122.3333
25	25	25	50	16.33333	100	33	200	66.33333	400	133
27	27	27	54	17.66667	108	35.66667	216	71.66667	432	143.6667
29	29	29	58	19	116	38.33333	232	77	464	154.3333
31	31	31	62	20.33333	124	41	248	82.33333	496	165

Table 6

Table 6 is a simple and effective way to write all the possible combinations for all ; 3n+1, 3n+1/2, 3n+1/4, 3n+1/8… and so on.

We know that 3n+1 ≠ 3x and we see the same in our table as there are no valid solutions for n in r [3,9,12,15,21,27]

Eliminating the same

r	co prime 2,3	reverse 3n+1 = (n-1)/3	x2	possible 2n	x4	possible 4n	x8	possible 8n	x16	possible 16n
1	1	1	2	0.333333	4	1	8	2.333333	16	5
2	2	2	4	1	8	2.333333	16	5	32	10.33333
3	5	5	10	3	20	6.333333	40	13	80	26.33333
4	7	7	14	4.333333	28	9	56	18.33333	112	37
5	11	11	22	7	44	14.33333	88	29	176	58.33333
6	13	13	26	8.333333	52	17	104	34.33333	208	69
7	17	17	34	11	68	22.33333	136	45	272	90.33333
8	19	19	38	12.33333	76	25	152	50.33333	304	101
9	23	23	46	15	92	30.33333	184	61	368	122.3333
10	25	25	50	16.33333	100	33	200	66.33333	400	133
11	29	29	58	19	116	38.33333	232	77	464	154.3333
12	31	31	62	20.33333	124	41	248	82.33333	496	165

Table 7

Eliminating invalid values for n

r	co prime 2,3	reverse 3n+1 = (n-1)/3	x2	valid 2n	x4	valid 4n	x8	valid 8n	x16	valid 16n
1	1	1	2		4	1	8		16	5
2	5	5	10	3	20		40	13	80	
3	7	7	14		28	9	56		112	37
4	11	11	22	7	44		88	29	176	
5	13	13	26		52	17	104		208	69
6	17	17	34	11	68		136	45	272	
7	19	19	38		76	25	152		304	101
8	23	23	46	15	92		184	61	368	
9	25	25	50		100	33	200		400	133
10	29	29	58	19	116		232	77	464	
11	31	31	62		124	41	248		496	165

Table 8

Now, we have very clear pattern that we have and can be extended because of their nature.

In co prime 2,3; we have all the values that are co prime 2&3. Co prime 2,3 = z

Every valid 2n, we have a gap of 4 units. logic:

Lets define n/2 set: 4m+2. '+2' element ensures that it is not divisible by 4 and only by 2.

When we solve 3n+1 = 4m+2 : n=(4m+1)/3. 4m+2 cannot have any value that is multiple of 3 as 3n+1 is not multiple of 3. So all the multiples of 6 are to be eliminated.

Upon solving we get: m = [5,8,11,14,17,20, …]

N= [3,7,11,15,19,23 …]

Every valid 4n, we have a gap of 8 units. Reason: same as above using 8m+2

Every valid 8n, we have a gap of 16 units. Reason: same as above using 16m+2… and so on

In terms of row: the first element of all the rows is just the value of valid n for our first table of 2Y → (n-1)/3

2^y	**Value**	**reverse transformation of 3n+1 = (n-1)/3**
1	2	0.333333333
2	4	**1**
3	8	2.333333333
4	16	**5**
5	32	10.33333333
6	64	**21**
7	128	42.33333333
8	256	**85**
9	512	170.3333333

Table 1

First element of each column h=1(defined below): The element column: r1w1 can be written as per the above table. First element of alternating columns which start from h=2, r2w2 can be written as w-1+ r1w1. We do need to keep in mind if y/4=integer , then first element = r1 else r2.

Example h=2. W-1=8, so r1w1 = 5; 5+8=13 which is r2w2

In terms of column: generalized form is: r(h)-r(h-2) =w+1, can be rewritten as : for any r; value(t) =value(t1) + (t-1)(w+1)

r(h) is the h^{th} term in r, r(h-2) is 2 number above r(h), t is the valid term for n ~r(h/2), value (t) is the value of the t^{th} term, w is the coefficient of n like w=16 for 16n

Example: r=9 and w = 4, t=5;

We are looking for the 5^{th} term, 1^{st} term =1. So 5^{th} term = 1+8x(5-1) =33

One could extend this table using the pattern or reverse 3n+1 transformation calculation as per table 6.

				gap 4		gap 8		gap 16		gap 32		gap 64		gap 128		gap 256		gap 512
r	coprime	**reverse 3n+1 = (n-1)/3**	x2	valid n	x4	valid n	x8	valid n	x16	valid n	x32	valid n	x64	valid n	x128	valid n	x256	valid n
1	1	1	2		4	1	8		16	5				21				85
2	5	5	10	3	20		40	13	80			53				213		
3	7	7	14		28	9	56		112	37				149				597
4	11	11	22	7	44		88	29	176			117				469		
5	13	13	26		52	17	104		208	69				277				1109
6	17	17	34	11	68		136	45	272			181				725		
7	19	19	38		76	25	152		304	101				405				1621
8	23	23	46	15	92		184	61	368			245				981		
9	25	25	50		100	33	200		400	133				533				2133
10	29	29	58	19	116		232	77	464			309				1237		
11	31	31	62		124	41	248		496	165				661				2645
12	35	35		23				93				373				1493		
13	37	37				49				197				789				3157
14	41	41		27				109				437				1749		
15	43	43				57				229				917				3669
16	47	47		31				125				501				2005		
17	49	49				65				261				1045				4181
18	53	53		35				141				565				2261		
19	55	55				73				293				1173				4693
20	59	59		39				157				629				2517		
21	61	61				81				325				1301				5205
22	65	65		43				173				693				2773		
23	67	67				89				357				1429				5717
24	71	71		47				189				757				3029		
25	73	73				97				389				1557				6229
26	77	77		51				205				821				3285		
27	79	79				105				421				1685				6741
28	83	83		55				221				837				3301		
29	85	85				113				453				1717				6773
30	89	89		59				237				853				3317		
31	91	91				121				485				1749				6805
32	95	95		63				253				869				3333		
33	97	97				129				517				1781				6837
34	101	101		67				269				885				3349		
35	103	103				137				549				1813				6869
36	107	107		71				285				901				3365		
37	109	109				145				581				1845				6901
38	113	113		75				301				917				3381		
39	115	115				153				613				1877				6933
40	119	119		79				317				933				3397		
41	121	121				161				645				1909				6965
42	125	125		83				333				949				3413		
43	127	127				169				677				1941				6997
44	131	131		87				349				965				3429		
45	133	133				177				709				1973				7029
46	137	137		91				365				981				3445		
47	139	139				185				741				2005				7061
48	143	143		95				381				997				3461		
49	145	145				193				773				2037				7093
50	149	149		99				397				1013				3477		
51	151	151				201				805				2069				7125
52	155	155		103				413				1029				3493		

Table 9

Of course, the table looks messy, but it doesn't matter as the number of are of little significance for this table. Initial part is the data generated by Table6 reverse 3n+1; (n-1)/3 computational method. The rest of the data has been filled only by the observed pattern. I could provide a table to prove that they are exactly the same but it is trivial. The understanding of this proof is not the numbers but the patterns in play. The pattern of 50% of the numbers of any continuous series being divisible by 2, the pattern of 50% of that 50% will be divisible by 4 and so on. The pattern that if for some series if 3 is divisible by 4 and 7 is divisible by 4 then the next term that would be divisible by 4 is 11(can be tested by 4m+2), which is just adding 4 in the said series as described by the earlier 4m+2 method. It is structured on every even number being divisible by 2; the most basic pattern in whole of mathematics.

The final table that would help us evaluate our analysis: Concerning only with valid values of n

r	coprime 2,3 /	valid 2n	valid 4n	valid 8n	valid 16n	valid 32n	valid 64n	valid 128n	valid 256n
1	1		1		5		21		85
2	5	3		13		53		213	
3	7		9		37		149		597
4	11	7		29		117		469	
5	13		17		69		277		1109
6	17	11		45		181		725	
7	19		25		101		405		1621
8	23	15		61		245		981	
9	25		33		133		533		2133
10	29	19		77		309		1237	
11	31		41		165		661		2645
12	35	23		93		373		1493	
13	37		49		197		789		3157
14	41	27		109		437		1749	
15	43		57		229		917		3669
16	47	31		125		501		2005	
17	49		65		261		1045		4181
18	53	35		141		565		2261	
19	55		73		293		1173		4693
20	59	39		157		629		2517	
21	61		81		325		1301		5205
22	65	43		173		693		2773	
23	67		89		357		1429		5717
24	71	47		189		757		3029	
25	73		97		389		1557		6229
26	77	51		205		821		3285	
27	79		105		421		1685		6741
28	83	55		221		885		3541	
29	85		113		453		1813		7253
30	89	59		237		949		3797	
31	91		121		485		1941		7765
32	95	63		253		1013		4053	

33	97		129		517		2069		8277
34	101	67		269		1077		4309	
35	103		137		549		2197		8789
36	107	71		285		1141		4565	
37	109		145		581		2325		9301
38	113	75		301		1205		4821	
39	115		153		613		2453		9813
40	119	79		317		1269		5077	
41	121		161		645		2581		10325
42	125	83		333		1333		5333	
43	127		169		677	64	2709	256	10837
44	131	87		349	32	1397	128	5589	512
45	133		177	16	709		2837		11349
46	137	91		365		1461		5845	
47	139		185		741		2965		11861
48	143	95		381		1525		6101	
49	145		193		773		3093		12373
50	149	99		397		1589		6357	
51	151		201		805		3221		12885
52	155	103		413		1653		6613	

Table 10

I have populated enough data to check existence of every odd number till 101 in the table.

One could create these tables on the basis of equations and all these tables are nothing but just equations and every step reorganizing the table is just the next step of the equation.

It is a simpler to visualize these equations and data and analyze it AS LONG AS THERE IS AN ABSOLUTE PATTERN THAT CAN BE ESHTABLISHED, which in this case is evident and obvious.

The pattern will be repeated for as long as we want and thus no matter what number it is, it will definitely be in this table. It gives us a sure shot method of being able to represent every number going through 3n+1 & n/2 transformation as 2^y. And, 2^y in the transformational rule always drops to 1

One could still be apprehensive regarding the validity of the above argument: all the odd numbers can be represented as per the above representation.

To complete the above argument

Lets consider valid2n, valid 4n. valid 8n, valid 16n … as sets and name them 2n, 4n, 8n, 16n…

We have the universal set of all the positive integers: U.

So, we have universal set of all the odd numbers: O

So, we have universal set of all the even numbers: e

We define a function number of elements: q

q(u) = q (o) + q(e). one may say q(o) = ½ q(u)

q(2n) : every element is 4 digits apart so = 1/4

q(4n): every element is 8 digits apart so =1/8

q(8n) : every element is 8 digits apart so =1/16

q(16n) : every element is 8 digits apart so =1/32 … and so on.

Table10 spits out only unique odds so, there is no issue of odds being repeated.

q(2n) + q(4n) + q(8n)+ q(16n) … = 1/4 +1/8 +1/16 +1/32

We know, 1/2 + 1/4 +1/8 +1/16 +1/32… = 1

So, 1/4 +1/8 +1/16 +1/32…= 1/2

Every form of 2n/4n…. spits out only odd numbers.

q(2n) + q(4n) + q(8n)+ q(16n) …=1/2 = q(o)

Thus, all the odd numbers are represented in the table10.

All the odd numbers can be transformed into values of 2Y with the defined transformations.

Thus, the conjecture is true.

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