Delayed-Choice Quantum Erasure Experiment: A Causal Explanation Using Wave-Particle Non-Duality

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According to the recently proposed *wave-particle non-dualistic interpretation of quantum mechanics*, the physical nature of Schrödinger's wave function is an 'instantaneous resonant spatial mode' in which a quantum flies akin to the case of a test particle moving along a geodesic in the curved space-time of general theory of relativity. By making use of this physical nature, a causal explanation is provided for the delayed-choice quantum erasure experiment.

I. INTRODUCTION

According to Bohr's principle of complementarity, *any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena* [1–5, 67, 70]. The wave-particle duality explains that a quantum behaves as a particle during an observation, but, as a wave when unobserved - suggesting to infer the quantum to posses both the behaviors simultaneously [6–10]. Which behavior becomes observable depends on the experimental configuration. For example, in the Young's double-slit experiment as shown in FIG. 1, the presence of a detection screen or a twin-telescope corresponds to the observation of wave behavior or particle behavior, respectively. Hence, the wave-particle duality forces to conclude that, after 'somehow' sensing the configuration of the measuring device, the quantum decides to behave accordingly - either as a wave or as a particle when the detection screen or twin-telescope is present, respectively - before it passes through the double-slit assembly [9, 10].

In Wheeler's delayed-choice experiment [9, 11–20, 66, 69, 76], the detection screen is quickly removed, exposing the twin-telescope, after the quantum has already passed through the double-slit assembly. Now, according to the wave-particle duality, the quantum retro-causally rearranges its past history of simultaneously passing through both the slits as a wave to that of passing through any one slit like a particle, yielding clump patterns at twin-telescope. Notice that, the principle of complementarity simply puts both wave and particle natures together, without providing any physical mechanism for such an union. Also, how the wave nature makes an instantaneous transition to the particle nature during the observation is unclear. In reality, the wave-particle duality may be the only one fundamental mystery of quantum mechanics [6], whose solution can even explain Einstein's spooky action-at-a-distance in the context of two or more entangled particles [21–30, 66, 68]. It is the root cause for all other paradoxes, like, Schrödinger's Cat [31, 32, 72] and for inferring the existence of retro-causal influences in Wheeler's delayed-choice experiment, delayed-choice quantum erasure experiments [33–42, 66], entanglement swapping experiments [43–50, 66, 68], etc. To resolve this mystery, various interpretations were proposed [51–65] and the present author also put forward the 'wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level' [66–77]. The delayed-choice quantum erasure experiment, a variant of double-slit experiment based on the concepts of both the quantum erasure [78–82] and Wheeler's delayed-choice experiments, is performed with pairs of entangled particles.

Either to erase or not the which-slit information in Young's double-slit experiment, first it's necessary for such an information to remain unmeasured, so that, later, during the joint-detection measurements, it can be either erased or not to obtain either interference or clump patterns, respectively (see FIG. 5 and FIG. 6). The photons are registered by photon-counting detector, D_0, much before their entangled photons are measured. The joint detection events of photons recorded later seem to retro-causally influence the behavior of their already registered entangled partners to behave either as a wave or as particles by exhibiting the corresponding interference or clump patterns, respectively. The key point here to note is that, the moment D_0 registers photons, the entanglement with their yet to be measured partner photons become instantaneously broken, making them as absolutely free photons. Hence, how come the later measurements on these disentangled free photons retro-causally influence the behavior of already recorded and no more existing photons is unclear. This clearly shows that the quantum mechanics is not in favor of retro-causality. In the present article, an unambiguous casual explanation for the delayed-choice quantum erasure experiment [33] is provided by using the wave-particle non-dualistic interpretation.

The present article is organized as follows: In Section-II, the "wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level" is briefed in the context of Young's double-slit experiment. In Section-III, the experiments done with or without delayed-choice of obtaining the which-slit information are described with respect to non-duality, based on the entanglement phenomenon during the measurements in Young's double-slit experiment. A causal explanation for the delayed-choice quantum erasure experiment is provided in Section-IV. Section-V contains the conclusion.

II. YOUNG'S DOUBLE-SLIT EXPERIMENT AND THE WAVE-PARTICLE NON-DUALITY

Both experimentally and theoretically, Young's double-slit experiment (YDE) is one of the most extensively studied physical system, which is supposed to contain at its heart the basic wave-particle mystery of quantum mechanics [6]. All material particles like photons, electrons, neutrons, atoms, molecules, etc., are known to exhibit this mystery [83–97]. In this section, the wave-particle non-duality ($WPND$) [66–77] is briefed using YDE as an example.

As shown in FIG. 1, consider a single-particle source in the YDE firing a particle onto a detection screen through double-slit assembly, only after the registration of previously fired particle. Classically, the particles were expected to leave two clump patterns on the screen, as some of them pass through slit-1 and the others through slit-2, because, they were thought to be moving in the 3-dimensional Euclidean space of classical mechanics. But, according to $WPND$, the particle actually moves in its own space - an *instantaneous resonant spatial mode* ($IRSM$) - obeying the Schrödinger wave equation and hence, an interference pattern occurs - this situation is analogous to a moving test particle along a geodesic in the curved space-time of general theory of relativity [98]. The complex vector space of quantum formalism is accepted by the WPND as the actual physical space of the Nature, where, all the quantum phenomena happen.

The moment a particle appears at the source, its space ($IRSM$) also appears instantaneously everywhere at the same moment and hence, through the entire experimental arrangement. The $IRSM$, being an eigenstate, has an eigenvalue carried by its particle and survives as long as the particle survives with the same eigenvalue. At the instant the particle disappears by some absorption at the detection screen, its $IRSM$ also disappears

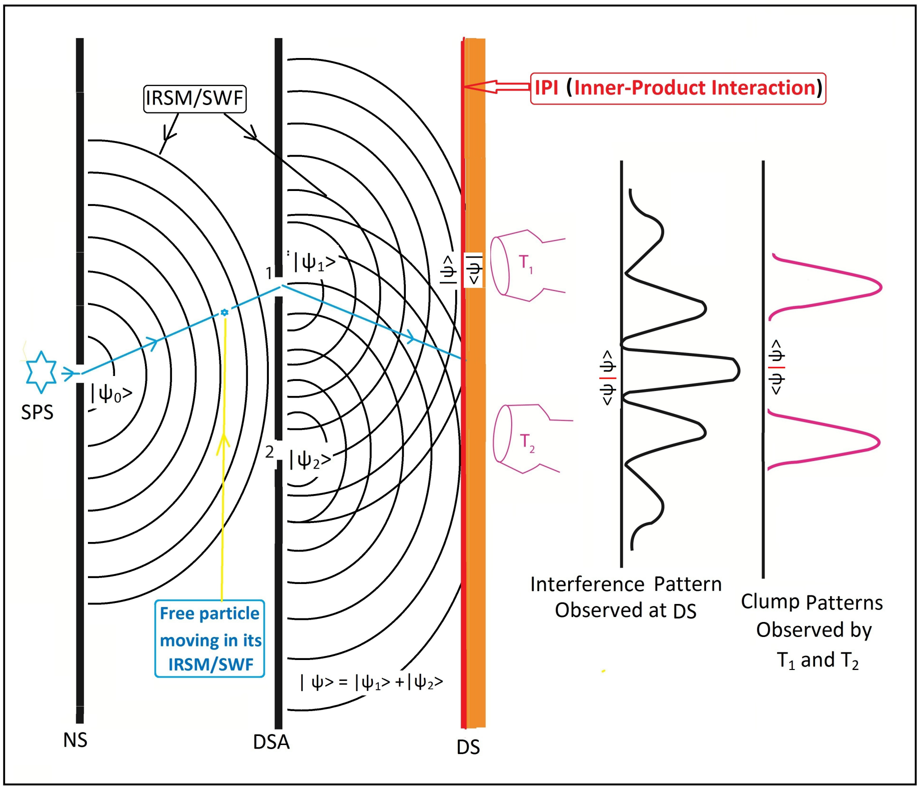

FIG. 1. **Schematic diagram of Young's Double-Slit Experiment with respect to the Wave-Particle Non-Duality**: SPS is a single-particle source and NS is a narrow slit. DSA stands for a double-slit assembly and DS, for a detection screen. According to non-duality, the *instantaneous resonant spatial mode* ($IRSM$) is a synonym for *Schrödinger's wave function* (SWF). $|\psi_0>$ is the state vector of a particle projected through the NA. $|\psi_1>$ and $|\psi_2>$ are projections of $|\psi_0>$ through the DSA. The superposed state, $|\psi>=|\psi_1>+|\psi_2>$, induces its dual, $<\psi|$, in DS and interacts according to the inner-product interaction (IPI) given by $<\psi|\psi>$. The moment the quantum appears at SPS, its space, IRSM/SWF, also appears everywhere at the same moment and hence, through the experimental arrangement, i.e., the projections $|\psi_0>$ and $|\psi>$ through NS and DSA, respectively, and the IPI, $<\psi|\psi>$, at DS - all happen at the same moment. The blue-line is a trajectory in the IRSM, formed by the eigenvalues of a particular position eigenstate evolving according to the Heisenberg equations of motion and is governed by the classical least action principal. The quantum resides in that particular position eigenstate which in turn depends upon the initial phase of its IRSM. T_1 and T_2 are telescopes tightly focused onto the slits 1 ans 2, respectively, which are added in this figure just to facilitate an explanation of Wheeler's delayed-choice experiment in Section-I.

everywhere at that same instant. Notice that, the appearance and disappearance of both the particle and its $IRSM$ together resembles a resonance phenomenon. That's why, IRSM is the actual physical meaning of Schrödinger's wave function (SWF) and hence, $IRSM$ is a synonym to SWF. The $IRSM$ being an eigenstate and its particle carrying the corresponding eigenvalues are inseparable and always live together as a single entity. This inseparability, living together and/or the united single entity is named as the $WPND$. This non-dualistic picture of a quantum particle moving in its $IRSM/SWF$ is further irreducible and is independent of any measuring device.

The intensity of a classical-wave is proportional to the square of its amplitude. But, according to $WPND$, the SWF can't be claimed to have such an intensity, because, it's an $IRSM$ and is unlike a propagating classical wave. When the particle hits the screen, then its $IRSM$, $|\psi>=|\psi_1>+|\psi_2>$, gets scattered into a new $IRSM$, say $|\psi'>$, which can be described by associating an operator, $\hat{O}_{DS}=|\psi'><\psi|$, to the detection screen:

$$\hat{O}_{DS}|\psi>=<\psi|\psi>|\psi'>, \tag{1}$$

where, $|\psi_1>$ and $|\psi_2>$ are the projections of the initial state, $|\psi_0>$, through the double-slit assembly. Notice that, $<\psi|$ is induced by $|\psi>$ and is analogous to an image in a mirror, totally confined only to the screen unlike $|\psi>$ as shown in FIG. 1. If the scattered state, $|\psi'>$, is a null-state, then it corresponds to the absorption process and the particle must have interacted somewhere in the region of inner-product, $<\psi|\psi>$.

When the particle first appears out of its source at some position eigenvalue, say $x_p(0)$, at time $t=0$, then the phase of its position eigenstate will be the same as that of its state vector $|\psi>$, resulting in the following phase-relation [67, 69, 76],

$$\begin{aligned}\text{phase of }\{|\psi>\} &= \text{phase of }\{<x_p(0)|\psi>\} = \text{phase of }\{<x_p(0)|x_p(t)><x_p(t)|\psi>\}\\ &= \text{phase of }\{<x_p(0)|x_p(t)>\} + \text{phase of }\{<x_p(t)|\psi>\};\end{aligned} \tag{2}$$

here, the subscript p stands for particle; $|x_p(0)>$ and $|x_p(t)>$ are the particular position eigenstates, where, the particle is residing at times $t=0$ and t, respectively. $|x_p(0)>$ evolves to $|x_p(t)>$ under the Heisenberg equations of motions, which, eventually results in the classical least action principle [67, 69, 76]:

$$\delta\int_0^t dt' L(\dot{x}_p(t'), x_p(t')) = 0. \tag{3}$$

The above equation explicitly shows that the position eigenvalues of a particle state always, as a function of time, lie on a classical path as shown by a blue-line in FIG. 1. This result is independent of whether the physical system is microscopic or macroscopic and proves that the same time parameter enters both classical and quantum mechanics.

As already mentioned, the moment the particle appears at source, its $IRSM/SWF$ also appears everywhere at the same moment which includes the projections $|\psi_0>$ and $|\psi>$ through a narrow slit and the double-slit assembly, the inner-product interaction (IPI) $<\psi|\psi>$ and the scattered state $|\psi'>$ at the screen, respectively. Now, the particle starts moving along the path of least action in its $IRSM$, obeying Eq. (3), and after some time t, it will be found on the screen at eigenvalue $x_p(t)$ in the region of IPI. The moment the particle gets scattered at $x_p(t)$ into the new $IRSM$, $|\psi'>$, the old $IRSM$, $|\psi>$, disappears resembling the wave function collapse advocated in the Copenhagen interpretation [51–53].

The next particle appears at the source along with its $IRSM$ whose initial phase will be different from the previous one and hence, takes a different path to the screen. However, its IPI, $<\psi|\psi>$, being independent of the initial phase, is the same as all previous particles. The hits of particles on the screen occur randomly at different locations due to different initial phases. This randomness in the phase is due to its dependence on the detailed nature of the source. After a large collection of detection events, an interference pattern emerges out, which is nothing but the construction of the function $|<x_p|\psi>|^2$ with individual points; here, the set of position eigenvalues, $\{x_p\}$, span the detecting region of the screen. No particle will be found in the regions of dark fringes because, $<\psi|\psi>$ vanishes there, which in turn implies that no classical paths, formed by the position eigenvalues of the particle states, are available from any slit to any dark fringe. Therefore, a moving particle itself never behaves like a wave though it is associated with de Broglie's wave nature (IRSM). It's needless to mention that, $|<x_p|\psi>|^2$ is the Born probability density to find a particle in an infinitesimal volume around x_p.

"One important conclusion of the present section is that, as already mentioned earlier, the moment the particle appears at the source, the scattered state $|\psi'>$ at the detection screen also appears at the same moment, though, the particle itself takes time t to reach and interact with the same screen - this also includes the appearance of the projections $|\psi_0>$ and $|\psi>$ by narrow slit and double-slit assembly, respectively, at the same moment. The $IRSM$ itself induces all possible interactions, because, it belongs to wave nature and exists everywhere [99, 100]. But, the particle itself will participate in one particular interaction, because, it's a localized entity and that particular interaction is determined by the initial phase of its $IRSM$" - Let's call this conclusion as "the criterion of section-II", or simply CS-II, so that it can be easily referred wherever it's used in the following sections.

Though the $WPND$ is presented here for the case of time-independent non-relativistic quantum mechanics, all its conclusion go through even for the cases of time-dependent non-relativistic and relativistic quantum mechanics, which will be reported elsewhere.

III. EXPERIMENTS WITHOUT AND WITH DELAYED-CHOICE

A. Young's Double-Slit Experiment with Entangled Quantum Particles

Nature maintains the conservation laws even in the absence of exchange-interactions via Einstein's spooky action-at-a-distance [21–30, 66, 68]. Exchange interactions, arising due to the exchange of material particles, are subjected to the Cosmic speed limit in accordance with the special theory of relativity, whereas, the spooky action, arising due to the instantaneous nature of the non-materialistic spatial mode ($IRSM/SWF$), is unbounded to any such speed limits.

Let's consider two independent free particles represented by the state vectors $|X_0>$ and $|Y_0>$, respectively, flying apart from each other after interacting for a brief time [21], i.e.,

$$|X_0>|Y_0> \xrightarrow{\text{Brief Interaction}} |X>|Y> \equiv |\Psi>>, \tag{4}$$

where, $|X>$ and $|Y>$ are the resultant state vectors of the particles after the brief interaction. $|\Psi>>$ is the joint quantum state of the same particles representing an entangled state due to some definite conservation law established during the brief interaction, which, in an appropriate reference frame, can be expressed as [66, 68],

$$(\hat{C}_X + \hat{C}_Y)|\Psi>> = 0, \tag{5}$$

here, the operators $\hat{C}_X$ & $\hat{C}_Y$ correspond to some conserved properties of the particles. The $IRSM$s, $|X>$ and $|Y>$, when represented using the position basis, are superimposed on top of each other so that the Eq. (5) holds at every position eigenvalue, resulting in the spooky action-at-a distance.

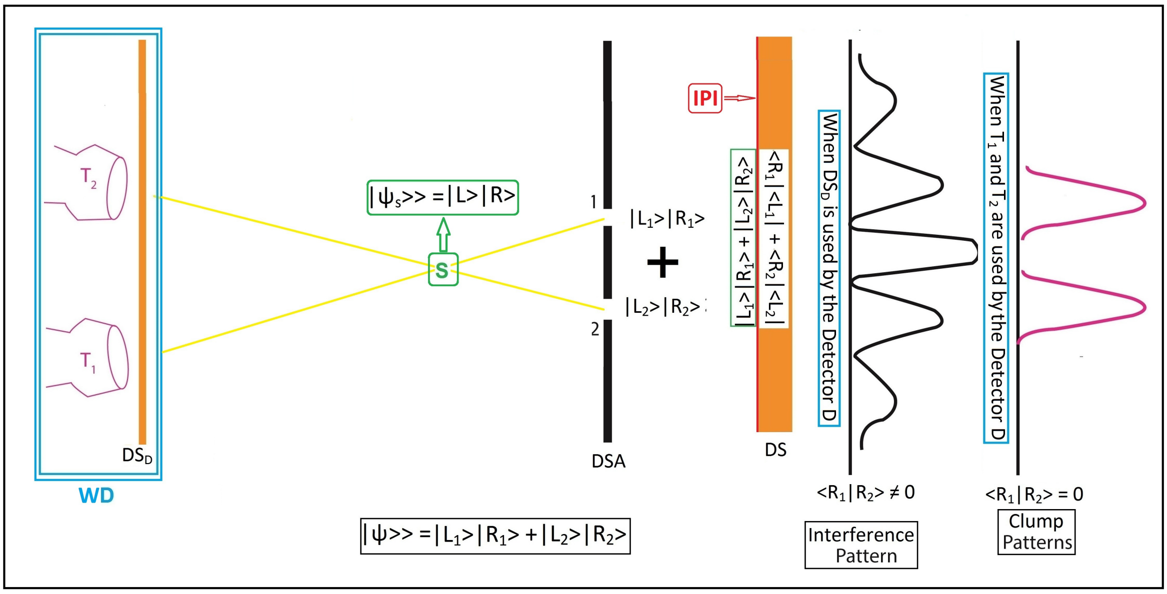

FIG. 2. **Young's Double-Slit Experiment with Entangled Quantum Particles**: The moment a pair of momentum-entangled particles, with total zero momentum, is created at the source, S, its IRSM, $|\psi_S>> (= |L>|R>)$ appears in the entire space and hence, through the double-slit assembly (DSA) as $|\psi>>$, which interacts with its induced dual $<<\psi|$ in the detection screen, DS. $|L>$ and $|R>$ stand for state vectors of left and right moving particles, respectively. The left-particle hits the screen in the region of inner-product interaction (IPI), $<<\psi|\psi>>$. Whether all such left-particles exhibit interference or clump patterns can be decided by an appropriate measurement on the right-particles by a Wheeler detector, WD. Another detection screen DS_D and a twin-telescope T_1 & T_2 are put together to make WD. T_1 and T_2 are tightly focused on the slits 1 and 2, respectively. WD can use either DS_D or T_1 & T_2 for the detection of the right-particles.

If $|X>$ is a superposition in some measurement basis:

$$|X> \longrightarrow |X_1> + |X_2>, \tag{6}$$

then, in actual reality, it is the $|\Psi>>$ becoming a superposition as,

$$|\Psi>> = |X_1>|Y_1> + |X_2>|Y_2>, \tag{7}$$

so that Eq. (5) holds for each component independently, i.e.,

$$(\hat{C}_X + \hat{C}_Y)|X_1>|Y_1> = (\hat{C}_X + \hat{C}_Y)|X_2>|Y_2> = 0. \tag{8}$$

Before analyzing the YDE with a source, S, emitting pairs of momentum-entangled particles with total zero momentum [100] as shown in FIG. 2, consider the following mappings:

$$|\Psi >> \longrightarrow |\psi_S >> \; ; \; |X > \longrightarrow |L > \; ; \; |Y > \longrightarrow |R > \; ; \; \hat{C}_X \longrightarrow \hat{P}_L \; ; \; \hat{C}_X \longrightarrow \hat{P}_Y \qquad (9)$$

such that $|\psi_S >>= |L > |R >$, obeys the law of conservation of momentum given by,

$$(\hat{P}_L + \hat{P}_R)|\psi_S >>= (\hat{P}_L + \hat{P}_R)|L > |R >= 0, \qquad (10)$$

where, $|L >$ and $|R >$ are the momentum eigenstates of the momentum operators $\hat{P}_L$ and $\hat{P}_R$ of the left and right moving particles, respectively.

Consider FIG. 2. A Wheeler detector and the usual YDE are placed at the left and right sides to the source, respectively. The Wheeler detector contains a detection screen akin to the one in YDE and a twin-telescope such that it can use either the screen or telescopes for detecting the left-particles; here, the telescopes are tightly focused on the slits 1 and 2, respectively.

The moment an entangled pair appears at the source, its IRSM, $|\psi_S >>$, appears in the entire space and hence, projected through the double-slit assembly as, say $|\psi >>$, forming an IPI, $<< \psi|\psi >>$, with its induced dual $<< \psi|$ in the the detection screen. The moment the right-particle hits the screen, the entanglement is spontaneously broken and the left-particle is thrown into an appropriate momentum eigenstate due to Einstein's spooky action-at-a-distance in accordance with the conservation law given in Eq. (10) - the same kind of situation exists in the case of entanglement swapping experiments which was explicitly worked out in references [66, 68]. Therefore, during the joint detection, the left-particle can also be described by the same $|\psi >>$ (after it looses entanglement) resulting in the same IPI, $<< \psi|\psi >>$, at the Wheeler detector. Using Eq. (7), the state $|\psi >>$ can be written as,

$$|\psi >>= \frac{1}{\sqrt{2}}(|L_1 > |R_1 > +|L_2 > |R_2 >), \qquad (11)$$

where, $|L_1 > |R_1 >$ and $|L_2 > |R_2 >$ are the projections of $|\psi_S >>$ through slit-1 and slit-2.

According to the CS-II of WPND, the IPIs at both Wheeler detector and YDE happen at the same moment of appearance of an entangled pair at the source, due to the instantaneous nature of the $IRSM$ and is given by,

$$\begin{aligned} << \psi|\psi >> = & \frac{1}{2} < L_1|L_1 >< R_1|R_1 > + \frac{1}{2} < L_2|L_2 >< R_2|R_2 > \\ & + \frac{1}{2} < L_1|L_2 >< R_1|R_2 > + \frac{1}{2} < L_2|L_1 >< R_2|R_1 >, \end{aligned} \qquad (12)$$

so that the actual distances of Wheeler detector and YDE from the source are immaterial for the observation of joint detection events. Irrespective of these distances, the particles start moving in their predetermined paths decided by the initial phases of their $IRSM$s, respecting conservation law (responsible for the entanglement) and causality.

As an example, consider a situation where the distance between Wheeler detector and source is much greater than the distance between source and YDE, an analogous situation in delayed-choice quantum erasure experiments [33–42]. Now, the Wheeler detector, by an appropriate joint measurements on the left-particles, appears to be deciding whether all the already registered right-particles in YDE exhibit interference or clump patterns. If it uses detector screen, then $< R_1|R_2 >\neq 0$ in Eq. (12) and hence, interference occurs. If it uses twin-telescope, then $< R_1|R_2 >= 0$, resulting in the clumps patterns.

Also notice that, by the following mapping,

$$|L_1 >\rightarrow |a >_1, |R_1 >\rightarrow |b >_2, |L_2 >\rightarrow |a' >_1, |R_2 >\rightarrow |b' >_2 \text{ and } |\psi >>\rightarrow |\psi >,$$

Eq. (11) becomes $|\psi >= \frac{1}{\sqrt{2}}(|a >_1 |b >_2 +|a' >_1 |b' >_2)$, which is the same state as the one considered in the reference [100].

B. Young's Double-Slit Experiment with Which-Slit Detector

As shown in FIG. 3, consider the YDE with a which-slit detector. The state vector of a single-particle emitted from the source is first projected through a narrow-slit as $|\psi_0 >$, which is then projected onto a detection screen through a double-slit assembly as $|\psi >= |\psi_1 > +|\psi_2 >$; here, $|\psi_1 >$ and $|\psi_2 >$ are the projected states through slits 1 and 2, respectively. The which-slit detector uses its prob vector $|D >$ to find out through which slit the particle is passing on. Using the CS-II of $WPND$, the interaction of $|D >$ with $|\psi >$ resulting in an entangled state $|\psi' >>$ can be written akin to the Eq. (11):

$$|\psi' >>= \frac{1}{\sqrt{2}}(|\psi'_1 > |D'_1 > +|\psi'_2 > |D'_2 >), \tag{13}$$

where, $|\psi'_1 >$ and $|\psi'_2 >$ are the scattered state vectors of the particle through slits 1 and 2 and $|D'_1 >$ and $|D'_2 >$ are the corresponding scattered probe states, respectively. The state

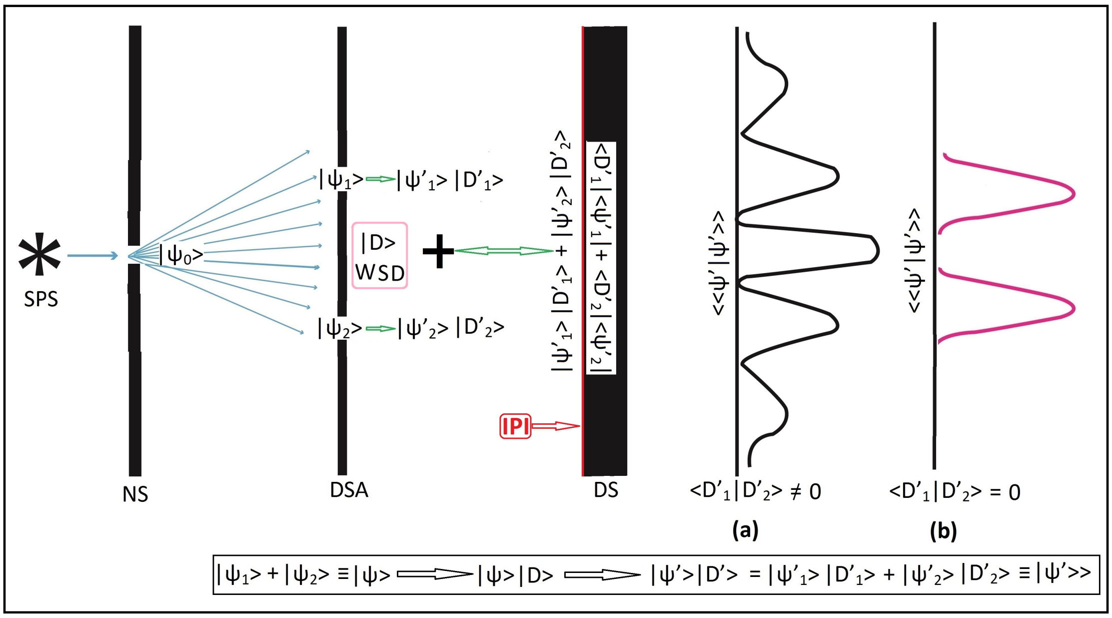

FIG. 3. **Young's Double-Slit Experiment with Which-Slit Detector**: SPS is a single-particle source. NS is a narrow-slit, DSA is a double-slit assembly and DS is a detection screen. WSD is a which-slit detector, using a state vector $|D>$ as a probe to find out through which slit a given particle is passing towards DS. The state vector $|\psi_0>$ through NS is projected through the DSA as $|\psi_1>$ and $|\psi_2>$ whose linear superposition, $|\psi>=|\psi_1>+|\psi_2>$, interacts with $|D>$ resulting in an entangled state $|\psi'>>$; $|\psi_1'>$ and $|\psi_2'>$ are the scattered state vectors of the particle and $|D_1'>$ and $|D_2'>$ are the corresponding scattered probe states. (a) If $|D_1'>$ and $|D_2'>$ are indistinguishable with respect to the vector space of WSD, i.e., $<D_1'|D_2'>\neq 0$, then an interference pattern results on the DS. (b) If $|D_1'>$ and $|D_2'>$ are distinguishable with respect to the vector space of WSD, i.e., $<D_1'|D_2'>=0$, then the clump patterns occur on the DS. IPI stands for inner-product interaction.

$|\psi'>>$ induces its dual $<<\psi'|$ in the screen and interacts according to the IPI:

$$\begin{aligned} <<\psi'|\psi'>> = \frac{1}{2}<\psi_1'|\psi_1'><D_1'|D_1'> + \frac{1}{2}<\psi_2'|\psi_2'><D_2'|D_2'> \\ +\frac{1}{2}<\psi_1'|\psi_2'><D_1'|D_2'> + \frac{1}{2}<\psi_2'|\psi_1'><D_2'|D_1'> . \end{aligned} \tag{14}$$

The above equation is identical to the Eq. (12), where the role of Wheeler detector is now played by the which-slit detector. Therefore, with respect to which-slit detector, if $|D_1'>$ and $|D_2'>$ are indistinguishable, i.e., $<D_1'|D_2'>\neq 0$, then an interference pattern is formed. On the other hand, if $|D_1'>$ and $|D_2'>$ are distinguishable, i.e., $<D_1'|D_2'>=0$, then Eq.

(14) yields clump patterns:

$$<<\psi'|\psi'>>=<\psi_1'|\psi_1'>+<\psi_2'|\psi_2'>, \tag{15}$$

where, $<D_1'|D_1'>=<D_2'|D_2'>=1$.

It's well-known that if the double-slit assembly in FIG. 3 is replaced by its complementary object, i.e., two scatterers like two localized atoms/ions at the double-slit, then, depending upon the measurement basis, either interference or two clump patterns can be observed [33, 101]. Consider the same YDE as in FIG. 3 with two differences, viz., (i) place two

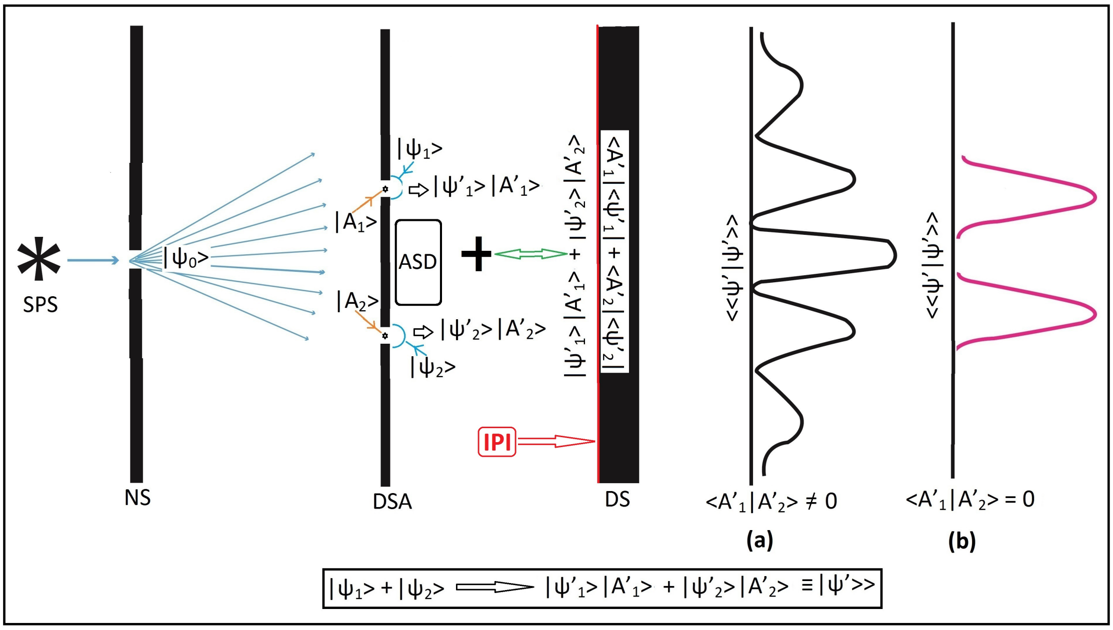

FIG. 4. **Which-Slit Detection with Atoms as a Probe**: All the details of this figure is the same as the one in FIG. 3 except for the replacement of WSD by an atomic-state detector, ASD, and the presence of identical atoms one at each slit, represented by the state vectors $|A_1>$ and $|A_2>$. The interaction of $|\psi_1>$ and $|\psi_2>$ with $|A_1>$ and $|A_2>$ results in the entangled states $|\psi_1'>|A_1'>$ and $|\psi_2'>|A_2'>$, respectively, whose superposition, say $|\psi'>>$, induces its dual in the DS and interacts according to the inner-product $<<\psi'|\psi'>>$. (a) If $|A_1'>$ and $|A_2'>$ are indistinguishable with respect to the vector space of ASD, i.e., $<A_1'|A_2'>\neq 0$, then an interference pattern results on the DS. (b) If $|A_1'>$ and $|A_2'>$ are distinguishable with respect to the vector space of WSD, i.e., $<A_1'|A_2'>=0$, then the clump patterns occur.

identical atoms, represented by the initial state vectors $|A_1>$ and $|A_2>$, one at each slit and (ii) replace the which-slit detector by an atomic-state detector which can measure the final states of the atoms, $|A_1' >$ and $|A_2' >$, as shown in the FIG. 4. Except these two differences, the actual physical process in both the cases are similar, which can be seen by the mapping, $|D_1' >\rightarrow |A_1' >$ and $|D_2' >\rightarrow |A_2' >$. Therefore, akin to Eq. (14), the IPI is given by,

$$\begin{aligned} << \psi'|\psi' >> = \frac{1}{2} < \psi_1'|\psi_1' >< A_1'|A_1' > +\frac{1}{2} < \psi_2'|\psi_2' >< A_2'|A_2' > \\ +\frac{1}{2} < \psi_1'|\psi_2' >< A_1'|A_2' > +\frac{1}{2} < \psi_2'|\psi_1' >< A_2'|A_1' > . \end{aligned} \tag{16}$$

Therefore, if the atomic-state detector can not distinguish $|A_1' >$ and $|A_2' >$, i.e., $< A_1'|A_2' >\neq 0$, then an interference pattern occurs. Otherwise, if it can distinguish, i.e., $< A_1'|A_2' >= 0$, then clump patterns appear.

In this kind of experiments described in FIG. 3 and FIG. 4, which-slit information is measured much before the detection of particles at the screen. But, the measurement of which-slit information can be delayed, so that, the particles are detected at the screen first. Moreover, these delayed-choice measurements can be made either by keeping which-slit information intact or by erasing the same. But, according to $WPND$, as explicitly shown in Section-III.A, the joint measurements without or with delayed-choice are on equal footing with respect to causality. A detailed causal explanation of the $DCQE$ [33] is provided in the following section.

IV. DELAYED-CHOICE QUANTUM ERASURE EXPERIMENT

As shown in FIG. 5, two identical atoms, A_1 and A_2, are placed at slits 1 and 2 of the YDE, respectively, and are excited by a single-photon pulse (not shown in FIG. 5), whose energy content is exactly sufficient to excite any one atom at a time. The excited atom can return to its initial state via atomic cascade decay, emitting a pair of entangled photons. See references [33, 78] for more details.

According to the CS-II of $WPND$ and also as explained in Section-III.A, the moment the photon pulse appears, its $IRSM$ induces two secondary $IRSM$s of entangled photons, $\frac{1}{\sqrt{2}}|L_1 > |R_1 >$ and $\frac{1}{\sqrt{2}}|L_2 > |R_2 >$ from A_1 and A_2 and forms the IPIs at the photon counting detectors D_a, D_b, D_c, D_d and D_0, after undergoing reflections and refractions at 50:50 beam splitters, BSA_1, BSA_2 and BS - all at once, respectively. The superposed

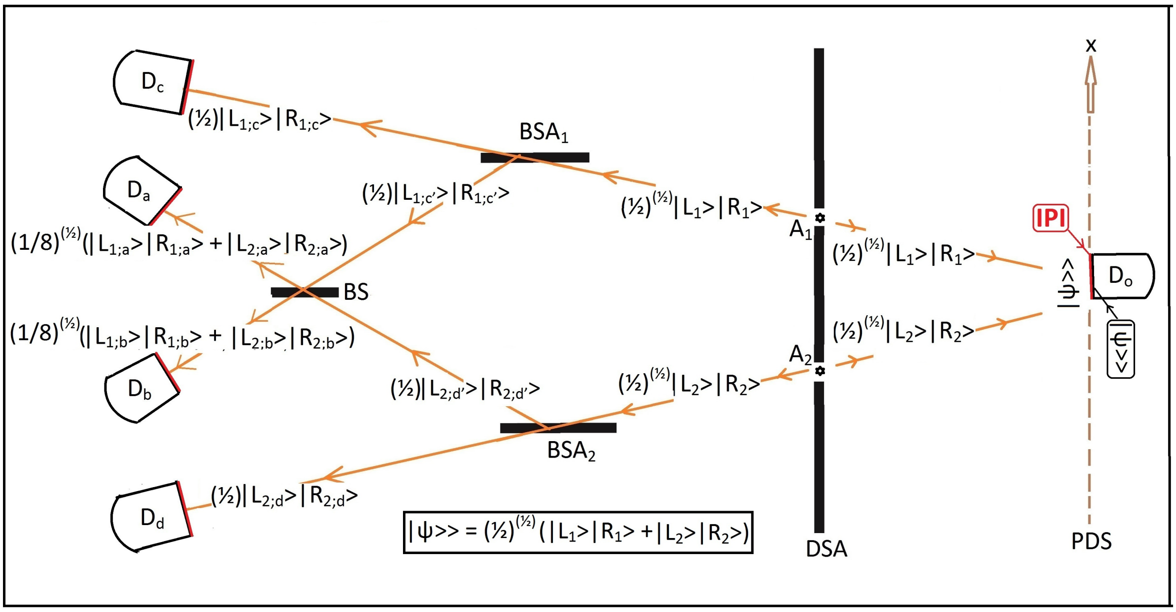

FIG. 5. **Quantum Erasure Experiment with a Pair of Entangled Photons Generated by Atomic Cascade Decay**: DSA is a double-slit assembly and PDS is a plane of detection screen. D_a, D_b, D_c, D_d and D_0 are photon counting detectors and BSA_1, BSA_2 and BS are 50:50 beam splitters. A_1 and A_2 are two identical atoms placed at slits 1 and 2 of DSA, respectively, and are simultaneously subjected to an excitation by a single-photon pulse (not shown in the figure), so that, any one of them emits a pair of entangled photons. The entangled pair from A_1 and A_2 are represented by their $IRSMs$, $\frac{1}{\sqrt{2}}|L_1>|R_1>$ and $\frac{1}{\sqrt{2}}|L_2>|R_2>$, where L and R stand for the left and right (moving particles). D_0 can be moved along x-direction and can be placed at different positions along PDS. In the absense of BSA_1 and BSA_2, D_c and D_d are equivalent to telescopes, T_1 and T_2, as in FIG. 2 and they will detect left-photons from A_1 and A_2 revealing which-slit information, respectively. If the optical path length of left-photon to reach D_a, D_b, D_c or D_d is much longer that that of right-photon to D_0, then this experimental arrangement corresponds to the delayed-choice quantum erasure.

entangled state is,

$$|\psi>>=\frac{1}{\sqrt{2}}(|L_1>|R_1>+|L_2>|R_2>). \qquad (17)$$

Akin to Eq. (7), the action of BSA_1 and BSA_2 on $\frac{1}{\sqrt{2}}|L_1>|R_1>$ and $\frac{1}{\sqrt{2}}|L_2>|R_2>$, yields,

$$\frac{1}{\sqrt{2}}|L_1>|R_1> = \frac{1}{2}(|L_{1;c}>|R_{1;c}> + |L_{1;c'}>|R_{1;c'}>)$$
$$\frac{1}{\sqrt{2}}|L_2>|R_2> = \frac{1}{2}(|L_{2;d}>|R_{2;d}> + |L_{2;d'}>|R_{2;d'}>), \tag{18}$$

with $\frac{1}{2}|L_{1;c}>|R_{1;c}>$ and $\frac{1}{2}|L_{2;d}>|R_{2;d}>$ as the refracted modes and $\frac{1}{2}|L_{1;c'}>|R_{1;c'}>$ and $\frac{1}{2}|L_{2;d'}>|R_{2;d'}>$ as the reflected modes, respectively. Again using Eq. (7), the act of BS on reflected modes can be found to result in the following superposition,

$$\frac{1}{2}(|L_{1;c'}>|R_{1;c'}> + |L_{2;d'}>|R_{2;d'}>) = \frac{1}{2\sqrt{2}}(|L_{1;a}>|R_{1;a}> + |L_{2;a}>|R_{2;a}>)$$
$$+\frac{1}{2\sqrt{2}}(|L_{1;b}>|R_{1;b}> + |L_{2;b}>|R_{2;b}>). \tag{19}$$

Therefore, the two-photon entangled state can be written as,

$$\begin{aligned}|\psi>> = &\frac{1}{2}|L_{1;c}>|R_{1;c}> + \frac{1}{2}|L_{2;d}>|R_{2;d}>\\ &+\frac{1}{2\sqrt{2}}(|L_{1;a}>|R_{1;a}> + |L_{2;a}>|R_{2;a}>)\\ &+\frac{1}{2\sqrt{2}}(|L_{1;b}>|R_{1;b}> + |L_{2;b}>|R_{2;b}>),\end{aligned} \tag{20}$$

whose IPI with its induced dual is given by,

$$\begin{aligned}<<\psi|\psi>> = &\frac{1}{4}<X_{1;c}|X_{1;c}><Y_{1;c}|Y_{1;c}> + \frac{1}{4}<X_{2;d}|X_{2;d}><Y_{2;d}|Y_{2;d}>\\ &+\frac{1}{8}(<L_{1;a}|L_{1;a}><R_{1;a}|R_{1;a}> + <L_{2;a}|L_{2;a}><R_{2;a}|R_{2;a}>)\\ &+\frac{1}{8}(<L_{1;a}|L_{2;a}><R_{1;a}|R_{2;a}> + <L_{2;a}|L_{1;a}><R_{2;a}|R_{1;a}>)\\ &+\frac{1}{8}(<L_{1;b}|L_{1;b}><R_{1;b}|R_{1;b}> + <L_{2;b}|L_{2;b}><R_{2;b}|R_{2;b}>)\\ &+\frac{1}{8}(<L_{1;b}|L_{2;b}><R_{1;b}|R_{2;b}> + <L_{2;b}|L_{1;b}><R_{2;b}|R_{1;b}>).\end{aligned} \tag{21}$$

From the above equation, it can easily be seen that, during the joint detection, D_c and D_d exhibit clump patterns and D_a and D_b exhibits interference patterns. Also, the same above equation shows that, with respect to the joint detection, how the full set of observed events at D_0 can be divided into smaller subsets of the events registered by D_a, D_b, D_c and D_d, i.e., $\{D_0\} = \{D_a\} \cup \{D_b\} \cup \{D_c\} \cup \{D_d\}$.

As already explained in Section-III.A, the left and right photons, depending upon the initial phases of their eigenstates, take predetermined paths and simply fly through their

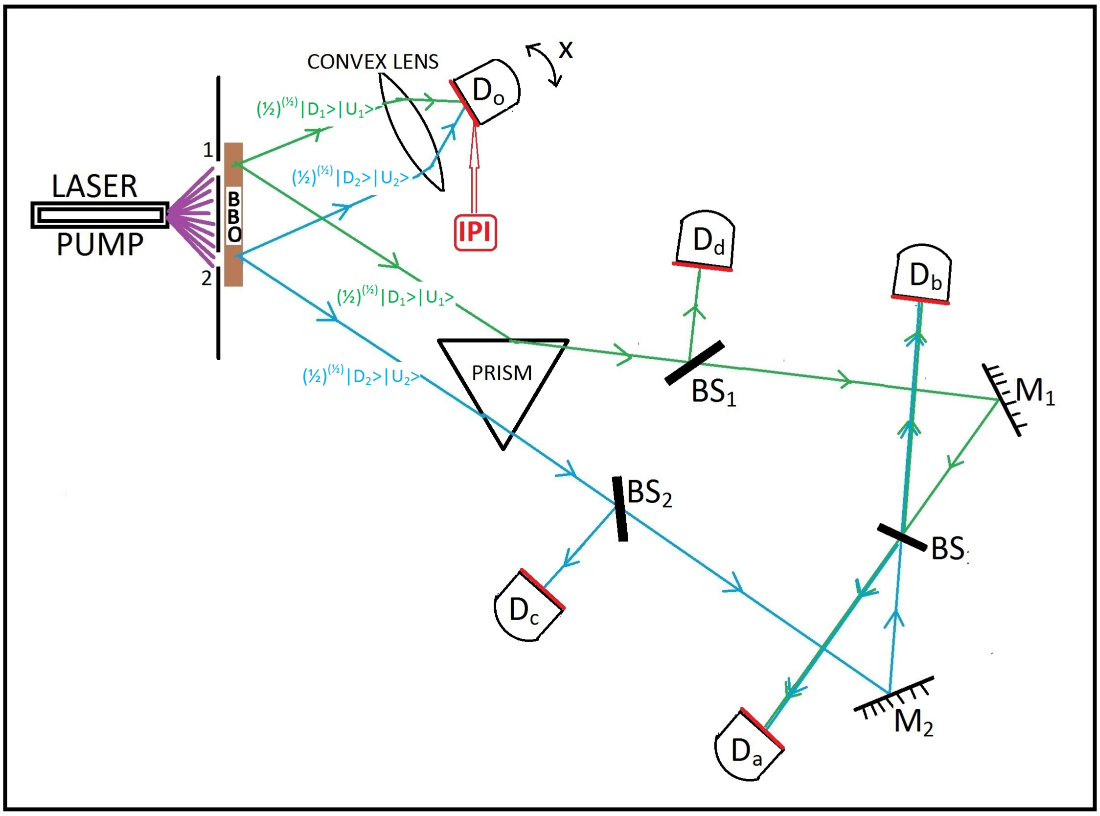

FIG. 6. **Quantum Erasure Experiment with a Pair of Entangled Photons Generated by Spontaneous Parametric Down Conversion**: A laser pump shoots a photon at the double-slit assembly. Immediately behind the slits, a beta barium borate (BBO) crystal is placed which can convert a single photon into two identical, orthogonally polarized entangled photons with twice the wave length of the original photon by a process known as the spontaneous parametric down conversion. Much before a photon passes through the double-slit, its IRSM induces two entangled states $\frac{1}{\sqrt{2}}|D_1 > |U_1 >$ and $\frac{1}{\sqrt{2}}|D_2 > |U_2 >$ from the slits 1 and 2, respectively. One photon from an entangled-pair reaches the movable detector D_0 while the other is directed towards a prism. The photons reaching D_0 are called signal photons and those reaching D_1, D_2, D_3 and D_4, via the 50:50 beam splitters BS_1, BS_2 and BS and 100% reflecting mirrors M_1 and M_2, are called idler photons. The optical path lengths are such that there is a 8ns time delay between first detecting a signal photon and later, its entangled idler photon.

entangled $IRSM$, $|\psi >>$, reaching the appropriate detector, D_1, D_2, D_3 or D_4, placed on the left side and D_0, on the right side, respectively. Therefore, according to $WPND$, it doesn't matter whether the right photon is detected before or after the detection of its entangled left photon. Occurrence of all detection events are causal and there are no such things as retro-causal influences, as it is the case in entanglement swapping experiments, as shown in references [66, 68].

The experiment in FIG. 5 is realized in the reference [33] by a similar experimental arrangement as shown in FIG. 6. In this experiment, a beta barium borate, BBO ($\beta - BaB_2O_4$), crystal is used instead of A_1 and A_2. BBO can convert a single photon into two identical, orthogonally polarized entangled photons with half the frequency of the original photon by a process known as the spontaneous parametric down conversion. The entangled photons from the regions of slit 1 and 2 are represented by $\frac{1}{\sqrt{2}}|D_1 > |U_1 >$ and $\frac{1}{\sqrt{2}}|D_2 > |U_2 >$ akin to $\frac{1}{\sqrt{2}}|L_1 > |R_1 >$ and $\frac{1}{\sqrt{2}}|L_2 > |R_2 >$; here, D and U stands for "down" and "up", respectively. In fact, by mapping $L \rightarrow D$ and $R \rightarrow U$ in Eq. (21), it can easily be seen that all the joint-detection rates in this experiment are exactly same as the one in FIG. 5.

V. CONCLUSION

The wave-particle non-dualistic interpretation of quantum mechanics treats both delayed-choice and without delayed-choice experiments on equal footing and hence, it naturally provides a causal explanation for the former ones.

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