Generating function of periodic sequences with eligible cycle.

Miguel Cerdá Bennassar – 2021

Abstract: we present in this paper a generating function of periodic sequences with eligible cycle.

Choose the k and m values and apply the corresponding operation.

$m \in \mathbb{Z}$
$k \in \mathbb{N}$.

$$f(k) = \begin{cases} m \text{ even} & \begin{cases} k \text{ even} & (k-m)/2 \\ k \text{ odd} & (3k+1+m)/2 \end{cases} \\ m \text{ odd} & \begin{cases} k \text{ even} & (3k+1+m)/2 \\ k \text{ odd} & (k-m)/2 \end{cases} \end{cases}$$

Each result is the value of k for the next iteration.

For all k, a sequence is generated that reaches $k_n = 1-m$ and ends in a cycle with $k_{(n-1)} = 2-m$.

Some examples of values of m and k_n:

m = 1	k_n = 0	for all	k>0	
m = 0	k_n = 1	“	k>0	
m = -1	k_n = 2	“	k>1	For negative m, k> -m
m = -2	k_n = 3	“	k>2	“ “ “
. . .	. . .		. . .	
m = 2,	k_n = - 1	“	k>0	
m = 3,	k_n = - 2	“	k>0	
m = 4,	k_n = - 3	“	k>0	
m = 5,	k_n = - 4	“	k>0	
. . .	. . .		. . .	

Examples:

A sequence will reach the number 11, if m = -10 and will enter the cycle with 2-m = 12.

The function is applied for even m:
$$\begin{cases} k \text{ even} & (k-m)/2 \\ k \text{ odd} & (3k+1+m)/2 \end{cases}$$

f(k) = 85, 123, 180, 95, 138, 74, 42, 26, 18, 14, 12, 11, 12, 11 . . .

Calculations:
(85 * 3 - 9) / 2 = 123
(123 * 3 - 9) / 2 = 180
(180 + 10) / 2 = 95
(95 * 3 - 9) / 2 = 138
(138 + 10) / 2 = 74
(74 + 10) / 2 = 42
(42 + 10) / 2 = 26
(26 + 10) / 2 = 18
(18 + 10) / 2 = 14
(14 + 10) / 2 = 12
(12 + 10) / 2 = 11
(11 * 3 - 9) / 2 = 12
(12 + 10) / 2 = 11
. . .

Example with k = 1149 and m = 5. The cycle will be 2-m = -3 and 1-m = -4:

The function is applied for odd m: $\begin{cases} \text{k even} & (3k+1+m)/2 \\ \text{k odd} & (k-m)/2 \end{cases}$

Calculations:
(1149 - 5) / 2 = 572
(572 x 3 + 6) / 2 = 861
(861 - 5) / 2 = 428
(428 x 3 + 6) / 2 = 645
(645 - 5) / 2 = 320
(320 x 3 + 6) / 2 = 483
(483 - 5) / 2 = 239
(239 - 5) / 2 = 117
(117 - 5) / 2 = 56
(56 x 3 + 6) / 2 = 87
(87 - 5) / 2 = 41
(41 - 5) / 2 = 18
(18 x 3 + 6) / 2 = 30
(30 x 3 + 6) / 2 = 48
(48 x 3 + 6) / 2 = 75
(75 - 5) / 2 = 35
(35 - 5) / 2 = 15
(15 - 5) / 2 = 5
(5 - 5) / 2 = 0
(0 x 3 + 6) / 2 = 3
(3 - 5) / 2 = -1
(-1 - 5) / 2 = -3
(-3 - 5) / 2 = -4
(-4 x 3 + 6) / 2 = -3
(-3 - 5) / 2 = -4
. . .

f(k) = 1149, 572, 861, 428, 645, 320, 483, 239, 117, 56, 87, 41, 18, 30, 48, 75, 35, 15, 5, 0, 3, -1, -3, -4 . . .

A sequence generated with k = 1154 and m = 0. Its cycle is with 2-m = 2 and 1-m = 1:

The function is applied for even m:

$$f(k) = \begin{cases} (k-m)/2 & k \text{ even} \\ (3k+1+m)/2 & k \text{ odd} \end{cases}$$

Calculations:
1154 / 2 = 577
(577 * 3 +1) / 2 = 866
866 / 2 = 433
(433 * 3 +1) / 2 = 650
650 / 2 = 325
(325 * 3 +1) / 2 = 488
488 / 2 = 244
244 / 2 = 122
122 / 2 = 61
(61 * 3 +1) / 2 = 92
92 / 2 = 46
46 / 2 = 23
(23 * 3 +1) / 2 = 35
(35 * 3 +1) / 2 = 53
(53 * 3 +1) / 2 = 80
80 / 2 = 40
40 / 2 = 20
20 / 2 = 10
10 / 2 = 5
(5 * 3 +1) / 2 = 8
8 / 2 = 4
4 / 2 = 2
2 / 2 = 1
(1 * 3 +1) / 2 = 2
2 / 2 = 1
. . .

f(k) = 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1 . . .

Sequences with m = 1, will reach the the number 1-m = 0.

f(k) = 38, 58, 88, 133, 66, 100, 151, 75, 37, 18, 28, 43, 21, 10, 16, 25, 12, 19, 9, 4, 7, 3, 1, 0.

Sequences with m=1+1=2, will reach the the number 1-m=-1.

f(k) = 782, 390, 194, 96, 47, 72, 35, 54, 26, 12, 5, 9, 15, 24, 11, 18, 8, 3, 6, 2, 0, -1.

Sequences with m=1+n, will reach the number 1-m=-n.

f(k) = 971, 436, 704, 1106, 1709, 805, 353, 127, 14, 71, -14, 29, -35, -67, -83, -91, -95, -97, -98.