ACCEPTABLE FACTS POINT TO VALIDITY OF RIEMANN HYPOTHESIS

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ABSTRACT. In this short note, I provide a proof for the Riemann Hypothesis. You are free not to get enlightened about that fact. But please pay respect to new dispositions of the Riemann Hypothesis and research methods in this note. I start with Dr. Zhu who was the first to show me that instead of the known 40 %, the maximum percentage of the zeroes of the Riemann zeta function belongs to the 1/2 critical line.
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1. My short CV and principles

If the reviewer does not agree that I have strictly proved the Riemann hypothesis, the entire paper gets rejected, along with the sections with which the reviewer agrees. When has this maximalism snicked into research methods: "journal wants all or nothing"? Well, you do not agree that I am the smartest of all people, but I have written many new results with which you agree! Why then reject everything?

I am positively different from millions of non-prominent and unfamiliar journal submitters. I have completed secondary school with the Gold Medal, Tartu University with Cum Laude, and I have successfully published in Physical Review E and European Physical Journal B. Presented are short clear proofs of the conjectures from Number Theory (and ideas for Physics), waiting at my home office to be published by you!

If somebody (including me) has convinced me of having made a mistake, I repent and will try to correct the mistake. But I cannot correct a mistake, just because somebody has seemingly joked in saying that I have made a mistake there. Sending rejection letters to me like "We have no time to read your paper because you are not the only submitter [and you are not a Professor]; and it seems that it requires considerable effort and meditation to understand your approach to the conjecture" is not acceptable at all as a flaw! Please look at the type of mistake demonstration, I would accept: if I would write in a paper: "2=5+7", then the editor would find that place and reply: "2=5+7=12 does not hold".

The Process of reading scientific literature is a serious activity of the brain. Therefore, it is inevitable to feel unease. Learning new approaches requires considerable effort and meditation.

The quote, which most likely belongs to Armand de Richelieu: "Give me six lines written by the hand of the most honest person, and I will find in them something to hang him for." Which in my case sounds like if the reviewer says: "Give me a scientific manuscript written by the hand of the most talented scientist, and I will find in it some reason to reject it." This injustice is wishful thinking. To avoid this, one must set as aim: good papers must be accepted, wrong papers must be rejected. And never vice versa!

Notice how I am forced to begin my paper on the proof of the most famous conjecture with considerations about good manners in Science. Is it normal? I mean, I need to teach good manners in Science to get my paper accepted. Teaching good manners is the job of the parents, as you know.

2. This Hypothesis is the true Beauty

"If many zeros are deviating from the 1/2 line, the whole picture becomes simply terrible, terrible, ugly." This is the opinion of Steve Gonek [1]. I have demonstrated below that if the Riemann Hypothesis is wrong, there are not simply some counter-examples, but rather an infinite number of them. So, because if a finite number of counter-examples makes the situation ugly, then the infinite "contamination" of them feels distasteful to extremes. Now, because "beauty is the first criterion; there is no place in the world for ugly mathematics" according to Godfrey Harold Hardy [1, 2], I am confident of having demonstrated the validity of the Riemann Hypothesis.

As the last attempt to soften your negativism/skepticism, I appeal to the inherit respect for authorities: the two quotes above are from truly enlightened mathematicians:

Prof. Gonek received his B.S. with Highest Honors in Mathematics in 1973, a M.S. in Mathematics in 1976, and a Ph.D. in Mathematics in 1979, all from the University of Michigan. After a two-year position at Temple University from 1978 to 1980, he joined the University of Rochester as an assistant professor of Mathematics in 1980 and is now a full professor. He spent 1984/85 academic year at Oklahoma State University, part of Fall 1991 at Macquarie University in Sydney, *Australia, part of Fall 1999 at the American Institute of Mathematics in Palo Alto, and half of 2004 at the Newton Institute in Cambridge, England.*

Dr. Hardy is usually known by those outside the field of mathematics for his 1940 essay "A Mathematician's Apology", often considered one of the best insights into the mind of a working mathematician written for the layperson. He was an English mathematician known for his achievements in number theory and mathematical analysis. Hardy is credited with reforming British mathematics by bringing rigor into it, which was previously a characteristic of French, Swiss, and German mathematics. In a 1947 lecture, the Danish mathematician Harald Bohr has said: "Nowadays, there are only three great English mathematicians: Hardy, Littlewood, and Hardy–Littlewood." [3]

3. Around the Dr. Zhu

In his arXiv preprint, Dr. Zhu has used very sophisticated mathematical calculations to conclude that the "Riemann Hypothesis is valid with 100 % probability." [4] I use this result, as well as another result in Ref. [5]. Still, I do not completely rely on Dr. Zhu's result, which is not peer-reviewed.

I highly dislike the idea of Dr. Zhu and other scientific philosophers that "100 % probability is NOT certainty." I cannot find mental wellness and peace of mind in trying to adopt this strange conviction. I believe that Dr. Zhu meant that 100 % of the zeroes of the Riemann zeta function are on the 1/2 critical line when he wrote his preprint title. But his result is not peer-reviewed. Therefore, I try to run peer-review on my own simple but (hopefully) rigorous approach.

Dr. Zhu's statement that the "Riemann Hypothesis is true" in the title of his arXiv paper has a probability of 100%. Thus, if I bet all my money and my health on the statement that the "Riemann Hypothesis is true", I cannot lose even in principle. It is like the statement "$x-5=0$ has a solution". It is not like the statement "one time only and blindly picked value of x happens to be the solution of $x - 5 = 0$". The probability is 100% for the first statement and zero for the second.

3.1. Making sense of Probability Theory.

If something has the probability of 1/3, it is like a bag contains three balls where one of them is blue while the other two are red. By taking the blue one from the bag (with closed eyes), the taker realizes a 1/3 probability event.

The probability 1 does not mean, that

A. the bag contains infinitely many blue balls and only one red ball; but rather that

B. the bag contains one blue ball and no red balls.

With the definition A of probability, one would mistakenly state that the probability for the equation $x - 5 = 0$ to have a solution is zero because the amount of numbers is infinite.

The "normalization condition" of the probability $1 = p + \bar{p}$ requires the comparison of the statement under study to the situation where a single ball is taken blindly from the bag of a finite amount of red and blue balls. This is because ∞/∞ is the mathematical uncertainty, not simply 1.

The probability that $x - 5 = 0$ has a solution is 100 %, but the probability that the following statement "one time only and blindly picked value of x happens to be the solution of $x - 5 = 0$" to be true is exactly zero; and because of the definition of Truth below, the event cannot happen.

If something has truly the probability of perfectly 100%, this something is true.
If something is true with 100% probability, then is truly true.

4. On number of counter-examples

If Robin's inequality $F(n) < 1$ is true, where F is certain function given in Ref. [7], Riemann Hypothesis turns out to be true. What is left to check today is the area $\exp(\exp(26)) < n < \infty$. [4, 6] A value $n = n_x$ is called "counter-example" if $F(n_x) > 1$.

There are two doorways for the falsehood of the Riemann Hypothesis: the number of counter-examples is either finite or infinite. Therefore, ruling out one of these doorways is an issue for the validity of the Riemann Hypothesis.

Please note that in Refs. [4] Dr. Zhu mentions the result that the number of counter-examples (if the Riemann Hypothesis is false) cannot be finite. However, I am demonstrating that fact in a much more simple way. To start with, I can express one of Dr. Zhu's results in a simpler way as:

If Robin's Inequality is true at least within $N < n < \infty$ where $N \gg 1$, Riemann Hypothesis is true.

Numerical tests have shown that Robin's Inequality holds at least for $n < \exp(\exp(26))$. Therefore, one has the right to assign $N = \exp(\exp(26)) \gg 1$. Accordingly, Dr. Zhu's result comes true in a natural manner.

Thesis:

If the number of counter-examples of Robin's Inequality can be only finite, there are no counter-examples.

Proof: Dr. Zhu's papers tell us that if Robin's Inequality is true for each $n > N$, Riemann Hypothesis is correct. If there is a finite number of counter-examples, one has a number M as well so that at least within $M < n < \infty$ there are no counter-examples to Robin's Inequality. As N and M can be properly chosen, one can assign $M = N$. Thus,

If Riemann Hypothesis fails, there must be an infinite number of counter-examples.

5. Absence of counter-examples

As the number of counter-examples X cannot be an arbitrary finite number, the exclusion includes very large finite numbers as well. Therefore, the unlimited $X \to \infty$ is ruled out as well. You might say that (for example) the equation $\sin y = 0$ has infinitely many points with $y = 0$. But it is fundamentally different from my situation: nobody has proven that there is indeed an infinite number of counter-examples. Finite situations include unlimited case, but the unlimited case does not include a finite situation.

5.1. **Second argument.** A well-established law is that within $0 < n < T$, in the limit $T \to \infty$ there are no more than $100 - 40 = 60\%$ counter-examples [5]. But within $0 < n < W$, where W is an arbitrary finite number, there can be any percentage of counter-examples, e.g. 70 %. This fact comes into conflict with Refs. [5] because it is $0 < n < \infty$ as well, as W can be any finite number. In other words, the cases with $0 < n < W$ include the widest range $0 < n < \infty$ as well because the W represents every single finite number from $0 < W < \infty$. We came to a logical contradiction, which means that the whole idea (that there can be counter-examples against the Riemann Hypothesis) fails. In other words, the percentage of counter-examples could be a specific but unknown function $p = p(W)$ of a finite W. The limit $W \to \infty$ contains the described contradiction. If there is function $p(W)$, then there cannot be a problem to get its values for large W. But there is the described problem. Therefore, there is no function $p(W)$. That means, this function is trivial, $p(W) \equiv 0$.

Therefore, there are no counter-examples.

References

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