Geometrisation of Electromagnetism

Alireza Jamali*

July 23, 2021

Abstract

After generalising the equivalence principle and introducing gravito-electric and gravito-magnetic transformations, we show that a metric which has the same form as the Kerr metric correctly describes the electrodynamics of a charged singularity.

Contents

1 Introduction

1.1 *Gravito-Electric transformation*

Shortly after Einstein's successful geometrisation of gravity attempts of geometrising electromagnetism began which had as their main goal unification of gravity and electromagnetism, not geometrisation of electromagnetism per se, for it was thought that a proper unification would automatically geometrise electromagnetism as well. The problem with such attempts soon turned out to be that they get easily misguided, for

*alireza.jamali.mp@gmail.com

there is no experimental nor theoretical clue as to what completion of electromagnetism one might expect from any such elusive unification. From a methodological point of view however, a complex task is often best done through decomposing it into simple parts. From this point of view any unification of general relativity and electromagnetism must first promote electromagnetism to the level of gravity by geometrising it. Only then, with two theories on equal grounds, we can proceed to unify them. The goal of this paper is to carry exactly such task of geometrising electromagnetism itself, without taking gravity into consideration. The higher goal of their unification will be pursued elsewhere in future.
According to our current understanding, proper treatment of situations involving both electromagnetism and gravity is carried by solving Einstein-Maxwell equations. The prototype solution of Einstein-Maxwell equations is the Reissner–Nordström metric,

$$ds^2 = -\left(1 - \frac{2Gm}{c^2 r} + \frac{Gq^2}{4\pi\epsilon_0 c^4 r^2}\right) c^2 dt^2 + \left(1 - \frac{2Gm}{c^2 r} + \frac{Gq^2}{4\pi\epsilon_0 c^4 r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

which describes spacetime around a spherically-symmetric charged source. Since according to the equivalence principle, spacetime and gravitational field are identical, an imporant consequence of Reissner–Nordström metric in the weak-field limit approximation, is that the electric charge q of a source contributes to the gravitational field of the source by the following gravitational potential

$$\phi_q = \frac{G}{8\pi\epsilon_0 c^2}\frac{q^2}{r^2},$$

On the other hand, in the Newtonian picture the source of gravity is mass. These considerations suggest an *electric charge-mass* equivalence, which can be clearly seen by observing how *gravito-electric transformation*,

$$m \to \frac{q}{\sqrt{4\pi\epsilon_0 G}} \quad \text{and} \quad \phi \to -\sqrt{4\pi\epsilon_0 G}\varphi \tag{1}$$

transforms Newtonian gravitational potential

$$\phi(r) = -\frac{Gm}{r},$$

into Coulomb potential

$$\varphi(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}.$$

This gravito-electric transformation suggests the explicit form of electric charge-mass equivalence, viz.

$$q \equiv m\sqrt{4\pi\epsilon_0 G}. \tag{2}$$

1.2 *Generalised* Equivalence Principle

But as

$$m\mathbf{a} = m\mathbf{g} + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{3}$$

the equivalence principle breaks in presence of electromagnetism. Consequently, *in presence of electromagnetism spacetime is not identical with* *the gravitational field*. On the other hand our current goal is to geometrise electromagnetism itself without taking gravity into consideration, we therefore neglect gravitational source m in (3), which yields an absurdity

$$0 = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

because it suggests either

$$\forall q = 0 : \quad \mathbf{E} + \mathbf{v} \times \mathbf{B} \neq 0$$

or

$$\forall q \neq 0 : \quad \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0,$$

Both cases are contradictory: According to the first case, in absence of gravity and its sources ($m = 0$), electromagnetic field can exist with no charged source (which, together with $m = 0$ means *no matter* whatsoever) in the world. According to the second case, in absence of gravity and its sources ($m = 0$), no electric field is created by presence of electric charge. The only way to resolve this paradox is to take into account the electric charge-mass equivalence and accordingly modify Newton's Second Law by addition of electric charge

$$\mathbf{F} = \left(m_i + \frac{q_i}{\sqrt{4\pi\epsilon_0 G}} \right) \mathbf{a}, \tag{4}$$

where q_i denotes *inertial* electric charge. Back to our goal of geometrisation of electromagnetism itself without taking gravity into consideration, neglecting gravity we now have

$$\frac{q_i}{\sqrt{4\pi\epsilon_0 G}} \mathbf{a} = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{5}$$

Assuming an analogue of equivalence principle holds for electric charge according to which $q_i = q_e$, we have

$$\mathbf{a} = \sqrt{4\pi\epsilon_0 G}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{6}$$

from which we conclude that *in absence of gravity, acceleration and electromagnetism are the same.* In other words, *in absence of gravity, spacetime continuum is identical to the electromagnetic field.* There is an apparent tension between the current interpretation 'acceleration and gravity are the same' and the one presented now, but it evaporates if we look from the right perspective of unification: electromagnetic field and gravitational field are one and the same and both are identical with spacetime. Accordingly we propose

Generalised Equivalence Principle *Spacetime continuum is identical with the totality of fundamental fields. It is the carrier of* ***all*** *fundamental forces of nature, not only gravity.*

This principle must be the foundation of any proper unification of gravity and electromagnetism; as such, it will not be used in this paper, for we have already restricted our work here to the approximation where gravity is absent. In this approximation we can safely assume that acceleration and electromagnetism are the same.

Now, similar to the way one motivates the passage from Newtonian gravity to its geometrisation, by comparing (6) and the geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau}$$

we conclude that

$$\frac{\sqrt{4\pi\epsilon_0 G}}{c^2}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \longleftrightarrow \Gamma^\mu_{\rho\sigma}$$

i.e. in geometrisation of electromagnetism (in absence of gravity), the electric and magnetic vector fields must be replaced by the affine (Levi-Civita) connection.

2 Geometrisation of Electricity

Assuming $\mathbf{B} = 0$, having introduced the gravito-electric transformation (1), we now how a shortcut to geometrise electricity alone. It sufficies to simply apply (1) to the Schwartzschild metric

$$ds^2 = -\left(1 - \frac{2GM/c^2}{r}\right)c^2dt^2 + \left(1 - \frac{2GM/c^2}{r}\right)^{-1}dr^2 + r^2d\Omega^2$$

$$ds^2 = -\left(1 + \frac{r_q}{r}\right)c^2dt^2 + \left(1 + \frac{r_q}{r}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{7}$$

where

$$r_q := \frac{2}{c^2}\sqrt{\frac{G}{4\pi\epsilon_0}}q. \tag{8}$$

Due to the appearance of velocity in (6), geometrising magnetism is a bit more subtle.

3 Geometrisation of Magnetism: Equivalence of rotation and magnetism

We claim that the electromagnetic analogue of the Kerr metric (which means only a change of its key parameters like J), represents the geometrised electromagnetic field in absence of gravity. To see this, note that *magnetism is not a genuine phenomenon*: it is created by 'movement of electricity' and it is electricity, which is caused by a fundamental propert of matter (electric charge) that is a fundamental force[1]. To proceed

[1] Analogues of electric charge for magnetism, i.e. magnetic charge (via magnetic monopoles) are proposed but they are not yet observed experimentally, but even if they are verified experimentally, including g (magnetic charge) in the metrics of GR will not reproduce Maxwell's electromagnetism in the weak-field limit, because it will be a copy of electrostatics without creating electro*dynamics*. So again that would be an 'artificial' unification, it will at best *defer* the problem, not solve it, because it will create a formally identical copy of electrostatics, but the intertwining dynamics is not geometrised.

with our proof, first note that the Kerr solution implies a *gravitational vector potential*[1] defined by

$$\mathbf{R} := \frac{G}{2c} \frac{\mathbf{J} \times \mathbf{r}}{|\mathbf{r}|^3} \tag{9}$$

where $\mathbf{J}$ is the angular momentum. This is similar in form to magnetic vector potential defined by

$$\mathbf{A} := \frac{\mu_0}{4\pi} \frac{\mathbf{d} \times \mathbf{r}}{|\mathbf{r}|^3} \tag{10}$$

where $\mathbf{d}$ is the magnetic dipole moment.
Since for a particle with electric charge q

$$d = \frac{1}{2} qrv,$$

if we compare d with

$$J = mrv,$$

using mass-charge equivalence we arrive at *gravito-magnetic transformation*

$$\mathbf{d} \equiv \sqrt{4\pi\epsilon_0 G}\mathbf{J} \tag{11}$$

$$\mathbf{A} \equiv \sqrt{\frac{\mu_0}{4\pi G}}\mathbf{R} \tag{12}$$

Applying gravito-electric and gravito-magnetic transformations *together*[2] to the Kerr parameter

$$a = \frac{J}{mc}$$

we arrive at

$$\boxed{\tilde{a} = \frac{1}{4\pi\epsilon_0 Gc} \frac{d}{q}} \tag{13}$$

Therefore by the substitutions

$$r_s \to r_q$$

and

$$a \to \tilde{a}$$

in the Kerr solution, we arrive at a metric representing *electrodynamics* around a charged singularity.

4 A long sweet dream or 'Do you *really* think Einstein could not think of what you say?'

It is now legit to ask for the *geometric* field equations of electromagnetism. For historical reasons and people's never-ending lust of slavish

[2]This is a clear sign that we are now doing electromagnetics.

pursuance of authority they expect something exotic from such a theory; (un)fortunately reality does not care about our wishes: the *bare mathematical* field equation of geometric electromagnetism are the same equations, only their physical interpretation changes. Everybody took Einstein's words for following some vague elusive idea about a dream, and this dream has become so sweet that people do not like to see it actuated. The first misconception began with the Einstein's GR article itself, in saying that

'We make a distinction between "gravitational field" and "matter" [...], that we denote *everything but the gravitational field* as "matter". Our use of the word therefore includes not only matter in the ordinary sense, *but the electromagnetic field as well.*'[2]

Here with 'everything but the gravitational field' began the first misconception by *singling out gravity* and people since then have started seeing matters in this way, but it is only the way things *are* not how they *should be*. This is the reason whenever I talk to people about comparing electric charge and mass they start talking mindlessly from gauge theory to QED and beyond without ever paying attention to the fundamental assumption that these theories are built upon: They are all based one way or another on $F = ma$, but singling out mass and gravity is built into this law without any objective reason: why electric charge is not included in $F = ma$? is it because people from the first days of the Enlightenment were aware of both mass and electric charge and somehow ruled out inclusion of electric charge? No, it is simply a consequence of history and people's lust for being followers, due to the inertia of consolidated ideas protected by authority and academia. The way things *should be* is that gravity and electromagnetism must be treated completely equally, but this simple guiding principle escaped even from Einstein. I am not afraid to say that Einstein, in his attempts for a unified field theory, did not have slightest idea of what he should pursue or what he was doing, and his confusion fed the prejudice that such a unification is almost inaccessible, 'If Einstein could not do it, who are we to do it?', a prejudice which still continues to harm scientific progress even to this day, being the very reason that this paper will be read with derision by those dogmatic academics who are feeding on this prejudicial sweet dream.

Let us enforce our guiding principle to make things the way they should be. We have already done what should be done about geometry by proposing the generalised equivalence principle and gravito-electromagnetic transformations. It only remains to resolve a misconception about the matter aspect, which again is rooted in Einstein's paper by including the electromagnetic field as matter. To achieve this goal, we need to first see how Maxwell equations arise as the weak-field limit of our proposed geometrisation of electromagnetism.

5 Maxwell equations as weak-field limit approximation

If we write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and $h_{\mu\nu}$ in our framework oughts to replace the electromagnetic four-potential. As we said earlier there is no new exotic equation and we must use the same equation that we use in geometrisation of gravity, namely Einstein Field Equations. In the weak-field limit (and Lorenz gauge) we therefore have

$$\Box h_{\mu\nu} = -2\tilde{\kappa} T^{\mathrm{EM}}_{\mu\nu} \tag{14}$$

which must be compared to

$$\Box A^{\mu} = \mu_0 J^{\mu}$$

which suggests the following definitions

$$\tilde{\kappa} := \frac{4\pi G}{c^4}$$

$$h_{0\nu} := -\frac{2\sqrt{4\pi\epsilon_0 G}}{c} A_{\nu}, \tag{15}$$

$$\boxed{T^{0\mu}_{\mathrm{EM}} := J^{\mu} \frac{c}{\sqrt{4\pi\epsilon_0 G}}} \tag{16}$$

and

$$A_0 = \varphi / c$$

To find the components of metric explicitly we pass to the dipole approximation of the vector potential

$$A_i = \frac{\mu_0}{4\pi} \frac{d^j x^k}{r^3} \epsilon_{ijk}.$$

where

$$d^j = \frac{1}{2} \int_V \epsilon^i{}_{jk} x^j J^k \, d^3x,$$

is the magnetic dipole moment.

It is now straightforward to show that by the following definitions

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

The metric

$$ds^2 = -\left(1 + \frac{2\sqrt{8\pi\epsilon_0 G}}{c^2}\varphi\right) c^2 dt^2 + \left(\frac{2\sqrt{8\pi\epsilon_0 G}}{c} A_{\nu}\right) dx^{\nu} dt + \left(1 - \frac{2\sqrt{8\pi\epsilon_0 G}}{c^2}\varphi\right) d\mathbf{x}^2 \tag{17}$$

yields Maxwell equations.

Lorentz force law will be naturally derived from the geodesic equation by the following approximate assumptions

$$\frac{dx^0}{d\tau} \approx 1, \quad \frac{dx^j}{d\tau} \approx v^i/c, \quad \text{static fields} : g_{\alpha\beta,0} = 0$$

6 Common misconception of $T_{\mu\nu}^{\mathbf{EM}}$

A new –and for academic mind unexpected– result of the previous section is the equation (16) which re-defines the electromagnetic energy-momentum tensor. Again, thinking by our guiding principle, there is little to explain here: just as the material source of gravity i.e. mass density ρ is continued to be present in the paradigm of general relativity by a change of dress, our guiding principle requires that the material source of electromagnetism i.e. current density four-vector must be present as well in the geometrisation of electromagnetism.

References

[1] Mashhoon, B. (2001). "Gravitomagnetism and the Clock Effect". arXiv:gr-qc/9912027

[2] Einstein, A. (1916). "The Foundation of the General Theory of Relativity". Collected Papers of Albert Einstein, Volume 6. pp. 146-200