A Simple Cellular Automaton Model of Visible and Dark Matter

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Abstract

We apply 6D phase space analysis and the deterministic Cellular Automaton Theory to propose a simple deterministic model for fundamental matter. Quantum mechanics, the Standard Model and supersymmetry are found to emerge from cell level. Composite states of three cells are manifested as quarks, leptons and the dark sector. Brief comments on unification, cosmology, and black hole issues are made.

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1 Introduction

We start from the familiar six dimensional phase space M of Hamiltonian mechanics and linearize a square form of **x** and **p** which have Born's reciprocity symmetry. We end up to isospin and hypercharge for particles. The eigenvalues of hypercharge turn out to be $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. We give these classical states a label: 1, 2, and 3.

The above results lead us to a three cell gear of the deterministic Cellular Automaton Theory (CAT) of ´t Hooft. Adopting Hilbert space methods in CAT leads to quantum mechanics and supersymmetry, like the Wess-Zumino supersymmetric model. Hence cells provide physical meaning to CAT. We will see also how quantum mechanics emerges from cells. Last, we adopt axion like particles (ALP) from string theory.

In our opinion, this cell scenario has properties required of a candidate for a model beyond the standard model (BSM), and it suggests a deterministic origin for quantum mechanics, long sought for by a few. We discuss whether our model is a candidate for ontological basis for the standard model and beyond. The key math points in favor of our scenario are available in the literature by various authors, but are consequently rather untethered. We collect them up into our scenario in a novel way into a model of visible and dark matter.

Unlike in Grand Unified Theories (GUT), unification is accomplished here in terms of small number of basic cells. Unification of gravity and electromagnetism may be done in 5D à la Kaluza-Klein but it is not discussed here. Strong and weak interactions are emergent and operate between SM particles only below a threshold value Λ_{cr}, which is near the GUT energy. As a bonus for cosmology, our scenario makes it possible to directly create the asymmetric visible universe from C-symmetric cells.

The article is organized as follows. In sections 2 we summarize briefly the concepts used to derive the main properties of our scenario: classical phase space, Born's reciprocity symmetry, Clifford algebra, deterministic quantum cells, emergent supersymmetry and very minimum of bosonic strings. Ontological questions are briefly touched. The structure of visible matter in terms of cells is presented in section 3. In section 4 candidates for dark matter are discussed. Gravitationally mediated supersymmetry breaking for SM particles is proposed in section 5. Delicate comments on black holes are made in section 6. Conclusions are given in section 7.

2 Theoretical Concepts

In this section we present a brief description of various, apparently unrelated theoretical concepts and methods, found in our exploration of literature. We go from Hamiltonian phase space M through cellular automata to Wess-Zumino supergravity. We end up in a simple CAT model. We do not give numerical predictions. Instead, our goal is finding a new kind of unity in physics. Namely constructing all matter from very few fundamental particles, or cells. We also present a derivation of quantum mechanics from deterministic cells.

2.1 Clifford Algebra, Hypercharge and Color

We start from non-relativistic phase space considerations and end up to spin and a formula for charge.[1] Born [1] studied the symmetrization of the roles of momenta and positions by the transformation $\mathbf{x} \to \mathbf{p}, \mathbf{p} \to -\mathbf{x}$. The symmetry holds in the zero mass limit. There are eight different orderings for the canonical positions and momenta. To us the interesting cases are the four even permutations shown in Table 1.

Position	Momentum
(x_1, x_2, x_3)	(p_1, p_2, p_3)
(x_1, p_2, p_3)	(p_1, x_2, x_3)
(p_1, x_2, p_3)	(x_1, p_2, x_3)
(p_1, p_2, x_3)	(x_1, x_2, p_3)

Table 1: Position-momentum even permutations.

Sixty years later Żenczykowski [2] (see also [3]) proposed the nimble conjecture that the four possibilities shown in Table 1 correspond to the first gen-

[1] We discuss spin and internal quantum numbers, which are valid concepts also non-relativistically.

eration leptons and three other states, which we call cells (see (2.6)). Let us see how this step was done. One may try the linearization of the 3D invariant $\mathbf{p}^2 = (\mathbf{p}\cdot\boldsymbol{\sigma})(\mathbf{p}\cdot\boldsymbol{\sigma})$. Linearization of the $x \leftrightarrow p$ symmetric expression $\mathbf{A}\cdot\mathbf{p}+\mathbf{B}\cdot\mathbf{x}$, where $\mathbf{A}$ and $\mathbf{B}$ are anticommuting objects, yields the result

$$\mathbf{A}\cdot\mathbf{p}+\mathbf{B}\cdot\mathbf{x} = \mathbf{p}^2+\mathbf{x}^2+R, \tag{2.1}$$

where the term R appears because x and p do not commute. A and B are eight-dimensional matrices

$$\begin{aligned} A_k &= \sigma_k \otimes \sigma_0 \otimes \sigma_1 \\ B_j &= \sigma_0 \otimes \sigma_j \otimes \sigma_2 \end{aligned} \tag{2.2}$$

R is the commutator of these matrices $R = -\frac{i}{2}\Sigma_k[A_k, B_k] = \Sigma_k\sigma_k \otimes \sigma_k \otimes \sigma_3$.

The seventh anticommuting element of the Clifford algebra in question is denoted as $B = iA_1A_2A_3B_1B_2B_3 = \sigma_0 \otimes \sigma_0 \otimes \sigma_3$. We define now

$$I_3 = \frac{1}{2}B, \quad Y = \frac{1}{3}RB \tag{2.3}$$

I_3 and Y commute with the operators describing ordinary 3D rotations and 3D reflections in phase space. The eigenvalues of I_3 and Y are

$$I_3 = \pm\frac{1}{2}, \quad Y = -1, +\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3} \tag{2.4}$$

I_3 and Y are candidates for two new quantum numbers. A reasonable conjecture is that (2.4) are identified with the Gell-Mann–Nishijima formula for charge Q

$$Q \equiv \frac{1}{6}\Big[(\mathbf{p}^2+\mathbf{x}^2)_{vac} + R\Big]B = I_3 + \frac{Y}{2} \tag{2.5}$$

where the first term denotes its the lowest eigenvalue of $\mathbf{p}^2+\mathbf{x}^2$, which is three. I_3 is the weak isospin and Y hypercharge. The eigenvalues of Q are therefore (0, +2/3, +2/3, +2/3, -1, -1/3, -1/3, -1/3). They are the charges of the Standard Model particles.

The correspondence between the phase space approach and the cell model is obtained from (2.4)

$$\begin{aligned} Y &= -1 \leftrightarrow m_1^0 m_2^0 m_3^0 \\ Y_R &= 1/3 \leftrightarrow m_1^+ m_2^+ m_3^0 \\ Y_G &= 1/3 \leftrightarrow m_2^+ m_3^0 m_1^+ \\ Y_B &= 1/3 \leftrightarrow m_3^0 m_1^+ m_2^+ \end{aligned} \tag{2.6}$$

where m's in (2.6) are classical particles, cells, with a label 1 for the first m^+, 2 for the second, and 3 for the m^0 (on lines 2-4). The subscript labels 1, 2, and 3 are independent of the physical properties of the cells. The lines 2-4 are distinguished by the position of the m_3^0 cell, see also Table 2.

There has been from time to time a hope to discover a deterministic theory behind quantum mechanics. We discuss this question in the next subsection 2.2. Deterministic cell behavior would release us from the requirements of uncertainty relations for almost pointlike particles (like perhaps $r \sim r_{\mathrm{Cartan}}$ of the electron or even l_{Pl}) implying high cell mass.

Finkelstein has given arguments, consistent with the ones in this subsection, for the possible existence of cell-like preons based on the quantum group $SLq(2)$ [4].

2.2 Deterministic Cells and Quantum Mechanics

What kind of equations of motion do cells obey? We adopt here the Cellular Automaton theory proposed by ´t Hooft [5, 6, 7] and use cells as the fundamental states of an automaton and matter.

A cellular automaton is a D-dimensional lattice in (D+1)-dimensional spacetime. Each cell, a line crossing, carries a limited amount of information, one or more numbers. The evolution law gives deterministically the values of the cells at time t+1 given the values of nearby cells at time t.

Quantum behavior enters when some information from the system is lost, of either position, momentum or due to a constraint. Referring to (2.4), we are interested here in three state systems.

Discrete time case. A simple prototype case is the D = 2 three-state gear system with a cyclic deterministic evolution of states $|1\rangle \to |2\rangle \to |3\rangle \to |1\rangle$, as indicated in Figure 1 (taken from [7]). It can be treated eithet classically or quantum mechanically without any modification of the physics.

This three gear system defines cell confinement inside quarks and leptons. This cinfinement took place in the early hot universe, before the known non-Abelian gauge interactions started to play a role.

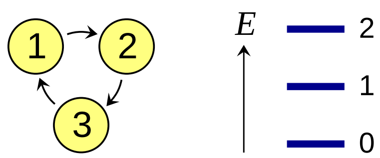

Figure 1: Gear model with three states and three energy levels.

It is advantageous to describe the automaton in terms of a Hilbert space. With this three state gear system the following Hilbert space is associated [7]

$$|\psi\rangle = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle \tag{2.7}$$

The time evolution $t_i \to t_{i+1}$ may be represented by the following unitary matrix U

$$\psi_{t+1} = U(t,t+1)\psi_t = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \psi_t \tag{2.8}$$

The probability of the system being in state $|i\rangle$ is defined as usually $P_i = |a_i|^2$. It is seen that conservation of probability corresponds to unitarity of the evolution matrix U.

In a basis in which U is diagonal, it has for a single time step the form

$$U(t+1,t) = \exp(-iH\Delta t) \tag{2.9}$$

where

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2\pi/3 & 0 \\ 0 & 0 & 2\pi/3 \end{pmatrix} \tag{2.10}$$

The eigenstates of this matrix. are

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{3}}\big(|0\rangle + |1\rangle + |2\rangle\big) \\ |1\rangle &= \frac{1}{\sqrt{3}}\big(|0\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}|1\rangle + \mathrm{e}^{-2\pi \mathrm{i}/3}|2\rangle\big) \\ |2\rangle &= \frac{1}{\sqrt{3}}\big(|0\rangle + \mathrm{e}^{-2\pi \mathrm{i}/3}|1\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}|2\rangle\big) \end{aligned} \tag{2.11}$$

and we have

$$U(\Delta(t))\begin{pmatrix} |0\rangle \\ |1\rangle \\ |2\rangle \end{pmatrix} = \begin{pmatrix} |0\rangle \\ \mathrm{e}^{-2\pi \mathrm{i}/3}|1\rangle \\ \mathrm{e}^{-4\pi \mathrm{i}/3}|2\rangle \end{pmatrix} \tag{2.12}$$

In this basis, we can write this as

$$U = \mathrm{e}^{-\mathrm{iH\Delta t}} \tag{2.13}$$

where $H = \frac{2\pi}{3\Delta t}$ diag(0, 1, 2).

At times $t = \text{const} \times \Delta\text{t}$ we have, in every basis,

$$U(t) = \mathrm{e}^{-\mathrm{iHt}} \tag{2.14}$$

In terms of the original states $|1\rangle, |2\rangle$ and $|3\rangle$, the Hamiltonian (2.15) reads

$$H = \frac{2\pi}{3\Delta t}\begin{pmatrix} 1 & \kappa & \kappa^* \\ \kappa^* & 1 & \kappa \\ \kappa & \kappa^* & 1 \end{pmatrix} \tag{2.15}$$

where $\kappa = -\frac{1}{2} + \frac{i\sqrt{3}}{6}$ and κ^* its complex conjugate.

Now we can conclude that a state (2.7) obeys the Schrödinger equation

$$\frac{d}{dt}|\psi\rangle = -iH|\psi\rangle \tag{2.16}$$

where H is defined by (2.15). It will be in the state described by the gear model at all times t that are an integral multiple of Δt. This is enough reason to claim that the model obeying this Schrödinger equation is mathematically equivalent to the deterministic gear model. The present cell cell model is a concrete implementation of the CAT program for the SM, and beyond.

The eigenvalues of the Hamiltonian are

$$E_n = \frac{2\pi}{3\Delta t} n, \quad n = 0, 1, 2 \tag{2.17}$$

as indicated in Figure 1. What are the energy states $\{E_0, E_1, E_2\}$? In atomic physics, an atom with spin one is subject to Zeeman splitting in a homogeneous magnetic field. Analogously, two cells with total spin one may experience the same splitting in the magnetic field of the third cell (like in a nucleon the quark-diquark effect) or rather some more complicated CA phenomenon. This kind of an effect may explain the three generations of the SM particles.

Terminology. At this point we have to clarify terminology. A quantum theory in the Heisenberg picture is deterministic if a complete set of operators $O_i(t)$ $(i = 1, .., N)$ exist, such that

$$[O_i(t), O_j(t')] = 0 \;\; \text{for all t, t}'; \;\text{i, j} = 1, .., \text{N} \tag{2.18}$$

These operators are called beables. The above three-state gear system is deterministic in this sense [5].

On classical and quantum mechanics we quote Witten *'... the symmetry groups of classical mechanics and quantum mechanics are different.'* [8]. In the former case the symmetry group is the group G of canonical transformations of the phase space M. In quantum mechanics the the group is the group U of unitary transformations of Hilbert space.

In [7] the deterministic gear system in Figure 1 is described by a Hilbert space vector (2.7), which has unitary time evolution as indicated in (2.8). The symmetry group of the {1, 2, 3} gear is the permutation group. Secondly, we believe the arguments of subsection 2.1 are at least approximately valid for our limited purposes.

Classical and quantum mechanics have an interesting cross section [7]: *Deterministic quantum mechanics is neither a modification of standard quantum mechanics, nor a modification of classical theory. It is a cross section of the two. This cross section is claimed to be much larger and promising than usually thought.*

Ontological system is defined as follows [7]: *"A physical state $|A\rangle$, where A may stand for any array of numbers, not necessarily integers or real numbers, is called an ontological state if it is a state our deterministic system can be in. These states themselves do not form a Hilbert space, since in a deterministic theory we have no superpositions, but we can declare that they form a basis for a Hilbert space ..."*

The cell model of this subsection would seem to be a simple candidate for the ontological basis for the standard models of particles and cosmology. Larger systems are beyond the scope of this note.[2]

[2]It would be, however, interesting to contemplate of a connection to Cellular Neural Networks [9] having the code for a mathematical solution.

Free Cells. During cosmological early times the temperature was very high and matter was in its most primitive form, which means cells in our scenario. Why cells form composite states or gears of three cells is a physical process but at present it is a postulate of the model. It may be considered as a phase transition for which details have to be found. A few words more are indicated in section 3.

Continuous time case. We follow in the rest of this subsection closely the treatment presented by Blasone, Jizba, and Kleinert in [10]. Classical systems of the form

$$H = p_a f^a(q) \tag{2.19}$$

evolve deterministically even after quantization [6, 7, 10]. This happens since in the Hamiltonian equations of motion

$$\begin{aligned} \dot{q}^a &= \{q^a, H\} = f^a(q) \\ \dot{p}_a &= \{p_a, H\} = -p_a \partial f^a(q)/\partial q^a \end{aligned} \tag{2.20}$$

the equation for the q^a does not contain p^a, making the q^a beables.

Now we have to stop because the Hamiltonian is not bounded from below. This defect can be revised by a constraint [6, 7, 10]. Consider a function $\rho(q_a) > 0$ with $[\rho, H] = 0$ and divide the Hamiltonian in two parts

$$\begin{aligned} H &= H_+ - H_- \\ H_+ &= \frac{1}{4\rho}(\rho + H)^2 \\ H_- &= \frac{1}{4\rho}(\rho - H)^2 \end{aligned} \tag{2.21}$$

where H_+ and H_- are positive definite operators satisfying

$$[H_+, H_-] = [\rho, H] = 0 \tag{2.22}$$

We may now enforce the following constraint to the Hamiltonian to get rid of the spectrum problem

$$H_-|\psi\rangle = 0 \tag{2.23}$$

Then the eigenvalues of H in $H|\psi\rangle = H_+|\psi\rangle = \rho|\psi\rangle$ are positive and the equation of motion

$$\frac{d}{dt}|\psi\rangle = -iH|\psi\rangle \tag{2.24}$$

has only positive frequencies. If there are stable orbits with period $T(\rho)$, then $|\psi\rangle$ satisfies

$$e^{-iHT}|\psi\rangle = |\psi\rangle, \quad \rho T(\rho) = 2\pi n,\ n \in \mathbb{Z} \tag{2.25}$$

so that the associated eigenvalues are discrete. 't Hooft motivated the constraint (2.23) by information loss. More details of information loss and periodicity, energy spectra, equivalence classes, limit cycles etc. are in [7].

Path Integral Quantization. A powerful technique for quantization is proposed by Faddeev and Jackiw in [11]. The authors start by observing that a Lagrangian for 't Hooft's equations of motion (2.20) can be simply taken as follows

$$L(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, \dot{\mathbf{p}}) = \mathbf{p} \cdot \dot{\mathbf{q}} - H(\mathbf{p}, \mathbf{q}) \tag{2.26}$$

with $\mathbf{q}$ and $\mathbf{p}$ being Lagrangian variables. Note that L does not depend on $\dot{\mathbf{p}}$. It is easily verified that the Euler-Lagrange equations for the Lagrangian (2.26) indeed coincide with the Hamiltonian equations (2.20). Thus given 't Hooft's Hamiltonian (2.19) one can always construct a first-order Lagrangian (2.26) whose configuration space coincides with the Hamiltonian phase space. By defining 2N configuration space coordinates as

$$\xi^a = p_a, \ a = 1, ..., N; \quad \xi^a = q_a, \ a = N + 1, ..., 2N \tag{2.27}$$

the Lagrangian (2.26) can be cast into the more expedient form, namely

$$L(\xi, \dot{\xi}) = \frac{1}{2}\xi^a \omega_{ab} \dot{\xi}^b - H(\xi) \tag{2.28}$$

where ω is the $2N \times 2N$ matrix

$$\omega_{ab} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}_{ab} \tag{2.29}$$

which has an inverse $\omega_{ab}^{-1} \equiv \omega^{ab}$. The equations of motion read

$$\dot{\xi}^a = \omega^{ab} \frac{\partial H(\xi)}{\partial \xi^b} \tag{2.30}$$

indicating that there are no constraints on ξ. Thus the procedure of [11] makes the system unconstrained, so that the path integral quantization may proceed in a standard way. The time evolution amplitude is [12]

$$\langle \xi_2, t_2 | \xi_1, t_1 \rangle = \mathcal{N} \int_{\xi_1}^{\xi_2} \mathcal{D}\xi \exp\left(\frac{i}{\hbar} \int_{t_1}^{t_2} dt L(\xi, \dot{\xi}) \right) \tag{2.31}$$

where $\mathcal{N}$ is a normalization factor. Since the Lagrangian (2.26) is linear in p, we may integrate these variables out and obtain

$$\langle q_2, t_2 | q_1, t_1 \rangle = \mathcal{N} \int_{q_1}^{q_2} \mathcal{D}q \prod_a \delta[\dot{q}^a - f^a(q)] \tag{2.32}$$

where $\delta[f] \equiv \Pi_t \delta(f(t))$ is the functional version of Dirac's δ-function. Hence the system described by the Hamiltonian (2.19) retains its deterministic character even after quantization. The paths are squeezed onto the classical trajectories determined by the differential equations $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q})$. The time evolution amplitude (2.37) contains a sum over only the classical trajectories. There are no quantum fluctuations driving the system away from the classical paths.

The equation (2.32) can be brought into more intuitive form by utilizing the identity

$$\delta[f(q) - \dot{q}] = \delta[q - q_{\rm cl}](\det(M)^{-1} \tag{2.33}$$

where where M is a functional matrix formed by the second functional derivatives of the action $\mathcal{A}[\xi] \equiv \int dt L(\xi, \dot{\xi})$

$$M_{a,b}(t,t') = \frac{\delta^2}{\delta\xi^a(t)\delta\xi^b(t')} \,|_{\mathbf{q}=\mathbf{q}_{\rm cl}} \tag{2.34}$$

Morse index theorem [13] ensures that for sufficiently short time intervals $t_2 - t_1$ (before the system reaches its first focal point), the classical solution with the initial condition $q(t_1) = q_1$ is unique. In such a case (2.32) can be brought in the form

$$\langle q_2, t_2 | q_1, t_1 \rangle = \frac{\mathcal{N}}{\det M} \int_{q_1}^{q_2} \mathcal{D}\delta(q - q_{\rm cl}) \tag{2.35}$$

indicating transparently the classical behavior.

2.3 Emergent Supersymmetry

We now turn to an interesting implication of the result (2.35) [10]. If we had started in (2.32) with an external current

$$\tilde{L}(\xi, \dot{\xi}) = L(\xi, \dot{\xi}) + i\hbar \mathbf{J} \cdot \mathbf{q} \tag{2.36}$$

integrated again over $\mathbf{p}$, and took the trace over $\mathbf{q}$, we would end up with a generating functional

$$\mathcal{Z}[\mathbf{J}] = \frac{\mathcal{N}}{\det M} \int \mathcal{D}\delta(q - q_{\rm cl}) \exp\left(\int_{t_1}^{t_2} dt \; \mathbf{J} \cdot \mathbf{q} \right) \tag{2.37}$$

The path integral (2.37) has an interesting mathematical structure. One may rewrite it as

$$\mathcal{Z}[\mathbf{J}] = \frac{\mathcal{N}}{\det M} \int \mathcal{D}\mathbf{q} \delta\left[\frac{\delta\mathcal{A}}{\delta\mathbf{q}}\right] \left| \frac{\delta^2 \mathcal{A}}{\delta q_a(t) \delta q_a(t')} \right| \times \exp\left[\int_{t_1}^{t_2} dt \mathbf{J} \cdot \mathbf{q} \right] \tag{2.38}$$

Introduce two real time dependent Grassman ghost variables $c_a(t)$ and $\bar{c}_a(t)$, fermion field λ_a, and two anticommuting coordinates θ and $\bar{\theta}$. The latter pair of variables extends the configuration space of $\mathbf{q}$ variables into superspace. The superfield is defined

$$\Phi_a(t, \theta, \bar{\theta}) = q_a(t) + i\theta c_a(t) - i\bar{\theta}\bar{c}_a(t) + i\bar{\theta}\theta\lambda_a(t) \tag{2.39}$$

Together with the identity $\mathcal{D}\Phi = \mathcal{D}q\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}\lambda$ we may therefore express the classical partition functions (2.37) and (2.38) as a supersymmetric path integral with fully fluctuating paths in superspace

$$\mathcal{Z}_{CM}[\mathbf{J}] = \int \mathcal{D}\Phi \exp\left\{ -\int d\theta d\bar{\theta} \mathcal{A}[\Phi](\theta, \bar{\theta}) \right\} \times \exp\left\{ \int dt d\theta d\bar{\theta}\, \Gamma(t, \theta, \bar{\theta}) \Phi(t, \theta, \bar{\theta}) \right\} \tag{2.40}$$

where the supercurrent is $\Gamma(t,\theta,\bar{\theta}) = \bar{\theta}\theta\mathbf{J}(t)$. A specific case of supersymmetry, namely Wess-Zumino supergravity, is discussed in the next section 2.4. There we consider the kinetic Lagrangians for our scenario.

2.4 Supergravity

We discuss tentatively a relativistic case for cells. Our previous preon scenario [14, 15] turned out to have close resemblance to the simplest N=1 globally supersymmetric 4D model, namely the free, massless Wess-Zumino model [16, 17] with the kinetic Lagrangian including three neutral fields m, s, and p with $J^P = \frac{1}{2}^+, 0^+$, and 0^-, respectively

$$\mathcal{L}_{\mathrm{WZ}} = -\frac{1}{2}\bar{m}\partial\!\!\!/ m - \frac{1}{2}(\partial s)^2 - \frac{1}{2}(\partial p)^2 \tag{2.41}$$

where m is a Majorana spinor, s and p are real fields (metric is mostly plus).

We assume that the pseudoscalar p is the axion [18], and denote it below as a. It has a fermionic superparther, the axino n, and a bosonic superpartner, the saxion s^0.

In order to have visible matter we assume the following charged chiral field Lagrangian

$$\mathcal{L}_{-} = -\frac{1}{2}m^{-}\partial\!\!\!/ m^{-} - \frac{1}{2}(\partial s_i^{-})^2, \quad i = 1,2 \tag{2.42}$$

The R-parity of cells is simply $P_R = (-1)^{2\times spin}$.

2.5 Bosonic String

Torsion of spacetime originates from General Relativity. Here we use a shorter introduction for it, as well as for the axion, from bosonic string theory. A point particle has one dimensional world line with a tangent vector $dx^\mu(\tau)/d\tau$, where τ is the world line parameter [19]. The tangent vector and the Maxwell field can be multiplied to form a Lorentz scalar. The interaction of a point particle of charge e with the Maxwell gauge field is written as $e\int \frac{dx^\mu(\tau)}{d\tau}A_\mu(x(\tau))d\tau$.

The endpoints of open strings may carry electric charge. But having two Lorentz indexes we hope to discover a new kind of charge that could be contracted with the string indexes. Such a field is the Kalb-Ramond antisymmetric tensor $B_{\mu\nu} = -B_{\nu\mu}$. It is a massless closed string. The obvious way to write a Lorentz scalar with two string tangent vectors of the form $\partial X^\lambda/d\rho$ is

$$-\int \frac{\partial X^\mu}{d\tau}\frac{\partial X^\nu}{d\sigma}B_{\mu\nu}\big(X(\tau,\sigma)\big)d\tau d\sigma \tag{2.43}$$

This describes how a string carrying electric Kalb-Ramond charge couples to the antisymmetric Kalb-Ramond field. The new field strength associated to $B_{\mu\nu}$ is $H_{\mu\nu\rho}$ is defined by

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \tag{2.44}$$

The $H_{\mu\nu\rho}$ plays the same role as torsion in General Relativity providing an anti-symmetric component to the affine connection.

The total action, analogous to the corresponding Maxwell action, is

$$S = S_{str} - \frac{1}{2}\int \frac{\partial X^{[\mu}}{d\tau}\frac{\partial X^{\nu]}}{d\sigma} B_{\mu\nu}\big(X(\tau,\sigma)\big) d\tau d\sigma + \int d^D x\Big(-\frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big) \quad (2.45)$$

where $x^{[\mu}y^{\nu]} \equiv x^\mu y^\nu - x^\nu y^\mu$. S_{str} includes General Relativity. In summary, the bosonic string oscillation include these (26D) quantum fields: the symmetric metric tensor $G_{\mu\nu}(X)$, the antisymmetric $B_{\mu\nu}(X)$, and the scalar $\phi(X)$.

In 4D the equations of motion imply that the dual of H field strength, $\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho}$ can be represented as $\partial^\sigma b(x)$, where $b(x)$ is a pseudoscalar, the Kalb-Ramond axion. It is a generalization of Peccei-Quinn axion. We will discuss axions and torsion in later sections.

3 Visible Matter

Visible, or the SM matter, has been discussed in detail in [14, 15], when 'preon' is interpreted as a 'cell'. The first generation standard model fermions are formed combinatorially (mod 3) of three cells, which are the charged $m^\pm$, with charge $\pm\frac{1}{3}$, and the neutral m^0, as composite states below an energy scale Λ_{cr} [15]. The new substance of this note is that the old 'preons' are now cells and obey the laws of CAT.

What happens at high temperatures at $t \gtrsim T_{\rm Pl}$ in the early universe? Most likely, the density of energy is so high that structure formation is not yet possible. The classical cell permutation is the simplest event, or interaction, which may form states obeying the laws of cellular automatons in discrete time. So, during the next phase, on logarithmic time scale, two cell and cell-anticell permutation pairs are formed. Two state automatons are not interesting unless they form a scalar or vector particle. Three cell permutation systems are important since they form quarks, leptons and dark fermions. These particles are behaving like quantum objects in continuous time, as described in subsection 2.2. Systems of more cells may form, leading to introduction of (local gauge) equivalence classes [7]. These questions are beyond the scope of this note. The deconfinement temperature of cells making composite states, $T = \Lambda_{cr}$, is for the moment a free parameter. Numerically $\Lambda_{cr} \sim 10^{10-16}$ GeV, somewhat above the reheating temperature since at reheating there must be SM particles, i.e. visible matter.

We clarify our earlier notation for cells, towards what is used in subsection 2.1, as indicated in Table 2. There we see that for the three u quarks, $u_{R,G,B}$, the m^0 is permuted on line two from position three to two and on the next line from position two to one. Similarly for the d quark the m^- is rotated through the same positions. Leptons consist of three like cells which can be identity rotated ($|1\rangle \to |1\rangle$) as labeled cells. In summary, quarks, leptons and the corresponding dark fermions o are considered in this model as consisting of cells obeying deterministic equation of motion discussed in subsection 2.2.

SM Matter	Cell state
ν_e	$m^0m^0m^0$
u_R	$m^+m^+m^0$
u_G	$m^+m^0m^+$
u_B	$m^0m^+m^+$
e^-	$m^-m^-m^-$
d_R	$m^0m^0m^-$
d_G	$m^0m^-m^0$
d_B	$m^-m^0m^0$
Dark Matter	Particle
boson(system)	axion(s), s^0, p
o (3 n composite)	n
black hole	any cell

Table 2: Visible and Dark Matter, and corresponding particles.

4 Dark Matter

Literature on dark matter, dark energy, and axions is extensive, see e.g. [20, 21, 22, 23, 24]. In this section we also patch our shortage in [15] to consider the pseudoscalar of (2.41). So we start from the Lagrangian (2.41).

As stated in the previous section 2.4, the superpartners of the axion a are the fermionic axino n, and the scalar saxion s^0, also indicated in Table 1.[3] Particle dark matter consists of all these three particles. The axino n may appear physically as single particle dust or three n composite o dust, gas, or a large astronomical object. The fermionic DM behaves naturally very differently from bosonic DM, which may form Bose-Einstein condensates.

Other candidate forms of DM include primordial black holes (PBH). They can be produced by gravitational instabilities induced from scalar fields such as axion-like particles or multi-field inflation. It is shown in [25] that PBH DM can be produced only in two limited ranges of 10^{-15} or 10^{-12} Solar masses (2×10^{30} kg). Dark photons open a rich phenomenology described [26]. We also mention another supergravity (the graviton-gravitino supermultiplet) based model [27], which may help to relieve the observed Hubble tension [28].

The axion was originally introduced to solve the strong CP problem in quantum chromodynamics (QCD) [18], see also [29, 30]. The Peccei-Quinn axion has a mass in the range 10^{-5} eV to 10^{-3} eV. Axions, or axion-like particles (ALP),

[3]In this note we mostly talk about all spin zero particles freely as scalars.

occur also in string theory in large numbers (in the hundreds), they form the axiverse.

The axion-like particle masses extend over many orders of magnitude making them distinct candidate components of dark matter. Ultra-light axions (ULA), with masses 10^{-33} eV $< M_a < 10^{-20}$ eV, roll slowly during inflation and behave like dark energy before beginning to oscillate (as we see below). The lightest ULAs with $M_a \lessapprox 10^{-32}$ eV are indistinguishable from dark energy. Higher mass ALPs, $M_a \gtrapprox 10^{-25}$ eV behave like cold dark matter [24]. Quantum mechanically, an axion of mass of, say 10^{-22} eV, has a Compton wavelength of 10^{16} m.

Ultra-light bosons with masses $\ll$ eV can form macroscopic Bose-Einstein condensates, such as axion stars [31, 32]. Due to the small mass the occupation numbers of these objects are large, and consequently, they can be described classically.

The fermionic axino n is supposed to appear, like the m cells, as free particle if $T > \Lambda_{cr}$ and when $T \lessapprox \Lambda_{cr}$ in composite states, gears. If the mass of the axino composite state o is closer to the electron mass rather than the neutrino mass it may form 'lifeless' dark stars in a wide mass range. In general, dark matter forms haloes with galaxies residing within.

To obtain a feeling of the possible roles of axions let us go briefly into the early universe. Axions, as well as the whole dark sector, are treated as spectator fields during and after inflation [20, 21, 22].[4] The axion is massless as long as non-perturbative effects are absent. When these effects are turned on the PQ symmetry is broken and the axion acquires a mass. A minimally coupled scalar field ϕ in General Relativity has an action

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\Big] \tag{4.1}$$

The cells m^+, m^-, m^0 form gears, i.e. quarks and leptons, during inflation. Unlike dark particles, the SM particles couple to the inflaton. When the inflaton potential reaches its minimum the high mass inflaton condensate oscillations cause reheating. So only the visible matter bangs due to coupling to the inflaton. The visible and dark matter are somewhat differently distributed in the universe because the latter interact only weakly.

In the Friedmann-Lemaitre-Robertson-Walker metric with potential $V = \frac{1}{2}M_a^2\phi^2$ [5] the axion equation of motion is

$$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}_0 + M_a^2 a^2 \phi_0 = 0 \tag{4.2}$$

where ϕ_0 is the homogeneous value of the scalar field as a function of the conformal time τ, a is here the cosmological scale factor, and dots denote derivatives with respect to conformal time.

[4] On the other hand, the axion can be modeled as causing the inflation [23].

[5] This is an adequate approximation over most of the parameter space observationally allowed provided $f_a < M_{\rm Pl}$. The potential is anyway unknown away from the minimum without a model for nonperturbative effects.

At an early time $t_i \gtrsim 10^{-36}$s, $M_a \ll H$ and the axion rolls slowly. If the initial velocity is zero it has equation of state $w_a \equiv P_a/\rho_a \simeq -1$. Consequently, the axion is a component of dark energy. With $t > t_i$ the temperature and H decrease and the axion field begins to oscillate coherently at the bottom of the potential. This happens when

$$M_a = 3H(a_{osc}) \tag{4.3}$$

which defines the scale factor a_{osc}. Now the number of axions is roughly constant and the axion energy density redshifts like matter with $\rho_a \propto a^{-3}$. The relic density parameter Ω_a is

$$\Omega_a = \left[\frac{1}{2a^2}\dot{\phi}_0^2 + \frac{M^2 a}{2}\phi_0^2\right]_{M_a^2=3H} a_{osc}^3/\rho_{crit} \tag{4.4}$$

where ρ_{crit} is the cosmological critical density today. Explicit estimates for the relic density are given in [24]. This applies to all axion-like particles, if there are many like in string theory.

When radiation and matter match in ΛCDM model the Hubble rate is $H(a_{eq}) \sim 10^{-28}$ eV. Axions with mass larger than 10^{-28} eV begin to oscillate in the radiation era and may provide for even all of dark matter. The upper limit of the ultralight axion mass fraction Ω_a/Ω_{DM}, where Ω_a is the axion relic density and Ω_{DM} is the total DM energy density parameter, varies from 0.6 in the low mas end 10^{-33} eV to 1.0 in the high mass limit 10^{-24} eV. In the middle region Ω_a/Ω_{DM} is constrained to be below about 0.05 [24].

In [33] it is proposed that the matter-antimatter asymmetry was manifested by direct production of asymmetric visible matter from C-symmetric cells.

5 Supersymmetry Breaking

There are several ways supersymmetry may get broken, and they are described extensively in a number of articles, reviews and textbooks [34, 35, 36]. To us an obvious method is the gravitationally mediated scenario. Supersymmetry is unbroken in the cell sector and is mediated by gravitational interaction to the visible minimal supersymmetric standard model (MSSM sector by soft term contributions, which means that the Lagrangian has two terms: symmetric and symmetry breaking

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{soft} \tag{5.1}$$

where $\mathcal{L}_{soft}$ violates supersymmetry but only by mass terms and coupling constants having positive mass dimension.

The brief description is that if supersymmetry is broken in the cell sector by a vev $\langle F \rangle$ then the soft terms in the visible sector are expected to be approximately $M_{soft} \sim \langle F \rangle / M_{\text{Pl}}$. For $M_{soft} \sim 200$ GeV one would estimate that the scale associated with supersymmetry breaking in the cell sector is about $\sqrt{\langle F \rangle} \sim 10^{10}$

or 10^{11} GeV, which must be below Λ_{cr} for consistency. This way the MSSM soft terms arise indirectly or radiatively, instead of tree-level renormalizable couplings to the supersymmetry breaking parameters. The gravitino mass is of the order of the masses of the MSSM sparticles. The gravitino in turn mediates the symmetry breaking with gravitational coupling to the MSSM. A gravitino mass of the order of TeV gives a lifetime 10^5 s, long enough not to disturb nucleosynthesis by decay products.

6 Black Holes

Here we make a brief remark concerning the quantum numbers of holes and cells. The quantum numbers of cells were chosen to be the same as those of black hole, namely mass, spin and charge. This has two consequences. In the early universe black holes were created just like cells or gears, depending on the energy and volume. Secondly, black holes can emit nearly massless cell pairs by the Hawking mechanism. If the hole is light enough the radiation is intensive producing clouds of cells which may combine into SM particles. Maybe cells are the nearly massless remnants of black holes.

Let us make a Gedanken experiment. A not too large amount of visible (or dark matter as well) matter has fallen into a black hole. The temperature of the matter increases substantially, above Λ_{cr} near the horizon. Consequently matter fallen in makes a phase transition into cell matter and looses all SM internal quantum numbers. Fermions at high density do not collapse into singularity because they form gears. Alternatively, torsion of spacetime causes a repulsive four fermion contact interaction, or some quantum effect takes place. The hole starts emitting massless cell-anticell pairs from near the horizon. The Hawking radiated cells will combine later into SM particles and form again visible and dark matter. Has there been information loss? If the particles fell into a classical singularity all information of them is obviously lost. On the other hand, if the singularity is smoothed out by some effect the situation is more interesting. The matter fallen in the hole cannot disappear. The radiated cells need not make quite the same celestial matter as fell in but it is matter of the same type. This would be in accordance with the common principle of cyclic processes of nature - like the cyclic universe.

The second Gedanken experiment is the following. A particle is Hawking radiated from a hole and its antiparticle falls into it. If the singularity is smoothed out the antiparticle does not disappear. If it would happen that the antiparticle of the antiparticle, i.e. the same type of particle, would happen to fall from the other side of the black hole into it and meet the antiparticle the situation would look like particle going through the black hole. It is deep elastic scattering off a black hole. If this would happen the singularity of the hole would be proven not to exist. Unfortunately, this may be impossible to verify experimentally, except perhaps in condensed matter physics or other analog experiments.

In our modest approach the question of quantum gravity is, at the moment at least, limited to the existence and nature of the black hole singularity. Our opinion is that there may not be such a physical singularity, though rigorously proven by general relativists. It may be that cells and black holes are on the same 'trajectory' but with the slope varying between zero and the overspinnig value of Kerr holes $J = M^2$, and a wholly radiated black hole may, if not must, leave a cell remnant.

7 Conclusions

By defining the fundamental fields as deterministic cells in Table 2 it has been possible to develop a model for visible matter as well as for the dark sector and sketch briefly how they behave during inflation. Theoretical arguments, if not partial proof, for the scenario were given in section 2. Further, we loft the idea whether the present model is a simple candidate for the ontological basis for the standard models of particles (and beyond) and cosmology.

The scenario is compatible with visible matter-antimatter asymmetry. The symmetric dark sector includes both fermionic and bosonic fields, which may conglomerate into objects of various sizes. The bosonic sector of (2.41) contains axion-like particles, a string theory concept. They are obvious candidates for bosonic dark matter are axions when $M_a \gtrapprox 10^{-25}$ eV and dark energy when $M_a \lessapprox 10^{-32}$ eV.

The deterministic nature of cells provides insight to the origin of the universe and nature of quantum mechanics. Cells had in the early universe, $T \gg \Lambda_{cr}$, no interactions (except what may be related to torsion). When $T \lessapprox \Lambda_{cr}$ during early inflation quarks and leptons are formed of cells by CAT gear forming mechanism, yet to be studied. Quantum nature of matter is mainly related to the standard model particles which obey the non-Abelian $SU(3)_C$ and $SU(2)_W \times U(1)_Y$ gauge symmetries. The non-Abelian interactions are essential for nuclear- and astrophysics, chemistry and biology. The two Abelian interactions are able to form a universe with less nuances.

We anticipate that the gear system, as a part of CAT, and unification require more work. In this scenario, unification takes place by all matter consisting of very few supersymmetric cells in 4D. Not by introducing unified gauge groups with large number of states, extra dimensions or multiverse. However, a common problem is the proton life time in $p \to e^+\pi^0$.

To build this scenario to a community deliberate level, or disprove it, simulations have to be done, more detailed Lagrangians be written and calculated. Phenomenological work is to be carried out with current data for many details like supersymmetry breaking and particle masses while waiting for future accelerator and celestial precision experiments to be carried out in the years, and a decade, to come. Machine learning and quantum computing may provide powerful methods for new type of quantitative studies.

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