EINSTEIN-ROSEN PROPOSITION (1935) REVISITED.

 ISBN 978-2-36923-114-1.

12 July 2021

This document discovers an important coincidence between a mathematical and a physical problem.

Contents

1 Chapter II

The first chapter has analyzed the dispersion relation of light in vacuum with the binoculars of the theory of deformed cross products.

1.1 The historical context.

1.1.1 The initial data problem in general relativity.

Einstein's master work [01-a; see a translation for example in 01-b] is published in 1916. In 1935, Einstein and Rosen propose in [02] a very original concept for the description of particles within a specific context which can be obtained in starting from the prescriptions exposed in [01]. The proposition was presumably supposed to allow a correct understanding of the atomic structures; at least the ones which was known at this time. In 1944, A. Lichnerowicz writes his famous equations [03-a]; see also [04-c; §8.2.4, pp. 130-131]. They are then reworked by J. York. Approximately thirty years later, Bowen and all. proposes solutions for the York-Lichnerowicz initial data problem (see [04-c; chapter 8; §8.2.6, pp. 136-139]).

1.1.2 Claim of the document.

The claim of this document is to prove the existence of a link between these Bowen solutions and non-trivial decompositions for deformed angular momentum obtained within a mathematical theory studying deformed Lie products.

1.2 Mathematical context.

1.2.1 Abbreviations and specificities.

The lecture will be easier if the following abbreviations are kept in mind:

- DCP: deformed cross product
- BH: black hole
- TEQ: theory of the (E) question

The decompositions of DCPs[1] have been extensively studied in [a], [b], [c] and [d]. There always exists at least one trivial decomposition for any given DCP; it is the representation of some rotation. Sometimes, there are also non-trivial ones. All of them (trivial or not) can be associated with a polynomial form of degree at most two which is nothing but the determinant of the difference between these two kinds of decompositions. These generic considerations apply in peculiar for DCPs of the following type (see explanation below[2]):

$$[d\mathbf{x}, ...]_{[A]} \tag{1.1}$$

The main part of its most trivial decomposition:

$$_{[A]}\Phi(d\mathbf{x})$$

is not always in coincidence with the main part [P] of some non-trivial decomposition; as consequence, the polynomial of interest is the difference:

$$\Lambda(d\mathbf{x}) \,=\, |_{[A]}\Phi(d\mathbf{x}) \,-\, [P]| \tag{1.2}$$

[1] The central topic of the TEQ

[2] Here "d" denotes an ordinary derivation (see Descartes or Leibniz), $\mathbf{x}$ is the spatial position for some event, $\boldsymbol{...}$ is any spatial vector and [A] represents either an anti-symmetric or a reduced cube with elements in M(3,C) or M(3,R).

ISBN 978-2-36923-114-1, v5, 12 July 2021.

1.2.2 Proposition

Provided:

1. That difference can be identified with at least one Taylor - Mac Laurin development until the second order for some numerical function f depending on the three spatial components of a given position (x, y, z) which I shall abusively write f($\mathbf{x}$);

2. The spatial gradient of that function depends on the inverse of the square of the Euclidean classical norm of the position:

$$\mathbf{Grad}f(\mathbf{x}) \sim \frac{1}{r^2} \,.\, \mathbf{e};\ r = ||\mathbf{x}|| = (< \mathbf{x}, \mathbf{x} >_{Id_3})^{1/2},\ ||\mathbf{e}|| = 1$$

then:

1. That difference is a non-degenerated polynomial of degree two;

2. Its singular vector, ${}_{\Lambda}\mathbf{s}$ coincides with the spatial position $\mathbf{x}$;

3. There exists a non-trivial decomposition of which the main part [P] is such that the spatial vector just below can be identified in a coherent way with a solution of [03-a], more precisely of the "Bowen-York type" (for the initial data problem) [04-c; §8.2.6, p. 136, (8.69)]:

$$k \,.\, {}^{(3)}[G]^{-1} \,.\, {}^{(3)}[P] \,.\, |{}^{(3)}\mathbf{p} >$$

Here: (i) k is some scalar of which I shall reveal the meaning during the demonstration; (ii) the inverse metric is spatial, local, conformally flat and degenerated; and (iii) $\mathbf{p}$ denotes a very classical Euclidean kinetic momentum.

1.3 Demonstration

1.3.1 Realizing the prerequisites of the proposition.

Let start a discussion about DCPs of the type which has been given with the Equ.(1.1). And let suppose *a priori* that they have non-trivial decompositions allowing:

$$|[d\mathbf{x},\ ...]_{[A]} >= [P]\,.\,|... > +|\mathbf{z}> \tag{1.3}$$

As consequence of the so-called *initial theorem* -read [a]- we now that this hypothesis gives a more precise visage to Equ.(1.2):

$$\Lambda(d\mathbf{x}) \;=\; |_{[A]}\Phi(d\mathbf{x}) \;-\; [P]| \;=\; d_{ij}\,.\,dx^i\,.\,dx^j \;+\; d_i\,.\,dx^i \;+\; d \tag{1.4}$$

Whatever the values of the coefficients are, this formalism evocates the one of a Taylor-Mac Laurin development. It is such a development each time there exists a numerical function f($\mathbf{x}$) such that:

$$f(\mathbf{x} + d\mathbf{x}) \;=\; \Lambda(d\mathbf{x}) \tag{1.5}$$

$$d \;=\; f(\mathbf{x})$$

$$d_i \;=\; \frac{\partial f(\mathbf{x})}{\partial x^i}$$

$$d_{ij} \;=\; \frac{1}{2}\,.\,\frac{\partial^2 f(\mathbf{x})}{\partial x^j \partial x^i} \;+\; 0(3)$$

Let a priori reduce the discussion to special situations such that:

$$d_i \;=\; -\frac{G\,.\,m}{r^3}\,.\,x^i \;\Longleftrightarrow\; \mathbf{d}^* \;=\; -\frac{G.m}{r^3}\,.\,\mathbf{x} \tag{1.6}$$

$$r \;=\; ||\mathbf{x}|| \;=\; (<\mathbf{x},\mathbf{x}>_{Id_3})^{1/2}$$

The Equ.(1.5) and (1.6) realize the prerequisites of proposition 1.2.2.

1.3.2 A criterion to know if the polynomial Λ is degenerated or not.

The coefficients of degree two can be put inside a (3-3) matrix:

$$[D] \;=\; [d_{ij}] \;=\; \frac{1}{2}\,.\,[\frac{\partial^2 f(\mathbf{x})}{\partial x^j \partial x^i}] \;+\; [0(3)]$$

In the case at hand, this matrix get a more precise visage because:

$$\frac{\partial^2 f(\mathbf{x})}{\partial x^j \partial x^i} \;=\; \frac{\partial d_i}{\partial x^j} \;=\; -\,G.m.\frac{\partial(\frac{x^i}{r^3})}{\partial x^j} \;=\; -\,\frac{G\,.\,m}{r^6}\,.\,(\delta^i_j\,.\,r^3 - x^i\,.\,3\,.\,r^2\,.\,\frac{\partial r}{\partial x^j})$$

Since:

$$\frac{\partial r}{\partial x^j} = \frac{x^j}{r} \tag{1.7}$$

It is easy to state that:

$$[Hess_{(\mathbf{x},\,0)} f(\mathbf{x})] = -\frac{G\,.\,m}{r^3}\,.\,\{Id_3 - \frac{3}{r^2}\,.\,\phi\} \tag{1.8}$$

with, per convention and for simplicity:

$$\phi = \begin{pmatrix} x^1\,.\,x^1 & x^2\,.\,x^1 & x^3\,.\,x^1 \\ x^1\,.\,x^2 & x^2\,.\,x^2 & x^3\,.\,x^2 \\ x^1\,.\,x^3 & x^2\,.\,x^3 & x^3\,.\,x^3 \end{pmatrix} = [x^i\,.\,x^j] = T_2(\otimes)(\mathbf{x},\mathbf{x}) \tag{1.9}$$

We also know that (see [a]):

$$[Hess_{(d\mathbf{x},\,0)} \Lambda(d\mathbf{x})] = [D] + [D]^t$$

and we remark that the matrix [D] is symmetric as long as the position is real; hence:

$$[Hess_{(d\mathbf{x},\,0)} \Lambda(d\mathbf{x})] = 2\,.\,[D] = [Hess_{(\mathbf{x},\,0)} f(\mathbf{x})] \tag{1.10}$$

The consequence of that relation is that the calculation of the determinant of the Hessian of f furnishes a criterium to know if the polynomial Λ is degenerated or not.

1.3.3 The polynomial is not degenerated.

Let now calculate the determinant of the Hessian of f and get:

$$|Hess_{(\mathbf{x},\,0)} f(\mathbf{x})| = -2.\,(\frac{G\,.\,m}{r^3})^3 \tag{1.11}$$

Except for vanishing sources (m = 0) or at infinity (r $\rightarrow \infty$), this quantity never vanishes. The Hessian of the Λ polynomial at hand is a non-degeneratd matrix and the singular vector can now be discovered.

1.3.4 The singular vector of the polynomial Λ.

The inverse of the Hessian is:

$$[Hess_{(d\mathbf{x},\,0)}\Lambda(d\mathbf{x})]^{-1} \;=\; -\frac{2\,.\,r^3}{G\,.\,m}\,.\,\{Id_3 \;-\; \frac{3}{2\,.\,r^2}\,.\,\phi\} \qquad (1.12)$$

As consequence, the singular vector is:

$$|_{\Lambda}\mathbf{s}>$$
$$=$$
$$-[Hess_{(d\mathbf{x},0)}\Lambda(d\mathbf{x})]^{-1}\,.\,|\mathbf{d}^*>$$
$$\downarrow\; Equ.(1.6)$$
$$=\frac{G\,.\,m}{r^3}\,.\,[Hess_{(d\mathbf{x},0)}\Lambda(d\mathbf{x})]^{-1}\,.\,|\mathbf{x}>$$
$$\downarrow\; Equ.(1.12)$$
$$=\;-2\,.\,\{Id_3 \;-\; \frac{3}{2\,.\,r^2}\,.\,\phi\}\,.\,|\mathbf{x}>$$

Since:

$$\phi\,.\,|\mathbf{x}>=\;r^2\,.\,|\mathbf{x}> \qquad (1.13)$$

This is in fact yielding the first important coincidence of this work:

$$|_{\Lambda}\mathbf{s}>=\;-2\,.\,|\mathbf{x}>\;+3\,.\,|\mathbf{x}>=\;|\mathbf{x}> \qquad (1.14)$$

The prerequisites of that demonstration have an important consequence: *the polynomial at hand is not degenerated and the spatial position coincides with a singular vector.* Recall that a singular vector is, per definition, minimizing the slopes.

1.3.5 The degree zero coefficient of the polynomial.

Recall the generic formula -see [a]:

$$-|P|\;=\;d\;=\;\frac{|Hess_{(d\mathbf{x},0)}\Lambda(d\mathbf{x})|}{8}\;+\;<_{\Lambda}\,\mathbf{s},\;_{\Lambda}\mathbf{s}>_{[D]} \qquad (1.15)$$

Due to Equ.(1.8), (10), (11) and (1.14), it can be rewritten as:

$$-|P|\;=\;d\;=\;\frac{-\,2.\,(\frac{G\,.\,m}{r^3})^3}{8}\;+\;\frac{1}{2}\,.\;<\mathbf{x},\;\mathbf{x}>_{[Hess_{(\mathbf{x},0)}\Lambda(\mathbf{x})]}$$

Let calculate the second term on the r.h.s. Since the Hessian is not coinciding with the identity matrix Id_3, that term is not the square of the Euclidean distance; in fact:

$$< \mathbf{x}, \mathbf{x} >_{[Hess_{(\mathbf{x},0)}\Lambda(\mathbf{x})]}$$

$$=$$

$$-\frac{G \,.\, m}{r^3} \,.\, < \mathbf{x}| \,.\, \{\{Id_3 - \frac{3}{r^2} \,.\, \phi\}.|\mathbf{x} >\}$$

$$=$$

$$\frac{2 \,.\, G \,.\, m}{r}$$

Hence:

$$-|P| = d = \frac{G \,.\, m}{r} - \frac{1}{4} \,.\, \frac{G^3 \,.\, m^3}{r^9} \tag{1.16}$$

The degree zero coefficient of the polynomial Λ is a modified expression of the newtonian gravitational potential.

1.3.6 The polynomial.

Assembling all parts of the discussion, we get:

$$\Lambda(d\mathbf{x}) \tag{1.17}$$

$$=$$

$$|_{[A]}\Phi(d\mathbf{x}) - [P]|$$

$$=$$

$$-\frac{G \,.\, m}{r^3} \,.\, \{\delta^i_j - \frac{3}{r^2} \,.\, x^i \,.\, x^j\} \,.\, dx^i \,.\, dx^j$$

$$- \frac{G \,.\, m}{r^3} \,.\, x^i \,.\, dx^i$$

$$+ \frac{G \,.\, m}{r} - \frac{1}{4} \,.\, \frac{G^3 \,.\, m^3}{r^9}$$

1.3.7 The main part of the decomposition.

In that context, the generic results which have been obtained in [a] have a peculiar visage:

$$[P]_{|A|} = |A| \,.\, \{[A]^t \,.\, [J]\} \,.\, \{-\frac{G\,.\,m}{2\,.\,r^3} \,.\, \{Id_3 - \frac{3}{r^2} \,.\, T_2(\otimes)(\mathbf{x},\, \mathbf{x})\} + {}_{[J]}\Phi(\mathbf{x})\} \tag{1.18}$$

Remark that the DCPs

$$[\mathbf{x}, ...]_{[A]}$$

have in general extrinsic decompositions of which the main part is -see [b]:

$$[Q]_{|A|} = {}_{[A]}\Phi(\mathbf{x}) +$$

And remark also that all DCPs are antisymmetric operations:

$$[\mathbf{x}, ...]_{[A]} = -[..., \mathbf{x}]_{[A]}$$

This statement is true for deformed angular momentum too:

$$[\mathbf{x},\, d\mathbf{x}]_{[A]} = -[d\mathbf{x},\, \mathbf{x}]_{[A]}$$

Anyway, the Equ.(1.18) can be detailled as:

$$_{|A|}p_{ij} = |A| \,.\, \{[A]^t \,.\, [J]\}^{ik} \,.\, \{-\frac{G\,.\,m}{2\,.\,r^3} \,.\, \{\delta^k_j - \frac{3}{r^2} \,.\, x^k \,.\, x^j\} + \epsilon_{klj} \,.\, x^l\}$$

1.3.8 Forming the Bowen solutions.

Let now form:

$$\frac{r^2}{6\,.\,G\,.\,m} \,.\, {}_{|A|}p_{ij} \,.\, p^j$$
$$=$$
$$\frac{r^2}{6\,.\,G\,.\,m} \,.\, \{[A]^t \,.\, [J]\}^{ik} \,.\, \{-\frac{G\,.\,m\,.\,|A|}{2.r^3} \,.\, \{\delta^k_j - \frac{3}{r^2} \,.\, x^k \,.\, x^j\} + \epsilon^k{}_{lj} \,.\, x^l\} \,.\, p^j$$
$$=$$
$$-\frac{|A|}{12.r} \,.\, \{[A]^t \,.\, [J]\}^{ik} \,.\, \{\delta^k_j - \frac{3}{r^2} \,.\, x^k \,.\, x^j\} \,.\, p^j$$
$$+ \frac{r^2}{6\,.\,G\,.\,m} \,.\, \{[A]^t \,.\, [J]\}^{ik} \,.\, \epsilon^k{}_{lj} . x^l \,.\, p^j$$

$$
\begin{aligned}
= \\
& -\frac{|A|}{12\,.\,r}\,.\,\{[A]^t\,.\,[J]\}^{ij}\,.\,p^j \\
& + \frac{|A|}{4\,.\,r^3}\,.\,\{[A]^t\,.\,[J]\}^{ik}\,.\,x^k\,.\,x^j\,.\,p^j \\
& + \frac{r^2}{6\,.\,G\,.\,m}\,.\,\{[A]^t\,.\,[J]\}^{ik}\,.\,\epsilon^k{}_{lj}\,.\,x^l\,.\,p^j
\end{aligned}
$$

Let now consider the Bowen-York solutions for Einstein's initial data problem - see for example [[04]; (b) §8.2.6, p.136, (8.69)], [[04]; (c) p. 23, (69)] and [[05]; p.3, (28) and (29)]:

$$X_i \;=\; -\frac{7}{4\,.\,r}\,.\,f^{ij}\,.\,p^j \;-\; \frac{1}{4\,.\,r^3}\,.\,x^i\,.\,x^j\,.\,p^j \;-\; \frac{1}{r^3}\,.\,\epsilon^i{}_{lj}\,.\,x^l\,.\,J^j$$

And state that both relations are identic:

$$\frac{r^2}{6\,.\,G\,.\,m}\,.\,{}_{|A|}p_{ij}\,.\,p^j \;=\; X_i \tag{1.19}$$

If, simultaneously:

$$
\begin{aligned}
-\frac{|A|}{12\,.\,r}\,.\,\{[A]^t\,.\,[J]\}^{ij}\,.\,p^j \;&=\; -\frac{7}{4\,.\,r}\,.\,f^{ij}\,.\,p^j && (1.20) \\
|A|\,.\,\{[A]^t\,.\,[J]\}^{ik}\,.\,x^k\,.\,x^j\,.\,p^j \;&=\; -x^i\,.\,x^j\,.\,p^j \\
\frac{r^2}{6\,.\,G\,.\,m}\,.\,\{[A]^t\,.\,[J]\}^{ik}\,.\,\epsilon^k_{lj}\,.\,x^l\,.\,p^j \;&=\; -\frac{1}{r^3}\,.\,\epsilon^i{}_{lj}\,.\,x^l\,.\,J^j
\end{aligned}
$$

1.4 Studying the coherence.

1.4.1 Whatever the pair (**x**, d**x**) is.

Let examine if these relations are coherent; in fact, they are in peculiar realized when:

$$
\begin{aligned}
\frac{r^2}{6\,.\,G\,.\,m}\,.\,{}_{|A|}p_{ij}\,.\,p^j \;&=\; X_i \\
\forall\,\mathbf{p}:\; |A|\,.\,\{[A]^t\,.\,[J]\}^{ij} \;&=\; 21\,.\,f^{ij}
\end{aligned}
$$

$$\forall\, \mathbf{x}, \mathbf{p} :\ |A| \,.\, \{[A]^t \,.\, [J]\}^{ik} \,.\, x^k \ =\ -x^i$$

$$\forall\, \mathbf{x} :\ \{[A]^t \,.\, [J]\}^{ik} \,.\, \epsilon^k_{lj} \,.\, p^j \ =\ -\,\frac{6 \,.\, G \,.\, m}{r^5} \,.\, \epsilon^i{}_{lj} \,.\, J^j$$

The coincidence:

$$|\mathbf{Bowen - York} >= \frac{r^2}{6 \,.\, G \,.\, m} \,.\, [P]_{-1} \,.\, |\mathbf{p} >$$

between:

- (i) the Bowen solutions and
- (ii) the vectors that can be built with the solutions of the (E) question studying the non-trivial decompositions of d$\mathbf{x}$ $\wedge$ **...** when the prerequisites exposed in the proposition 1.2.2 are realized:

is effective for any pair ($\mathbf{x}$, d$\mathbf{x}$) when:

$$\forall\, \mathbf{p} :\ [A]^t \,.\, [J] \ =\ -21 \,.\, [f^{ij}]$$

$$\forall\, \mathbf{x}, \mathbf{p} :\ \{[A]^t \,.\, [J] \ -\ Id_3\} \,.\, |\mathbf{x} >=\ |\mathbf{0} >$$

$$\forall\, \mathbf{x} :\ \{[A]^t \,.\, [J]\} \,.\, {}_{[J]}\Phi(\mathbf{p}) \ =\ -\,\frac{6.G.m}{r^5} \,.\, {}_{[J]}\Phi(\mathbf{J})$$

This means that:

1. The product between the transposed of the deforming matrix and the generator of the cyclic group C6 is a peculiar representation of some metric; precisely here, the spatial, local, conformally flat and degenerated metric which is forming the geometric background for the Bowen solutions:

$$^{(3)}[A]^t \,.\, [J] \ =\ ^{(3)}[G]^{-1} \ =\ -21 \,.\, [f]^{-1}$$

2. An important characteristic of that peculiar metric can be proved in writing the second relation of coherence with the first one:

$$\forall\, \mathbf{x}, \mathbf{p} :\ \{[G]^{-1} \ -\ Id_3\} \,.\, |\mathbf{x} >=\ |\mathbf{0} >$$

Recall that the analysis yielding the Bowen solutions is done within an ADM context. In that context, **p** is the constant ADM kinetic momentum and **J** is the constant ADM angular momentum of the source.

In a classical three-dimensional Euclidean geometry, the angular momentum is orthogonal to the kinetic momentum since, per definition:

$$\mathbf{J} = \mathbf{x} \wedge \mathbf{p}$$

This unavoidable necessity suggests that the classical Euclidean three-dimensional geometry characterized by [G] = Id_3 cannot belong to the Bowen solutions; indeed:

$$[G] = Id_3 \Rightarrow [G]^{-1} = Id_3$$

In injecting this relation into the third condition, we get:

$$\forall \mathbf{x}, \mathbf{p} : {}_{[J]}\Phi(\mathbf{p}) = \frac{6 \,.\, G \,.\, m}{r^5} \cdot {}_{[J]}\Phi(\mathbf{J})$$

And, since ${}_{[J]}\Phi$ is an isomorphism:

$$\mathbf{p} = \frac{6 \,.\, G \,.\, m}{r^5} \,.\, \mathbf{J} = \frac{6 \,.\, G \,.\, m}{r^5} \,.\, \mathbf{x} \wedge \mathbf{p}$$

This obviously is a mathematical non-sense, the consequences of which are:

$$[G]^{-1} = {}^{(3)}[A]^t \,.\, [J] \neq Id_3$$

and:

$$\forall \mathbf{x}, \mathbf{p} : |[G]^{-1} - Id_3| = 0$$

This constraint is equivalent to:

$$\begin{aligned}
&(g^{11} \,.\, g^{22} \,.\, g^{33} + g^{12} \,.\, g^{31} \,.\, g^{23} + g^{13} \,.\, g^{21} \,.\, g^{32}) \\
&- (g^{11} \,.\, g^{32} \,.\, g^{23} + g^{22} \,.\, g^{31} \,.\, g^{13} + g^{33} \,.\, g^{21} \,.\, g^{12}) \\
&+ (g^{12} \,.\, g^{21} + g^{23} \,.\, g^{32} + g^{13} \,.\, g^{21}) \\
&- (g^{11} \,.\, g^{22} + g^{22} \,.\, g^{33} + g^{33} \,.\, g^{11}) \\
&+ (g^{11} + g^{22} + g^{33}) - 1 = 0
\end{aligned}$$

3. The components of the angular momentum can be isolated:

$$\epsilon_{ikr} \,.\, g^{im} \,.\, \epsilon_{mjk} \,.\, p^j \;=\; \frac{6\,.\,G\,.\,m}{r^5} \,.\, J^r$$

Starting from this equation, the quantified norm of the angular momentum can be calculated if the ADM metric is not degenerated (|G| $\neq 0$):

$$g_{rt} \,.\, J^r \,.\, J^t \;=\; n\,.\,(n\,+\,1)\,.\,\hbar^2;\; g_{rt} \;\neq\; \delta_{rt},\; n \;=\; 0,\; 1,\; 2,\; 3...$$

4. Supposing that the induced metric is not degenerated (|G| $\neq 0$), it is possible to rewrite the ADM kinetic momentum as:

$${}_{[J]}\Phi(\mathbf{p}) \;=\; -\,\frac{6.G.m}{r^5} \,.\, [G] \,.\, {}_{[J]}\Phi(\mathbf{J})$$

From which:

$${}_{[J]}\Phi^t(\mathbf{p}) \;=\; -\,\frac{6.G.m}{r^5} \,.\, {}_{[J]}\Phi^t(\mathbf{J}) \,.\, [G]^t$$

But:

$${}_{[J]}\Phi^t(\mathbf{p}) \;=\; -\,{}_{[J]}\Phi(\mathbf{p})$$

As consequences:

$$[G] \,.\, {}_{[J]}\Phi(\mathbf{J}) \;-\; {}_{[J]}\Phi(\mathbf{J}) \,.\, [G]^t \;=\; {}^{(3)}[0]$$

$${}_{[J]}\Phi(\mathbf{p}) \;=\; -\,\frac{3\,.\,G\,.\,m}{r^5} \,.\, \{[G] \,.\, {}_{[J]}\Phi(\mathbf{J}) \;+\; {}_{[J]}\Phi(\mathbf{J}) \,.\, [G]^t\}$$

- When the admissible induced metrics [G] are symmetric, they commute with the rotations related to the ADM angular momentum $\mathbf{J}$:

$$\forall\, r,\, m,\, \mathbf{J},\, [G] \;=\; [G]^t \,:\, [[G]\,,\, {}_{[J]}\Phi(\mathbf{J})] \;=\; {}^{(3)}[0]$$

and:

$$\forall\, r,\, m,\, \mathbf{J},\, [G] \;=\; [G]^t \,:\, {}_{[J]}\Phi(\mathbf{p}) \;=\; -\,\frac{3\,.\,G\,.\,m}{r^5} \,.\, \{[G],\, {}_{[J]}\Phi(\mathbf{J})\}$$

- When the admissible induced metrics [G] are anti-symmetric, they anti-commute with the rotations related to the ADM angular momentum **J**:

$$\forall\, r,\, m,\, \mathbf{J},\, [G] \;=\; [G]^t \,:\, \{[G]\,,\,{}_{[J]}\Phi(\mathbf{J})\} \;=\; {}^{(3)}[0]$$

and:

$$\forall\, r,\, m,\, \mathbf{J},\, [G] \;=\; -[G]^t \,:\, {}_{[J]}\Phi(\mathbf{p}) \;=\; -\,\frac{3.G.m}{r^5}\,.\,[[G]\,.\,{}_{[J]}\Phi(\mathbf{J})]$$

These relations and their formalism suggest a possible link with two important concepts: (i) the one of spinor which has been introduced by E. Cartan in [06]; and (ii) the other one of propagator which has been developed by A. Lichnerowicz too, e.g. in [[03]-b] a long time ago (1964).

1.4.2 The case of the classical cross product.

The cross product corresponds to the matrix [A] = [J] and, because of that, to: |A| = -1. This is the situation we should concentrate on and start with because it is the classical one in which we live in. Previous results prove clearly that the classical Euclidean three-dimensional configuration is never exactly realized within the ADM context allowing the expected coincidence.

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