Theory about rounding of real numbers.

Juan Elias Millas Vera

juanmillaszgz@gmail.com

Zaragoza (Spain)

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0- Abstract:

With a very intuitive notation, we are going to see in this paper a solution of the approximation to a determinate position digit in whole and fractional numbers. We are going to use ceiling and floor functions to give precision to the rounding.

1- Introduction:

Firstly, we should see the main notation for the different digits number following the decimal positioning system, we are going to name <a> to the whole part and <b> to the fractional part.

$$(1) \quad a_m a_{(m-1)} \dots a_2 a_1 a_0 . b_1 b_2 b_3 \dots b_{(n-1)} b_n$$

Where:

a_0 correspond to ones, a_1 to the tens and a_2 to the hundreds.
b_1 correspond to tenths, b_2 to the hundredths and b_3 to the thousandths.

Now with a given number $a_m . b_n$ we will use the ceiling ($\lceil a_m . b_n \rceil$) or the floor ($\lfloor a_m . b_n \rfloor$) function. We will use a positional determinant s in the super index, this will show us which is the relevant last number in the approximation. The super index will be an indeterminate variable a_m or b_n and it will precise which determinate position we should choose end with.

2- Theory:

Theorem 1: The main rounding of the number could be in the whole part, and it will be cut in a determinate last right position number a_x , we should substitute all numbers cutting off in the right part of the determinate number, with zeros until the position of ones a_0 . If the last number chosen are in the fractional part we just delete (substituting for zeros) all numbers in the right part and rounding to the determinate b_x .

Proposition 1: The possible combinations of the method are four of them: 1- $(\lceil a_m . b_n \rceil)^{(ax)}$ 2- $(\lfloor a_m . b_n \rfloor)^{(ax)}$ 3- $(\lceil a_m . b_n \rceil)^{(bx)}$ and 4- $(\lfloor a_m . b_n \rfloor)^{(bx)}$. The first one express a ceiling function with and ending in the whole part, the second one a floor function with an ending in the whole part, the third a ceiling function with and ending in the fractional part and the fourth a floor function with and ending in the fractional part.

Proposition 2: In the ceiling function the rounding will be plus one (+1) to the determinate a_x or b_x not depending which is the $a_{(x-1)}$ or the $b_{(x+1)}$ previous or next. All $a_{(x-n)}$ until a_0 will be zeros (0s) and all $b_{(x+n)}$ will be zero until $-\infty$. Except when all the right numbers until $-\infty$ are zero (0) then we should add zero (+0) to the chosen number.

- **Lemma 1:** If the chosen determinate variable is in a determinate case a number nine (9) in a_x instead of plus one (Proposition 2), we should substitute with a zero (0) and add one (+1) in $a_{(x+1)}$.
- **Lemma 1.1:** If $a_{(x+1)}$ is also a nine (9) we should substitute with a zero (0) $a_{(x+1)}$ and add one (+1) to $a_{(x+2)}$. If we have an iteration we should do this steps until $a_{(x+n)}$ is not a nine and then add one (+1) to these number.
- **Lemma 2:** If the chosen determinate variable is in a determinate case a number nine (9) in b_x instead of plus one (Proposition 2) we should substitute with a zero (0) and add one (+1) in $b_{(x-1)}$.
- **Lemma 2.1:** If $b_{(x-1)}$ is also a nine (9) we should substitute with a zero (0) $b_{(x-1)}$ and add one (+1) to $b_{(x-2)}$. In this case too if we have an iteration we should continue doing this until $b_{(x-m)}$ is not a nine.
- **Lemma 2.2:** If we have a necessary iteration of substitutions of nines to zeros until b_1 and we pass to a_0 we should continue doing substitutions in the whole part and add one (+1) to the non-nine number. (Ex.11)

It will be interesting to notice that in some cases there are possibilities to obtain the same result in a different ceiling functions. ($(\lceil a_m . b_n \rceil)^{(ax)}$ or $(\lceil a_m . b_n \rceil)^{(bx)}$). This happens in the exposed situations of Lemmas 1 and 2. In this situations $(\lceil a_m . b_n \rceil)^{(ax)}$ = $(\lceil a_m . b_n \rceil)^{(ax+1)}$, when $a_x = 9$ and $(\lceil a_m . b_n \rceil)^{(bx)}$ = $(\lceil a_m . b_n \rceil)^{(bx-1)}$, when $b_x = 9$. We will see in the examples part in (Ex.16 and Ex.17)

Proposition 3: The rounding will be plus zero (+0) in the a_x or b_x in the floor function. All $a_{(x-n)}$ until a_0 will be zeros (0s) and all $b_{(x+n)}$ will be zero until $-\infty$.

3-Examples:

1. $\lceil 32.0 \rceil^{(a0)} = 32.0$ 2. $\lceil 32.0 \rceil^{(b1)} = 32.0$ 3. $\lceil 32.0 \rceil^{(a1)} = 40.0$

4. $\lceil 32.2 \rceil^{(a0)} = 33.0$ 5. $\lceil 32.2 \rceil^{(b1)} = 32.2$ 6. $\lceil 32.2 \rceil^{(a1)} = 40.0$

7. $\lfloor 538.79 \rfloor^{(b1)} = 538.7$ 8. $(\lfloor 9.3873 \rfloor)^{(b2)} = 9.38$ 9. $\lceil 38.37521 \rceil^{(a0)} = 39$

10. $\lceil 3259.395 \rceil^{(b2)} = 3259.4$ 11. $\lceil 59.9953 \rceil^{(b2)} = 60$ 12. $\lfloor 7853.39 \rfloor^{(a2)} = 7800$

13. $\lfloor 325379839.235 \rfloor^{(a6)} = 325000000$ 14. $\lfloor 3.141592 \rfloor^{(b5)} = 3.14159$

15. $\lceil 3.141592 \rceil^{(b5)} = 3.1416$ 16. $\lceil 89.59 \rceil^{(a0)} = 90$ 17. $\lceil 89.59 \rceil^{(a1)} = 90$

4- Conclusions:

We have seen the main theorem and its related ideas and exemplified some key cases for understanding the theory. This theory can be interesting in applied sciences such as physics, chemistry or economics. It is a clear example of how an elementary idea that we all learned as children can be taken further in a precise and rigorous way.