Motion From a Particle's Point of View: an Interpretation of the Double Slit Experiment

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June 2021

Abstract

Traditionally, motion has been defined *extrinsically*, as the change in the position of an object, such as a particle, over time. This definition presupposes the existence of an "observer" external to the particle who measures the position of the particle as a function of time. Yet, in the double slit experiment, we have experimental proof that a particle can travel unobserved between a source and a detector. It does so in such a way that we cannot, even in principle, describe its motion by a position changing in time. We propose that the correct interpretation of this experiment is that the traditional extrinsic definition of motion is incomplete. We propose a more general *intrinsic* definition of motion, in which the role of observer is played by the particle itself, and which presupposes no external observer. We discuss the double slit experiment including the nature of measurement and the particle-wave duality. We show that using the intrinsic point of view allows us to avoid the "weirdness" that tends to crop up when these topics are analysed from the extrinsic point of view.

1. Extrinsic and Intrinsic Points of View

Suppose we wish to study the motion of a car moving along a network of roads. Following Newton, we set up some system of coordinates (e.g. latitude and longitude) and represent the position of the car by a position variable $\boldsymbol{x}$. At each point in time t, the position of the car is given by the function $\boldsymbol{x}(t)$. Newton's laws let us compute this function given the starting conditions and thus predict where the car will go before it gets there. We call this traditional approach to studying motion the *extrinsic* point of view. In it we imagine we are hovering somewhere above the roads and car, as if in a helicopter, and can see where the car is at each point in time.

But this is not the only way to study the motion of the car. Instead, we could imagine we are sitting in the car, and study our motion through the road network from this *intrinsic* point of view.

If we are sitting in a car, the most direct way we are aware that we are in motion is by seeing the scenery moving past us through the car windows. Perhaps there are lampposts all along the roads. As we see them go past us, we conclude we are moving. We can gauge how fast we are moving by timing how long it takes to get from one lamppost to the next. The greater this time, the lower we conclude our speed is, so that doubling the time interval halves our perceived speed. In accordance with Newton's first law, this speed must remain constant as long as our car is not disturbed in any way. We can therefore justify the following two postulates which form the foundation of the intrinsic approach to motion:

Postulate 1 (P1): To be in motion is to see scenery (such as lampposts) repeating with a time period τ. The speed v of the motion is given by the reciprocal of the time period τ :

$$v = \frac{1}{\tau} \tag{1}$$

Postulate 2 (P2): Newton's first law: if we are not disturbed in any way in our motion, our speed v (and therefore τ) must remain constant.

Note that P1 defines motion *only* in terms of the repetition of scenery. More abstractly, for any particle to be in undisturbed motion, it only needs to "see" that it is crossing the graduations of a coordinate scale at a constant rate. When thinking about the car and road analogy, we should ignore that in a real car we have other clues as to our motion, such as wind and road noise, the speedometer and the inertia we feel when turning a corner. Also, the units of the intrinsic speed, in our example, are "lampposts per second."

The distinction between studying motion from the extrinsic and intrinsic points of view is similar to, and is in part inspired by, the distinction between extrinsic and intrinsic (Gaussian) curvature of a surface. The former imagines a surface to be embedded in an outside higher-dimensional space. The curvature of the surface at a given point can be measured using the radius of a circle in this exterior space whose circumference curves at the same rate as the surface. Gauss' insight was that such an embedding of a surface in an exterior space was not necessary. Instead, its curvature could be measured intrinsically, never leaving the surface, by measuring the ratio of the circumference to the radius of a small circle on the surface.

With Gauss' idea in mind, to illustrate the difference between extrinsic and intrinsic definition of motion, consider an ant crawling on a flat sheet of paper with a coordinate grid drawn on it. Intrinsically, the ant can measure its speed by counting how many lines it crosses per unit of time. Extrinsically, if we are looking from above the ant and paper, we can measure the extrinsic velocity by, first, counting how many lines the ant crosses in unit time. We then additionally use the location and orientation of the paper, and the fact that it is flat, to define the velocity of the ant in the coordinate system of the lab we are in. In this case, both the extrinsic and intrinsic speeds are well defined.

But this need not always be the case. Instead, the ant could be crawling along a flexible paper ribbon with lines across it, but this ribbon may not have a defined embedding in any outside space. We can imagine the ribbon free to flop around freely in our lab, and not following any defined curve in the lab coordinate system. So the ant can measure an intrinsic speed, but any outside observer is unable to define an extrinsic velocity, as the curve the paper ribbon follows in any outside space is undefined.

In the next section we build on postulates P1 and P2 to the case where multiple different motions are possible between the same two places. In section 3, we discuss the nature of measurement, and in section 4 we consider the double slit experiment from the intrinsic point of view. In the final section, section 5, we replace the discrete scenery composed of the abrupt passing of discrete lampposts with continuous scenery composed of gradual periodic transitions between two states and thereby make a connection with the Schrödinger equation of quantum mechanics.

2. Multiple Motions

P1 defines motion in terms of repeating scenery. It is sufficient that the scenery is composed of two alternating states. In the car and road analogy, we can use the lamppost as one state, and imagine that in between the lampposts there is a line of paint alongside the road which constitutes the other state. These states alternate with period τ when seen from inside our car, from which we can infer our speed by (1).

When driving a car, it may be possible to travel from A to B by more than one road. Lets consider the case where there are just two different roads, road 1 and road 2, connecting A to B. These roads must be distinguishable in some way, for example we can make the lampposts and line along road 1 red (R), and along road 2 green (G).

If there were no way to distinguish these roads, then travelling along either one we would see the exact same scenery, so we couldn't speak of these as being *different* roads in the intrinsic point of view.

We use the term *motion* specifically to refer to the alteration of two given states, that is a specific color of lamppost and a specific color of line. The term *road* is more general, and it may have multiple differently colored lampposts and lines along it.

Roads 1 and 2 both start at A and end at B. Therefore the scenery along A and B must be the union of the scenery along roads 1 and 2. We can state this principle as "same place, same scenery." So there will be an R and G lamppost and an R and a G line along A and B. We illustrate this in figure 1.

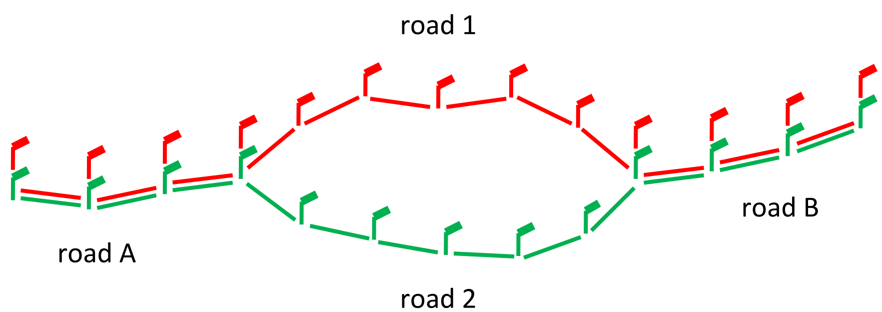

Fig. 1: There are two roads, road 1 and road 2, from road A to road B. All roads are drawn as arbitrary curves to emphasize they have no defined embedding in any outside space (see section 1).

In order to be in motion, P1 only requires us to experience the alteration of two states. Now, along road A there is an R and a G lamppost and an R and a G line, giving a total of four possible motions to choose from, which we label by the color of the lamppost followed by the color of the line: RR, RG, GR, GG. Now, for all motions, P2 must hold. Referring to figure 1, for motions RR and GG we can travel from A to B with P2 satisfied. But for the motions RG and GR, P2 is violated where A splits into roads 1 and 2. After this point, these motions abruptly terminate, not continuing on along either road 1 or road 2, in violation of P2. So, as drawn, the road network of figure 1 does not satisfy P2. We can modify it to do so by adding a G line to road 1 and an R line to road 2 as is shown in figure 2. Now all motions RR, RG, GR, GG satisfy P2 from A to B.

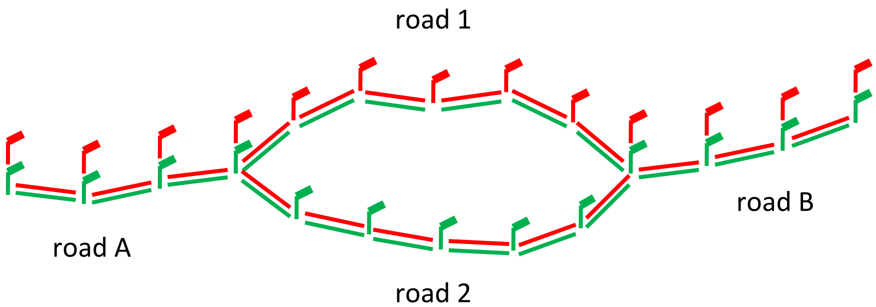

Fig. 2: The road network of figure 1 modified so it satisfies P2.

We call the number of different colored lampposts along a road its *amplitude*. We can generalize our discussion to the case of N different roads between A and B, each with amplitude 1. The amplitude of roads A and B will be N and the number of different motions possible from A to B will be equal to N^2.

In general, it is possible that it takes a different amount of time to travel from A to B via road 1 and road 2. P2 requires that the R and G lampposts along A and B line up with each other, as must do the R and G lines. That is, we cannot have the situation pictured in figure 3, where the red and green line segments along roads 1 and 2 do not line up with each other where these roads join into road B. Instead, they overlap each other, forming one continuous line of each color along road B. Thus P2 will be violated along road B, as we will see things like an R lamppost and R line simultaneously, which differs from the undisturbed motions along roads A, 1 and 2. This implies that the difference between the time it takes to travel from A to B along two different roads must be an exact integer multiple of τ.

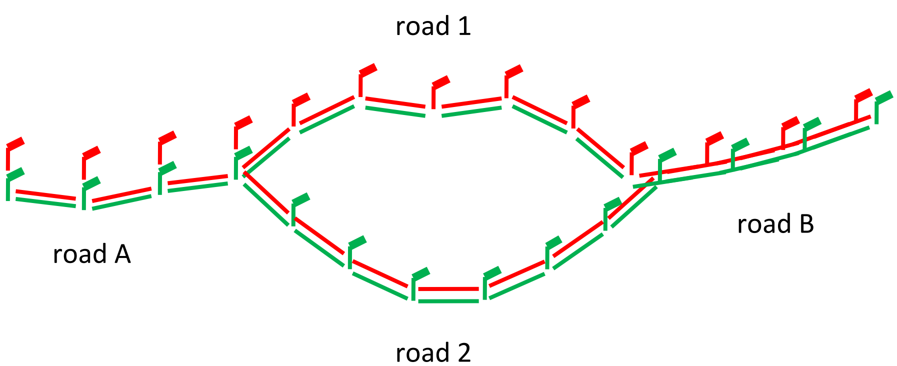

Fig. 3: If it takes different times to travel road 1 and road 2, P2 requires that the lampposts and lines at A and B line up. If not, as is shown here, P2 is violated.

We can summarize these results in the following three lemmas:

Lemma 1 (L1): A road of amplitude N can split into N different roads of amplitude 1 which then join back together into a road of amplitude N.

Lemma 2 (L2): There are N^2 different motions possible along a road of amplitude N.

Lemma 3 (L3): The difference in the duration of any pair of different roads which are joined at their starts and ends must be an exact integer multiple of τ.

3. Measurement

Consider the road network in figure 2. Now imagine we have two delicate balloons, one filled with red (R) paint, the other with green (G) paint. We hang the R balloon somewhere along road 1 and the G balloon somewhere along road 2, in such a way that a passing car will hit them and make them burst. Now lets investigate what happens when a car travels from A to B from both the extrinsic and intrinsic points of view.

In the extrinsic point of view, we can say that if the R balloon burst, the car went along road 1, and if the G balloon burst, the car went along road 2. We can say this even though neither road has a defined embedding in any outside space. Further, we are always guaranteed that only one paint balloon will burst, never both. We equate the *measurement* or *detection* of the car with a bursting of one of the balloons.

In the intrinsic point of view, we start our journey at A along one of the possible motions there: RR, RG, GR or GG. If we are travelling along the RR or RG motions, we will hit the R balloon. As soon as we hit a balloon, we are disturbed in our motion, in the sense that P2 no longer applies. While we have used a somewhat fanciful image of a car and paint balloons, the argument is completely general. In any measurement or detection we have the *interaction* of the particle (or "car") being measured and the particles of which the measuring apparatus is composed of (the "paint balloons"). Now it is in the nature of interaction of two particles that, from the point of view of either particle, the interaction is a disturbance, an out-of-the-ordinary event which differs in nature from motion as defined by P1, and thus violates P2.

By the above, we justify the following postulate:

Postulate 3 (P3): A *measurement* of a particle in motion is the interaction of the particle with any other particle and is a disturbance to both particles.

We also make the following postulate, which we justify by symmetry:

Postulate 4 (P4): Following a disturbance, if subsequent motion for a particle is possible along a set of different roads, the particle will randomly and with equal probability continue along one of the motions available to it. It follows from L2 that the probability of a particle continuing on after a disturbance along a road of amplitude N is proportional to N^2.

Note that looking intrinsically, the choice of which motion to continue after a disturbance is made locally, at the disturbance. We therefore say we have *intrinsic locality*. However, looking extrinsically, as soon as one of the paint balloons has burst, we are *instantaneously* guaranteed that the other paint balloon will not burst. This holds no matter how far apart these balloons are, as if they were in faster-than-light communication with each other. We therefore say we have *extrinsic non-locality*. (Strictly speaking, in the extrinsic case, the distance between the balloons is undefined, but we can define a "pseudoextrinsic" distance, see (4) below, and note non-locality with respect to this distance.)

It is important to emphasize that the behavior seen extrinsically can be wildly non-local. A particle travelling along a road only "cares" that P1 and P2 hold. It cares nothing about where the road leads, whether to a junction with a given road, or another given road, or whether a paint balloon hangs, or does not hang, somewhere further along it. This means the road network seen extrinsically can be modified while a particle is in motion through it, provided P2 is not violated, that is, no road is abruptly cut or terminated. So, a road that currently ends at a junction with a given road can be moved to join to a different road instead. Also, paint balloons can be hung and removed along any road while the particle is already in motion.

Such changes to the road network can, seen extrinsically, have puzzling non-local effects, while intrinsically locality is maintained. Changes in one part of the road network may necessitate corresponding changes to other parts of the road network so that P2 holds, such as when a road is moved from one junction to another, even if these parts are located very far apart. This, we believe, is the mechanism behind a lot of quantum "weirdness": as seen in extrinsically non-local behavior in the Wheeler delayed choice experiment [1], the EPR-Bohm experiment [2, 3] and associated Bell inequality [4] and other "retrocausal" experiments [5].

4. The Double Slit Experiment

Let's first consider the double slit experiment from the extrinsic point of view, as illustrated in figure 4.

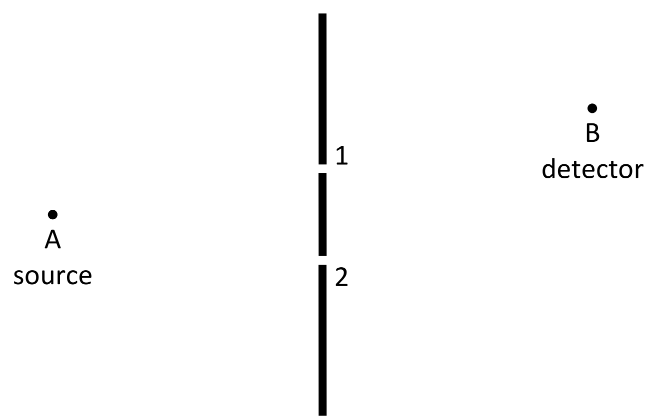

Fig. 4: The double slit experiment shown from the extrinsic point of view.

Referring to figure 4, we see a source of particles is placed at A and it fires particles at a constant rate one at a time towards a barrier. The barrier has two slits in it, labelled 1 and 2, through which particles may pass. The source and barrier are fixed in space, however the particle detector B can be moved around anywhere on the right side of the barrier. We count how many particles the detector detects per unit time at different positions of the detector. When we perform this experiment, we find that the detector always detects discrete particles, so that the detection is an "all-or-nothing" event.

However, if we plot the detection rate as a function of detector position, we find a wave-like pattern where particles are detected at some places much more often than at other places. The overall pattern of detection resembles the pattern seen when waves from two nearby sources interfere on the surface of water.

In the double slit experiment, it is very unclear exactly what path the particle takes from A to B. We cannot, even in principle, plot the position of the particle as a function of time in the extrinsic point of view. Any attempt to do so leads to paradoxes and a lot of "weirdness", see for example [6].

Now let's study this experiment from the intrinsic point of view. The particle (our "car") can travel from A to B in one of two possible ways: through slit 1 or 2, that is along one of two roads, road 1 or road 2, from A to B. This is just the situation we had in figure 2 of section 2. The source of particles, located where road A splits into roads 1 and 2, disturbs the particles (because of the interaction when a photon or electron is ejected by an atom) and thus by P4 we randomly start our journey along one of the four possible motions at A. The role of the detector B is played by a paint balloon hanging at the junction of roads 1 and 2 into road B.

Now L3 tells us that the difference in times T_1 and T_2 to travel from A to B via roads 1 and 2 respectively must be an integer multiple of τ:

$$T_1 - T_2 = n\tau \qquad n = 0, \ \pm 1, \ \pm 2, \ \ldots \tag{2}$$

We thus know how the times to travel roads 1 and 2 must be related. But say we are interested in the "lengths" of these roads. Thus far, in the intrinsic point of view, we have only measured time and, via (1), speed. We have not made any reference to the concepts of length or distance.

One way we could define the length of a road is to simply multiply its duration in time T by the speed we perceive when travelling along it in our car given by (1), giving a definition of *intrinsic distance* x_I:

$$x_I = vT \tag{3}$$

This is a valid intrinsic definition of distance, and it is equal to the number of lampposts we see along the road. But say we wanted to draw a scale diagram or map of the road network in figure 2. If we were to use this definition of distance, we would have to redraw the map depending on what speed we travelled at. This is not what we mean by "distance" when thinking in the traditional extrinsic view, where distance does not depend on speed (at least, in the non-relativistic mechanics which we are considering here). We can define a *pseudoextrinsic distance* x_P as the intrinsic distance at a given fixed reference speed c, that is:

$$x_P = cT \tag{4}$$

This is in accordance with how distance is traditionally defined in extrinsic non-relativistic mechanics, as the distance travelled by a particle (e.g. photon) moving at a constant reference speed c in a given length of time T.

We can now use this pseudoextrinsic distance (*pseudodistance* for short) to draw a scale diagram of the road network, which we call a *pseudospace* diagram. This "diagram" may be multidimensional, and we use an ordinary flat, Euclidean n-dimensional space to build this "diagram" in. This diagram is the best "map" a cartographer can make of the road network given only the pseudodistances between all the junctions of the roads, that is, based on purely intrinsic measurements. Thinking back to section 1, where we used the image of an ant crawling along lined ribbons of paper to define intrinsic motion, here we imagine our ant is a cartographer. It keeps track of the time it takes it to travel between junctions of ribbons, and based on the pseudodistances (4) obtained from the measurements, it triangulates the relative locations of all the junctions in pseudospace. In this way, it builds a model of the layout of the road network. Note that the roads still do not have any fixed embedding in any outside space—the pseudospace is not "real" and is only a map, an inferred layout of roads that the ant uses to navigate and reason about the network it is in.

When travelling through a road network, by P1 we only experience motion through the repetition of scenery. We have no sense of the "direction" we are travelling in, or any changes to it; as far as we are concerned, our motion is one-dimensional, and the speed (1) is a scalar. So if we, or our cartographer ant, measure a road to have a duration in time T, that is a pseudolength of $x_P = cT$, we must draw this road as a straight line segment in our pseudospace diagram, as long as we have no other information about this road. This requirement is so that the pseudospace diagram is uniquely defined (up to overall rotations and translations) given a specific road network. In the case of a road network of multiple roads which split and join at junctions, we may indeed have some other information which necessitates a road to not be drawn as a straight line segment. This information is in the list of all pseudodistances between all junctions, from which we triangulate the locations of all the junctions in pseudospace. Based on these locations, a given road may need to be drawn as a series of straight line segments between successive junctions along it. We summarize this in the following postulate:

Postulate 5 (P5): Roads in pseudospace diagrams are drawn as straight line segments, unless other information about them is available from purely intrinsic measurements.

Now the pseudolengths l_1 and l_2 of roads 1 and 2, that is the pseudodistances from A to B along each road will be given by:

$$\begin{aligned} l_1 &= cT_1 \\ l_2 &= cT_2 \end{aligned} \tag{5}$$

And so from (2) we have:

$$l_1 - l_2 = cn\tau \qquad n = 0,\ \pm 1,\ \ \pm 2,\ \ldots \tag{6}$$

Only for roads 1 and 2 with pseudolengths related by (6) is motion possible from A to B. The locations of the roads at source A, detector B and where they pass through slit 1 and 2 are fixed in pseudospace by extra roads which join these roads which each other in such a way that from purely intrinsic measurements it is possible to fix their relative locations in pseudospace by triangulation. The detector B will only ever detect a particle if it is located at such a position on the right of the barrier that the difference in pseudolengths of the two roads from A to B through the slits 1 and 2 is related by (6). That is, if the detector is placed at one of these locations, it will eventually detect a particle. It will never do so at any other location. The locations where detections can occur are illustrated in figure 5.

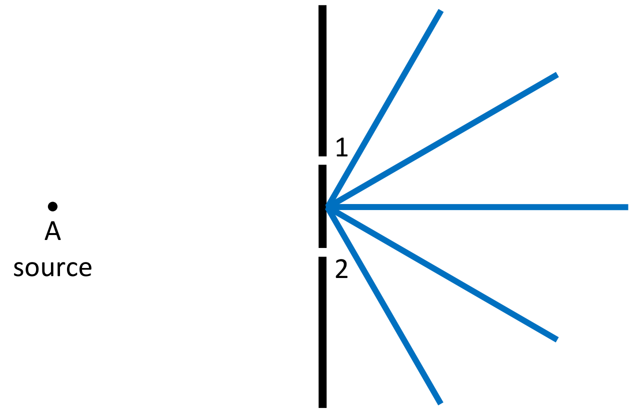

Fig. 5: Schematic illustration of the double slit experiment apparatus in pseudospace, with locations of possible particle detections on the right of the barrier indicated in blue.

The paint balloon hanging at the start of road B, our detector, will always either burst or not. So we will always detect a discrete *particle*-like unit. Yet, the locations in pseudospace where these detections can occur follow a *wave*-interference-like pattern from the two slits. That is, our postulates P1 and P2 lead us to recover the dual nature of quantum motion, known as the wave-particle duality. This states that motion cannot be explained solely in terms of discrete particles or continuous waves, but both concepts are needed. Our moving "car" is a discrete indivisible unit, the paint balloon either bursts or not, and the detector either makes a detection or not. Yet, the intrinsic definition of motion P1 in terms of repetition of scenery, together with P2 requiring the period of repetition to be constant, implies that if multiple motions between two places are possible, the difference in the pseudolengths of the different motions can only take a discrete set of values. These values are integer multiples of the *wavelength*:

$$\lambda = c\tau \tag{7}$$

Or, using (1):

$$\lambda \propto \frac{1}{v} \tag{8}$$

Which we recognize as the de Broglie wavelength in quantum mechanics, which is inversely proportional to the speed of the particle.

Note that the scenery we have used thus far consists of abrupt discrete transitions between two states, the lamppost and the line. Therefore, the locations of possible detections in figure 5 are also discrete. We could instead use scenery where the transitions between the two states are gradual, but still periodic with a constant period. Doing so would yield the classic continuous double slit interference pattern. We develop such continuous scenery in the next section, section 5, but we will continue to use discrete scenery for the remainder of this section because of its conceptual simplicity.

If we modify our experiment by closing one of the slits, say slit 2, we will now only have one possible road connect A to B. The time it takes to travel from A to B can take any value at all.

That means the pseudolength of the road from A to B can take any value, and we can detect a particle whatever location we place the detector B at. This is illustrated in figure 6.

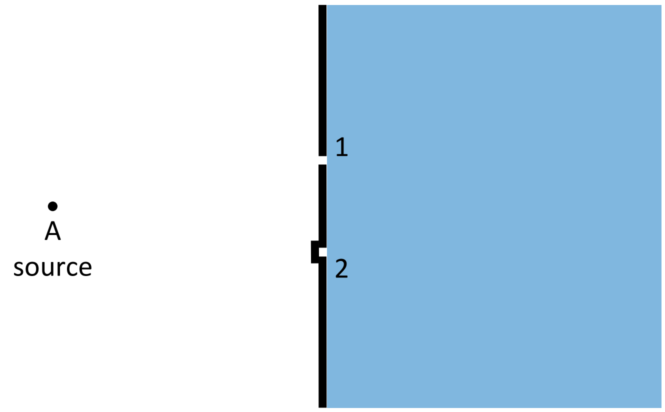

Fig. 6: Schematic illustration of the double slit experiment apparatus in pseudospace, with slit 2 closed. Particles may be detected at any location to the right of the barrier, indicated by the blue shading of the region.

Now lets open slit 2 again, but instead place particle detectors D1 and D2 at slits 1 and 2 respectively. That is, we imagine we hang paint filled balloons along roads 1 and 2 of figure 2, as we discussed in section 3 on measurement. Looking intrinsically, the particle (our "car") will burst one of the balloons, thereby constituting a detection of the particle at one of the slits, and by P3 the particle will be disturbed. Assuming the particle continues in motion, by P4 the particle will continue on a randomly chosen motion out of all the motions available to it. Since there are no more barriers to the right of the barrier with the two slits and detectors, motion is possible for the particle in any direction. We therefore may detect the particles at detector B whenever it is placed to the right of the barrier after we first detect it at either D1 or D2. This is illustrated in figure 7.

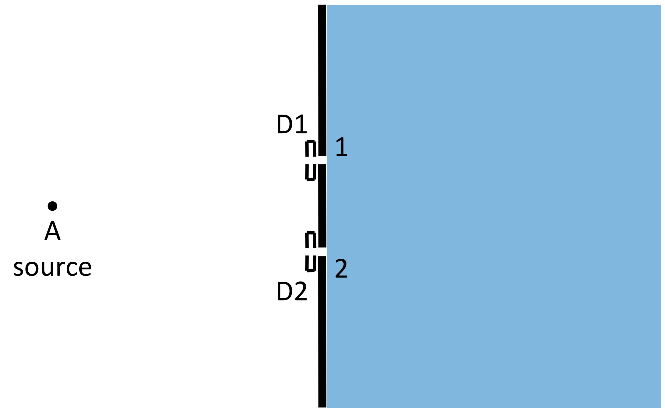

Fig. 7: Schematic illustration of the double slit experiment apparatus in pseudospace with both slits open and detectors placed at the slits. The detectors disturb the particle in motion, meaning P2 is violated, therefore by P4 the particle may continue along any motion available to it. Hence, it may be detected at any possible location of the detector, indicated by the shaded blue region.

By analyzing these three versions of the double slit experiment, shown in figures 5, 6 and 7, using the intrinsic point of view, we have deduced the actual behaviours observed when this experiment is performed, which behaviors are unexplainable if we maintain a traditional extrinsic definition of motion.

5. Continuous Scenery

Thus far we have considered discrete scenery wherein the transition between two states (lamppost and line) is abrupt. Here we consider the case when these transitions are gradual. We replace the lampposts by a real-valued periodic function $\Psi(t)$ of time which has maxima at times when travelling in our car we would be next to a lamppost, and minima halfway between these maxima. We replace the lines by a function $\Psi'(t)$ which has maxima midway between the lampposts, and minima at the lampposts. If the amplitude of a road is 1, we fix the value of $\Psi(t)$ and $\Psi'(t)$ at their maxima to be +1, and set their minima to be -1. The following functions satisfy these conditions:

$$\begin{aligned} \Psi(t) &= \cos(2\pi v t) \\ \Psi'(t) &= -\cos(2\pi v t) \end{aligned} \tag{9}$$

These functions are plotted in figure 8 along with the discrete lampposts and lines they replace.

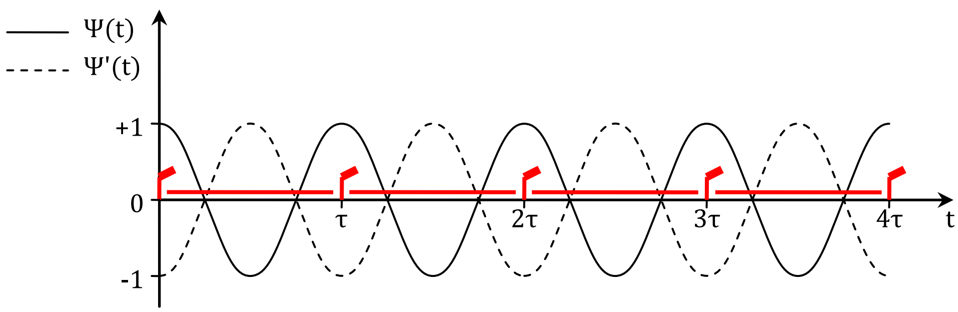

Fig. 8: In continuous scenery the abrupt transitions between lamppost and line states are replaced with gradual transitions between two abstract states, $\Psi(t)$ and $\Psi'(t)$.

Now let us re-analyze the case of multiple motions of section 2, figure 2, this time using continuous scenery. We replace the R lampposts and lines by two functions $\Psi_R(t)$ and $\Psi_R{}'(t)$, respectively, and the G lampposts and lines by $\Psi_G(t)$ and $\Psi_G{}'(t)$, respectively. We retain the principle of "same place, same scenery," that the scenery where multiple roads join is the union, that is the sum, of the scenery of all the roads there. Let's assume for now the times T_1 and T_2 to travel roads 1 and 2 to be equal. We can then describe the scenery along roads A, 1, 2 and B by the following functions:

$$
\begin{aligned}
&\Psi_{\mathrm{A}}(t) = \Psi_{\mathrm{R}}(t) + \Psi_{\mathrm{G}}(t)\\
&\Psi'_{\mathrm{A}}(t) = \Psi'_{\mathrm{R}}(t) + \Psi'_{\mathrm{G}}(t)\\
&\Psi_1(t) = \Psi_{\mathrm{R}}(t)\\
&\Psi'_1(t) = \Psi'_{\mathrm{R}}(t) + \Psi'_{\mathrm{G}}(t)\\
&\Psi_2(t) = \Psi_{\mathrm{G}}(t)\\
&\Psi'_2(t) = \Psi'_{\mathrm{R}}(t) + \Psi'_{\mathrm{G}}(t)\\
&\Psi_{\mathrm{B}}(t) = \Psi_{\mathrm{R}}(t) + \Psi_{\mathrm{G}}(t)\\
&\Psi'_{\mathrm{B}}(t) = \Psi'_{\mathrm{R}}(t) + \Psi'_{\mathrm{G}}(t)
\end{aligned} \tag{10}
$$

So, in the case when $T_1 = T_2$, the functions $\Psi_{\mathrm{A}}(t)$, $\Psi'_{\mathrm{A}}(t)$, $\Psi_{\mathrm{B}}(t)$ and $\Psi'_{\mathrm{B}}(t)$ all will range from +2 to -2. As in section 2, we define the amplitude of a road only in terms of the number of lampposts, here replaced by half of the peak-to-peak amplitude of the unprimed functions $\Psi_{\mathrm{A}}(t)$, $\Psi_1(t)$, $\Psi_2(t)$ and $\Psi_{\mathrm{B}}(t)$. So, the amplitude of roads A and B will be 2, and of roads 1 and 2 will be 1, just as we had in section 2. By L2, the number of different motions along A and B is $2^2 = 4$.

Now, if $T_1 \neq T_2$, that is, the difference in travel times along roads 1 and 2:

$$\Delta T = T_1 - T_2 \tag{11}$$

is nonzero, the scenery functions $\Psi_{\mathrm{R}}(t)$ and $\Psi_{\mathrm{G}}(t)$ (and $\Psi'_{\mathrm{R}}(t)$ and $\Psi'_{\mathrm{G}}(t)$) will be out of phase with each other. Note that due to P2, the amplitudes of A and B must be equal. If this weren't the case, P2 would be violated as a motion would have to abruptly end or begin between A and B, not allowing undisturbed motion between A and B.

It follows therefore that the total phase shift must be divided equally at both A and B, but in opposite directions. Half of the total phase shift is given by:

$$\Delta\phi = \pi v \Delta \mathrm{T} \tag{12}$$

So now the unprimed scenery functions at A and B are given by:

$$
\begin{aligned}
\Psi_{\mathrm{A}}(t) &= \cos(2\pi vt) + \cos(2\pi vt + \Delta\phi) = 2\cos\left(\frac{\Delta\phi}{2}\right)\cos\left(2\pi vt + \frac{\Delta\phi}{2}\right)\\
\Psi_{\mathrm{B}}(t) &= \cos(2\pi vt) + \cos(2\pi vt - \Delta\phi) = 2\cos\left(\frac{\Delta\phi}{2}\right)\cos\left(2\pi vt - \frac{\Delta\phi}{2}\right)
\end{aligned} \tag{13}
$$

And so the amplitudes of roads A and B will be given by:

$$N = \left|2\cos\left(\frac{\Delta\phi}{2}\right)\right| \tag{14}$$

When working with summing out-of-phase waves, it is often more convenient to work with imaginary exponential functions than with real trigonometric functions. We can replace the functions of (9) by:

$$
\begin{aligned}
\Psi(t) &= e^{i2\pi vt}\\
\Psi'(t) &= -e^{i2\pi vt}
\end{aligned} \tag{15}
$$

The amplitude of a road where two roads with amplitudes 1 and a phase difference $\Delta\phi$ join is now given by:

$$N = \left|e^{i2\pi\nu t} + e^{i(2\pi\nu t + \Delta\phi)}\right| = \left|1 + e^{i\Delta\phi}\right|$$

$$= \sqrt{2 + 2\cos(\Delta\phi)} = \sqrt{4\cos^2\left(\frac{\Delta\phi}{2}\right)}$$

$$= \left|2\cos\left(\frac{\Delta\phi}{2}\right)\right| \tag{16}$$

which is identical to (14).

Now we know by L2 that the number of motions along a road of amplitude N is given by N^2, and by P3 we know that this is proportional to the probability P that a particle will continue on this road following a disturbance. We can therefore write:

$$P \propto |\Psi(t)|^2 \tag{17}$$

which we recognize as the Born rule of quantum mechanics, with $\Psi(t)$ being the wavefunction. Note that in section 2, where we dealt with discrete scenery, the amplitudes of roads were always discrete positive integers. Here, in the case of continuous scenery, the amplitudes, and thus the number of motions, can take on non-integer values. If this bothers us conceptually, we can always multiply all the scenery functions by a fixed large integer, and round their amplitudes to the nearest integer. In any case, since the car must always travel on *some* road, the sum of all possible probabilities given by (17) must sum to one, from which requirement we can compute the proportionality coefficient in (17), called the *normalization* coefficient. If we choose to multiply our scenery functions by the large integer the normalization coefficient will cancel this out, giving the same numerical predictions for the probabilities.

Now let us make one final connection with the formalism of quantum mechanics, the Schrödinger equation. The function $\Psi(t)$ of (15) has period τ in t, and so using (4) has a wavelength λ in pseudospace given by (7). We can therefore write it as a function of pseudospace position x_P:

$$\Psi(x_P) = e^{i2\pi\frac{\nu}{c}x_P} \tag{18}$$

This describes motion with a constant speed v, for which the traditional extrinsic equation of motion is:

$$x = x_0 + vt \tag{19}$$

in which x_0 is the extrinsic position of the particle at time zero, that is an initial position derived from a "map" or scale diagram of the experiment, which in out approach to motion is replaced by the pseudospace position x_P. This allows us to combine (18) and (19) by setting x_0 = x_P to obtain:

$$\Psi(x,t) = e^{i2\pi\frac{\nu}{c}(x - vt)} \tag{20}$$

which is a solution of the Schrödinger equation with zero potential, mass $m = 1$, and Planck's constant $h = 2c$.

6. Conclusion

By starting with the intrinsic definition of motion P1 we have deduced the behaviors seen in the double slit experiment, which are so paradoxical when analyzed using the traditional extrinsic definition of motion. We have shown wave-particle duality, the de Broglie wavelength, the Born rule and the Schrödinger equation all follow naturally from considering motion from the intrinsic point of view.

The core idea in our approach can be summarized as follows. For a particle to be in constant speed motion, it just needs to be crossing the graduations of a coordinate scale (our "lampposts") at a constant rate. But it does *not* follow from this, as is widely assumed, that there must be a global continuous coordinate system, or grid of intersecting coordinate scale gradations, for motion to be possible. It suffices for a coordinate scale to be established along each possible motion, and that the graduations of these scales line up where the motions split from, and join with, each other.

In other words, the manifold in which motion takes place need not be continuous. Instead it can be torn or shredded into interconnecting ribbons (our "roads"). Only along such ribbons is the metric defined. There is no direct metric relationship defined between different ribbons, only the one obtained by integrating distance while following a sequence of motions from one ribbon to the other. This, together with P1 and P2, gives rise to, and explains, the necessity for having a phase associated with each possible motion in quantum mechanics.

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