

**On the Higher Spins and strings - Supersymmetry Breaking revisited.  
Mathematical connections with various parameters of Particle Physics and some  
sectors of Number Theory**

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**Abstract**

*In this revisited research thesis, we analyze some equations concerning the Higher Spins and Strings - Supersymmetry Breaking and obtain various mathematical connections with various parameters of Particle Physics and several sectors of Number Theory.*

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**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

From:

### **String Lessons for Higher-Spin Interactions**

*A. Sagnotti and M. Taronna - arXiv:1006.5242v2 [hep-th] 31 Aug 2010*

We have that:

$$\begin{aligned}
\mathbf{Z} = & i g_o \frac{(2\pi)^d}{\alpha'} \delta^{(d)}(p_1 + p_2 + p_3) \\
& \times \exp \left\{ \xi_1^{(1)} \cdot \xi_2^{(1)} + \xi_2^{(1)} \cdot \xi_3^{(1)} + \xi_3^{(1)} \cdot \xi_1^{(1)} - 2 \xi_1^{(1)} \cdot \xi_2^{(2)} \frac{y_{23}}{y_{31}} - 2 \xi_2^{(1)} \cdot \xi_3^{(2)} \frac{y_{31}}{y_{12}} - 2 \xi_3^{(1)} \cdot \xi_1^{(2)} \frac{y_{12}}{y_{23}} \right. \\
& + 2 \xi_1^{(2)} \cdot \xi_2^{(1)} \frac{y_{31}}{y_{23}} + 2 \xi_2^{(2)} \cdot \xi_3^{(1)} \frac{y_{12}}{y_{31}} + 2 \xi_3^{(2)} \cdot \xi_1^{(1)} \frac{y_{23}}{y_{12}} + 6 \xi_1^{(2)} \cdot \xi_2^{(2)} + 6 \xi_2^{(2)} \cdot \xi_3^{(2)} + 6 \xi_3^{(2)} \cdot \xi_1^{(2)} \\
& + \sqrt{\frac{\alpha'}{2}} \left[ \xi_1^{(1)} \cdot \left( p_{23} + p_1 \frac{y_{31} + y_{21}}{y_{23}} \right) + \xi_2^{(1)} \cdot \left( p_{31} + p_2 \frac{y_{12} + y_{32}}{y_{13}} \right) \right. \\
& + \xi_3^{(1)} \cdot \left( p_{12} + p_3 \frac{y_{23} + y_{13}}{y_{12}} \right) + \xi_1^{(2)} \cdot \left( p_{23} \frac{y_{31} + y_{21}}{y_{23}} + p_1 \frac{y_{31}^2 + y_{21}^2}{y_{23}^2} \right) \\
& \left. + \xi_2^{(2)} \cdot \left( p_{31} \frac{y_{12} + y_{32}}{y_{13}} + p_2 \frac{y_{12}^2 + y_{32}^2}{y_{13}^2} \right) + \xi_3^{(2)} \cdot \left( p_{12} \frac{y_{23} + y_{13}}{y_{12}} + p_3 \frac{y_{23}^2 + y_{13}^2}{y_{12}^2} \right) \right] \left. \right\} . \tag{4.57}
\end{aligned}$$

where  $\alpha'$ , the so-called Regge slope,  $g_0$  denotes the open string coupling,  $y$ 's are real variables,  $p$  denotes the string momentum

Planck momentum

$$m_P c = \frac{\hbar}{l_P} = \sqrt{\frac{\hbar c^3}{G}} = 6.5249 \text{ kg}\cdot\text{m/s} ;$$

$$g_0 = e^\phi = e^{(0.9991104684)} = 2.715864906; \quad \alpha' = 0.915$$

$$p = 6, 7, 8 \quad y = 1, 2, 4 \quad \xi = 3, 5, 13$$

$$y_{12} = 1, \quad y_{23} = 2, \quad y_{31} = 4$$

where 0.9991104684 is the the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

While  $\alpha'$  is the Regge slope (string tension) of Omega mesons:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

Now, we have:

$$\begin{aligned} \mathbf{Z} = & i g_0 \frac{(2\pi)^d}{\alpha'} \delta^{(d)}(p_1 + p_2 + p_3) \\ & \times \exp \left\{ \xi_1^{(1)} \cdot \xi_2^{(1)} + \xi_2^{(1)} \cdot \xi_3^{(1)} + \xi_3^{(1)} \cdot \xi_1^{(1)} - 2 \xi_1^{(1)} \cdot \xi_2^{(2)} \frac{y_{23}}{y_{31}} - 2 \xi_2^{(1)} \cdot \xi_3^{(2)} \frac{y_{31}}{y_{12}} - 2 \xi_3^{(1)} \cdot \xi_1^{(2)} \frac{y_{12}}{y_{23}} \right. \\ & \left. + 2 \xi_1^{(2)} \cdot \xi_2^{(1)} \frac{y_{31}}{y_{23}} + 2 \xi_2^{(2)} \cdot \xi_3^{(1)} \frac{y_{12}}{y_{31}} + 2 \xi_3^{(2)} \cdot \xi_1^{(1)} \frac{y_{23}}{y_{12}} + 6 \xi_1^{(2)} \cdot \xi_2^{(2)} + 6 \xi_2^{(2)} \cdot \xi_3^{(2)} + 6 \xi_3^{(2)} \cdot \xi_1^{(2)} \right\} \end{aligned}$$

$$i*2.715864906 * (((2\pi)^4)/(0.915)))*(6+7+8)*\exp[15+65+39-2*15*1/2-2*65*4-2*39*1/2+2*15*2+2*65*1/4+2*39*2+6*15+6*65+6*39]$$

$$e^{(0.9991104684)}$$

### Input interpretation:

$$e^{0.9991104684}$$

### Result:

$$2.715864906\dots$$

$$2.715864906\dots$$

### Alternative representation:

$$e^{0.99911} = \exp^{0.99911}(z) \text{ for } z = 1$$

### Series representations:

$$e^{0.99911} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{0.99911}$$

$$e^{0.99911} = 0.500308 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.99911}$$

$$e^{0.99911} = \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.99911}$$

### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From

$$\begin{aligned}
\mathbf{Z} &= i g_o \frac{(2\pi)^d}{\alpha'} \delta^{(d)}(p_1 + p_2 + p_3) \\
&\times \exp \left\{ \xi_1^{(1)} \cdot \xi_2^{(1)} + \xi_2^{(1)} \cdot \xi_3^{(1)} + \xi_3^{(1)} \cdot \xi_1^{(1)} - 2 \xi_1^{(1)} \cdot \xi_2^{(2)} \frac{y_{23}}{y_{31}} - 2 \xi_2^{(1)} \cdot \xi_3^{(2)} \frac{y_{31}}{y_{12}} - 2 \xi_3^{(1)} \cdot \xi_1^{(2)} \frac{y_{12}}{y_{23}} \right. \\
&\quad \left. + 2 \xi_1^{(2)} \cdot \xi_2^{(1)} \frac{y_{31}}{y_{23}} + 2 \xi_2^{(2)} \cdot \xi_3^{(1)} \frac{y_{12}}{y_{31}} + 2 \xi_3^{(2)} \cdot \xi_1^{(1)} \frac{y_{23}}{y_{12}} + 6 \xi_1^{(2)} \cdot \xi_2^{(2)} + 6 \xi_2^{(2)} \cdot \xi_3^{(2)} + 6 \xi_3^{(2)} \cdot \xi_1^{(2)} \right. \\
&\quad \left. + \sqrt{\frac{\alpha'}{2}} \left[ \xi_1^{(1)} \cdot \left( p_{23} + p_1 \frac{y_{31} + y_{21}}{y_{23}} \right) + \xi_2^{(1)} \cdot \left( p_{31} + p_2 \frac{y_{12} + y_{32}}{y_{13}} \right) \right. \right. \\
&\quad \left. \left. + \xi_3^{(1)} \cdot \left( p_{12} + p_3 \frac{y_{23} + y_{13}}{y_{12}} \right) + \xi_1^{(2)} \cdot \left( p_{23} \frac{y_{31} + y_{21}}{y_{23}} + p_1 \frac{y_{31}^2 + y_{21}^2}{y_{23}^2} \right) \right. \right. \\
&\quad \left. \left. + \xi_2^{(2)} \cdot \left( p_{31} \frac{y_{12} + y_{32}}{y_{13}} + p_2 \frac{y_{12}^2 + y_{32}^2}{y_{13}^2} \right) + \xi_3^{(2)} \cdot \left( p_{12} \frac{y_{23} + y_{13}}{y_{12}} + p_3 \frac{y_{23}^2 + y_{13}^2}{y_{12}^2} \right) \right] \right\}. \tag{4.57}
\end{aligned}$$

For:  $p = 6, 7, 8$      $\xi = 3, 5, 13$      $y_{12} = 1, y_{23} = 2, y_{31} = 4$

$$\begin{aligned}
&[15+65+39-2*15*1/2-2*65*4- \\
&2*39*1/2+2*15*2+2*65*1/4+2*39*2+6*15+6*65+6*39+\text{sqrt}(0.915/2) * \\
&((((3*(7+6*5/2)+5(8+7*3/4)+13(6+8*6))+3(7*5/2+6*25/4)+5(8*3/4+7*5/16)+13(6*6 \\
&+8*20)))]
\end{aligned}$$

**Input:**

$$\begin{aligned}
&15 + 65 + 39 - 2 \times 15 \times \frac{1}{2} - 2 \times 65 \times 4 - 2 \times 39 \times \frac{1}{2} + \\
&2 \times 15 \times 2 + 2 \times 65 \times \frac{1}{4} + 2 \times 39 \times 2 + 6 \times 15 + 6 \times 65 + 6 \times 39 + \\
&\sqrt{\frac{0.915}{2}} \left( 3 \left( 7 + 6 \times \frac{5}{2} \right) + 5 \left( 8 + 7 \times \frac{3}{4} \right) + 13 (6 + 8 \times 6) + 3 \left( 7 \times \frac{5}{2} + 6 \times \frac{25}{4} \right) + \right. \\
&\quad \left. 5 \left( 8 \times \frac{3}{4} + 7 \times \frac{5}{16} \right) + 13 (6 \times 6 + 8 \times 20) \right)
\end{aligned}$$

**Result:**

2934.51...

2934.51...

From which:

$[15+65+39-2*15*1/2-2*65*4-$   
 $2*39*1/2+2*15*2+2*65*1/4+2*39*2+6*15+6*65+6*39+\text{sqrt}(0.915/2)*((3*(7+6*5/$   
 $2)+5(8+7*3/4)+13(6+8*6)+3(7*5/2+6*25/4)+5(8*3/4+7*5/16)+13(6*6+8*20)))]+47$   
 $-1/\text{golden ratio}$

**Input:**

$$\left( 15 + 65 + 39 - 2 \times 15 \times \frac{1}{2} - 2 \times 65 \times 4 - 2 \times 39 \times \frac{1}{2} + \right. \\
 \left. 2 \times 15 \times 2 + 2 \times 65 \times \frac{1}{4} + 2 \times 39 \times 2 + 6 \times 15 + 6 \times 65 + 6 \times 39 + \right. \\
 \left. \sqrt{\frac{0.915}{2}} \left( 3 \left( 7 + 6 \times \frac{5}{2} \right) + 5 \left( 8 + 7 \times \frac{3}{4} \right) + 13 (6 + 8 \times 6) + 3 \left( 7 \times \frac{5}{2} + 6 \times \frac{25}{4} \right) + \right. \right. \\
 \left. \left. 5 \left( 8 \times \frac{3}{4} + 7 \times \frac{5}{16} \right) + 13 (6 \times 6 + 8 \times 20) \right) \right) + 47 - \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

2980.89...

2980.89... result practically equal to the rest mass of Charmed eta meson 2980.3

**Series representations:**

$$\left( 15 + 65 + 39 - \frac{2 \times 15}{2} - 2 \times 65 \times 4 - \frac{2 \times 39}{2} + 2 \times 15 \times 2 + \frac{2 \times 65}{4} + 2 \times 39 \times 2 + 6 \times 15 + \right. \\
 \left. 6 \times 65 + 6 \times 39 + \sqrt{\frac{0.915}{2}} \left( 3 \left( 7 + \frac{6 \times 5}{2} \right) + 5 \left( 8 + \frac{7 \times 3}{4} \right) + 13 (6 + 8 \times 6) + \right. \right. \\
 \left. \left. 3 \left( \frac{7 \times 5}{2} + \frac{6 \times 25}{4} \right) + 5 \left( \frac{8 \times 3}{4} + \frac{7 \times 5}{16} \right) + 13 (6 \times 6 + 8 \times 20) \right) \right) + \\
 47 - \frac{1}{\phi} = \frac{1109}{2} - \frac{1}{\phi} + \frac{57411}{16} \sum_{k=0}^{\infty} \frac{(-1)^k (-0.5425)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\left( 15 + 65 + 39 - \frac{2 \times 15}{2} - 2 \times 65 \times 4 - \frac{2 \times 39}{2} + 2 \times 15 \times 2 + \frac{2 \times 65}{4} + 2 \times 39 \times 2 + 6 \times 15 + \right. \\ \left. 6 \times 65 + 6 \times 39 + \sqrt{\frac{0.915}{2}} \left( 3 \left( 7 + \frac{6 \times 5}{2} \right) + 5 \left( 8 + \frac{7 \times 3}{4} \right) + 13 (6 + 8 \times 6) + \right. \right. \\ \left. \left. 3 \left( \frac{7 \times 5}{2} + \frac{6 \times 25}{4} \right) + 5 \left( \frac{8 \times 3}{4} + \frac{7 \times 5}{16} \right) + 13 (6 \times 6 + 8 \times 20) \right) \right) + 47 - \frac{1}{\phi} = \\ \frac{1109}{2} - \frac{1}{\phi} - \frac{57411 \sum_{j=0}^{\infty} \text{Res}_{s=-j} (-0.5425)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{32 \sqrt{\pi}}$$

$$\left( 15 + 65 + 39 - \frac{2 \times 15}{2} - 2 \times 65 \times 4 - \frac{2 \times 39}{2} + 2 \times 15 \times 2 + \frac{2 \times 65}{4} + 2 \times 39 \times 2 + 6 \times 15 + \right. \\ \left. 6 \times 65 + 6 \times 39 + \sqrt{\frac{0.915}{2}} \left( 3 \left( 7 + \frac{6 \times 5}{2} \right) + 5 \left( 8 + \frac{7 \times 3}{4} \right) + 13 (6 + 8 \times 6) + \right. \right. \\ \left. \left. 3 \left( \frac{7 \times 5}{2} + \frac{6 \times 25}{4} \right) + 5 \left( \frac{8 \times 3}{4} + \frac{7 \times 5}{16} \right) + 13 (6 \times 6 + 8 \times 20) \right) \right) + 47 - \frac{1}{\phi} = \\ \frac{1109}{2} - \frac{1}{\phi} + \frac{57411}{16} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.4575 - z_0)^k z_0^{-k}}{k!}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

Thence:

$$i \times 2.715864906 * (((((2\pi)^4)/(0.915)))) * (6+7+8) * \exp(2934.51)$$

**Input interpretation:**

$$i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6 + 7 + 8) \exp(2934.51)$$

$i$  is the imaginary unit

**Result:**

$$2.68489... \times 10^{1279} i$$

**Polar coordinates:**

$$r = 2.684886475559863 \times 10^{1279} \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$2.684886475559863 * 10^{1279}$$

**Alternate form:**

$$2.684886475559863 \times 10^{1279} i$$

From which:

$$\ln(\left(i \cdot 2.715864906 \cdot \frac{(2\pi)^4}{0.915} \cdot (6+7+8) \cdot \exp(2934.51)\right))$$

**Input interpretation:**

$$\log\left(i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6 + 7 + 8) \exp(2934.51)\right)$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

**Result:**

$$2945.99\dots + 1.57080\dots i$$

**Polar coordinates:**

$$r = 2945.99 \text{ (radius)}, \quad \theta = 0.03055^\circ \text{ (angle)}$$

2945.99

**Alternative representations:**

$$\log\left(\frac{(i(2\pi)^4 \cdot 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \log_e\left(\frac{57.0332 i \exp(2934.51) (2\pi)^4}{0.915}\right)$$

$$\log\left(\frac{(i(2\pi)^4 \cdot 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \log(a) \log_a\left(\frac{57.0332 i \exp(2934.51) (2\pi)^4}{0.915}\right)$$



### Series representations:

$$\log\left(\frac{(i(2\pi)^4 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \log(-1 + 997.301 i \pi^4 \exp(2934.51)) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 997.301 i \pi^4 \exp(2934.51))^{-k}}{k}$$

$$\log\left(\frac{(i(2\pi)^4 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = 2\pi \mathcal{A} \left[ \frac{\arg(-x + 997.301 i \pi^4 \exp(2934.51))}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + 997.301 i \pi^4 \exp(2934.51))^k}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{(i(2\pi)^4 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \left[ \frac{\arg(997.301 i \pi^4 \exp(2934.51) - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(997.301 i \pi^4 \exp(2934.51) - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (997.301 i \pi^4 \exp(2934.51) - z_0)^k z_0^{-k}}{k}$$

### Integral representations:

$$\log\left(\frac{(i(2\pi)^4 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \int_1^{997.301 i \pi^4 \exp(2934.51)} \frac{1}{t} dt$$

$$\log\left(\frac{(i(2\pi)^4 2.71586)(6+7+8)\exp(2934.51)}{0.915}\right) = \frac{1}{2\pi \mathcal{A}} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{(-1 + 997.301 i \pi^4 \exp(2934.51))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

And:

$$\left(\left(\left(i \times 2.715864906 \times \left(\frac{(2\pi)^4}{0.915}\right) \times (6+7+8) \times \exp(2934.51)\right)\right)\right)^{1/6144}$$

**Input interpretation:**

$$\sqrt[6144]{i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6+7+8) \exp(2934.51)}$$

$i$  is the imaginary unit

**Result:**

$$1.61525... + 0.000412961... i$$

**Polar coordinates:**

$$r = 1.61525 \text{ (radius), } \theta = 0.0146484^\circ \text{ (angle)}$$

1.61525 result that is a very good approximation to the value of the golden ratio  
1.618033988749...

$$\left(\left(\left(i \times 2.715864906 \times \left(\frac{(2\pi)^4}{0.915}\right) \times (6+7+8) \times \exp(2934.51)\right)\right)\right)^{1/395} - 5$$

**Input interpretation:**

$$\sqrt[395]{i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6+7+8) \exp(2934.51) - 5}$$

$i$  is the imaginary unit

**Result:**

$$1729.03... + 6.89576... i$$

**Polar coordinates:**

$$r = 1729.05 \text{ (radius), } \theta = 0.228507^\circ \text{ (angle)}$$

1729.05

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

$$[\left(\left(\left(i \cdot 2.715864906 \cdot \left(\frac{(2\pi)^4}{0.915}\right)\right) \cdot (6+7+8) \cdot \exp(2934.51)\right)\right)^{1/395} - 5]^{1/15}$$

**Input interpretation:**

$$\sqrt[15]{\sqrt[395]{i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6 + 7 + 8) \exp(2934.51) - 5}}$$

$i$  is the imaginary unit

**Result:**

$$1.64382... + 0.000437058... i$$

**Polar coordinates:**

$$r = 1.64382 \text{ (radius), } \theta = 0.0152338^\circ \text{ (angle)}$$

$$1.64382 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$\left(\left(\left(i \cdot 2.715864906 \cdot \left(\frac{(2\pi)^4}{0.915}\right)\right) \cdot (6+7+8) \cdot \exp(2934.51)\right)\right)^{1/597}$$

**Input interpretation:**

$$\sqrt[597]{i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6 + 7 + 8) \exp(2934.51)}$$

$i$  is the imaginary unit

**Result:**

$$139.026... + 0.365799... i$$

**Polar coordinates:**

$$r = 139.026 \text{ (radius), } \theta = 0.150754^\circ \text{ (angle)}$$

139.026 result practically equal to the rest mass of Pion meson 139.57 MeV

$$\left(\left(\left(i \times 2.715864906 \times \left(\frac{(2\pi)^4}{0.915}\right) \times (6+7+8) \times \exp(2934.51)\right)\right)\right)^{1/610}$$

**Input interpretation:**

$$\sqrt[610]{i \times 2.715864906 \times \frac{(2\pi)^4}{0.915} (6+7+8) \exp(2934.51)}$$

$i$  is the imaginary unit

**Result:**

$$125.148... + 0.322266... i$$

**Polar coordinates:**

$$r = 125.148 \text{ (radius)}, \quad \theta = 0.147541^\circ \text{ (angle)}$$

125.148 result very near to the Higgs boson mass 125.18 GeV

Now, we have that:

$$\begin{aligned} \mathcal{A} = & \left\{ \mathbf{Z} \star_{\xi_i^{(2)}} \left[ 1 - \frac{1}{\hat{s}_1 - 1} \sqrt{\frac{\alpha'}{2}} (p_1 \cdot \partial_{\xi_1^{(1)}}) (\xi_1^{(2)} \cdot \partial_{\xi_1^{(1)}}) \right] \right. \\ & \times \left[ 1 - \frac{1}{\hat{s}_2 - 1} \sqrt{\frac{\alpha'}{2}} (p_2 \cdot \partial_{\xi_2^{(1)}}) (\xi_2^{(2)} \cdot \partial_{\xi_2^{(1)}}) \right] \left. \left[ 1 - \frac{1}{\hat{s}_3 - 1} \sqrt{\frac{\alpha'}{2}} (p_3 \cdot \partial_{\xi_3^{(1)}}) (\xi_3^{(2)} \cdot \partial_{\xi_3^{(1)}}) \right] \right\} \\ & \star_{\xi_i^{(1)}} \phi_1(p_1, \xi_1^{(1)}) \phi_2(p_2, \xi_2^{(1)}) \phi_3(p_3, \xi_3^{(1)}) , \quad (4.58) \end{aligned}$$

For:  $p = 6, 7, 8$  ;  $\xi = 3, 5, 13$  ;  $y_{12} = 1, y_{23} = 2, y_{31} = 4$

$$\left(\left(\left(2.684886475559863 \times 10^{1279} \left[1 - \frac{1}{2} \sqrt{0.915/2} \times (6 \times 3)\right] \left[1 - \frac{1}{2} \sqrt{0.915/2} \times (7 \times 5)\right] \left[1 - \frac{1}{2} \sqrt{0.915/2} \times (8 \times 13)\right]\right)\right)\right) \times 0.9991104684^3 \times (6 \times 3)(7 \times 5)(8 \times 13)$$

**Input interpretation:**

$$\left( (2.684886475559863 \times 10^{1279}) \left( 1 - \frac{1}{2} \sqrt{\frac{0.915}{2}} (6 \times 3) \right) \left( 1 - \frac{1}{2} \sqrt{\frac{0.915}{2}} (7 \times 5) \right) \right. \\ \left. \left( 1 - \frac{1}{2} \sqrt{\frac{0.915}{2}} (8 \times 13) \right) \right) \times 0.9991104684^3 (6 \times 3) (7 \times 5) (8 \times 13)$$

**Result:**

$$-3.30534... \times 10^{1287}$$

**Result:**

$$-3.30534070003323106962086485545353233286719828377661... \times 10^{1287}$$

$$-3.3053407... * 10^{1287}$$

From the ratio between the two results, we obtain:

$$-(-3.3053407000332310696208648 \times 10^{1287} / 2.684886475559863 \times 10^{1279})$$

**Input interpretation:**

$$-\left( \frac{3.3053407000332310696208648 \times 10^{1287}}{2.684886475559863 \times 10^{1279}} \right)$$

**Result:**

$$1.23109141862096367956022282523896097632703058363412541... \times 10^8$$

**Decimal form:**

$$123109141.862096367956022282523896097632703058363412541$$

$$123109141.862$$

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$\text{sqrt}(\text{golden ratio}) * \exp(\text{Pi} * \text{sqrt}(n/15)) / (2 * 5^{(1/4)} * \text{sqrt}(n))$  for  $n = 792$  , we obtain:

$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{792}{15}}\right)}{2 \sqrt[4]{5} \sqrt{792}} - 710647 - 167761 - 5778 - 2207 - 843 - 123 - 47 - 18$

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{792}{15}}\right)}{2 \sqrt[4]{5} \sqrt{792}} - 710647 - 167761 - 5778 - 2207 - 843 - 123 - 47 - 18$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{2\sqrt{66/5} \pi} \sqrt{\frac{\phi}{22}}}{12 \sqrt[4]{5}} - 887424$$

**Decimal approximation:**

1.23109141068486494619810773693019522319117401602488681...  $\times 10^8$

123109141.06848

**Property:**

$-887424 + \frac{e^{2\sqrt{66/5} \pi} \sqrt{\frac{\phi}{22}}}{12 \sqrt[4]{5}}$  is a transcendental number

**Alternate forms:**

$$\frac{1}{24} \sqrt{\frac{1}{55} (5 + \sqrt{5})} e^{2\sqrt{66/5} \pi} - 887424$$

$$\frac{\sqrt{\frac{1}{11} (1 + \sqrt{5})} e^{2\sqrt{66/5} \pi}}{24 \sqrt[4]{5}} - 887424$$

$$\frac{5^{3/4} \sqrt{11 (1 + \sqrt{5})} e^{2\sqrt{66/5} \pi} - 1171399680}{1320}$$

## Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{792}{15}}\right)}{2 \sqrt[4]{5} \sqrt{792}} - 710647 - 167761 - 5778 - 2207 - 843 - 123 - 47 - 18 =$$

$$\left( -8874240 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (792 - z_0)^k z_0^{-k}}{k!} + 5^{3/4} \right.$$

$$\left. \exp\left[ \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{264}{5} - z_0\right)^k z_0^{-k}}{k!} \right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) /$$

$$\left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (792 - z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{792}{15}}\right)}{2 \sqrt[4]{5} \sqrt{792}} - 710647 - 167761 - 5778 - 2207 - 843 - 123 - 47 - 18 =$$

$$\left( -8874240 \exp\left(i\pi \left\lfloor \frac{\arg(792 - x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (792 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right.$$

$$5^{3/4} \exp\left(i\pi \left\lfloor \frac{\arg(\phi - x)}{2\pi} \right\rfloor\right) \exp\left[ \pi \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{264}{5} - x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{264}{5} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right] \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) /$$

$$\left( 10 \exp\left(i\pi \left\lfloor \frac{\arg(792 - x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (792 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{792}{15}}\right)}{2 \sqrt[4]{5} \sqrt{792}} - 710647 - 167761 - 5778 - 2207 - 843 - 123 - 47 - 18 = \\
& \left( \left(\frac{1}{z_0}\right)^{-1/2 [\arg(792-z_0)/(2\pi)]} z_0^{-1/2 [\arg(792-z_0)/(2\pi)]} \right. \\
& \quad \left. \left( -8874240 \left(\frac{1}{z_0}\right)^{1/2 [\arg(792-z_0)/(2\pi)]} z_0^{1/2 [\arg(792-z_0)/(2\pi)]} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (792-z_0)^k z_0^{-k}}{k!} + 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(\frac{264}{5}-z_0)/(2\pi)]}\right) \right. \right. \\
& \quad \left. \left. z_0^{1/2 (1+[\arg(\frac{264}{5}-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{264}{5}-z_0\right)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \left. \left(\frac{1}{z_0}\right)^{1/2 [\arg(\phi-z_0)/(2\pi)]} z_0^{1/2 [\arg(\phi-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (792-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

Now, we have that

$$\hat{\mathcal{P}} = -\frac{1}{p^2} \left[ a \left( \frac{1}{2} \xi \cdot \lambda + \frac{1}{2} \sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2} \right) + a \left( \frac{1}{2} \xi \cdot \lambda - \frac{1}{2} \sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2} \right) - a_0 \right], \quad (5.37)$$

For:

$$\xi = 2; \quad \lambda = 3; \quad p = 5; \quad a_0 = 8; \quad \text{and} \quad a = 13,$$

we obtain:

$$-1/16 [13(((1/2*2*3+1/2*\sqrt{((2*3)^2-2^2*3^2)})))+13(((1/2*2*3-1/2*\sqrt{((2*3)^2-2^2*3^2)})))-8]$$



**Input:**

$$-\frac{1}{16} \left( 13 \left( \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \sqrt{(2 \times 3)^2 - 2^2 \times 3^2} \right) + 13 \left( \frac{1}{2} \times 2 \times 3 - \frac{1}{2} \sqrt{(2 \times 3)^2 - 2^2 \times 3^2} \right) - 8 \right)$$

**Exact result:**

$$-\frac{35}{8}$$

**Decimal form:**

-4.375  
**-4.375**

Now, we have that:

$$M_s^2 = \frac{s-1}{\alpha'}$$

(2-1)/0.915

**Input:**

$$\frac{1}{0.915}$$

**Result:**

1.092896174863387978142076502732240437158469945355191256830...  
**1.0928961748.....**

$$\hat{p} = \sum_{s=0}^{\infty} -\frac{a_s}{p^2 + M_s^2} \left\{ \frac{\Gamma(\beta)}{\Gamma(\beta + s)} \left( \frac{\xi^2 \lambda^2}{4} \right)^{s/2} G_s^{[\beta]} \left( \frac{\xi \cdot \lambda}{\sqrt{\xi^2 \lambda^2}} \right) \right\}, \quad (5.40)$$

For  $s = 2$ ;  $p = 5$ ;  $\beta > 0$ ;  $\beta = 3$ ;  $\alpha' = 0.915$ ;  $\xi = 2$ ;  $\lambda = 3$ ; we obtain:

$-8/(25+1.0928961748) [\text{gamma}(3) / \text{gamma}(5) ((2^2 \cdot 3^2)/4) * (2 \cdot 3) / (\text{sqrt}(2^2 \cdot 3^2))]$

### Input interpretation:

$$-\frac{8}{25 + 1.0928961748} \left( \frac{\Gamma(3)}{\Gamma(5)} \left( \frac{1}{4} (2^2 \times 3^2) \right) \times \frac{2 \times 3}{\sqrt{2^2 \times 3^2}} \right)$$

$\Gamma(x)$  is the gamma function

### Result:

-0.22994764397961620788903948649455766660593442281311598728...

-0.229947643...

### Rational approximation:

$$-\frac{1098}{4775}$$

### Alternative representations:

$$\frac{(\Gamma(3) (2^2 \times 3^2) (2 \times 3)) (-8)}{(\Gamma(5) 4 \sqrt{2^2 \times 3^2}) (25 + 1.09289617480000)} = -\frac{3456}{\frac{4}{12} \times 26.09289617480000 \times 288 \sqrt{36}}$$

$$\frac{(\Gamma(3) (2^2 \times 3^2) (2 \times 3)) (-8)}{(\Gamma(5) 4 \sqrt{2^2 \times 3^2}) (25 + 1.09289617480000)} = -\frac{1728 \times 2!}{4 \times 26.09289617480000 \times 4! \sqrt{36}}$$

$$\frac{(\Gamma(3) (2^2 \times 3^2) (2 \times 3)) (-8)}{(\Gamma(5) 4 \sqrt{2^2 \times 3^2}) (25 + 1.09289617480000)} = \frac{1728 e^{\log(2)}}{4 \times 26.09289617480000 e^{-\log(12)+\log(288)} \sqrt{36}}$$

### Series representations:

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} =$$

$$\frac{16.556230366532367 \Gamma(3)}{\exp\left(i\pi \left\lfloor \frac{\arg(36-x)}{2\pi} \right\rfloor\right) \Gamma(5) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (36-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} =$$

$$\frac{16.556230366532367 \Gamma(3) \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(36-z_0)/(2\pi) \rfloor} z_0^{-1/2-1/2 \lfloor \arg(36-z_0)/(2\pi) \rfloor}}{\Gamma(5) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (36-z_0)^k z_0^{-k}}{k!}}$$

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} =$$

$$\frac{16.556230366532367 \sum_{k=0}^{\infty} \frac{(3-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\sqrt{35} \left( \sum_{k=0}^{\infty} 35^{-k} \binom{\frac{1}{2}}{k} \right) \sum_{k=0}^{\infty} \frac{(5-z_0)^k \Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

### Integral representations:

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} =$$

$$\frac{16.556230366532367 \int_0^1 e^{-(-2+x^3+x^4)/\log(x)} dx}{\sqrt{36}}$$

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} = \frac{16.556230366532367 \exp\left(2\gamma + \int_0^1 \frac{x^3 - x^5 - \log(x^3) + \log(x^5)}{(-1+x)\log(x)} dx\right)}{\sqrt{36}}$$

$$\frac{(\Gamma(3)(2^2 \times 3^2)(2 \times 3))(-8)}{(\Gamma(5)4\sqrt{2^2 \times 3^2})(25 + 1.09289617480000)} = - \frac{16.556230366532367 \int_0^1 \log^2\left(\frac{1}{t}\right) dt}{\sqrt{36} \int_0^1 \log^4\left(\frac{1}{t}\right) dt}$$

$\gamma$  is the Euler-Mascheroni constant

From the two results -0.229947643 and -4.375 , performing the  $\pi^{\text{th}}$  root and changing the sign, we obtain:

$$-[-4.375 + (((-8 / (25 + 1.0928961748)) [\text{gamma}(3) / \text{gamma}(5) ((2^2 \times 3^2) / 4) * (2 \times 3) / (\text{sqrt}(2^2 \times 3^2))])])^{1/\pi}$$

**Input interpretation:**

$$\sqrt[\pi]{- \left( -4.375 - \frac{8}{25 + 1.0928961748} \left( \frac{\Gamma(3)}{\Gamma(5)} \left( \frac{1}{4} (2^2 \times 3^2) \right) \times \frac{2 \times 3}{\sqrt{2^2 \times 3^2}} \right) \right)}$$

$\Gamma(x)$  is the gamma function

**Result:**

1.625964194649730699381553290257912994507373811030175551121...

1.62596419464..... result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternative representations:**

$$\sqrt[\pi]{-\left(-4.375 - \frac{8(\Gamma(3)(2^2 \times 3^2)(2 \times 3))}{(25 + 1.09289617480000)(\Gamma(5)4\sqrt{2^2 \times 3^2})}\right)} = \sqrt[\pi]{4.375 + \frac{3456}{\frac{4}{12} \times 26.09289617480000 \times 288 \sqrt{36}}}$$

$$\sqrt[\pi]{-\left(-4.375 - \frac{8(\Gamma(3)(2^2 \times 3^2)(2 \times 3))}{(25 + 1.09289617480000)(\Gamma(5)4\sqrt{2^2 \times 3^2})}\right)} = \sqrt[\pi]{4.375 + \frac{1728 \times 2!}{4 \times 26.09289617480000 \times 4! \sqrt{36}}}$$

$$\sqrt[\pi]{-\left(-4.375 - \frac{8(\Gamma(3)(2^2 \times 3^2)(2 \times 3))}{(25 + 1.09289617480000)(\Gamma(5)4\sqrt{2^2 \times 3^2})}\right)} = \sqrt[\pi]{4.375 + \frac{1728 e^{\log(2)}}{4 \times 26.09289617480000 e^{-\log(12)+\log(288)} \sqrt{36}}}$$

**Series representations:**

$$\sqrt[\pi]{-\left(-4.375 - \frac{8(\Gamma(3)(2^2 \times 3^2)(2 \times 3))}{(25 + 1.09289617480000)(\Gamma(5)4\sqrt{2^2 \times 3^2})}\right)} = \sqrt[\pi]{4.375 + \frac{16.556230366532367 \Gamma(3)}{\exp\left(i\pi \left[\frac{\arg(36-x)}{2\pi}\right]\right) \Gamma(5) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (36-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt{\pi} \sqrt{-\left( -4.375 - \frac{8 (\Gamma(3) (2^2 \times 3^2) (2 \times 3))}{(25 + 1.09289617480000) (\Gamma(5) 4 \sqrt{2^2 \times 3^2})} \right)} =$$

$$\sqrt{\pi} \sqrt{4.375 + \frac{16.556230366532367 \Gamma(3) \left(\frac{1}{z_0}\right)^{-1/2 [\arg(36-z_0)/(2\pi)]} z_0^{-1/2-1/2 [\arg(36-z_0)/(2\pi)]}}{\Gamma(5) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (36-z_0)^k z_0^{-k}}{k!}}$$

$$\sqrt{\pi} \sqrt{-\left( -4.375 - \frac{8 (\Gamma(3) (2^2 \times 3^2) (2 \times 3))}{(25 + 1.09289617480000) (\Gamma(5) 4 \sqrt{2^2 \times 3^2})} \right)} =$$

$$\sqrt{\pi} \sqrt{4.375 + \frac{16.556230366532367 \sum_{k=0}^{\infty} \frac{(3-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\sqrt{35} \left( \sum_{k=0}^{\infty} 35^{-k} \binom{\frac{1}{2}}{k} \right) \sum_{k=0}^{\infty} \frac{(5-z_0)^k \Gamma^{(k)}(z_0)}{k!}}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

**Integral representations:**

$$\sqrt{\pi} \sqrt{-\left( -4.375 - \frac{8 (\Gamma(3) (2^2 \times 3^2) (2 \times 3))}{(25 + 1.09289617480000) (\Gamma(5) 4 \sqrt{2^2 \times 3^2})} \right)} =$$

$$\sqrt{\pi} \sqrt{4.375 + \frac{16.556230366532367 \int_0^1 e^{-(-2+x^3+x^4)/\log(x)} dx}{\sqrt{36}}}$$

$$\sqrt{\pi} \sqrt{-\left( -4.375 - \frac{8 (\Gamma(3) (2^2 \times 3^2) (2 \times 3))}{(25 + 1.09289617480000) (\Gamma(5) 4 \sqrt{2^2 \times 3^2})} \right)} =$$

$$\sqrt{\pi} \sqrt{4.375 + \frac{16.556230366532367 \int_0^1 \log^2\left(\frac{1}{t}\right) dt}{\sqrt{36} \int_0^1 \log^4\left(\frac{1}{t}\right) dt}}$$

$$\sqrt[3]{\left(-4.375 - \frac{8(\Gamma(3)(2^2 \times 3^2)(2 \times 3))}{(25 + 1.09289617480000)(\Gamma(5)4\sqrt{2^2 \times 3^2})}\right)} = \sqrt[3]{4.375 + \frac{16.556230366532367 \exp\left(2\gamma + \int_0^1 \frac{x^3 - x^5 - \log(x^3) + \log(x^5)}{(-1+x)\log(x)} dx\right)}{\sqrt{36}}}$$

From

$$\hat{p} = -\frac{1}{p^2} \left[ e^{\frac{1}{2}\xi \cdot \lambda + \frac{1}{2}\sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2}} \sum_{i=0}^{\beta-1} \binom{\beta-1}{i} g_{-}^{(i)} \left( \frac{1}{2} \xi \cdot \lambda + \frac{1}{2}\sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2} \right) + e^{\frac{1}{2}\xi \cdot \lambda - \frac{1}{2}\sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2}} \sum_{i=0}^{\beta-1} \binom{\beta-1}{i} g_{+}^{(i)} \left( \frac{1}{2} \xi \cdot \lambda - \frac{1}{2}\sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2} \right) \right]. \quad (5.56)$$

For:  $\beta = 3$  ;  $\alpha' = 0.915$ ;  $\xi = 2$ ;  $\lambda = 3$   $g_0 = 2.715864906$ ;  $p = 8$  , we obtain:

$$-1/64 * [\exp(((1/2*2*3+1/2*\sqrt{6^2-2^2*3^2}))) * \{2, 1\} * 2.715864906 + \exp(((1/2*6+1/2*\sqrt{6^2-2^2*3^2})) + \exp(((1/2*6-1/2*\sqrt{6^2-2^2*3^2}))) * \{2, 1\} * 2.715864906 (1/2*6-1/2*\sqrt{6^2-2^2*3^2}))]$$

**Input interpretation:**

$$-\frac{1}{64} \left( \exp\left(\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \{2, 1\} \times 2.715864906 \left(\frac{1}{2} \times 6 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \exp\left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \{2, 1\} \times 2.715864906 \left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right)$$

**Result:**

$$\{-10.2281, -5.11403\}$$

$$-10.2281, -5.11403$$

**Difference:**

$$-5.11403 - -10.2281 = 5.11403$$

**Ratio:**

$$\frac{-10.2281}{-5.11403} = 2$$

**Total:**

$$-10.2281 - 5.11403 = -15.3421$$

**Vector length:**

11.4353

11.4353

**Normalized vector:**

(-0.894427, -0.447214)

**Angles between vector and coordinate axes:**

horizontal: 153.435° | vertical: 116.565°

**Polar coordinates:**

$r = 11.4353$  (radius),  $\theta = -153.435^\circ$  (angle)

11.4353

From which:

$$(11.4353)^{1/5}$$

**Input interpretation:**

$$\sqrt[5]{11.4353}$$

**Result:**

1.627982...

1.627982.... result very near to the mean between  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  and the value of the golden ratio 1.618033988749...



We note that:

$$-1/64 * [\exp(((1/2*2*3+1/2*\sqrt{6^2-2^2*3^2}))) * 2.715864906 \\ (1/2*6+1/2*\sqrt{6^2-2^2*3^2}) + \exp(((1/2*6-1/2*\sqrt{6^2-2^2*3^2}))) * \\ 2.715864906 (1/2*6-1/2*\sqrt{6^2-2^2*3^2})]$$

**Input interpretation:**

$$-\frac{1}{64} \left( \exp\left(\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \times 2.715864906 \left(\frac{1}{2} \times 6 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \\ \left. \exp\left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \times 2.715864906 \left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right)$$

**Result:**

-5.114025454...

-5.114025454...

**Series representations:**

$$\frac{1}{64} \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \\ \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) (-1) = \\ 0.0212177 \left( -6 \exp\left(3 - \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) - \right. \\ \left. 6 \exp\left(3 + \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) + \right. \\ \left. \exp\left(3 - \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} - \right. \\ \left. \exp\left(3 + \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \left( \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right) \right)$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\begin{aligned}
& \frac{1}{64} \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \\
& \quad \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) (-1) = \\
& 0.0212177 \left( -6 \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) - \\
& 6 \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\
& \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right) \\
& \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right) \\
& \left( \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\
& \left. \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{64} \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \\
& \quad \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) (-1) = 0.0212177 \\
& \left( -6 \exp\left[3 - \frac{1}{2} \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right) - \right. \\
& 6 \exp\left[ \right. \\
& \quad \left. 3 + \frac{1}{2} \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right] + \\
& \exp\left[3 - \frac{1}{2} \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right] \\
& \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 + 1/2 [\arg(-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} - \\
& \exp\left[3 + \frac{1}{2} \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right] \\
& \left. \left(\frac{1}{z_0}\right)^{1/2 [\arg(-z_0)/(2\pi)]} z_0^{1/2 + 1/2 [\arg(-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

From which:

$$1 + 1 / \left( \left( -\frac{1}{64} \left[ \exp\left(\left(\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right)\right) \times 2.715864906 \right. \right. \right. \\ \left. \left. \left( \frac{1}{2} \times 6 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2} \right) + \exp\left(\left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right)\right) \times \right. \right. \\ \left. \left. \left. 2.715864906 \left( \frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2} \right) \right] \right) \right) \right)^{1/4}$$

**Input interpretation:**

$$1 + 1 / \left( \left( -\frac{1}{64} \left( \exp\left(\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \times 2.715864906 \right. \right. \right. \\ \left. \left. \left( \frac{1}{2} \times 6 + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2} \right) + \exp\left(\frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \times \right. \right. \\ \left. \left. \left. 2.715864906 \left( \frac{1}{2} \times 6 - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2} \right) \right) \right) \right)^{(1/4)}$$

**Result:**

1.6649810642...

1.6649810642..... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

## Series representations:

$$\begin{aligned}
& 1 + 1 / \left( \left( -\frac{1}{64} (-1) \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \right. \right. \\
& \quad \left. \left. \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) \right) \right)^{\wedge (1/4)} \\
& (1/4) = \left( 2\sqrt{2} + \left( \exp\left(3 - \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right. \right. \\
& \quad \left. \left( 8.14759 - 1.35793 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) + \right. \\
& \quad \left. \exp\left(3 + \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right. \\
& \quad \left. \left( 8.14759 + 1.35793 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right)^{\wedge (1/4)} / \\
& \left( \left( \exp\left(3 - \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right. \right. \\
& \quad \left. \left( 8.14759 - 1.35793 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) + \right. \\
& \quad \left. \exp\left(3 + \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right. \\
& \quad \left. \left( 8.14759 + 1.35793 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-z_0)^k z_0^{-k}}{k!}\right) \right)^{\wedge (1/4)} \right)
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\begin{aligned}
& 1 + 1 / \left( \left( -\frac{1}{64} (-1) \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \right. \right. \\
& \quad \left. \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \right. \right. \\
& \quad \left. \left. \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) \right)^{(1/4)} = \\
& \left( 2\sqrt{2} + \left( \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left( 8.14759 - \right. \right. \\
& \quad \left. \left. 1.35793 \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\
& \quad \left( 8.14759 + 1.35793 \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right)^{(1/4)} / \\
& \left( \left( \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left( 8.14759 - \right. \right. \\
& \quad \left. \left. 1.35793 \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\
& \quad \left( 8.14759 + 1.35793 \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right)^{(1/4)} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& 1 + 1 / \left( \left( -\frac{1}{64} (-1) \left( \exp\left(\frac{2 \times 3}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \left(\frac{6}{2} + \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) + \right. \right. \right. \\
& \quad \left. \left. \exp\left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) 2.71586 \right. \right. \\
& \quad \left. \left. \left(\frac{6}{2} - \frac{1}{2} \sqrt{6^2 - 2^2 \times 3^2}\right) \right) \right)^{(1/4)} = \\
& \left( 2\sqrt{2} + \left( 2.71586 \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right. \\
& \quad \left( 3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) + \\
& \quad 2.71586 \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) \\
& \quad \left( 3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) \right)^{(1/4)} / \\
& \left( \left( 2.71586 \exp\left[3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right. \\
& \quad \left( 3 - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) + \\
& \quad 2.71586 \exp\left[3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) \\
& \quad \left( 3 + \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg) \right)^{(1/4)} \\
& \left. \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

Now, we have that:

$$\hat{P} = -\frac{1}{p^2} \left( 1 - \xi \cdot \lambda + \frac{\xi^2 \lambda^2}{4} \right)^{-\beta},$$

For:  $\xi = 2$ ;  $\lambda = 3$ ;  $p = 8$ ;  $\beta = 3$ , we obtain:

$$-1/64 * (1 - 6 + (3^2 * 2^2) / 4)^{-3}$$

**Input:**

$$-\frac{1}{64\left(1-6+\frac{1}{4}(3^2 \times 2^2)\right)^3}$$

**Exact result:**

$$-\frac{1}{4096}$$

**Decimal form:**

-0.000244140625

**-0.000244140625**

$$-1/\left(\left(-1/64*(1-6+(3^2*2^2)/4)^{-3}\right)\right)$$

**Input:**

$$\frac{-1}{64\left(1-6+\frac{1}{4}(3^2 \times 2^2)\right)^3}$$

**Exact result:**

4096

$$4096 = 64^2$$

We note how with the data we entered, we get 1/4096 as a result and that, inverting the expression, it is precisely  $4096 = 64^2$

**Ramanujan mathematics: Class Invariants.**

From:

**Ramanujan's Notebooks Part V – Bruce C. Berndt**

We have that:

$$\sqrt{2 + \sqrt{5}} \left( \frac{21 + \sqrt{445}}{2} \right)^{1/4} \left( \sqrt{\frac{13 + \sqrt{89}}{8}} + \sqrt{\frac{5 + \sqrt{89}}{8}} \right)$$

and obtain:

$$(2 + \sqrt{5})^{0.5} (1/2 * (21 + \sqrt{445}))^{0.25} \\ (((1/8 * (13 + \sqrt{89}))^{0.5} + (1/8 * (5 + \sqrt{89}))^{0.5}))$$

**Input:**

$$\sqrt{2 + \sqrt{5}} \left( \frac{1}{2} (21 + \sqrt{445}) \right)^{0.25} \left( \sqrt{\frac{1}{8} (13 + \sqrt{89})} + \sqrt{\frac{1}{8} (5 + \sqrt{89})} \right)$$

**Result:**

13.3037...  
13.3037...

From which:

$$(((2 + \sqrt{5})^{0.5} (1/2 * (21 + \sqrt{445}))^{0.25} \\ ((1/8 * (13 + \sqrt{89}))^{0.5} + (1/8 * (5 + \sqrt{89}))^{0.5}))))^{1/(2e)}$$

**Input:**

$$\sqrt[2e]{\sqrt{2 + \sqrt{5}} \left( \frac{1}{2} (21 + \sqrt{445}) \right)^{0.25} \left( \sqrt{\frac{1}{8} (13 + \sqrt{89})} + \sqrt{\frac{1}{8} (5 + \sqrt{89})} \right)}$$

**Result:**

1.609694362999406759194213494996586800380495971257469538801...

1.609694362..... result that is a good approximation to the value of the golden ratio  
1.618033988749...



**Series representations:**

$$\begin{aligned}
 & \sqrt[2e]{\sqrt{2+\sqrt{5}} \left(\frac{1}{2}(21+\sqrt{445})\right)^{0.25} \left(\sqrt{\frac{1}{8}(13+\sqrt{89})} + \sqrt{\frac{1}{8}(5+\sqrt{89})}\right)} = \\
 & \sqrt[2e]{0.840896} \left( \sqrt{2+\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}} \left( \frac{\sqrt{5+\sqrt{88} \sum_{k=0}^{\infty} 88^{-k} \binom{\frac{1}{2}}{k}}}{2\sqrt{2}} + \right. \right. \\
 & \left. \left. \frac{\sqrt{13+\sqrt{88} \sum_{k=0}^{\infty} 88^{-k} \binom{\frac{1}{2}}{k}}}{2\sqrt{2}} \right) \left( 21+\sqrt{444} \sum_{k=0}^{\infty} 444^{-k} \binom{\frac{1}{2}}{k} \right)^{0.25} \right) \wedge \left( \frac{1}{2e} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \sqrt[2e]{\sqrt{2+\sqrt{5}} \left(\frac{1}{2}(21+\sqrt{445})\right)^{0.25} \left(\sqrt{\frac{1}{8}(13+\sqrt{89})} + \sqrt{\frac{1}{8}(5+\sqrt{89})}\right)} = \\
 & \sqrt[2e]{0.840896} \left( \sqrt{2+\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right. \\
 & \left( \frac{\sqrt{5+\sqrt{88} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{88}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}} + \frac{\sqrt{13+\sqrt{88} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{88}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}} \right) \\
 & \left. \left( 21+\sqrt{444} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{444}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^{0.25} \right) \wedge \left( \frac{1}{2e} \right)
 \end{aligned}$$

$$\begin{aligned}
& \sqrt[2e]{\sqrt{2+\sqrt{5}} \left(\frac{1}{2}(21+\sqrt{445})\right)^{0.25} \left(\sqrt{\frac{1}{8}(13+\sqrt{89})} + \sqrt{\frac{1}{8}(5+\sqrt{89})}\right)} = \\
& \sqrt[2e]{0.840896} \\
& \left( \sqrt{2+\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} \left( \frac{\sqrt{5+\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (89-z_0)^k z_0^{-k}}{k!}}}{2\sqrt{2}} + \right. \right. \\
& \left. \left. \frac{\sqrt{13+\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (89-z_0)^k z_0^{-k}}{k!}}}{2\sqrt{2}} \right) \right. \\
& \left. \left( 21+\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (445-z_0)^k z_0^{-k}}{k!} \right)^{0.25} \right)^{\wedge} \left( \frac{1}{2e} \right)
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

### Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \text{ for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

We have:

$$\left(\frac{3\sqrt{3} + \sqrt{23}}{2}\right)^{1/4} (78\sqrt{2} + 23\sqrt{23})^{1/12} \left(\sqrt{\frac{5+2\sqrt{6}}{4}} + \sqrt{\frac{1+2\sqrt{6}}{4}}\right)^{1/2}$$

$$\begin{aligned}
& (1/2*(3\sqrt{3}+\sqrt{23}))^{0.25} (78\sqrt{2}+23\sqrt{23})^{(1/12)} \\
& (((((1/4(5+2\sqrt{6}))^{0.5}+(1/4(1+2\sqrt{6}))^{0.5})))^{0.5}
\end{aligned}$$

**Input:**

$$\left(\frac{1}{2} (3\sqrt{3} + \sqrt{23})\right)^{0.25} \sqrt[12]{78\sqrt{2} + 23\sqrt{23}} \sqrt{\sqrt{\frac{1}{4}(5+2\sqrt{6})} + \sqrt{\frac{1}{4}(1+2\sqrt{6})}}$$

**Result:**

3.91352...

**3.91352...**

From which:

$$\left(\left(\left(\frac{1}{2}(3\sqrt{3}+\sqrt{23})\right)^{0.25} (78\sqrt{2}+23\sqrt{23})^{1/12}\right.\right. \\ \left.\left.\left(\left(\left(\frac{1}{4}(5+2\sqrt{6})\right)^{0.5}+(\frac{1}{4}(1+2\sqrt{6}))^{0.5}\right)\right)^{0.5}\right)\right)^{1/e}$$

**Input:**

$$\sqrt[14]{\left(\frac{1}{2} (3\sqrt{3} + \sqrt{23})\right)^{0.25} \sqrt[12]{78\sqrt{2} + 23\sqrt{23}} \sqrt{\sqrt{\frac{1}{4}(5+2\sqrt{6})} + \sqrt{\frac{1}{4}(1+2\sqrt{6})}}}$$

**Result:**

1.65194...

1.65194.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

**Integral representation:**

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From the mean between the two results, we obtain:

$$\frac{1}{2}[1.6096943629+\left(\left(\left(\frac{1}{2}(3\sqrt{3}+\sqrt{23})\right)^{0.25} (78\sqrt{2}+23\sqrt{23})^{1/12}\right.\right. \\ \left.\left.\left(\left(\left(\frac{1}{4}(5+2\sqrt{6})\right)^{0.5}+(\frac{1}{4}(1+2\sqrt{6}))^{0.5}\right)\right)^{0.5}\right)\right)^{1/e}]$$

**Input interpretation:**

$$\frac{1}{2} \left( 1.6096943629 + \left( \left( \frac{1}{2} (3\sqrt{3} + \sqrt{23}) \right)^{0.25} \sqrt[12]{78\sqrt{2} + 23\sqrt{23}} \right. \right. \\ \left. \left. \sqrt{\sqrt{\frac{1}{4}(5+2\sqrt{6})} + \sqrt{\frac{1}{4}(1+2\sqrt{6})}} \right)^{(1/e)} \right)$$

**Result:**

1.630815328825611765214442994211733576342274345730037661006...

1.630815328..... result very near to the mean between  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  and the value of the golden ratio 1.618033988749...

From

$$G_{117} = \frac{1}{2} \left( \frac{3 + \sqrt{13}}{2} \right)^{1/4} (2\sqrt{3} + \sqrt{13})^{1/6} \left( 3^{1/4} + \sqrt{4 + \sqrt{3}} \right). \quad (3.23)$$

$$1/2(1/2(3+\text{sqrt}13))^{0.25} (2\text{sqrt}3+\text{sqrt}13)^{(1/6)} ((3^{(1/4)}+(4+\text{sqrt}3)^{0.5}))$$

**Input:**

$$\left( \frac{1}{2} \left( \frac{1}{2} (3 + \sqrt{13}) \right)^{0.25} \right)^6 \sqrt[6]{2\sqrt{3} + \sqrt{13}} \left( \sqrt[4]{3} + \sqrt{4 + \sqrt{3}} \right)$$

**Result:**

3.46464...

3.46464...

From which

$$1+1/(((1/2(1/2(3+\text{sqrt}13))^{0.25} (2\text{sqrt}3+\text{sqrt}13)^{(1/6)} ((3^{(1/4)}+(4+\text{sqrt}3)^{0.5}))))^{1/3}$$

**Input:**

$$1 + \frac{1}{\sqrt[3]{\left(\frac{1}{2}\left(\frac{1}{2}(3 + \sqrt{13})\right)^{0.25}\right)^6 \sqrt{2\sqrt{3} + \sqrt{13}} \left(\sqrt[4]{3} + \sqrt{4 + \sqrt{3}}\right)}}$$

**Result:**

1.660867...

1.660867.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

Now, we have that:

$$\left(\sqrt{748 + 432\sqrt{3}} + \sqrt{747 + 432\sqrt{3}}\right)^2 = \left(\frac{5 + \sqrt{23}}{\sqrt{2}}\right)^2 \left(\frac{3\sqrt{3} + \sqrt{23}}{2}\right)^3.$$

$$(1/(\text{sqrt}2)*(5+\text{sqrt}23))^2 (1/2*(3\text{sqrt}3+\text{sqrt}23))^3$$

**Input:**

$$\left(\frac{1}{\sqrt{2}}(5 + \sqrt{23})\right)^2 \left(\frac{1}{2}(3\sqrt{3} + \sqrt{23})\right)^3$$

**Result:**

$$\frac{1}{16}(5 + \sqrt{23})^2 (3\sqrt{3} + \sqrt{23})^3$$

**Decimal approximation:**

5982.983628338331952051281609163182789919354901178275259041...

**5982.983628****Alternate forms:**

$$\frac{1495 + 864\sqrt{3} + 312\sqrt{23} + 180\sqrt{69}}{(24 + 5\sqrt{23})(36\sqrt{3} + 13\sqrt{23})}$$

$$1495 + 864\sqrt{3} + 12\sqrt{23(1351 + 780\sqrt{3})}$$

**Minimal polynomial:**

$$x^4 - 5980x^3 - 17850x^2 - 5980x + 1$$

and:

$$(((748+432\sqrt{3})^{0.5}+(747+432\sqrt{3})^{0.5}))^2$$

**Input:**

$$\left(\sqrt{748 + 432\sqrt{3}} + \sqrt{747 + 432\sqrt{3}}\right)^2$$

**Decimal approximation:**

5982.983628338331952051281609163182789919354901178275259041...

5982.983628...

**Alternate forms:**

$$1495 + 864\sqrt{3} + 12\sqrt{23(1351 + 780\sqrt{3})}$$

$$\left(3\sqrt{83 + 48\sqrt{3}} + \sqrt{748 + 432\sqrt{3}}\right)^2$$

$$\left(3\sqrt{83 + 48\sqrt{3}} + 2\sqrt{187 + 108\sqrt{3}}\right)^2$$

**Minimal polynomial:**

$$x^4 - 5980x^3 - 17850x^2 - 5980x + 1$$

From which:

$$[(((748+432\sqrt{3})^{0.5}+(747+432\sqrt{3})^{0.5}))^2]^{1/18}$$

**Input:**

$$\sqrt[18]{\left(\sqrt{748 + 432\sqrt{3}} + \sqrt{747 + 432\sqrt{3}}\right)^2}$$

**Exact result:**

$$\sqrt[9]{\sqrt{747 + 432\sqrt{3}} + \sqrt{748 + 432\sqrt{3}}}$$

**Decimal approximation:**

1.621170780216268732746613462696659272226199873369094626261...

1.6211707802.... result that is a good approximation to the value of the golden ratio

1.618033988749...

**Alternate forms:**

$$\sqrt[18]{1495 + 864\sqrt{3} + 12\sqrt{23(1351 + 780\sqrt{3})}}$$

$$\sqrt[9]{3\sqrt{83 + 48\sqrt{3}} + \sqrt{748 + 432\sqrt{3}}}$$

$$\sqrt[9]{3\sqrt{83 + 48\sqrt{3}} + 2\sqrt{187 + 108\sqrt{3}}}$$

**Minimal polynomial:**

$$x^{72} - 5980x^{54} - 17850x^{36} - 5980x^{18} + 1$$

From

$$G_{65} = \left(\sqrt{\frac{\sqrt{65} + 9}{8}} + \sqrt{\frac{\sqrt{65} + 1}{8}}\right)^{1/2} \left(\sqrt{\frac{\sqrt{65} + 7}{8}} + \sqrt{\frac{\sqrt{65} - 1}{8}}\right)^{1/2}$$

we obtain, after some calculations:

1.5204308681639869376735244300731 \* 1.588939028253301759079475888573

**Result:**

2.415871946186809362816339075281483267763757252039583025844...

2.415871946.....

$1 + \frac{1}{4}(1.5204308681639 * 1.5889390282533)$

**Input interpretation:**

$1 + \frac{1}{4}(1.5204308681639 \times 1.5889390282533)$

**Result:**

1.6039679865466671373987789675

1.60396798654..... result that is a good approximation to the value of the golden ratio 1.618033988749...

Now, we have:

**Entry 8.2 (p. 316, NB 1).** *We have*

$$G_{175} = \frac{3 \cdot 2^{1/4}}{\frac{\sqrt{5}-1}{2} + \left(\frac{5-\sqrt{5}}{4}\right)^{1/3} \left(\sqrt[3]{8-3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8-3\sqrt{5}-3\sqrt{21}}\right)} \quad (8.6)$$

and

$$G_{257} = \frac{3 \cdot 2^{1/4}}{\frac{\sqrt{5}+1}{2} + \left(\frac{5+\sqrt{5}}{4}\right)^{1/3} \left(\sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}}\right)} \quad (8.7)$$



$$(3 \cdot 2^{0.25}) / [((\sqrt{5}-1)/2) + (1/4(5-\sqrt{5}))^{1/3}] * (((8-3\sqrt{5}+3\sqrt{21})^{1/3}) + (8-3\sqrt{5}-3\sqrt{21})^{1/3})]$$

**Input:**

$$\frac{3 \times 2^{0.25}}{\frac{1}{2}(\sqrt{5}-1) + \sqrt[3]{\frac{1}{4}(5-\sqrt{5})} \left( \sqrt[3]{8-3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8-3\sqrt{5}-3\sqrt{21}} \right)}$$

**Result:**

$$0.7674879... - 0.3561169... i$$

**Polar coordinates:**

$$r = 0.846083 \text{ (radius), } \theta = -24.8915^\circ \text{ (angle)}$$

**0.846083**

$$(3 \cdot 2^{0.25}) / [((\sqrt{5}+1)/2) + (1/4(5+\sqrt{5}))^{1/3}] * (((8+3\sqrt{5}+3\sqrt{21})^{1/3}) + (8+3\sqrt{5}-3\sqrt{21})^{1/3})]$$

**Input:**

$$\frac{3 \times 2^{0.25}}{\frac{1}{2}(\sqrt{5}+1) + \sqrt[3]{\frac{1}{4}(5+\sqrt{5})} \left( \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} \right)}$$

**Result:**

$$0.5454916...$$

**0.5454916...**

From the two expression, we obtain also:

$$1 + 1 / (((0.7674879 - 0.3561169i) / (((3 \cdot 2^{0.25}) / [((\sqrt{5}+1)/2) + (1/4(5+\sqrt{5}))^{1/3}] * (((8+3\sqrt{5}+3\sqrt{21})^{1/3}) + (8+3\sqrt{5}-3\sqrt{21})^{1/3}))))))$$

**Input interpretation:**

$$1 + \frac{1}{\frac{0.7674879 + i \times (-0.3561169)}{3 \times 2^{0.25}} \left( \frac{1}{2}(\sqrt{5}+1) + \sqrt[3]{\frac{1}{4}(5+\sqrt{5})} \left( \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} \right) \right)}$$

*i* is the imaginary unit

**Result:**

$$1.58483... + 0.271365... i$$

**Polar coordinates:**

$r = 1.6079$  (radius),  $\theta = 9.71632^\circ$  (angle)

1.60769 result that is a good approximation to the value of the golden ratio

1.618033988749...

From:

0.846083+0.5454916

We obtain also:

$\text{sqrt}[2*(0.846083+0.5454916)]$

**Input interpretation:**

$\sqrt{2(0.846083 + 0.5454916)}$

**Result:**

1.668277315076842972295075281776177089802990219682740326270...

1.668277315..... result very near to the 14th root of the following Ramanujan's

class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

We have that:

$$\alpha_{40} = \left(2\sqrt{2} + \sqrt{5} - \sqrt{4(3 + \sqrt{10})}\right)^4 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^4,$$

$$(2\text{sqrt}2+\text{sqrt}5-(4(3+\text{sqrt}10))^{0.5})^4 (\text{sqrt}2+\text{sqrt}5-(2(3+\text{sqrt}10))^{0.5})^4$$

**Input:**

$$\left(2\sqrt{2} + \sqrt{5} - \sqrt{4(3 + \sqrt{10})}\right)^4 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^4$$

**Exact result:**

$$\left(2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}}\right)^4 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^4$$

**Decimal approximation:**

3.7587742834282045212695462270758171424627846216734342... × 10<sup>-8</sup>  
 3.7587742834282... \* 10<sup>-8</sup>

**Alternate forms:**

root of  $x^8 - 26\,604\,424x^7 + 175\,226\,140x^6 - 376\,661\,176x^5 + 457\,127\,494x^4 - 376\,661\,176x^3 + 175\,226\,140x^2 - 26\,604\,424x + 1$  near  $x = 3.75877 \times 10^{-8}$

$$\left(\sqrt{2} - \sqrt{3-i} - \sqrt{3+i} + \sqrt{5}\right)^4 \left(-2\sqrt{2} - \sqrt{5} + 2\sqrt{3 + \sqrt{10}}\right)^4$$

$$3\,325\,553 + 2\,351\,520\sqrt{2} + 1\,487\,232\sqrt{5} + 1\,051\,632\sqrt{10} - 90\,504\sqrt{3 + \sqrt{10}} -$$

$$98\,256(3 + \sqrt{10})^{3/2} - 68\,928\sqrt{2}(3 + \sqrt{10})^{3/2} - 43\,584\sqrt{5}(3 + \sqrt{10})^{3/2} -$$

$$31\,080\sqrt{10}(3 + \sqrt{10})^{3/2} - 14\,848(3 + \sqrt{10})^{5/2} - 10\,688\sqrt{2}(3 + \sqrt{10})^{5/2} -$$

$$66\,88\sqrt{5}(3 + \sqrt{10})^{5/2} - 4\,736\sqrt{10}(3 + \sqrt{10})^{5/2} - 256(3 + \sqrt{10})^{7/2} -$$

$$256\sqrt{2}(3 + \sqrt{10})^{7/2} - 128\sqrt{5}(3 + \sqrt{10})^{7/2} - 128\sqrt{10}(3 + \sqrt{10})^{7/2} -$$

$$65\,232\sqrt{2(3 + \sqrt{10})} - 41\,256\sqrt{5(3 + \sqrt{10})} - 28\,620\sqrt{10(3 + \sqrt{10})}$$

**Minimal polynomial:**

$$x^8 - 26\,604\,424x^7 + 175\,226\,140x^6 - 376\,661\,176x^5 + 457\,127\,494x^4 - 376\,661\,176x^3 + 175\,226\,140x^2 - 26\,604\,424x + 1$$

From which, we obtain also:

$$\left[1/\left(\left(\left(\left(2\sqrt{2} + \sqrt{5} - (4(3 + \sqrt{10}))^{0.5}\right)^4 (\sqrt{2} + \sqrt{5} - (2(3 + \sqrt{10}))^{0.5})^4\right)\right)\right)\right)^{1/34}$$

**Input:**

$$\sqrt[34]{\frac{1}{\left(2\sqrt{2} + \sqrt{5} - \sqrt{4(3 + \sqrt{10})}\right)^4 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^4}}$$

**Exact result:**

$$\sqrt[34]{\frac{1}{\left(2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}}\right)^4 \left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^4}}$$

**Decimal approximation:**

1.653411647646244515183442637713657849580489589599182827229...

1.6534116476.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

**Minimal polynomial:**

$$x^{136} - 5160x^{119} + 10588x^{102} + 6120x^{85} - 19002x^{68} + 6120x^{51} + 10588x^{34} - 5160x^{17} + 1$$

**Alternate forms:**

$$\sqrt[17]{\frac{1}{\left(2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}}\right)\left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^{2/17}}}$$

root of  $x^8 - 5160x^7 + 10588x^6 + 6120x^5 - 19002x^4 + 6120x^3 + 10588x^2 - 5160x + 1$  near  $x = 5157.95$

$$\sqrt[34]{\frac{1}{\left(\sqrt{2} - \sqrt{3-i} - \sqrt{3+i} + \sqrt{5}\right)^4 \left(-2\sqrt{2} - \sqrt{5} + 2\sqrt{3 + \sqrt{10}}\right)^4}}$$

**All 34th roots of  $1/((2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}}))^4 (\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10}}))^4$ :**

$$\frac{e^0}{\left(2\sqrt{2} + \sqrt{5} - 2\sqrt{3 + \sqrt{10}}\right)\left(\sqrt{2} + \sqrt{5} - \sqrt{2(3 + \sqrt{10})}\right)^{2/17}} \approx 1.653$$

(real, principal root)

$$\frac{e^{(i\pi)/17}}{\left( \left( 2\sqrt{2} + \sqrt{5} - 2\sqrt{3+\sqrt{10}} \right) \left( \sqrt{2} + \sqrt{5} - \sqrt{2(3+\sqrt{10})} \right) \right)^{2/17}} \approx 1.625 + 0.3038i$$

$$\frac{e^{(2i\pi)/17}}{\left( \left( 2\sqrt{2} + \sqrt{5} - 2\sqrt{3+\sqrt{10}} \right) \left( \sqrt{2} + \sqrt{5} - \sqrt{2(3+\sqrt{10})} \right) \right)^{2/17}} \approx 1.542 + 0.597i$$

$$\frac{e^{(3i\pi)/17}}{\left( \left( 2\sqrt{2} + \sqrt{5} - 2\sqrt{3+\sqrt{10}} \right) \left( \sqrt{2} + \sqrt{5} - \sqrt{2(3+\sqrt{10})} \right) \right)^{2/17}} \approx 1.406 + 0.870i$$

$$\frac{e^{(4i\pi)/17}}{\left( \left( 2\sqrt{2} + \sqrt{5} - 2\sqrt{3+\sqrt{10}} \right) \left( \sqrt{2} + \sqrt{5} - \sqrt{2(3+\sqrt{10})} \right) \right)^{2/17}} \approx 1.222 + 1.114i$$

## Mathematical connections with some sectors of String Theory

From:

**Modular equations and approximations to  $\pi$  - Srinivasa Ramanujan**  
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64 \left\{ \left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

From:

## An Update on Brane Supersymmetry Breaking

*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left( p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left( 7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$  instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning  $p$ ,  $C$ ,  $\beta_E$  and  $\phi$  correspond to the exponents of  $e$  (i.e. of exp). Thence we obtain for  $p = 5$  and  $\beta_E = 1/2$ :

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 642, while  $-6C+\phi$  is equal to  $-\pi\sqrt{18}$ . From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp((-Pi*\text{sqrt}(18))$  we obtain:

**Input:**

$$\exp\left(-\pi \sqrt{18}\right)$$

**Exact result:**

$$e^{-3\sqrt{2}\pi}$$

**Decimal approximation:**

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

**Property:**

$e^{-3\sqrt{2}\pi}$  is a transcendental number

**Series representations:**

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \binom{-1/2}{k}}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:



$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

**Input interpretation:**

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

**Result:**

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp((-Pi*\sqrt{18})))))))*1/0.000244140625$$

**Input interpretation:**

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

**Result:**

0.00666501785...

0.00666501785...

**Series representations:**

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$\begin{aligned}e^{-6C+\phi} &= 0.0066650177536 \\ \exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625} &= \\ e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625} &= \\ &= 0.00666501785\dots\end{aligned}$$

From:

$\ln(0.00666501784619)$

**Input interpretation:**

$\log(0.00666501784619)$

**Result:**

-5.010882647757...

-5.010882647757...

**Alternative representations:**

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$

### Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2i\pi \left[ \frac{\arg(0.006665017846190000 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

### Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for  $C = 1$ , we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \phi$$

Note that the values of  $n_s$  (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

Also performing the 512<sup>th</sup> root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

**Input interpretation:**

$$\sqrt[512]{\frac{1}{139.57}}$$

**Result:**

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 =  $\phi$**  and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}}} - \varphi + 1$$

## Observations

We note that, from the number 8, we obtain as follows:

$$8^2$$

$$64$$

$$8^2 \times 2 \times 8$$

$$1024$$

$$8^4 = 8^2 \times 2^6$$

True

$$8^4 = 4096$$

$$8^2 \times 2^6 = 4096$$

$$2^{13} = 2 \times 8^4$$

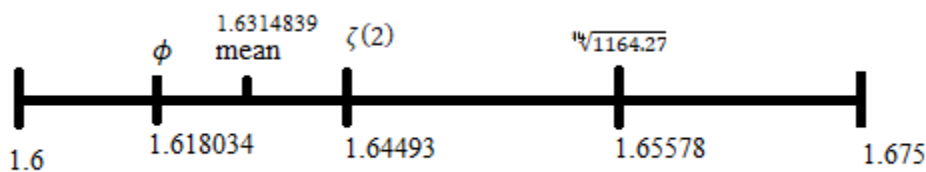
True

$$2^{13} = 8192$$

$$2 \times 8^4 = 8192$$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact  $8^2 = 64$ ,  $8^3 = 512$ ,  $8^4 = 4096$ . We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6, a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

### “Golden” Range



Finally we note how  $8^2 = 64$ , multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to  $\zeta(2)$  that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding  $64 = 8^2$ , one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good

approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## **Conclusions**

With regard the dimensions, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for  $\alpha$ ”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and  $\zeta(2)$ , (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally  $e^{\pi\sqrt{22}}$



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