A New Simultaneity Method for Accelerated Observers in Special Relativity

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Abstract:

In this paper, I have defined and explained a new simultaneity method for accelerated observers in special relativity, which I call Fontenot's simultaneity method. And I have derived an equation I call Fontenot's equation, which makes it easy to determine current age at a distance. My method focuses on determining by how much a distant person (she) ages during the transit of images (which show her age) that she transmits to an observer who sometimes accelerates. Unlike in the well-known "co-moving inertial frames" (CMIF) simultaneity method, in my simultaneity method, the accelerating traveler (he) in the twin paradox does *not* conclude that the home twin (she) instantaneously gets older (or younger) during the traveler's instantaneous turnaround. Instead, he concludes that, after his instantaneous turnaround, she ages faster than he does, at a linear finite rate, for a well-defined number of years, and then she ages more slowly than he does for the last portion of the return trip.

Section 1. Introduction

It has been more than a hundred years since Einstein published his special theory of relativity. Many consider it to be a completed discipline. But there is one aspect of special relativity that is still controversial: there is still disagreement about the answer to the following question: how does an observer, who sometimes accelerates, answer the question “What is the current age of that distant person *right now*? I.e., the controversy is about how an accelerated observer should determine simultaneity at a distance.

The issue is most starkly illustrated by the famous twin ‘paradox’. There is no controversy about which twin is the older at the reunion: it is the ‘home’ twin. And the home twin (whom I will refer to throughout as “she”, for brevity) can very easily determine that outcome: being perpetually inertial, she is clearly entitled to use the famous time-dilation equation, which says that any given perpetually inertial observer will conclude that any other observer (accelerating or not), who is moving at a non-zero relative velocity v, is aging more *slowly* than the given perpetually inertial observer, by the well-known factor *gamma* > 1, where

gamma = 1 / sqrt (1 - v * v),

where the asterisk denotes multiplication, and where I have used dimensionless units for distance and time, with the speed of light being numerically equal to 1 lightyear per year, and the magnitude of v is less than 1.

But the ‘traveling’ twin (whom I’ll refer to throughout as ‘he’) is *not* perpetually inertial, and so he is not always entitled to use the time-dilation equation. He *might* believe that he can use the time dilation equation whenever he is not accelerating, and he *might* believe that nothing happens to their respective ages during his instantaneous turnaround. If so, he will conclude that she will be the younger at the reunion, which we know is incorrect. He will find out at the reunion that his prediction was wrong … thus the apparent paradox. How does he resolve it? To resolve it, he knows that she must age by a large amount at some other time during his trip. He *may* conclude that the only remaining time for her extra aging is during his instantaneous turnaround. So in the most commonly given resolution of the apparent paradox in the twin ‘paradox’ scenario, he concludes that she *instantaneously* ages by a large amount during his instantaneous turnaround. This way of thinking produces an even more startling conclusion in some modified scenarios, where the traveler also accelerates in the opposite direction: he can conclude that she can instantaneously get *younger*. That possibility is repugnant to many physicists.

The simultaneity method used in the above resolution of the twin ‘paradox’ is usually referred to as the ‘Co-Moving Inertial Frames’ (CMIF) method. But there have been other methods proposed and published. I will discuss two of them. And I will define and discuss my new simultaneity method, which I call Fontenot’s simultaneity method.

Section 2. The Dolby and Gull Simultaneity Method

In arXiv:gr-qc/0104077v2, “On Radar Time and the Twin ‘Paradox’ ” [1], Dolby and Gull gave their answer to the question “How old is the home twin, at each instant in the traveling twin’s life, according to the traveling twin?”. Basically, Dolby and Gull’s method says that, according to the ‘traveling twin’ (he), the home twin (she) ages more slowly than he does, as given by the time-dilation equation, for the *first* portion of the outbound leg. Then, part-way to the turnaround, she starts aging *faster* than he does, at a constant rate that continues through the initial portion of the return leg. (The constant faster aging is symmetric about the turnaround point). And then, on the final portion of the return leg, her aging again becomes slower than his, as given by the time-dilation equation.

Note that Dolby and Gull’s simultaneity method violates the principle of *causality*: it says that, according to the the traveling twin, the home twin starts to age faster than the traveler does *well* *before* the traveler does his turnaround. At that point in his trip, there's no way to know if he even *will* turn around. So their method is clearly non-causal.

Section 3. The Minguzzi Simultaneity Method

In arXiv:physics/0411233v1, "Differential Aging from Acceleration, an Explicit Formula" [2], E. Minguzzi defines a simultaneity method which differs from either the CMIF method or the Dolby and Gull method. Minguzzi's method makes use of an additional (imaginary) twin, born when the home twin (she) and the traveling twin (he) are born. I'll sometimes refer to the imaginary twin as *it* (as distinct from *he* or *she*, to prevent confusion), perhaps because its gender was ambiguous at birth. When he instantaneously changes his velocity, relative to *her*, from zero to v when all three of the twins were born, the imaginary twin (*it*) also changes its velocity, to a (usually) different constant velocity, *v_i*. The velocity *v_i* is chosen so that the imaginary twin will be momentarily co-located with him at the exact instant in his life when he wants to determine her current age. Minguzzi *defines* her current age, according to him, at any given instant in his life, *to be the age of the imaginary twin when it passes him*. (So, if he wants to know her current age at more than one instant of his life, there will need to be more than one imaginary twin, because their constant velocities *v_i* will all need to be different).

It is not difficult to show that Minguzzi's above simultaneity definition violates the principle of causality (as does the Dolby and Gull method). To do that, we will examine the case where he wants to know her current age immediately before he turns around. In that case, the imaginary twin (*it*) will have the *same* velocity that he has on his outbound leg: he and *it* will be co-located and mutually stationary during the entire outbound leg. So *it* and he will be exactly the same age immediately before the turnaround. What does Minguzzi's simultaneity method say is her current age (according to him), immediately before he turns around? Minguzzi says that her current age, according to him, is *equal* to *its* age then. *It's* age then is the same age as his age. So Minguzzi says that her current age is the same as his age then. I.e., at this particular instant in his life, *all three of them have the same age,* according to Minguizzi.

The above situation is contrary to the principle of causality. By causality, his decision to change his velocity, or *not* change his velocity, *can't* have any effect on what happens *before* the instant the decision is made. But we *know* what happens when he *doesn't* ever accelerate: he is just a perpetually-inertial observer in that case, and he is entitled to use the time dilation equation (TDE). The TDE says that she ages gamma times slower than he does. So, for example, if gamma equals 2.0, she ages half as fast as he does. So she is only half as old as he is. Minguzzi says she and he are the *same* age. Therefore Minguizzi's simultaneity method violates the principle of causality.

If the reader is concerned that the instantaneous velocity change by him and *it* at birth disqualifies them from being perpetually-inertial observers, there are two rebuttals to that objection. The first rebuttal is that, because all three twins are co-located with each other when they are born, any acceleration there has no effect on their ages. That can be confirmed, because they are all there together, looking at each other ... any sudden age changes would be visible. The second rebuttal is that we *could* replace all three twins with babies born from three different mothers, *each* of whom are perpetually inertial, and who are each moving at the assumed relative velocities. They are therefore *all* entitled to use the time dilation equation.

Section 4. A New Simultaneity Method

I have discovered *another* simultaneity method, which I call Fontenot's simultaneity method. It significantly differs from the above three methods. I believe that special relativity assumes at the outset that the principle of causality is valid: i.e., that causes *precede* effects. Therefore, the non-causality of the Dolby and Gull method, and of the Minguzzi method, disqualifies them as simultaneity methods in special relativity. Einstein certainly believed that the principle of causality was sacrosanct. His lifelong refusal to accept quantum mechanics was based on the fact that quantum mechanics violates causality.

So what is my new simultaneity method? Suppose the traveler is continually receiving a TV image transmitted by the home twin. Because of the finite speed of light, it is obvious that the image the traveler sees is "out of date" … the *current* age of the home twin is *greater* than the age shown on the image, because she has aged during the transit of the image. My simultaneity method is based on *explicitly* determining by how much the home twin has aged during the transit of the image. Once the traveler knows that, he can add to it the age shown in the image, to get her *current* age when he received the image.

Before going any further, it will be helpful to fully specify a particular scenario. Choose the velocity *v* of the traveler (his velocity) relative to the home twin, on the outbound leg, to be 0.57735 lightyears per year (ly/y), positive when they are moving apart. (I've chosen that particular velocity because it results in a 30-degree angle between his worldline and her age axis in a Minkowski diagram, which makes it easy to draw the diagram). That velocity results in a gamma factor of 1.2247. Choose her age at the reunion to be 80 years old. According to her, she will thus be 40 years old at the turnaround. She uses the well-known time-dilation equation to conclude the he will be

40/gamma = 32.66 years old

when he turns around. The turnaround is an *event*, so everyone must agree about his age at the turnaround. Since he ages by the same amount on his return leg as he does on his outbound leg, he will be 65.32 years old at their reunion. Alternatively, we can also get that number directly from the time-dilation equation:

80/gamma = 65.32 years old.

Section 5. The Minkowski Diagram

I will now describe in detail the Minkowski diagram for the above scenario. I prefer to draw Minkowski diagrams with the home twin's time axis *horizontal*, and her spatial axis *vertical*. The reader should draw the diagram as I describe it … that will result in the best understanding of what I'm about to describe. It is best to take time to draw it accurately, but the reader in a hurry can just make a rough sketch if necessary. By my describing the diagram *verbally*, which the reader then draws or sketches as we go along, the reader will get a better understanding of my method than would be obtained if I just included a drawing in the paper.

On a letter-size piece of paper, orient the paper in "landscape" mode, with the long length horizontal. Draw a horizontal line about an inch or so above the bottom of the page, starting about an inch or so from the left edge of the paper. Draw that line to within about an inch of the right edge of the paper. Label the right end of that line "tau", corresponding to the age of the home twin. That line is the horizontal (*time*) axis of the Minkowski diagram, and it corresponds to the worldline of the home twin.

I like to use a ruler with a centimeter scale to establish a correspondence between a year of her life and some specific number of centimeters (I use the ratio 4 years per centimeter). So the point on the horizontal axis where she is 80 years old is about an inch or so from the right edge of the paper, at 20 cm from the origin of the diagram (where she is zero years old). Draw a small vertical "tic" mark at the 80 year point on the horizontal axis, and write "80" under it. Also, locate the point on the horizontal axis where she is 40 years old … it is at the 10cm point. Draw a vertical "tic" there, and write "40" under the tic. And under the left end of the horizontal axis, write "0", since she has just been born at the beginning of the scenario.

Through the left end of the horizontal axis, above the "0" point, draw a vertical line, ending about 2 inches from the top of the page. That line is the *spatial* axis of the Minkowski diagram, giving the distance (according to the home twin) from the home twin to various objects. It has units of lightyears (ly), so that the speed of light is numerically equal to 1. Label the top end of that axis "X". The scale on that vertical axis (in ly per cm) will be the *same* as the scale of the horizontal axis (4 ly/cm), since the speed of light is equal to 1 ly/y. To the left of the bottom of the vertical axis, write "0", indicating zero distance from the home twin. The intersection of the horizontal and vertical axes is the origin of the Minkowski diagram.

Draw a vertical line through the 40-year tic mark on the horizontal axis, extending more than half way as high as the vertical axis. That line corresponds to the home twin's line of simultaneity at the turnaround point.

Next, draw a line sloping upwards to the right, through the origin of the diagram. Its slope is equal to the velocity of the traveling twin on his outbound leg (0.57735 ly/y). The line makes an angle of 30 degrees with respect to the horizontal axis. That makes it easy to draw if you have a 30-60-90-degree plastic drafting triangle; if you don't, you can establish the slope using a centimeter ruler. Or if you are just sketching, just estimate 30 degrees by eye. Extend that line an inch or so past the vertical line you drew through the 40-year point on the horizontal axis. That line is the traveler's worldline on his *outbound* leg. Above the intersection of his outbound worldline and the vertical line through the 40-year point on the horizontal axis, write a "T", denoting the traveler's turnaround point. And put a "tic" at turnaround point on his outbound worldline, perpendicular to his outbound worldline. Above that "tic" (and written parallel to the outbound worldline), write the traveler's age immediately *before* his turnaround: 32.66 years.

Next, draw a straight line from the intersection "T" to the 80-year point on the horizontal axis. That is the traveler's worldline on the *inbound* leg of his trip. Put a "tic" on at the beginning of his outbound worldline, perpendicular to his inbound worldline. Above that "tic" (and parallel to the inbound worldline), write the traveler's age immediately *after* his turnaround: 32.66 years. Those two times are the same, because the scenario assumes an instantaneous turnaround.

We can also get the distance from the home twin to the traveler, according to the home twin, when the traveler is at the turnaround. She says that while she is aging by 40 years, the traveler travels a distance

d3 = v * 40 = 0.57735 * 40 = 23.094 ly,

where the asterisk denotes multiplication.

So draw a horizontal line through the turnaround point, going left to the vertical axis (and extend that line for an inch or so to the right of the turnaround point). Put a "tic" at the point where that horizontal line intersects the vertical axis, and write the distance 23.094 there. (That point occurs at the 5.8 cm point on the vertical axis).

For those readers who are making an accurate drawing, I recommend that the reader make several photo copies of the diagram as described so far, and save them to add to in future.

Section 6. Description of My Simultaneity Method

At the beginning of the standard twin 'paradox' scenario, when the twins are born, the traveling twin (he) instantaneously changes his velocity with respect to the home twin from zero to *v* ly/y (0.57735 in my example). This produces no change in the ages of the either twin at that instant … they can see that, because they are co-located at that instant and can directly observe each other. If this still troubles the reader, as I explained earlier, we *could* revise the scenario so that two pregnant mothers, who are *perpetually* inertial and moving at a relative velocity *v*, happen to be momentarily co-located at the instant that their babies are born. So those two babies are certainly entitled to use the time dilation equation to determine how the other baby's aging compares to their own: *each baby will say that the other baby is aging slower than they themselves are, by the gamma factor*. From here on out though, I will *still* refer to them as actual twins.

Section 7. Image Pulses Contained Entirely In the Left Half of the Minkowski Diagram

Now, we need to talk about a very special image pulse that she sends to him during his outbound leg. My method focuses on considering by how much she ages while her images are in transit, according to *him*. She sends that special image (giving her exact age when the image is transmitted) so that it reaches him at the instant immediately *before* he turns around. On the Minkowski diagram, we can easily determine how old she was when she transmitted the pulse (which is an *event*, that everyone must agree about), and how much she ages during the transit of that pulse, according to *him*. Draw a 45-degree line through the turnaround point, descending to the left from the turnaround point, until it intersects the horizontal axis. That line is the worldline of the special pulse that she transmits. It is a 45-degree line because the speed of the pulse is 1 ly/y, so its slope (which is equal to its velocity) is equal to 1. It intersects the horizontal axis at the point where she transmits the pulse. We can determine her age when she transmits that pulse by noting that the combination of (1) the worldline of the pulse, and (2) the vertical line from the turnaround point, and (3) the horizontal axis, *defines* a triangle whose vertical and horizontal sides have equal lengths of numerical value 23.094. Therefore her age when she transmits the pulse has to be

tau_1 = 40 - 23.094 = 16.91 years old.

Write that label, tau_1, and its value, 16.91, on the diagram.

Next, we need to determine how much she ages during the transit of that pulse, according to *him*. To do that, we just need to determine how old she is when he receives her message, *according to him*. We get that that by using his line-of-simultaneity (LOS) that passes through the turnaround point. Her age then, according to him, is her age tau_2 where that LOS intersects the horizontal axis. The combination of (1) that LOS, together with (2) the vertical line descending from the turnpoint, together with (3) the horizontal axis, defines a right triangle whose height has length 23.094, and whose width has a length which must equal

delta_tau = tau_3 - tau_2 = 40 - tau_2.

We can compute the value of delta_tau since we know that his LOS has a slope of

tan(60) = 1.7321.

So

23.094 / delta_tau = 23.094 / (40 - tau_2) = 1.7321.

Therefore

(40 - tau_2) = 23.094 / 1.7321 = 13.333

or

tau_2 = 40 - 13.333 = 26.67.

We've determined that she was 16.91 years old when she transmitted the pulse, and she was 26.67 years old when he received her pulse (according to *him*), so therefore he says that she aged

26.67 - 16.91 = 9.76 years

while the pulse was in transit.

So, when he receives her message immediately before his turnaround, we've determined that he knows that she aged 9.76 years during the pulse's transit, and that she is currently 26.67 years old.

Note that we could have gotten the result that she is 26.67 years old (according to him), at the instant before he reverses his velocity, more easily from the time dilation equation (TDE). Even though he instantaneously changed his velocity from zero to 0.57735 ly/y right after he was born, their separation was zero then, and we can treat him as if he were a perpetually inertial observer. So he is entitled to use the TDE. The TDE says that she will age gamma times slower than he does (according to him). He is 32.66 years old at the turnaround, so he says she is
32.66 / 1.2247 = 26.67
years old then.

In the next section, I need to define my method for all image pulses that are transmitted *after* the pulse analyzed above, but *before* the turnaround. None of those pulses will reside *entirely* in the left half, or *entirely* in the right half, of the Minkowski diagram.

Section 8. Pulses Partly in Both Halves of the Minkowski Diagram

First, I need some definitions. Call the perpetually-inertial observer who is co-located and stationary with the traveler on the *outbound* leg 'the Outbound Co-Moving Inertial Observer', abbreviated as the OCMIO. And call the perpetually-inertial observer who is co-located and stationary with the traveler on the *inbound* leg 'the Inbound Co-Moving Inertial Observer', abbreviated as the ICMIO. The OCMIO operates in the *left* half of the Minkowski diagram, i.e, for ages of the home twin *less than* 40 years old in the specific example we've chosen. The ICMIO operates in the *right* half of the Minkowski diagram, i.e, for ages of the home twin *greater than* 40 years old in our chosen example. Remember these two terms *well*; they will be used extensively in what follows.

The pulse analyzed in the last section was special, in that it was the *last* pulse that is *wholly contained* within the left half of the Minkowski diagram. So it was the last pulse that could be analyzed in its *entirety* by the OCMIO. A pulse transmitted *after* that special pulse, and *before* she is 40 years old, will spend the *first* part of its trip in the *left* half of the Minkowski diagram, and the *latter* part of its trip in the *right* half of the diagram, so it can't be analyzed in its *entirety* by *either* the OCMIO *or* by the ICMIO. The *first* portion of those pulses' trips will need to be analyzed by the OCMIO, and the *last* portion of those pulses' trips will need to be analyzed by the ICMIO. The OCMIO can determine by how much the home twin ages while the pulse is in the *left* half of the Minkowski diagram, and the ICMIO can determine by how much the home twin ages while the pulse is in the *right* half of the Minkowski diagram. In my method, those two amounts of aging by the home twin are *added together* by him, in order to get the total aging by the home twin, according to him, during the *entire* transit of the pulse.

The above process is needed for all pulses that are partly in the left half and partly in the right half of the Minkowski diagram. The first pulse that is *entirely* within the *right* half of the Minkowski diagram is the pulse that the home twin transmits immediately *after* (according to *her*) the traveler's turnaround. From there until the end of the scenario, the aging of the home twin during the complete transit of any pulse can be determined *entirely* by the ICMIO, and that process is essentially the same as has been described for the *last* pulse that is *entirely* within the *left* half of the Minkowski diagram (what I called the special pulse) … the only difference is that in the *right* half of the diagram, the slope of the traveler's worldline is *negative* (-0.57735 in this example), and the slope of the ICMIO's line of simultaneity (LOS) is also *negative* (-1.73205 in this example).

Now, I will explicitly demonstrate the way the OCMIO and the ICMIO each determine the amount of aging by the home twin, during the portion of a pulse that is in their respective domains. In this demonstration, I will choose to do that for the particular pulse that travels an *equal* distance in each domain (according to the home twin). According to the home twin, the traveler is a distance

$$d_3 = 23.094 \text{ ly}$$

from her when he is at the turnaround. Find the point midway along the vertical line connecting the turnaround point (point T) to the horizontal axis on the Minkowski diagram. That point is at the distance

$$d_3 / 2 = 11.547 \text{ ly}$$

from the home twin (according to the home twin). Label that point as point R on the Minkowski diagram. Now, draw a 45 degree line through point R, starting at the horizontal axis and extending just past the traveler's worldline on his inbound leg. Label the point where that line intersects his worldline as point Q. And label the point where the 45 degree line intersects the horizontal axis as tau_7. We need to know the value of tau_7. That's easy, because the 45-45-90 triangle formed by (1) the pulse, (2) the vertical line descending from the turnaround point, and (3) the horizontal axis, has equal vertical and horizontal sides, of length equal to d_3 / 2, which we know is 11.547 ly. So tau_7 must equal

$$\tau_7 = \tau_3 - d_3 / 2 = 40 - 11.547 = 28.453 \text{ years.}$$

Draw the OCMIO's line of simultaneity (LOS) that passes through point R. It slopes downward to the left, with slope

$$\tan(60) = 1.73205.$$

Label the intersection of that LOS with the horizontal axis as tau_8.

Consider the triangle formed by (1) that LOS, and (2) the vertical line from point R to the horizontal axis, and (3) the horizontal axis. That triangle's height is 11.547. So 11.547, divided by the base of that triangle's length, must equal the slope of the hypotenuse, which is 1.73205. The base of that triangle is

$$\tau_3 - \tau_8 = 40 - \tau_8.$$

So

$$11.547 / (40 - \tau_8) = 1.73205$$

or

$$40 - \tau_8 = 11.547 / 1.73205 = 6.66667.$$

Therefore

$$tau\_8 = 40 - 6.66667 = 33.33333.$$

So the age of the home twin, according to the OCMIO, when the pulse reaches point R, is 33.33 years old. She was 28.45 years old when she sent that pulse (that's tau_7 on the diagram), so during the portion of the pulse's trip that was in the *left* half of the diagram, she aged by

$$33.33 - 28.45 = 4.88 \text{ years.}$$

That completes the work that has to be done by the OCMIO. Now, we need to do the ICMIO's calculations for determining by how much she ages during the portion of the pulse that is in the *right* half of the diagram. We already have identified the point Q on the traveler's (his) worldline where he receives her pulse, and the point R where the pulse enters the right half of the diagram. Draw a vertical line from Q down to the horizontal axis, and label the intersection of that line with the horizontal axis as her time tau_6. We need to determine the value of tau_6. We do that as follows. Point Q is the intersection of his worldline on his inbound leg with the worldline of the pulse. The equation of his inbound worldline is

$$X(tau) = (tau - 80) * (v) = (tau - 80) * (-0.57735).$$

The equation of the worldline of the pulse is

$$X(tau) = tau - tau\_7 = tau - 28.453.$$

(We determined the value of tau_7 previously).

Equating the right-hand-sides of those two equations gives

$$(tau - 80) * (-0.57735) = tau - 28.453$$

or

$$-0.57735 * tau - 80 * (-5.7735) = tau - 28.453$$

or

$$tau * (-0.57735 - 1) = -28.453 - 46.188$$

or

$$-1.57735 * tau = -74.641$$

or

$$tau = tau\_6 = 47.321.$$

Once we have tau_6, we can determine his age t_7 when he receives her message at point Q. She says that when she is 47.32, he is

$$t\_7 = 47.32 / gamma = 47.32 / 1.2247 = 38.64,$$

using the time-dilation equation. Write the symbols tau_6, t_7, and their values, on the diagram.

What does the ICMIO say about her age when the pulse reaches point Q? We get that from the ICMIO's LOS through point Q. That LOS slopes downward and to the right with slope

$$-tan(60) = -1.7321.$$

Draw that LOS on the diagram. Denote the point on the horizontal axis where the LOS intersects the horizontal axis as tau_9. So tau_9 is her age when the pulse reaches Q, according to the ICMIO. We need to determine the value of tau_9.

The combination of (1) the LOS, and (2) the vertical line extending downward from point Q, and (3) the horizontal axis, define an important triangle. We need to determine the numerical height, d_4, of that triangle. His age at point Q, where he receives her message, is 38.64 years old. So his distance from her then, according to her, is (remembering that v on the inbound leg is *negative*)

$$d\_4 = d\_3 + v * (tau\_6 - tau\_3) = 23.094 - 0.57735 * (47.32 - 40)$$

$$= 23.094 - 0.57735 * 7.32 = 23.094 - 4.226$$

$$d\_4 = 18.868.$$

Label that distance, and that value, on the diagram.

Now that we have d_4, we can compute the numerical length of the base of that triangle, because we know that the slope of the hypotenuse of that triangle is -1.7321. The numerical length of the base of the triangle is equal to

$$tau\_9 - tau\_6.$$

So we have

$$1.7321 = d_4 / (tau_9 - tau_6) = 18.868 / (tau_9 - 47.32)$$

or

$$tau_9 - 47.321 = 18.868 / 1.7321 = 10.893,$$

so

$$tau_9 = 47.321 + 10.893 = 58.214.$$

Write the value of tau_9 on the diagram.

So her age, according to the ICMIO, when the pulse reaches point Q (when he receives the pulse), is 58.214. We need to know what her age was when the pulse reached point R. We get that from the ICMIO's LOS through point R. The slope of the ICMIO's LOS is equal to -1.7321. The combination of (1) that LOS, and (2) the vertical line below point R, and (3) the horizontal axis, define a 30-60-90 triangle. The height of that triangle is equal to

$$d_r = d_3 / 2 = 23.094 / 2 = 11.547.$$

Denote the point where the LOS intersects the horizontal axis as tau_10. tau_10 is her age when the pulse reaches point R, according to the ICMIO. Label that on the diagram.

So the numerical length of the base of that triangle is

$$tau_10 - tau_3 = tau_10 - 40.$$

From the known slope of the LOS, we get

$$1.7321 = d_r / (tau_10 - 40) = 11.547 / (tau_10 - 40)$$

or

$$tau_10 - 40 = 11.547 / 1.7321 = 6.666$$

so

$$tau_10 = 46.67.$$

Write that value for tau_10 on the diagram.

So her age, according to the ICMIO, when her pulse reaches point R, is 46.67. We already determined that her age when he receives the pulse is 58.21. Therefore, the ICMIO says that she aged by

$$58.21 - 46.67 = 11.54 \text{ years}$$

during the transit of the pulse from point R to point Q.

So the total amount of her aging, according to *him* (the accelerating traveler), during the entire transit of the pulse, is the sum of her aging while the pulse is in the *left* half of the diagram (4.88 years, according to the OCMIO), *plus* her aging while the pulse is in the *right* half of the diagram (11.54 years, according to the ICMIO):

$$4.88 + 11.54 = 16.42.$$

Since she was 28.45 years old when she transmitted the pulse, she is

$$28.45 + 16.42 = 44.87 \text{ years old}$$

when he receives her pulse (according to *him*), and he is 38.64 years old then.

Section 9. Pulses Entirely in the Right Half of the Minkowski Diagram

Finally, we need do the analysis for the *first* pulse she transmits, according to her, after the turnaround. That is the *first* pulse that can be handled *entirely* by the ICMIO. (And it will be of prime importance when we construct the age correspondence diagram later). The analysis of this pulse is very similar to the analysis that we did for the *last* pulse what could be handled *entirely* by the OCMIO: the only difference is that the slope of his worldline is now *negative*, and the slope of the ICMIO's LOS is also now *negative*.

From the point tau_3 on the horizontal axis of the Minkowski diagram (whose value is 40 years, her age when he turns around, according to her), draw a 45-degree line sloping upward to the right, until it intersects his inbound worldline. That is the worldline of the pulse. Label the point of intersection of the pulse with his worldline as point P, and draw a vertical line from there down to the horizontal axis. Label the point of that vertical line's intersection with the horizontal as tau_4. tau_4 is her age when he receives the pulse, according to her.

The equation of the pulse's worldline is

$$X(tau) = (tau - tau_3) = (tau - 40)$$

because the velocity of the pulse is +1.

The equation of his inbound worldline is

$$X(tau) = (tau - tau_r) * v = (tau - 80) * (-0.5773503) = -0.5773503 * tau + (80 * 0.5773503),$$

where tau_r is her age at the reunion.

Equating the right hand sides of the two equations gives

$$(tau - 40) = -0.5773503 * tau + (80 * 0.5773503) = -0.5773503 * tau + 46.18802$$

so

$$1.57735 * tau = 86.18802$$

so

$$tau = 54.641 \text{ years.}$$

That value of tau is the value of tau_4, her age when he receives the pulse, according to her:

$$tau_4 = 54.6410 \text{ years old.}$$

Using the time dilation equation, she computes that his age t_4 when he receives the pulse is

$$t_4 = tau_4 / gamma = tau_4 / 1.2247 = 54.6410 / 1.224745,$$

so

$$t_4 = 44.6142.$$

On the Minkowski diagram, label her age when he receives the pulse, according to her, as tau_4, with value 54.641. And label his age when he receives the pulse as t_4, with value 44.61.

Next, we need to determine her age, *according to him*, when he receives the pulse. To get that, we use the ICMIO's LOS through the point P (where he received the pulse). That LOS slopes downward to the right from point P, making an angle of 60 degrees with the horizontal axis. The slope of that LOS is

$$-\tan(60) = -1.732051.$$

Draw that LOS on the Minkowski diagram, and label its intersection with the horizontal axis as tau_5. We need to determine the value of tau_5, because tau_5 is her age, according to *him*, when he receives the pulse.

The combination of (1) LOS, and (2) the vertical line extending downward from point P, and (3) the horizontal axis, define an important triangle. The height of that triangle is the value of X at point P. That is the value of X when he receives the pulse. We already wrote the equation of that pulse:

$$X(tau) = tau - 40.$$

So when tau equals tau_4, the value of X is

$$X = tau_4 - 40 = 54.6410 - 40$$

so

$$X = 14.6410.$$

The numerical height of the important triangle is therefore 14.6410. The hypotenuse has a slope of -1.73205, so the ratio of the height of the triangle to the base of the triangle must equal 1.73205. The base of that triangle has the numerical value

$$tau_5 - tau_4 = tau_5 - 54.6410,$$

so we get

$$1.73205 = 14.6410 / (tau_5 - 54.6410)$$

or

$$tau_5 - 54.6410 = 14.6410 / 1.73205 = 8.45299.$$

so

$$tau_5 = 63.09399.$$

Therefore her age, according to *him*, when he receives the pulse, is 63.09. He is 44.61 years old then. Those two ages determine a *very* important point on the age correspondence diagram to be described in the next section.

Section 10. The Age Correspondence Diagram (ACD) for My Simultaneity Method

By definition, the age-correspondence diagram (ACD) plots the curve giving the current age of the home twin (on the vertical axis), according to the traveler, versus the age of the traveler (on the horizontal axis). That is the ultimate objective, when discussing simultaneity according to an observer who sometimes changes his velocity. The Minkowski diagram has just been the means to get there.

On a sheet of paper oriented with the long edge vertical, draw a horizontal line starting from an inch or so from the bottom of the page, and extending to a little less than inch from the right edge of the page. Label that line with a "t", which is the traveler's (his) age at the instants of his life during his trip.

Next, draw a vertical line from the left edge of the horizontal line, up to within about an inch from the top of the page. Label that line with a "tau_hat", which is the home twin's (her) age, *according to him*, when he is age *t*. (On Minkowski diagrams, I often use tau *with a circumflex*, to denote *his* view of her age when he is age *t*, versus tau *without* the circumflex to denote her age *according to her* when he is age *t*. I'll use that convention here.)

As soon as the particular scenario we're using was defined, one point on the age correspondence curve was already known: the twins are both zero years old at the start of the trip, so we know that the curve will start from the origin. So her age when he is age zero is already known to be zero. We can also easily determine their respective ages at the *end* of the scenario, by making use of the time dilation equation for the home twin (since she is perpetually inertial, and is *always* entitled to use it). We know from the scenario specification that she is 80 years old at the reunion, so she says that he is

80 / gamma = 80 / 1.2247 = 65.32 years old

at the reunion. And since they must agree with each other at the reunion (because they are standing right there looking at each other), that is his conclusion also. So we can plot the point on the curve representing the reunion. If you have a centimeter ruler, put a "tic" at the 20cm point on the *tau_hat* (vertical) axis, and write "80" to the left of that point. So the scale factor is

80 years / 20 cm = 4 years/cm,

and so his age of 65.32 years corresponds to

65.32 years / 4 years/cm = 16.3 cm.

Measure off 16.3 cm on the *t* axis (the horizontal axis), put a "tic" mark there, and write 65.32 below the "tic". Then draw a vertical line up from that "tic" until it intersects the horizontal line you drew from the 80 year point on the vertical axis. The intersection of those two lines is the reunion point on the age correspondence curve.

So we have the beginning point and the final point of the curve plotted on the age correspondence diagram. Now we need to fill in all the points of the curve in between that show us how her age (according to him) changes as his age increases.

From Section 6, we know that for the *initial* part of the age correspondence curve (up to and including the turnaround), the traveler completely agrees with the OCMIO … i.e., we can treat him as a perpetual observer throughout that portion of his trip. Therefore he concludes that *she* is aging gamma times *slower* than *he* is during that portion. So for that portion, the age correspondence curve is a *straight* line, starting at the origin, and sloping upward and to the right with a slope of

1 / gamma = 1 / 1.2247 = 0.8165.

That is an angle of about

arctan(0.8165) = 39 degrees.

That initial straight line terminates when he is 32.66 years old, and she is 26.67 years old (according to him). Locate these ages (with "tic" marks) on the axes of the age correspondence diagram (using the years per centimeter scale factor), and label them with the above values. Then (lightly) draw the horizontal and vertical lines from there to locate the corresponding point on the age correspodence curve. That point marks the end of the *first* section of the age correspondence curve, and the beginning of the *middle* section of the curve.

To get the next (middle) section of the age correspondence curve, we need to use the results obtained in Section 8 (pulses partly in both halves of the Minkowski diagram), and also the results obtained in Section 9 (pulses entirely in the right half of the Minkowski diagram). This middle section of the age correspondence curve starts at the end of the first section, when he is 32.66 years old and she is 26.67 years old. The middle section ends when the final (third) section begins. In Section 9, we determined that he is 44.61 years old then, and she is 63.09 years old then (according to him), so those are coordinates of the end of the middle section of the curve. Use the years-per-centimeter scale factor to plot those ages on the axes of the diagram, and lightly draw horizontal and vertical lines to determine the corresponding point on the age correspondence

curve that marks the end of the middle section.
The results obtained in Section 8 allow us to determine the mid-point of the middle section of the age correspondence curve. Section 8 showed (near the end of that section) that he was 38.64 years old then, and she was 44.87 years old then (according to him). So locate that mid-point of the middle section on the curve. You should see that that mid-point lies on a *straight line* between the beginning and end of the middle section. That's *not* a quirk or a coincidence: That the middle section of the age correspondence curve *is* a straight line can be confirmed by carrying out the procedure detailed in Section 8 for pulses other than that mid-point pulse. If you do that, you will find that the *entire* middle section of the curve is a straight line. Unlike the straight line of the first section of the curve (whose slope is *less* than one), the slope of the middle straight line is *greater* than one.
From the endpoints of the middle section of the curve, we can calculate that the slope of that straight line is

(63.09 - 26.67) / (44.61 - 32.66) = 36.42 / 11.95 = 3.048.

So in this middle section of the age correspondence curve, according to him she is aging *faster* than he is, in contrast to the first section, where, according to him, she is aging *slower* than he is. I'll sometimes refer to the middle section as the "fast aging" section. The angle that the middle section makes to the horizontal axis is about

arctan(3.048) = 72 degrees.

Now, how about the last section of the age correspondence curve? If you look at the Minkowski diagram, you can see that sending pulses after the last one we analyzed doesn't result in any radical new behavior: those additional pulses continue to be contained entirely in the right half of the diagram, and his worldline continues on the same straight line, going downward to the right without any change in its slope. That is true all the way to the reunion. So the upper (third) section of the age correspondence curve is just a straight line, for which the change in her age is

80 - 63.09 = 16.91,

and the change in his age is

65.32 - 44.61 = 20.71,

so she ages slower than he does, by the ratio

16.91 / 20.71 = 0.8165.

That ratio is exactly what we got for the slope of the *first* section of the age correspondence curve, and we got that ratio then because we had shown that it was equal to

1 / gamma = 1 / 1.2247 = 0.8165.

So for the third section of the age correspondence curve, his conclusions about her current age are *exactly* the same as the conclusions of a perpetually inertial observer, traveling along with him during that section, would be. The first and third sections of the age correspondence curve are essentially the same, except that the third section isn't quite as long as the first section is. The age correspondence curve turns out to not acually be curved anywhere: it is piece-wise linear. That is a *very* nice outcome, which *greatly* simplifies the construction of age correspondence diagrams in my method.

Note that this age correspondence diagram has given us a warning about the unqualified use of the term "inertial observer". The term "inertial observer" is often considered in special relativity to just mean an observer who is *not currently* accelerating. That is certainly the view in the CMIF simultaneity method. But it is definitely *not* what happens in the middle section of the age correspondence curve for *my* simultaneity method. There, he spends *years* of his life (during which he is not accelerating) in which he would *completely disagree* with a perpetually inertial observer who happened to be riding along with him that whole time! So by *no means* can he be considered to be "an inertial observer" during that time. The question "When does an observer who has accelerated the past become an inertial observer?" isn't trivial to answer. In the present twin 'paradox' example, one answer to the question is "He becomes an inertial observer immediately after the end of the fast-aging section of the age correspondence diagram". Another answer to the question (which is consistent with the answer immediately above) is "He becomes an inertial observer immediately after the end of the middle section of the Minkowski diagram in which the pulses are not entirely contained in the left half of the diagram, and are not entirely contained in the right half of the diagram". Or a third equivalent answer to the question is "He becomes an inertial observer at the beginning of the third section of the Minkowski diagram, where the pulses are completely contained in the right half of the Minkowski diagram".

The results obtained already, from both the Minkowski diagram analysis and from the above results for the age correspondence diagram (ACD), can be used to derive an explicit equation that directly gives the slope of the middle section (the section with a slope greater than 1) of the ACD curve. It allows the slope of that middle section of the curve to be obtained analytically, rather than graphically.

Denote the slope of the middle section of the ACD curve as S. Then the S equation is

S = (1 / gamma_2) + gamma_2 * (1 - v_2) * (v_1 - v_2),

where v_1 is the relative velocity *before* the velocity change, v_2 is the relative velocity *after* the velocity change, and gamma_2 is the gamma factor corresponding to v_2. Velocities are positive when the twins are diverging, and negative when the twins are converging. It is possible to show,

by using the S equation, that my simultaneity method never produces a *negative* value for S. I.e., in my method, the traveling twin never says that the home twin is getting *younger*. That is in contrast to the CMIF method, which predicts negative aging of the home twin for some scenarios.
I call the above S equation Fontenot's equation.
There is one question that has not been addressed yet: How does the curve of the age correspondence diagram, for my simultaneity method, change for *finite* accelerations by the traveler? In the instantaneous velocity-change case, the linear increase in her age during the fast-aging portion of the curve is spread over a large number of years (about twelve years in this particular example). Finite accelerations that only last a few years don't change the ACD curve very much. Small finite accelerations that last for the entire round trip DO change the ACD quite a bit, however. It is possible to calculate the ACD curve for finite accelerations using a numerical integration procedure which can be done on a computer. I have done that for an example where there is a constant 0.2g acceleration during the whole trip (but whose *direction* is initially directed *away* from the home twin, then switched to *toward* her, half way to the turnaround, and finally switched to *away* from her again, half way back on the return trip (so as to bring their relative velocity to zero at the reunion). I doubt that it is possible to get an *analytical* solution for the finite acceleration case (even for piecewise-constant finite accelerations), but that is the subject of my current research.

Section 11. Velocities, According to the Accelerated Observer

In the first and third sections of the ACD, the accelerating observer (he) agrees with a co-located and co-stationary perpetually-inertial observer (the PIO). So he says that his velocity relative to the home twin (her) is the same as what the PIO says it is (0.57735 ly/y in the first section, and -0.57735 ly/y in the third section). Any perpetually-inertial observer agrees with any other perpetually-inertial observer about their relative velocity. So the observer who sometimes accelerates (he) agrees with the home twin (she) about their relative velocity whenever he agrees with his co-located and co-stationary PIO. But in the middle section of the ACD, for some scenarios, he does NOT always agree with the PIO about their relative velocity with the home twin.

However, he CAN compute the average relative velocity with respect to her during the entire middle section. He can determine his distance from her immediately before he changes his velocity. That is the distance from her, immediately before he accelerates, according to his PIO then (provided he DOES agree with his PIO then). Then, he can do the same thing when he first agrees with his new PIO (at the beginning of the third section of the ACD). Subtracting those two distances tells him how much the distance to the twin has changed over the course of the middle section. He then divides that change in distance by his change in age over the course of the middle section, which gives the average relative velocity with respect to her during the middle section.

For the standard twin paradox scenario, with v1 = 0.57735 and v2 = -0.57735, it turns out that he agrees with her about their relative velocity. That is also true for the scenario with v1 = 0.866 and v2 = -0.866. (It may be true for all cases where v2 = -v1, but I haven't verified that.) But for the scenario with v1 = 0.57735 and v2 = 0.0, he says their relative velocity is 0.18347 ly/y ... he does NOT agree with her.

Also, he often disagrees with her about the speed of distant light pulses. In the v1 = 0.57735 and v2 = -0.57735 case, for the pulse that leaves her when she is 26.6667 years old, and is received by him when he is 37.7138, he says its AVERAGE velocity over its complete transit is 3.155 ly/y, NOT 1.0 ly/y. But for the pulse that leaves her when she is 40 years old, and is received by him when he is 44.6, he says the average pulse speed is 2.49 ly/y.

In the v1 = 0.57735 and v2 = 0.0 case, he agrees with her about the pulse that begins when she is 40 (that its speed is 1.0). But for the pulse that begins when she is 26.6667, he says its average speed is 2.115 ly/y.

Section 12. More than One Velocity Change during the Trip

All of the above sections have assumed that the traveler (he) makes only one velocity change during the trip. What happens if he decides to instantaneously change his velocity again, at some time after the first velocity change? Can he still use the S equation?

The answer depends on how much time in his life passes between the two velocity changes. Recall that after the first velocity change, we drew a pulse on the Minkowski diagram, sent from her immediately after the velocity change. While that pulse is in transit, he doesn't agree with the PIO who is riding along with him. But when he receives that pulse, he again agrees with the PIO, and he continues to agree with the PIO thereafter (unless he decides later to accelerate a third time). Call that interval during the pulse transit "the disagreement interval", or the "DI" ... it's just an interval in his
life during which he disagrees with the PIO riding along with him.

The answer to the last question in the first paragraph in this section is this:

If he changes his velocity again before the DI is over, he can't use the S equation to determine the slope of that next section of the age correspondence diagram (ACD). Instead, he uses the Minkowski diagram to determine how old she would have been at the end of the DI (if he hadn't decided to do another velocity change), and determines the slope by dividing what HER age change would have been during the DI by what HIS age change would have been during the DI. Note that, by causality, the slope of the ACD after the first velocity change can't depend on whether of not he decides to change his velocity again during the DI.

Section 13. Some Philosophical Thoughts

I've never been able to adopt the "simultaneity at a distance is meaningless" view, mainly for philosophical reasons (which are supposed to be off-limits in physics, but I think everyone is influenced by philosophical thoughts to some extent). I don't believe that my home twin *ceases to exist* whenever we are separated. If she *does* still exist "right now", she must be *doing something specific* right now. And if she is doing something specific right now, she must be a *specific age* right now (because at each instant of a person's life, their brain at that instant is in a state that is consistent with their actions at that instant). So I believe she *must* have some specific current age. Her current age is *not* just one of a set of equally good "conventions" of simultaneity, as many physicists believe. Therefore there *must* be a single, correct simultaneity method. Since I believe that Dolby and Gull simultaneity, and Minguizzi simultaneity are both disqualified because they are non-causal, that means that the correct simultaneity is either CMIF simultaneity, or my (Fontenot's) simultaneity. It may be impossible to prove which one is correct and which one is not.

Section 14. Conclusions, and Final Remarks

In this paper, I have defined and explained a new simultaneity method (Fontenot's simultaneity method) for accelerated observers in special relativity. And I have derived Fontenot's equation, which allows the age correspondence diagram (ACD) for my simultaneity method to be quickly and easily determined.

Until very recently, I have been a strong proponent of the CMIF simultaneity method. In 1999, I published a paper (Fontenot, Michael L., "Accelerated Observers in Special Relativity", Physics Essays, December 1999, pp. 629-648 [3]) in which I claimed to prove that the CMIF method was the *only* method that was consistent with the accelerating observer's own elementary observations and elementary calculations. But recently, I discovered an error my proof. So I took a fresh look at the simultaneity issue, and the result is the new simultaneity method that I have defined and explained in this paper.

I have endeavored in this paper, to the maximum extent possible, to "show my work". If any readers believe they have found any errors (either logical or calculational) in this paper, I would very much like to hear from them. I ask that they specifically identify the location of the error, and that they also "show *their* work". I will take their concerns seriously. I can be reached at PhysicsFiddler@gmail.com.

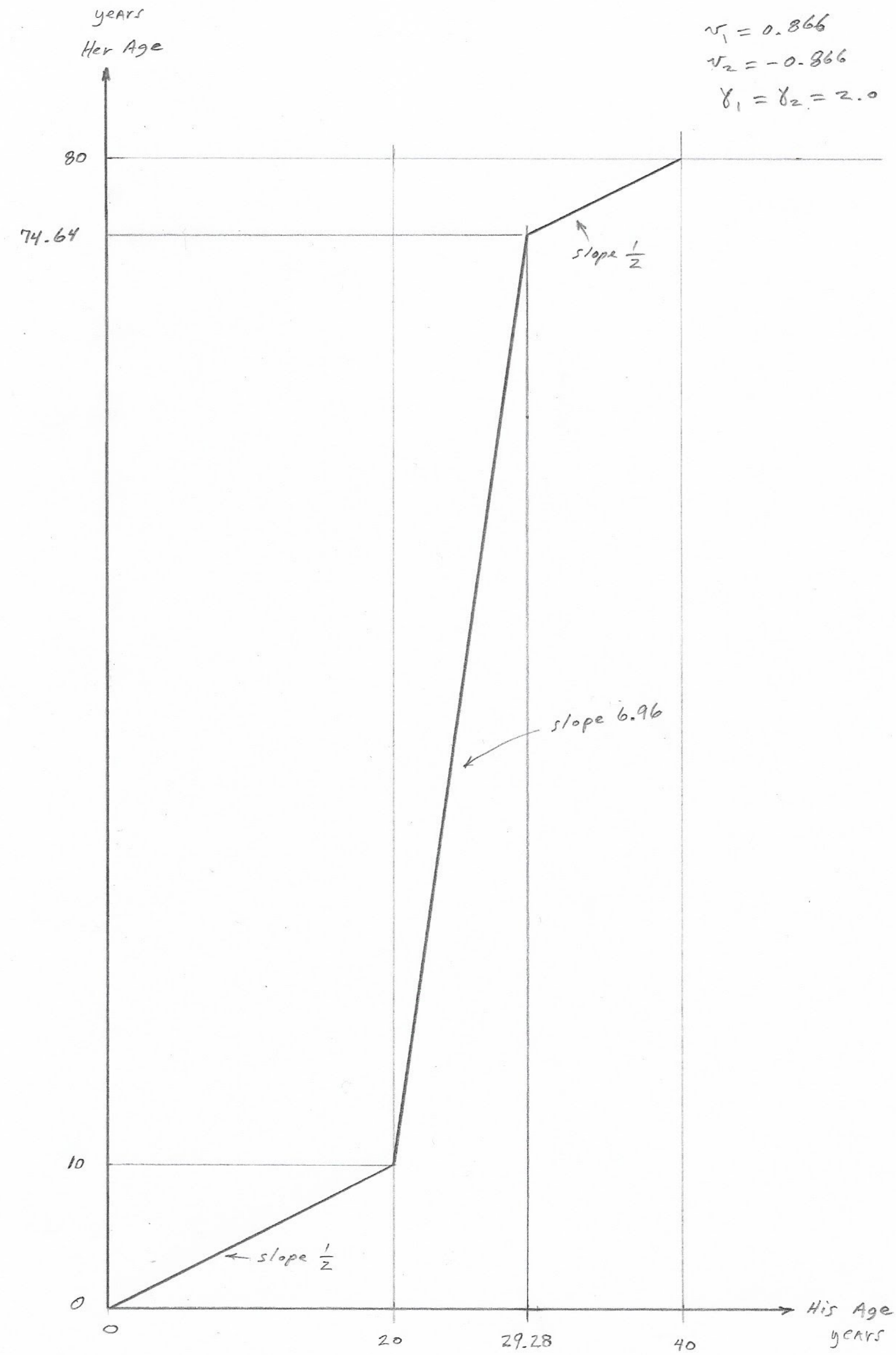
References:

1. Dolby, Carl E. and Gull, Stephen F, "On Radar Time and the Twin 'Paradox' ", Am.J.Phys. 69 (2001) pp. 1257-1261.

2. Minguzzi, E., "Differential Aging from Acceleration, an Explicit Formula", Am.J.Phys. 73 (2005) pp. 876-880.

3. Fontenot, Michael L., "Accelerated Observers in Special Relativity", Physics Essays, December 1999, pp. 629-648.

Here is the ACD for my simultaneity method, for the case where gamma = 2.0, v = +-0.866 ly/y, and the traveling twin is 20 years old at the turnaround:

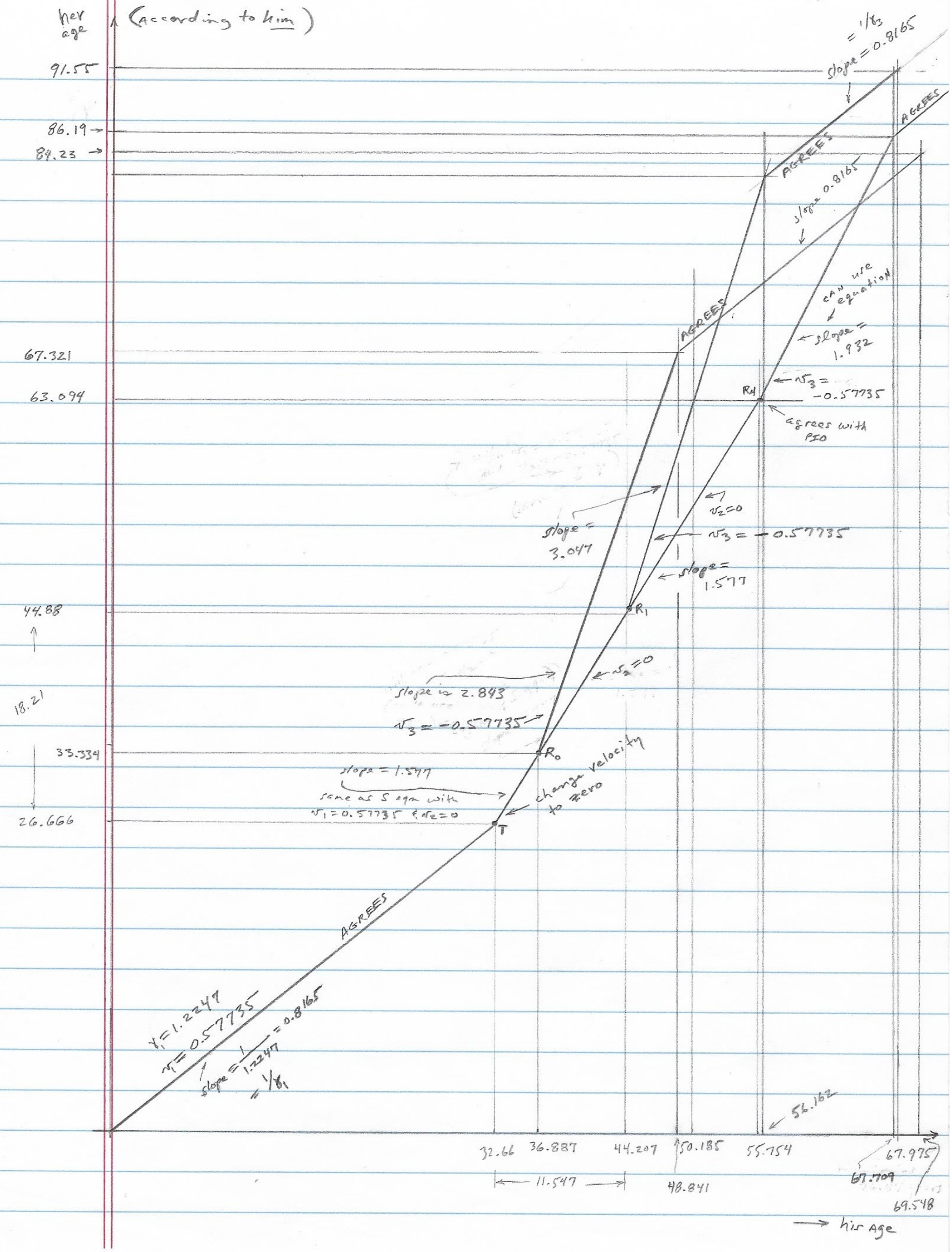


years
Her Age
$v_1 = 0.866$
$v_2 = -0.866$
$\gamma_1 = \gamma_2 = 2.0$
80
74.64
slope $\frac{1}{2}$
slope 6.96
10
slope $\frac{1}{2}$
0
0
20
29.28
40
His Age
years

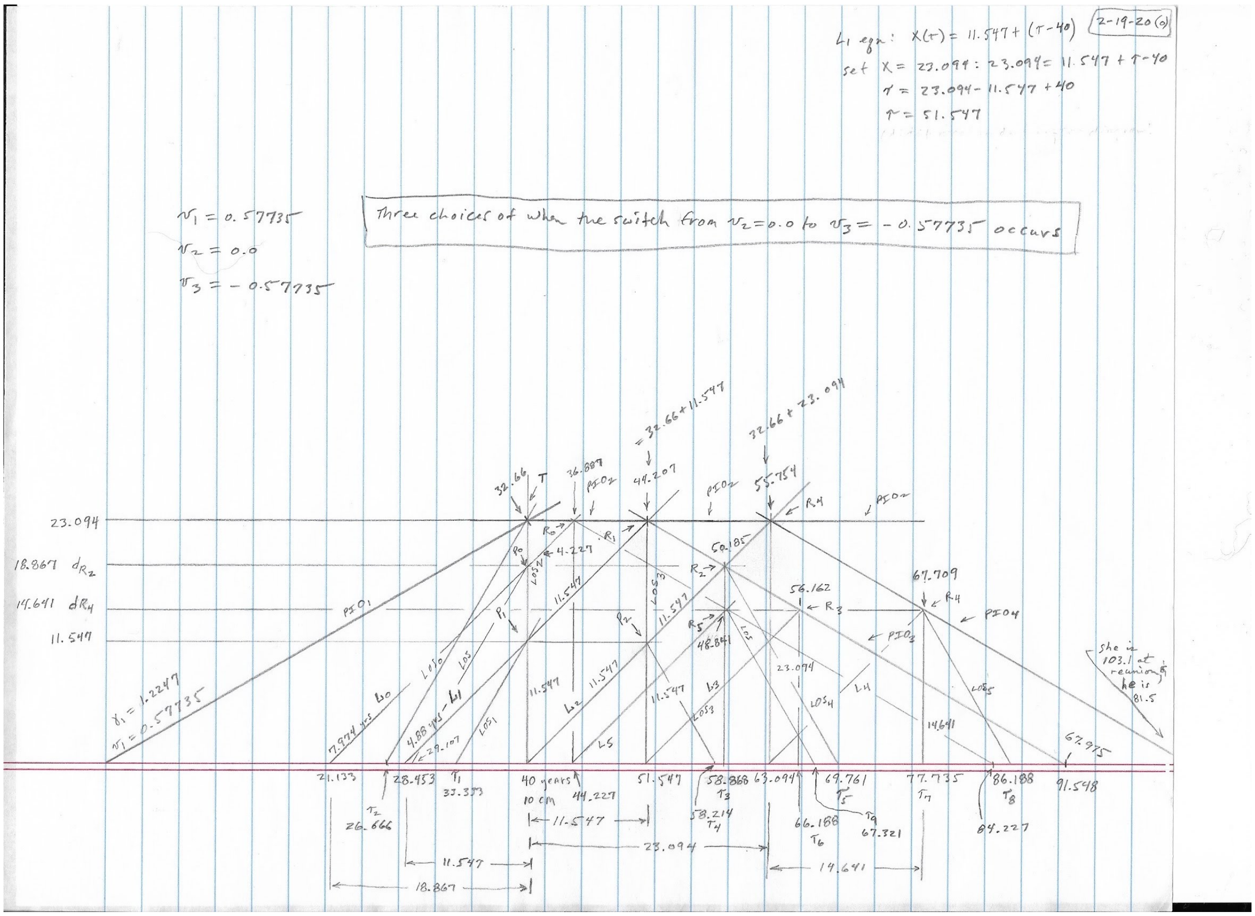
The above diagram is the age correspondence diagram (ACD) that my simultaneity method produces. By comparison, the CMIF ACD would have the same first segment that mine has, but then would have a VERTICAL line segment going upward when he is 20, and intersecting the extension of my third line segment (of slope 1/2). The Dolby and Gull ACD would look somewhat like mine, except that it would begin the steep middle line segment WELL BEFORE the velocity change. And the Minguzzi ACD would have a slope of 1.0 for the first segment (until he is 20), and then a curved line going upward to the right, first with an increasing slope, and then with a decreasing slope. Note that the ACD diagram shows her age versus his age, ACCORDING TO HIM. Her age versus his age, ACCORDING TO HER, is just a single straight line, going upward to the right, with a constant slope of 2.0.

I've just worked out the age correspondence diagram (ACD) and the Minkowski diagram, using my simultaneity method, for a scenario with two separated velocity changes: the initial v1 = 0.57735 when the twins are born, then a velocity change to 0.0 when the traveler (he) is 32.66 years old, and then later a velocity change to -0.57735. I do three different spacings between those last two velocity changes, when he is 36.887, 44.207, and 55.754 years old.

Here is the ACD:


her age
(According to him)
91.55
86.19
84.23
67.321
63.094
44.88
18.21
33.334
26.666
slope = 0.8165
= 1/γ3
AGREES
slope 0.8165
AGREES
can use equation
slope = 1.932
R4
v3 = -0.57735
agrees with P50
AGREES
slope = 3.047
v2=0
v3 = -0.57735
slope = 1.577
R1
v2=0
slope is 2.843
v3 = -0.57735
R0
change velocity to zero
slope = 1.577
same as 5 eqn with
v1 = 0.57735 & v2 = 0
T
AGREES
γ1 = 1.2247
v1 = 0.57735
slope = 1/1.2247 = 0.8165
= 1/γ1
56.162
32.66
36.887
44.207
50.185
55.754
67.975
11.547
48.841
67.709
69.548
his Age

Here is the Minkowski diagram:



In the Minkowski diagram above, the amount of her ageing during the upper portion of the L0 pulse is 4.227 years. That is what the PIO (perpetually-inertial observer) AFTER the velocity change calculates. The PIO BEFORE the velocity change determined that her ageing during the lower portion of the pulse (up to the point P0) is 7.974 years. So the traveler concludes that her aging during the entire pulse is 7.974 + 4.227 = 12.201 years. And she was 21.133 years old when she transmitted the pulse. So he concludes that she was 21.133 + 12.201 = 33.334 years old when he received her pulse. He was 36.887 years old then. The fact that he ADDS the amounts of her ageing during the two portions of the pulse (as determined by the two PIO's), to determine her current age when he receives her pulse, is the HEART of the definition of my simultaneity method ... everything follows from that.

What I found from this analysis is that when there are two velocity changes (both occurring when there is a non-zero separation between the twins), my equation for the slope of the ACD after the velocity change doesn't work for any periods when the traveling twin (he) doesn't agree with the perpetually-inertial observer (PIO) who is currently riding along with him. In general, after he instantaneously changes his velocity, there will be a period of time (sometimes lasting years) when he disagrees with the PIO about the current age of the home twin (her). But eventually (if he doesn't accelerate again), he will start agreeing with the PIO. That delay before agreement can be easily determined from the Minkowski diagram ... you just draw a pulse starting from the horizontal axis directly below the point where the velocity changes, sloping upward to the right with slope 1.0, and terminating on his worldline. At that point of intersection with his worldline, he begins to agree with the PIO, and my slope equation begins to work there, and works unless and until he changes his velocity again.