Bulk Branes in Supergravity Theories

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Abstract

We construct a consistent supersymmetric action for brane chiral and vector multiplets in a six-dimensional chiral gauged supergravity. A nonzero brane tension can be accommodated by allowing for a brane-localized Fayet-Iliopoulos term proportional to the brane tension. When the brane chiral multiplet is charged under the bulk $U(1)_R$, we obtain a nontrivial coupling to the extra component of the $U(1)_R$ gauge field strength and a singular scalar self-interaction term. In the context of $D = 5, N = 2$, Yang-Mills Supergavity compactified on S^1/Z_2 we consider the supersymmetric coupling of matter fields propagating on the brane. To solve the discrepancy between the bulk actions with limited field content and the wide range of brane actions that involve all the possible R-R forms, we have constructed a new formulation of IIA/IIB supergravity up to quartic order in fermions.

1 Introduction

There has been a lot of interest in brane world models in higher dimensions with the hope to solve the particle physics problems and give a hint for physics beyond the Standard Model (SM) in a different context. Particularly, in order to ameliorate the hierarchy problem of the Higgs mass, models with extra dimensions compactified on a large flat or small warped space were suggested as an alternative to the weak-scale supersymmetry (SUSY). Furthermore, regarding the cosmological constant problem which has been one of the most notorious problems as dictated by a no-go theorem, the self-tuning mechanism in higher dimensions was suggested. This may give a better understanding of the cosmological constant problem, although one has only the SM quantum corrections confined on a brane under control. In particular, brane world models in six dimensions have drawn much attention because the brane tension generates a nonzero deficit angle in extra dimensions without curving the 4D spacetime. This feature has been first pursued in the framework of spontaneous compactification due to gauge fluxes in 6D Einstein-Maxwell theory, but ended up with a fine-tuning condition for the brane tension due to flux quantization or conservation. Furthermore, we still need some symmetry to ensure that a bulk tuning condition is stable against the quantum corrections.

Six dimensional supergravity provides a fascinating laboratory for investigating the issues which underly the cosmological constant problem, largely because six dimensions is both simple enough to allow the construction of explicit solutions, yet rich enough to exhibit an interesting variety of properties. In particular, it provides the simplest setting within which a collection of positive-tension branes can combine to produce vanishing 4D curvature. This makes it a very fruitful arena in which to explore how natural are the choices which must be made in order to ensure acceptably flat 4D worlds. The Salam-Sezgin supergravity has drawn a renewed interest due to the possibility of attacking both brane and bulk fine-tuning problems encountered in the non-supersymmetric models. In this model, Salam and Sezgin obtained a spontaneous compactification on a sphere with $U(1)_R$ flux to get the 4D Minkowski spacetime and showed that 4D $\mathcal{N}=1$ SUSY survives or there is a massless chiral gravitino in four dimensions. The most general warped non-singular solutions with 4D maximal symmetry have been recently found to be a warped product of the 4D Minkowski space and a two dimensional compact manifold. Nonetheless, there is still a fine-tuning between brane tensions due to the flux quantization. It has been shown, on the other hand, that there are warped singular solutions with 4D curved spacetime. The stability analysis of the warped background has been done for scalar perturbations and bulk gauge fields and fermions.

In this paper, we consider the supersymmetrisation of the brane tension action in a way compatible with the bulk SUSY in 6D Salam-Sezgin supergravity. We find that a brane-localised Fayet-Ilioupolos (FI) term proportional to each brane tension must be introduced to cancel the SUSY variation of the brane tension term. With a nonzero FI term, we should also add in the action the brane-localised bilinear fermion terms that couple to the $U(1)_R$ field strength. Furthermore, we should modify the SUSY transformation of the $U(1)_R$ gaugino with a singular term. The Z_2 orbifold boundary conditions on the branes are also required to project out half of the bulk SUSY. In order to get the right Bianchi identities with the modified gauge field strengths, we also need to add a localised correction to the Chern-Simons term in the field strength for the Kalb-Ramond field appearing in the action and the SUSY transformation.

We discuss a generalized form of IIA/IIB supergravity depending on R-R potentials $C^{(p)}$ ($p = 0, 1, \ldots 9$) as the effective field theory of Type IIA/IIB superstring theory. For the IIA case we explicitly break this R-R democracy to either $p \leq 3$ or $p \geq 5$ which allows us to write a new bulk action that can be coupled to $N = 1$ supersymmetric brane actions. Our purpose is to construct supersymmetric domain walls of string theory in $D = 10$ which may shed some light on the stringy origin of the brane world scenarios. In the process of pursuing this goal we have realized that all descriptions of the effective field theory of Type IIA/IIB string theory available in the literature are inefficient for our purpose. This has led us to introduce new versions of the effective supergravities corresponding to Type IIA/IIB string theory. The standard IIA massless supergravity includes the $C^{(1)}$ and $C^{(3)}$ R-R potentials and the corresponding $G^{(2)}$ and $G^{(4)}$ gauge-invariant R-R forms. Type IIB supergravity includes the $C^{(0)}$, $C^{(2)}$ and $C^{(4)}$ R-R potentials and the corresponding $G^{(1)}$, $G^{(3)}$ and $G^{(5)}$ gauge-invariant R-R forms. The realization of the total system in supergravity is rather obscure.

2 Brane Multiplets in $D = 5\,,\ N = 2$ Supergravity

In the on-shell formulation of $D = 5\,,\ N = 2$ supergravity, compactified on S^1/Z_2, we extend the results describing the interaction of the bulk fields with matter which is assumed to be confined on the brane. We consider a five-dimensional Yang-Mills supergravity model. The field content of the model is

$$\{e_{\tilde{\mu}}^{\tilde{m}}, \Psi_{\tilde{\mu}}^i, A_{\tilde{\mu}}^I, \lambda^{ia}, \phi^x\} \tag{1}$$

where $\tilde{\mu} = (\mu, 5)$ are curved and $\tilde{m} = (m, \dot{5})$ are flat five-dimensional indices, with μ, m their corresponding four dimensional indices. The remaining indices are $I = 0, 1, \ldots, n$, $a = 1, \ldots, n$ and $x = 1, \ldots, n$. The supergravity multiplet consists of the fünfbein $e_{\tilde{\mu}}^{\tilde{m}}$, two gravitini $\Psi_{\tilde{\mu}}^i$ and the graviphoton $A_{\tilde{\mu}}^0$, where $i = 1, 2$ is the symplectic $SU(2)_R$ index. Moreover, there exist n vector multiplets, counting the Yang-Mills fields ($A_{\tilde{\mu}}^a$). The spinor and the scalar fields included in the vector multiplets are denoted by λ^{ia}, ϕ^x respectively.

The bulk Lagrangian is

$$\begin{aligned} \mathcal{L}_0/e^{(5)} &= -\frac{1}{2}R^{(5)} + \frac{i}{2}\bar{\Psi}_{i\tilde{\mu}}\gamma^{\tilde{\mu}\tilde{\nu}\tilde{\rho}}\nabla_{\tilde{\nu}}\Psi_{\tilde{\rho}}^i - \frac{1}{4}\mathring{a}_{IJ}F_{\tilde{\mu}\tilde{\nu}}^I F^{I\,\tilde{\mu}\tilde{\nu}} - \frac{1}{2}g_{xy}(\mathcal{D}_{\tilde{\mu}}\phi^x)(\mathcal{D}^{\tilde{\mu}}\phi^y) \\ &\quad + \ \mathrm{Fermion} \ + \ \mathrm{Chern-Simons\ \ terms} \end{aligned} \tag{2}$$

Recalling the linearized supersymmetry transformations of the bulk fields

$$\delta e_{\tilde{\mu}}^{\tilde{m}} = i\bar{\epsilon}_i\gamma^{\tilde{m}}\Psi_{\tilde{\mu}}^i$$

$$\begin{aligned}
\delta\Psi^i_{\tilde\mu} &= 2\nabla_{\tilde\mu}(\omega)\epsilon^i - \frac{h_I}{2\sqrt{6}}\gamma_{\tilde\mu}^{\tilde\nu\tilde\rho}F^I_{\tilde\nu\tilde\rho}\epsilon^i - \frac{2h_I}{\sqrt{6}}\gamma^{\tilde\rho}F^I_{\tilde\mu\tilde\rho}\epsilon^i \\
\delta A^I_{\tilde\mu} &= -ih^I_a\bar\epsilon_i\gamma_{\tilde\mu}\lambda^{ai} - \frac{i\sqrt{6}}{2}h^I\bar\Psi_{\tilde\mu i}\epsilon^i \\
\delta\lambda^{ai} &= -f^a_x\gamma^{\tilde\mu}D_{\tilde\mu}\phi^x\epsilon^i - \frac{1}{2}h^a_I\gamma^{\tilde\mu\tilde\nu}\epsilon^i F^I_{\tilde\mu\tilde\nu} \\
\delta\phi^x &= -if^x_a\bar\epsilon_i\lambda^{ai}
\end{aligned} \tag{3}$$

The matter fields are considered to be localized on the branes at the fixed points $x^5 = 0$ and $x^5 = \pi R$. For the purposes of this work it suffices to consider only the brane at $x^5 = 0$. The treatment of fields living on the brane at $x^5 = \pi R$ is done similarly.

The requirement of $N = 1$ local supersymmetry invariance on the branes determines the on-shell couplings of these fields to the gravity and gauge multiplets. These can be found following Nöther's procedure. This procedure is used in the on-shell formulation of local supersymmetry, where the role of the gauge field is played by the gravitino, while the gauge current is the supercurrent. However in the case of supersymmetry besides the modification of the Lagrangian the transformation laws should be also modified accordingly. This is well understood since the on-shell formulation follows from the off-shell after eliminating the auxiliary fields by solving the equations of motion which are modified upon changing the Lagrangian at each step.

The original Lagrangian is

$$\mathcal{L}_{\rm orig} = \mathcal{L}_0 + \mathcal{L}_b \tag{4}$$

with $\mathcal{L}_b$ the brane part including the interactions of the matter fields, localized on the brane, with the projections of the bulk fields, gravity and gauge fields, on the brane. The original SUSY transformations will be denoted by δ_0. $\mathcal{L}_0$ is invariant under δ_0, $\delta_0\mathcal{L}_0 = 0$, but not $\mathcal{L}_b$ that is $\delta_0\mathcal{L}_b \neq 0$. As already stated we must modify the original theory by adding new terms, $\Delta\mathcal{L}$, so that the total Lagrangian

$$\mathcal{L}_s = \mathcal{L}_0 + \mathcal{L}_b + \sum_k \Delta_k\mathcal{L}$$

is invariant under the modified SUSY transformations denoted by δ_s,

$$\delta_s = \delta_0 + \sum_k \delta_k$$

that is $\delta_s\mathcal{L}_s = 0$. We will proceed iteratively and in the above sums k denotes the iteration step.

In order to derive the gravitational couplings we ignore for the moment the gauge interactions and consider for simplicity just one chiral multiplet on the brane at $x^5 = 0$. Thus we start from $\mathcal{L}_b$ which for one chiral multiplet, (φ, χ), has the form

$$\mathcal{L}_b = -e^{(5)}\Delta_{(5)}\left(\partial_\mu\varphi\partial^\mu\varphi^* + i\bar\chi\bar\sigma^\mu D_\mu\chi\right). \tag{5}$$

In order to facilitate the discussion we have ignored at this stage superpotential and gauge interactions. Since $\mathcal{L}_b$ includes dependencies on the vierbein it facilitates to write

$$\delta_0 = \delta_0^{(e)} + \delta_0^{(\text{rest})} \tag{6}$$

with $\delta_0^{(e)}$, $\delta_0^{(\text{rest})}$ denoting variations acting on the vierbein and the remaining fields respectively. With this we get from (5)

$$\delta_0 \mathcal{L}_b = \delta_0^{(e)} \left[-e^{(5)} \Delta_{(5)} \left(\partial_\mu \varphi \partial^\mu \varphi^* + i \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi \right) \right] + e^{(5)} \Delta_{(5)} \left(J_\mu \partial^\mu \varepsilon + h.c. \right) \tag{7}$$

where J^μ in (7) is the (Nöther) supercurrent given by $J^\mu = \sqrt{2} \chi \sigma^\mu \bar{\sigma}^\nu \partial_\nu \varphi^*$. According to Nöther's procedure in order to eliminate the last term in (7) we must add a term $\Delta_1 \mathcal{L}$ while no change of SUSY transformations is required at this stage. Thus

$$\delta_1 = 0 \ , \ \ \Delta_1 \mathcal{L} = -\frac{1}{2} e^{(5)} \Delta_{(5)} \left(J_\mu \psi^\mu + h.c. \right) \tag{8}$$

since in our conventions $\delta_0 \psi_\mu \sim 2 D_\mu \varepsilon + ...$ Next we have to check the invariance of the so constructed Lagrangian $\mathcal{L}_0 + \mathcal{L}_b + \Delta_1 \mathcal{L}$ and modify it accordingly if it happens to be non-invariant under the new SUSY transformation law $\delta_s = \delta_0 + \delta_1$, which however at this stage, due to the vanishing of δ_1 coincides with the original transformation δ_0. Using the gravitino transformation law one gets

$$\begin{aligned} \delta_s \left(\mathcal{L}_0 + \mathcal{L}_b + \Delta_1 \mathcal{L} \right) &= \delta_0^{(e)} \left[-e^{(5)} \Delta_{(5)} \left(\partial_\mu \varphi \partial^\mu \varphi^* + i \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi \right) \right] - \frac{1}{2} e^{(5)} \Delta_{(5)} \left[\left(\delta_0 J_\mu \right) \psi^\mu + h.c \right] \\ &+ \frac{1}{\sqrt{6}} e^{(5)} \Delta_{(5)} \ F^0_{\mu \dot{5}} \ \delta_0 \left(J^{(\varphi)\, \mu} - \frac{1}{2} J^{(\chi)\, \mu} \right) + \delta_0^{(e)} \left(-\frac{1}{2} e^{(5)} \Delta_{(5)} J_\mu \psi^\mu + h.c. \right) \end{aligned} \tag{9}$$

The third term in (9) follows from the second term of the gravitino transformation in (**??**), as can be verified by a straightforward algebra, and $J^{(\varphi)}$, $J^{(\chi)}$ denoting the $U_R(1)$ currents of φ and χ fields, given by

$$J_\mu^{(\varphi)} = -i \varphi^* \overset{\leftrightarrow}{\partial}_\mu \varphi \ , \quad J_\mu^{(\chi)} = \chi \sigma_\mu \bar{\chi} \tag{10}$$

Note also that we have not included the spin connection $\omega_{\mu\, mn}$ for lack of space. Its contribution at each stage is determined by fully covariantizing the results.

The first three terms in (9) are cancelled if we modify the SUSY transformations as it appears below

$$\begin{aligned} \delta_2 \, \varphi \ &= \ 0, \quad \delta_2 \chi \ = -i \sigma^\mu \bar{\varepsilon} \left(\psi_\mu \chi \right) \\ \delta_2 e^m_\mu \ &= \ 0, \quad \delta_2 \psi_\mu = \frac{i}{2} \Delta_{(5)} \left(J_\mu^{(\varphi)} \varepsilon - \sigma_{\mu\nu} \varepsilon J^{(\chi)\, \nu} \right) \end{aligned} \tag{11}$$

and add a term to the Lagrangian given by

$$\begin{aligned} \Delta_2 \mathcal{L} \ &= \ e^{(5)} \Delta_{(5)} \Big\{ \ \frac{1}{4} J_\sigma^{(\chi)} \left[\ i E^{\mu\nu\rho\sigma} \left(\psi_\mu \sigma_\nu \bar{\psi}_\rho \right) + \left(\psi_\mu \sigma^\sigma \bar{\psi}^\mu \right) \right] \\ &- \ \frac{i}{4} \ E^{\mu\nu\rho\sigma} J_\sigma^{(\varphi)} \psi_\mu \sigma_\nu \bar{\psi}_\rho - \frac{1}{\sqrt{6}} \left(J^{(\varphi)\, \mu} - \frac{1}{2} J^{(\chi)\, \mu} \right) F^0_{\mu \dot{5}} \ \Big\} \ , \end{aligned} \tag{12}$$

where $E^{\mu\nu\rho\sigma}$ is the four dimensional antisymmetric tensor. The last term in (12) is needed for the cancellation of the $F^0_{\mu\dot{5}}\left(-J^{(\varphi)\,\mu}+\cdots\right)$ term in (9). The need of introducing the remaining terms will be clarified in the following.

We next have to check the invariance of $\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L}$ under δ_s transformations. Since $\delta_1=0$ we have

$$\delta_s(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L})=\delta_0(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L})+\delta_0(\Delta_2\mathcal{L})+\delta_2(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L}) \quad (13)$$

As we have already discussed, from the transformation $\delta_0(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L})+\delta_0(\Delta_2\mathcal{L})$ only the term $\delta_0^{(e)}\left(\frac{e^{(5)}}{e^{\dot{5}}_{5}}J_\mu\psi^\mu+h.c.\right)$ survives. In fact the variation $\delta_0(\Delta_2\mathcal{L})$ is given by

$$\begin{aligned}\delta_0(\Delta_2\mathcal{L})= \; & \tfrac{1}{\sqrt{6}}\left[-\delta_0\left(e^{(5)}\Delta_{(5)}F^0_{\mu\dot{5}}\right)\left(J^{(\varphi)\,\mu}-\tfrac{1}{2}J^{(\chi)\,\mu}\right)-e^{(5)}\Delta_{(5)}F^0_{\mu\dot{5}}\delta_0\left(J^{(\varphi)\,\mu}-\tfrac{1}{2}J^{(\chi)\,\mu}\right)\right]\\ & +\delta_0^{(e)}\left\{\tfrac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right)\left(J^{(\chi)}_\sigma-J^{(\varphi)}_\sigma\right)+\left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)J^{(\chi)}_\sigma\right]\right\}\\ & +\tfrac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right)\delta_0\left(J^{(\chi)}_\sigma-J^{(\varphi)}_\sigma\right)+\left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)\delta_0 J^{(\chi)}_\sigma\right]\\ & +\tfrac{1}{4}e^{(5)}\Delta_{(5)}\left[iE^{\mu\nu\rho\sigma}\left(J^{(\chi)}_\sigma-J^{(\varphi)}_\sigma\right)\delta_0\left(\psi_\mu\sigma_\nu\bar{\psi}_\rho\right)+J^{(\chi)}_\sigma\delta_0\left(\psi_\mu\sigma^\sigma\bar{\psi}^\mu\right)\right] \end{aligned}\quad (14)$$

The term $\sim\ F^0_{\mu\dot{5}}\delta_0\left(J^{(\varphi)\,\mu}-\frac{1}{2}J^{(\chi)\,\mu}\right)$ in (14) cancels the corresponding term in eq. (9). Also the term in the last line cancels the first two terms of eq. (9) along with δ_2 variations of the gravitino ψ_μ and the fermion χ kinetic terms occuring within $\mathcal{L}_0+\mathcal{L}_b$, that is $\delta_2\left(\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma-ie^{(5)}\Delta_{(5)}\bar{\chi}\bar{\sigma}^\mu D_\mu\chi\right)$. Actually that was the reason nonvanishing variations δ_2 had to be introduced for the χ and ψ_μ fields. However not all of the terms in $\delta_s(\mathcal{L}_0+\mathcal{L}_b+\Delta_1\mathcal{L}+\Delta_2\mathcal{L})$ are completely cancelled. Among those terms that survive is the δ_2 variation of $\Delta_1\mathcal{L}$ in (13) which reveals an interesting feature that needs be discussed. In fact

$$\delta_2(\Delta_1\mathcal{L})=-\frac{1}{2}e^{(5)}\Delta_{(5)}J_\mu\delta_2\psi^\mu+\delta_2\left(-\frac{1}{2}e^{(5)}\Delta_{(5)}J_\mu\right)\psi^\mu+h.c. \quad (15)$$

and the first term in (15), after some straightforward algebra, is brought into the form

$$-\frac{i}{2\sqrt{2}}e^{(5)}{\Delta_{(5)}}^2\left[\left(\chi\sigma^\mu\bar{\sigma}^\nu\varepsilon\right)J^{(\varphi)}_\mu\partial_\nu\varphi^*-\frac{\sqrt{2}}{2}\chi\sigma^\mu\bar{\sigma}^\rho\sigma_{\mu\nu}\varepsilon J^{(\chi)\nu}\partial_\rho\varphi^*\right]+h.c. \quad (16)$$

due to the variation $\delta_2\psi_\mu$ (see eq. (11)). Since $i\sqrt{2}\bar{\sigma}^\nu\varepsilon\partial_\nu\varphi^*$ is actually $\delta_0\bar{\chi}$ we have from the expression (16) a contribution $-\frac{1}{4}e^{(5)}{\Delta_{(5)}}^2\chi\sigma^\mu\left(\delta_0\bar{\chi}\right)\left(J^{(\varphi)}_\mu+\frac{1}{4}J^{(\chi)}_\mu\right)+h.c.$ Due to the appearance of this we need to add a new term in the Lagrangian which includes, among others, the aforementioned contribution that is

$$\Delta_3\mathcal{L}=\frac{1}{4}e^{(5)}{\Delta_{(5)}}^2J^{(\chi)}_\mu\left(J^{(\varphi)\,\mu}+\frac{1}{4}J^{(\chi)\,\mu}\right)+\cdots \quad (17)$$

In (17) the ellipsis denote additional terms. The terms in (17) are not new. Such terms do appear in the derived Lagrangian completing previous derivations. In that work the $\delta^2(x^5)$ terms complete a perfect square. This is not the case in our approach.

The above results are easily extended in the case that the original brane action has the structure of a general σ-model

$$\begin{aligned}\mathcal{L}_b = \; & - \; e^{(5)}\Delta_{(5)}\Big[K_{ij^*}D_\mu\varphi^i D^\mu\varphi^{*j} + (\frac{i}{2}\; K_{ij^*}\chi^i\sigma^\mu D_\mu\bar{\chi}^j \; + \; h.c) \\ & + \; \frac{1}{2}\,(D_iD_jW\;\chi^i\chi^j \; + \; h.c.) + \; K^{ij^*}D_iW\; D_{j^*}W^* - \frac{1}{4}R_{ij^*kl^*}\;\chi^i\chi^k\;\bar{\chi}^j\bar{\chi}^l\Big] \end{aligned} \quad (18)$$

where K_{ij^*} is the Kähler metric. In this equation $D_\mu\bar{\chi}^j$ is covariant under both spacetime and Kähler transformations. The superpotential and Yukawa terms, are also included. In the flat case $D_iW = \partial_iW$, $D_iD_jW = \partial_i\partial_jW - \Gamma^k_{ij}\;\partial_kW$. Later when considering the curved case it turns out that these include additional terms so that they are covariant with respect to the Kähler function K as well.

The coupling of the brane fields to the gauge and the gaugino fields propagating in the bulk is known from the flat case, so that here we will only outline the steps we follow. The transformation of λ^a, stemming from the five dimensions is not exactly that of a gaugino. The extra variation requires the addition to the Lagrangian of a term $g\;\Delta_{(5)}\;D^{(a)}f^a_x\partial_5\phi^x$ while it is known that the variation of the gaugino-fermions Yukawa terms, given by $\Delta_{(5)}\;\big(-ig\sqrt{2}D^{(a)}{}_{,j^*}\;\bar{\chi}^j\bar{\lambda}^a + h.c.\big)$, requires the modification of the gaugino transformation rule by adding a term $\delta'_\varepsilon\lambda^a = -ig\Delta_{(5)}\;D^{(a)}\varepsilon$ and supersymmetry invariance is finally restored by adding the term

$$-\frac{g^2}{2}{\Delta_{(5)}}^2\;D^{(a)}D^{(a)}. \quad (19)$$

As far as the presence of the superpotential is concerned, we already know from the flat case it modifies the fermions supersymmetry transformation law according to

$$\delta'_\varepsilon\chi^i = -\sqrt{2}\;K^{ij^*}D_{j^*}W^*\varepsilon\,, \quad (20)$$

The extra variation of the fermion fields applied to the coupling of the Nöther current with the gravitino field $\sim J^\mu\psi_\mu$, see eq. (8), leads to modification of the gravitino transformation law as $\delta'_\varepsilon\psi_\mu = i\Delta_{(5)}\;W\sigma_\mu\bar{\varepsilon}$, and the addition to the Lagrangian of the term

$$\mathcal{L}' = e^{(5)}\;\Delta_{(5)}\;\left[W^*\psi_\mu\sigma^{\mu\nu}\psi_\nu + W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu\right]\,. \quad (21)$$

We can see in turn that its variation, due to $\delta'_\varepsilon\psi_\mu$ above and the supersymmetry transformation for $e^{(4)}$ is

$$\delta'_\varepsilon\mathcal{L}' = -3\Delta_{(5)}\;\delta\left(e^{(5)}\Delta_{(5)}\right)\mid W\mid^2 \quad (22)$$

which is cancelled by the addition of the known $\mid W\mid^2$ term of the supergravity potential which however in our case it appears multiplied by ${\Delta_{(5)}}^2$. Variations of the potential terms $K^{ij^*}D_iW\;D_{j^*}W^*$ are cancelled by the Yukawa terms $\sim D_iD_jW\chi^i\chi^j + h.c.$ and those of the $\mid W \mid^2$ require the appearence of terms $\sim D_i W \chi^i \sigma^\mu \bar{\psi}_\mu + h.c.$ in the Lagrangian for their cancellation. This procedure can be continued and in the following steps the wellknown Kählerian exponents $e^{K/2}$ appearing in the ordinary D=4 supergravity start showing up accompanying each power of the superpotential W, or derivative of it, both in the Lagrangian and the transformation laws. However the Kähler function K in the exponent appears multiplied by $\Delta_{(5)} = e^{\dot 5}_{5}\, \delta(x^5)$ as shown in the Lagrangian given below. In conjuction with this we point out that the covariant derivatives of the superpotential W are also found to depend on the Kähler function through the combination $\Delta_{(5)}\, K$, rather than K itself, so that Kähler invariance is indeed maintained.

Summarizing, the interactions of a set of chiral multiplets localized on the brane designated by the index i, with the bulk gravity and gauge fields are found to be

$$
\begin{aligned}
\mathcal{L}^{(4)} \; = e^{(5)}\Delta_{(5)} \Big[& - K_{ij^*} D_\mu \varphi^i D^\mu \varphi^{*j} - \left(\tfrac{i}{2} K_{ij^*} \chi^i \sigma^\mu D_\mu \bar{\chi}^j + h.c \right) - ig\sqrt{2} \left(D^{(a)}{}_{,j^*}\, \bar{\chi}^j \bar{\lambda}^a - h.c. \right) \\
& - \tfrac{g}{2} D^{(a)} \left(\psi_\mu \sigma^\mu \bar{\lambda}^a - \bar{\psi}_\mu \bar{\sigma}^\mu \lambda^a \right) - \tfrac{1}{\sqrt{2}} K_{ij^*} \left(D_\mu \varphi^{*j} \chi^i \sigma^\nu \bar{\sigma}^\mu \psi_\nu + D_\mu \varphi^i \bar{\chi}^j \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_\nu \right) \\
& + \tfrac{i}{4} E^{\mu\nu\rho\sigma} \left(J_\sigma^{(\chi)} - J_\sigma^{(\varphi)} \right) \psi_\mu \sigma_\nu \bar{\psi}_\rho + \tfrac{1}{4} J_\sigma^{(\chi)} \psi_\mu \sigma^\sigma \bar{\psi}^\mu + \tfrac{1}{4} \Delta_{(5)}\, J^{(\chi)\,\mu} \left(J_\mu^{(\varphi)} + \tfrac{1}{4} J_\mu^{(\chi)} \right) \\
& - \tfrac{1}{8}\, R_{ij^*kl^*}\, \chi^i \sigma^\mu \bar{\chi}^j\, \chi^k \sigma_\mu \bar{\chi}^l \; - \tfrac{1}{4} \left(J_\mu^{(\varphi)} - \tfrac{1}{2} J_\mu^{(\chi)} \right) \lambda^a \sigma^\mu \bar{\lambda}^a \\
& + \tfrac{1}{\sqrt{6}} \left(-J^{(\varphi)\,\mu} + \tfrac{1}{2} J^{(\chi)\,\mu} \right) F^0_{\mu\dot 5} - \tfrac{1}{2} g^2 \Delta_{(5)}\; D^{(a)} D^{(a)} + g D^{(a)} f^a_x \partial_5 \phi^x \\
& - e^{\Delta_{(5)}\, K/2} \left(W^* \psi_\mu \sigma^{\mu\nu} \psi_\nu + \tfrac{i}{\sqrt{2}} D_i W \chi^i \sigma^\mu \bar{\psi}_\mu + \tfrac{1}{2} D_i D_j W \chi^i \chi^j + h.c. \right) \\
& - e^{\Delta_{(5)}\, K} \left(K^{ij^*}\, D_i W\, D_{j^*} W^* - 3 \Delta_{(5)}\, |W|^2 \right) \Big] + \cdots
\end{aligned}
\qquad (23)
$$

where in the general case

$$
J_\mu^{(\varphi)} = -i \left(K_i \partial_\mu \varphi^i - K_{i^*} \partial_\mu \varphi^{*i} \right) \;, \quad J_\mu^{(\chi)} = K_{ij^*} \chi^i \sigma_\mu \bar{\chi}^j \,. \qquad (24)
$$

The ellipsis in (23) stand for couplings of the brane fields with the radion multiplet, which is even, and other even combinations of odd fields which are not presented here. The prefactor $e^{(5)}\, \Delta_{(5)}$ in the Lagrangian above provides $e^{(4)}$ upon integration with respect x^5. Note that the terms $D^{(a)} D^{(a)}$, $J^{(x)}(\cdots)$, $\mid W \mid^2$ and the exponents involving the Kähler function appear multiplied by an extra power of $\Delta_{(5)}$ whose argument can be put to zero, due to the overall $\Delta_{(5)}$ multiplying the Lagrangian, which is proportional to $\delta(x^5)$. Since $\int_{-\pi R}^{\pi R} dx^5 e^{\dot 5}_5 \Delta_{(5)}(x^5) = 1$ and $\int_{-\pi R}^{\pi R} dx^5 e^{\dot 5}_5 = L$ is the volume of the fifth dimension, we are tempted to interpret $\Delta_{(5)}(0) \simeq 1/L \equiv M_L$. Replacing then $\Delta_{(5)}(0)$ by M_L and reestablishing units we find that M_L enters in our formulae only through the ratio M_5^3/M_L where M_5 is related to the 5 - D gravitational coupling through $k^2_{(5)} = 1/M_5^3$. The 4 - D gravitational constant is $k^2_{(4)} = k^2_{(5)}/L$ and the aforementioned ratio is related to the Planck scale via $M_5^3/M_L = M^2_{Planck}$. In doing all this the gravitino, gauge boson and gaugino, as well as the five dimensional gauge coupling should scale appropriately as $\psi_\mu = L^{-1/2} \hat{\psi}_\mu$, $A_\mu^{(a)} = L^{-1/2} \hat{A}_\mu^{(a)}$, $\lambda_\mu^{(a)} = L^{-1/2} \hat{\lambda}_\mu^{(a)}$ and $g = L^{1/2} g_{(4)}$, as dictated by the kinetic terms of these fields, in order for them to have the right normalization and the appropriate dimensions in four dimensions. It then turns that with this interpretation the terms in (23) are exactly those encountered in the ordinary D=4 supergravity involving the interactions of the chiral fields φ^i, χ^i among themselves and their interactions with the gravity and gauge multiplets. Exception to it are additional terms where bulk fields, involving $F^0_{\mu\dot{5}}, \partial_5\phi^x$, the radion multiplet etc., interact with the multiplets on the brane. This rather rough qualitative argument is only used to show the correctness of our results. In a decent mathematical way this can be seen after replacing the bulk fields which interact with the brane chiral multiplets by their classical equations of motion as was first done in the model. This is the case for instance with the D - terms which complete a perfect square, as in the flat case, involving the derivative $\partial_5\phi^x$. Eliminating this by its classical equation of motion results to the ordinary four dimensional D - terms. The importance of the $\Delta_{(5)}(0)$ terms at the quantum level has been discussed.

3 Bulk Branes in Salam-Sezgin Supergravity

We present a new anomaly-free gauged $N = 1$ supergravity model in six dimensions. We construct a consistent supersymmetric action for brane chiral and vector multiplets in a six-dimensional chiral gauged supergravity. When the brane chiral multiplet is charged under the bulk $U(1)_R$, we obtain a nontrivial coupling to the extra component of the $U(1)_R$ gauge field strength and a singular scalar self-interaction term.

The six-dimensional Salam-Sezgin supergravity consists of gravity coupled to a dilaton field ϕ, a Kalb-Ramond(KR) field B_{MN}, along with the SUSY fermionic partners, the gravitino ψ_M, the dilatino χ. Moreover, it also contains a bulk $U(1)_R$ vector multiplet (A_M, λ) that gauges the R-symmetry of six-dimensional supergravity. All the bulk fermions are 6D Weyl. In order to do this analysis, we need the spinor part of the action and in particular the part that is quadratic in fermionic terms. This is given by

$$\begin{aligned} e^{-1}\mathcal{L}_f &= \bar{\psi}_M\Gamma^{MNP}\mathcal{D}_N\psi_P + \bar{\chi}\Gamma^M\mathcal{D}_M\chi + \bar{\lambda}\Gamma^M\mathcal{D}_M\lambda + \bar{\lambda}\Gamma^M\mathcal{D}_M\lambda \\ &+\frac{1}{4}(\partial_M\phi)(\bar{\psi}_N\Gamma^M\Gamma^N\chi + \text{h.c.}) + \sqrt{2}ge^{-\frac{1}{4}\phi}(i\bar{\psi}_M\Gamma^M\lambda - i\bar{\chi}\lambda + \text{h.c.}) \\ &-\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\Big\{F_{MN}(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda + \bar{\chi}\Gamma^{MN}\lambda) + F^I_{MN}(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda^I + \bar{\chi}\Gamma^{MN}\lambda^I) + \text{h.c.}\Big\}. \end{aligned} \tag{25}$$

The complete bulk Langrangian up to four fermion terms is

$$\begin{aligned} e_6^{-1}\mathcal{L}_{\text{bulk}} &= R - \frac{1}{4}(\partial_M\phi)^2 - \frac{1}{12}e^{\phi}G_{MNP}G^{MNP} - \frac{1}{4}e^{\frac{1}{2}\phi}F_{MN}F^{MN} - 8g^2e^{-\frac{1}{2}\phi} \\ &+\bar{\psi}_M\Gamma^{MNP}\mathcal{D}_N\psi_P + \bar{\chi}\Gamma^M\mathcal{D}_M\chi + \bar{\lambda}\Gamma^M\mathcal{D}_M\lambda + \frac{1}{4}(\partial_M\phi)(\bar{\psi}_N\Gamma^M\Gamma^N\chi + \bar{\chi}\Gamma^N\Gamma^M\psi_N) \\ &+\frac{1}{24}e^{\frac{1}{2}\phi}G_{MNP}(\bar{\psi}^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R\Gamma^{MNP}\Gamma^R\chi - \bar{\chi}\Gamma^R\Gamma^{MNP}\psi_R - \bar{\chi}\Gamma^{MNP}\chi \\ &+\bar{\lambda}\Gamma^{MNP}\lambda) - \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{MN}(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda + \bar{\lambda}\Gamma^Q\Gamma^{MN}\psi_Q + \bar{\chi}\Gamma^{MN}\lambda - \bar{\lambda}\Gamma^{MN}\chi) \\ &+i\sqrt{2}ge^{-\frac{1}{4}\phi}(\bar{\psi}_M\Gamma^M\lambda + \bar{\lambda}\Gamma^M\psi_M - \bar{\chi}\lambda + \bar{\lambda}\chi). \end{aligned} \tag{26}$$

The field strengths of the gauge and Kalb-Ramond(KR) fields are defined as

$$F_{MN} = \partial_M A_N - \partial_N A_N, \qquad G_{MNP} = 3\partial_{[M}B_{NP]} + \frac{3}{2}F_{[MN}A_{P]}, \tag{27}$$

and satisfy the Bianchi identities

$$\partial_{[Q}F_{MN]} = 0, \qquad \partial_{[Q}G_{MNP]} = \frac{3}{4}F_{[MN}F_{QP]}. \tag{28}$$

For $\delta_\Lambda A_M = \partial_M \Lambda$ under the $U(1)_R$, the field strength for the KR field is made gauge invariant by allowing for B_{MN} to transform as

$$\delta_\Lambda B_{MN} = -\frac{1}{2}\Lambda F_{MN}. \tag{29}$$

All the spinors have the same R charge $+1$, so the covariant derivative of the gravitino, for instance, is given by

$$\mathcal{D}_M \psi_N = (\partial_M + \frac{1}{4}\omega_{MAB}\Gamma^{AB} - igA_M)\psi_N. \tag{30}$$

The local $\mathcal{N}=2$ SUSY transformations are

$$\begin{aligned}
\delta e^A_M &= -\frac{1}{4}\bar{\varepsilon}\Gamma^A\psi_M + \text{h.c.}, \qquad \delta\phi = \frac{1}{2}\bar{\varepsilon}\chi + \text{h.c.}, \\
\delta B_{MN} &= A_{[M}\delta A_{N]} + \frac{1}{4}e^{-\frac{1}{2}\phi}(\bar{\varepsilon}\Gamma_M\psi_N - \bar{\varepsilon}\Gamma_N\psi_M + \bar{\varepsilon}\Gamma_{MN}\chi + \text{h.c.}), \\
\delta\chi &= -\frac{1}{4}(\partial_M\phi)\Gamma^M\varepsilon + \frac{1}{24}e^{\frac{1}{2}\phi}G_{MNP}\Gamma^{MNP}\varepsilon, \\
\delta\psi_M &= \mathcal{D}_M\varepsilon + \frac{1}{48}e^{\frac{1}{2}\phi}G_{PQR}\Gamma^{PQR}\Gamma_M\varepsilon, \\
\delta A_M &= \frac{1}{2\sqrt{2}}e^{-\frac{1}{4}\phi}(\bar{\varepsilon}\Gamma_M\lambda + \text{h.c.}), \qquad \delta\lambda = \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{MN}\Gamma^{MN}\varepsilon - i\sqrt{2}g\,e^{-\frac{1}{4}\phi}\varepsilon.
\end{aligned} \tag{31}$$

The above spinors are chiral with handednesses

$$\Gamma^7\psi_M = +\psi_M, \quad \Gamma^7\chi = -\chi, \quad \Gamma^7\lambda = +\lambda, \quad \Gamma^7\varepsilon = +\varepsilon. \tag{32}$$

Taking into account that $\Gamma^7 = \sigma^3 \otimes \mathbf{1}$, the 6D (8-component) spinors can be decomposed to 6D Weyl (4-component) spinors as

$$\psi_M = (\tilde{\psi}_M, 0)^T, \quad \chi = (0, \tilde{\chi})^T, \quad \lambda = (\tilde{\lambda}, 0)^T, \quad \varepsilon = (\tilde{\varepsilon}, 0)^T. \tag{33}$$

We decompose the 6D Weyl spinor $\tilde{\psi}$ to $\tilde{\psi} = (\tilde{\psi}_L, \tilde{\psi}_R)^T$, satisfying $\gamma^5(\tilde{\psi}_L, 0)^T = +(\tilde{\psi}_L, 0)^T$ and $\gamma^5(0, \tilde{\psi}_R)^T = -(0, \tilde{\psi}_R)^T$. Henceforth we drop the tildes for simplicity.

We can show that the action for the Lagrangian (26) is invariant under the above SUSY transformations up to the trilinear fermion terms and the Bianchi identities as follows,

$$\begin{aligned}
\delta\mathcal{L}_{\text{bulk}} &= e_6\Big[-\frac{1}{24}e^{\frac{1}{2}\phi}\Big(\partial_S G_{MNP} - \frac{3}{4}F_{MN}F_{SP}\Big)\Big(\bar{\psi}_R\Gamma^{RMNPS}\varepsilon - \bar{\chi}\Gamma^{SMNP}\varepsilon + \text{h.c.}\Big) \\
&\quad + \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\Big(\partial_Q F_{MN}\bar{\lambda}\Gamma^{QMN}\varepsilon + \text{h.c.}\Big)\Big].
\end{aligned} \tag{34}$$

Thus, as will be seen later, the SUSY variation of the brane action can be cancelled with the bulk variation (34) by modifying the Bianchi identities (**??**) and (28).

We consider a nonzero brane tension as well as brane matter multiplets: a brane chiral multiplet (Q, ψ_Q), the superfield of which has an R charge $-r$, and a brane vector multiplet (W_μ, Λ). The 4D chirality of the fermion in the brane chiral multiplet is taken to be right-handed in contrast to the Z_2-even gravitino and the Z_2-even gaugino and the brane gaugino. So, the conventional chiral superfield containing a left-handed fermion, $(Q^*, (\psi_Q)^c)$, should have an opposite R charge, namely, r for Q^* and $r-1$ for $(\psi_Q)^c$. Then, by employing the Noether method for the local SUSY, we find that the supersymmetric action for the bulk-brane system (up to four fermion terms) is composed of the original bulk action (26) with the field strength tensors G_{MNP} and F_{MN} being replaced by the modified ones $\hat{G}_{MNP}$ and $\hat{F}_{MN}$, respectively, and the brane action as follows,

$$\mathcal{L} = \mathcal{L}_{\rm bulk}(G \to \hat{G}, F \to \hat{F}) + \delta^2(y)\mathcal{L}_{\rm brane} \tag{35}$$

with

$$\begin{aligned}\mathcal{L}_{\rm brane} = e_4\Big[&e^{\frac{1}{2}\phi}\Big(-(D^\mu Q)^\dagger D_\mu Q + \frac{1}{2}\bar{\psi}_Q\gamma^\mu D_\mu\psi_Q + {\rm h.c.}\Big) + \sqrt{2}irge^{\frac{1}{4}\phi}\bar{\psi}_Q\lambda_+ Q + {\rm h.c.}\\ &-4rg^2|Q|^2 - T + e^{\frac{1}{2}\phi}\Big(\frac{1}{2}\bar{\psi}_{\mu+}\gamma^\nu\gamma^\mu\psi_Q(D_\nu Q)^\dagger + \frac{1}{2}\bar{\psi}_Q\gamma^\mu\chi_+ D_\mu Q + {\rm h.c.}\Big)\\ &-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}\bar{\Lambda}\gamma^\mu D_\mu\Lambda + {\rm h.c.} - ie\sqrt{2}e^{\frac{1}{2}\phi}Q\bar{\psi}_Q\Lambda + {\rm h.c.} - \frac{1}{2}e^2|Q|^4 e^\phi\\ &-\frac{1}{4\sqrt{2}}\bar{\Lambda}\gamma^\mu\gamma^{\nu\rho}\psi_{\mu+}W_{\nu\rho} - \frac{i}{2\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\bar{\Lambda}\gamma^\mu\psi_{\mu+} + {\rm h.c.}\\ &-\frac{i}{\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\bar{\chi}_+\Lambda + {\rm h.c.}\Big].\end{aligned} \tag{36}$$

The SUSY transformations of the brane chiral multiplet are

$$\delta Q = \frac{1}{2}\bar{\varepsilon}_+\psi_Q\,, \qquad\qquad \delta\psi_Q = -\frac{1}{2}\gamma^\mu\varepsilon_+ D_\mu Q\ . \tag{37}$$

On the other hand, the SUSY transformations of the brane vector multiplet are

$$\delta W_\mu = \frac{1}{2\sqrt{2}}\bar{\varepsilon}_+\gamma_\mu\Lambda + {\rm h.c.}, \qquad \delta\Lambda = \frac{1}{4\sqrt{2}}\gamma^{\mu\nu}\varepsilon_+ W_{\mu\nu} + \frac{i}{2\sqrt{2}}e|Q|^2 e^{\frac{1}{2}\phi}\varepsilon_+. \tag{38}$$

Here the brane gauge field strength is $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and the covariant derivatives of the brane multiplets are

$$D_\mu Q \;=\; (\partial_\mu + irgA_\mu - ieW_\mu)Q, \tag{39}$$

$$D_\mu\psi_Q \;=\; (\partial_\mu + i(r-1)gA_\mu - ieW_\mu + \frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta})\psi_Q, \tag{40}$$

$$D_\mu\Lambda \;=\; (\partial_\mu - igA_\mu + \frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta})\Lambda. \tag{41}$$

We note that the R charges of the component fields in the brane chiral multiplet are different by $+1$ as known to be the case in 4D local SUSY. The gaugino of a brane vector multiplet also has the same R charge $+1$ as the bulk gravitino.

The modified field strength tensors are

$$\hat{G}_{\mu mn} = G_{\mu mn} + \Big(J_\mu - \xi A_\mu\Big)\epsilon_{mn}\frac{\delta^2(y)}{e_2}, \tag{42}$$

$$\hat{G}_{\tau\rho\sigma} = G_{\tau\rho\sigma} + J_{\tau\rho\sigma}\frac{\delta^2(y)}{e_2}, \tag{43}$$

$$\hat{F}_{mn} = F_{mn} - (rg|Q|^2 + \xi)\epsilon_{mn}\frac{\delta^2(y)}{e_2} \tag{44}$$

where $\xi = \frac{T}{4g}$ is the localized FI term, ϵ_{mn} is the 2D volume form and

$$J_\mu = \frac{1}{2}i\Big[Q^\dagger D_\mu Q - (D_\mu Q)^\dagger Q + \frac{1}{2}\bar{\psi}_Q\gamma_\mu\psi_Q - \frac{1}{2}e^{-\frac{1}{2}\phi}\bar{\Lambda}\gamma_\mu\Lambda\Big], \tag{45}$$

$$J_{\tau\rho\sigma} = -\frac{1}{4}\bar{\psi}_Q\gamma_{\tau\rho\sigma}\psi_Q - \frac{1}{8}e^{-\frac{1}{2}\phi}\bar{\Lambda}\gamma_{\tau\rho\sigma}\Lambda. \tag{46}$$

Here in order to cancel the variation of the brane tension action, we needed to modify the gauge field strength with the localized FI term proportional to the brane tension. Moreover, the modified field strength for the KR field contains a gauge non-invariant piece proportional to the localized FI term so the gauge transformation of the KR field needs to be modified to

$$\delta_\Lambda B_{mn} = \Lambda\Big(-\frac{1}{2}F_{mn} + \xi\epsilon_{mn}\frac{\delta^2(y)}{e_2}\Big). \tag{47}$$

On the other hand, the SUSY transformations of the bulk fields are the same as eqs. (31)-(32) with G_{MNP} and F_{MN} being replaced by $\hat{G}_{MNP}$ and $\hat{F}_{MN}$, respectively, and the gauge field A_M being kept the same as in the no-brane case, with an exception that the SUSY transformation of the extra components of the KR field has an additional term as

$$\delta B_{mn} = \frac{1}{4}i\bar{\psi}_Q\varepsilon_+ Q\epsilon_{mn}\frac{\delta^2(y)}{e_2} + \text{h.c.}. \tag{48}$$

Furthermore, for the modified field strength tensors, we obtain the Bianchi identities as follows,

$$\partial_{[\mu}\hat{G}_{\nu mn]} = \frac{3}{4}\hat{F}_{[\mu\nu}\hat{F}_{mn]} + \Big[\frac{i}{2}(D_{[\mu}Q)^\dagger(D_{\nu]}Q) + \frac{1}{4}e|Q|^2W_{\mu\nu}\Big]\epsilon_{mn}\frac{\delta^2(y)}{e_2}, \tag{49}$$

$$\partial_{[\mu}\hat{F}_{mn]} = -\frac{1}{3}rg\partial_\mu|Q|^2\epsilon_{mn}\frac{\delta^2(y)}{e_2}. \tag{50}$$

Then, by using (34) with the modified Bianchi identities (49) and (50), we are able to cancel all the remaining variations of the brane action given in (36).

We can extend the result to the more general case with multiple branes. When all the branes preserve the same 4D $\mathcal{N}=1$ SUSY, we only have to replace the delta terms appearing in the action and the SUSY/gauge transformations: $T\delta^2(y)$ with $\sum_i T_i\delta^2(y-y_i)$, and $f(Q)\delta^2(y)$ with $\sum_i f(Q_i)\delta^2(y-y_i)$.

We introduce a gravitino mass term on the brane. Then, the brane action is supplemented by the supersymmetric gravitino mass terms as

$$\mathcal{L}_{\rm gmass} = -e_4\frac{1}{2}W_0 e^{\frac{1}{2}\psi}(\bar{\psi}_{\mu+}\gamma^{\mu\nu}C\bar{\psi}^T_{\nu+}+\bar{\psi}_1\gamma^\mu C\bar{\psi}^T_{\mu+}+\bar{\psi}_2\gamma^\mu C\bar{\psi}^T_{\mu+}+\bar{\lambda}_+C\bar{\lambda}^T_+)+\text{h.c.} \tag{51}$$

where W_0 is a constant parameter and

$$\psi_1=\psi_{5+}+i\psi_{6+},\quad \psi_2=\psi_{5+}-i\psi_{6+}. \tag{52}$$

We also need to modify the SUSY transformations of the extra components of the gravitino as follows,

$$\delta\psi_+ = W_0 e^{\frac{1}{2}\psi}C\bar{\varepsilon}^T_+\frac{\delta^2(y)}{e_2}\ ,\qquad \delta\psi_- = -W_0 e^{\frac{1}{2}\psi}C\bar{\varepsilon}^T_+\frac{\delta^2(y)}{e_2}\ . \tag{53}$$

Here e^ψ is the volume modulus of the extra dimensions. Thus, similarly to the ungauged supergravity, the brane gravitino mass has a nontrivial coupling to the volume modulus of the extra dimensions. Under the modified gravitino variations, the variation of the bulk gravitino linear terms would have induced singular terms for a nonzero background gauge flux. So, in order to cancel them, we needed to introduce the brane-localized gaugino mass, which is the same as the gravitino mass. When the superpotential depends on the brane chiral multiplets, we can infer the form of the brane F-term as

$$\mathcal{L}_F=-e_4e^{\psi-\frac{1}{2}\phi}|F_Q|^2 \tag{54}$$

with $F_Q=\frac{\partial W}{\partial Q}$. Consequently, similarly to the ungauged supergravity case, we show that the F-term has a nontrivial coupling to the dilaton as well as the volume modulus.

4 Supersymmetrising the Brane Tension Action

In this section, we will add in the previous action codimension-two branes with nonzero tension. With this addition, the total action is no longer invariant under the transformations (31)-(32). We will, thus, modify our action and SUSY transformations, so that the brane-bulk system is rendered supersymmetric. With the modification that we propose, we show that the bulk action remains supersymmetric while the brane action preserves $\mathcal{N}=1$ SUSY.

4.1 Requirements for the Supersymmetric Brane Action

Let us add to the bulk Lagrangian a term for a brane located at the position $y=y_i$, where y is the internal space 2D coordinate. This brane Lagrangian will be given by

$$\mathcal{L}_{\rm brane}=-e_4T_i\delta^{(2)}(y-y_i), \tag{55}$$

where T_i is the brane tension and the $2D$ delta function is defined as $\int d^2y\delta^{(2)}(y-y_i)=1$.

The SUSY transformation of the brane action is non-vanishing as follows,

$$\delta\mathcal{L}_{\rm brane}=-e_4\frac{1}{4}T_i\delta^{(2)}(y-y_i)(\bar\psi_\mu\Gamma^\mu\varepsilon+{\rm h.c.}). \tag{56}$$

On the other hand, because the gravitino is charged under $U(1)_R$, varying the gravitino kinetic term under (31), it contains a piece of the gauge field strength as

$$\begin{aligned}\delta\mathcal{L}_{\rm gravitino} &\supset e_6\bar\psi_M\Gamma^{MNP}\mathcal{D}_N\mathcal{D}_P\varepsilon \\ &= -\frac{i}{2}e_6 g\bar\psi_M\Gamma^{MNP}\varepsilon F_{NP}+\cdots. \end{aligned}\tag{57}$$

We can utilise the above term of the gravitino vatiation to cancel the brane tension term as following. The $U(1)_R$ field can have in principle FI localised terms parameterized by constants ξ_i. We can then define a hatted field strength $\hat F_{MN}$

$$\hat F_{\mu\nu} = F_{\mu\nu},\quad \hat F_{\mu m}=F_{\mu m}, \tag{58}$$

$$\hat F_{mn} = F_{mn}-\epsilon_{mn}\xi_i\,\frac{\delta^{(2)}(y-y_i)}{e_2}, \tag{59}$$

where ϵ_{mn} is the 2D volume form, and rewrite the variation of the gravitino kinetic term as

$$\begin{aligned}\delta\mathcal{L}_{\rm gravitino} &\supset -\frac{i}{2}e_6 g\bar\psi_M\Gamma^{MNP}\varepsilon\hat F_{NP} \\ &\quad +e_4 g\xi_i\delta^{(2)}(y-y_i)\bar\psi_\mu\Gamma^\mu\gamma^5\varepsilon+\cdots, \end{aligned}\tag{60}$$

where use is made of $\Gamma^{mn}\epsilon_{mn}=2\Gamma^{56}=2i\sigma^3\otimes\gamma^5$, the 6D chirality condition, $\sigma^3\otimes\mathbf{1}\varepsilon=\varepsilon$, and $\frac{e_6}{e_2}=e_4$. Then, the first term cancels the variation of the bulk fermion bilinear term, if the F_{MN} in the fermion bilinear term is replaced with $\hat F_{MN}$. Most importantly, the second term has the right form to cancel the variation of the brane tension term. The condition for this to happen is that,

$$\Big(\gamma_5-\frac{T_i}{4g\xi_i}\Big)\varepsilon(y_i)=0. \tag{61}$$

In other words, decomposing the SUSY variation spinor as $\varepsilon=(\tilde\varepsilon,0)^T$ with $\tilde\varepsilon=(\tilde\varepsilon_L,\tilde\varepsilon_R)^T$, the following should be satisfied,

$$\Big(1-\frac{T_i}{4g\xi_i}\Big)\tilde\varepsilon_L(y_i) = 0, \tag{62}$$

$$\Big(1+\frac{T_i}{4g\xi_i}\Big)\tilde\varepsilon_R(y_i) = 0. \tag{63}$$

Thus, fixing the FI terms with the brane tensions as $\xi_i=\frac{T_i}{4g}$ or $-\frac{T_i}{4g}$, one needs to impose that either $\tilde\varepsilon_R$ or $\tilde\varepsilon_L$ vanish on the brane. Therefore, only $\mathcal{N}=1$ SUSY can be preserved on the brane. For other values of ξ_i, both $\tilde{\varepsilon}_L$ and $\tilde{\varepsilon}_R$ must vanish at the brane, so there would be no SUSY left. In the bulk action and the SUSY transformations, when F_{MN} is replaced by $\hat{F}_{MN}$, we also need to modify the field strength G_{MNP} by $\hat{G}_{MNP}$ as

$$\hat{G}_{\mu\nu\lambda} = G_{\mu\nu\lambda}, \tag{64}$$

$$\begin{aligned}\hat{G}_{\mu mn} &= 3\partial_{[\mu}B_{mn]} + \frac{3}{2}F_{[mn}A_{\mu]} - \xi_i A_\mu \epsilon_{mn}\frac{\delta^{(2)}(y-y_i)}{e_2} \\ &= \hat{G}_{mn\mu} = \hat{G}_{n\mu m}.\end{aligned} \tag{65}$$

On the other hand, keeping the form of terms A_M to be the same as in the case with no branes, the modified bulk action is supersymmetric up to four fermion terms.

From now on, we choose $\xi_i = \frac{T_i}{4g}$ for all branes present in the internal space, so that there is $\mathcal{N} = 1$ SUSY remaining in the brane action with a SUSY parameter $\tilde{\varepsilon}_L$ non vanishing on the branes. This choice is made to agree with the no-brane Salam-Sezgin vacuum where a constant $\tilde{\varepsilon}_L$ is a Killing spinor.

4.2 The Supersymmetric Brane-Bulk Coupling

As a consequence of introducing the localised FI terms, we have seen that the brane tension action is made compatible with the bulk SUSY transformations. The supersymmetric action of the brane-bulk system up to four fermion terms is

$$\begin{aligned}e_6^{-1}\mathcal{L}_{\rm SUSY} = {}& R - \frac{1}{4}(\partial_M\phi)^2 - \frac{1}{12}e^{\phi}\hat{G}_{MNP}\hat{G}^{MNP} - \frac{1}{4}e^{\frac{1}{2}\phi}\hat{F}_{MN}\hat{F}^{MN} - 8g^2 e^{-\frac{1}{2}\phi} \\ &+\bar{\psi}_M\Gamma^{MNP}\mathcal{D}_N\psi_P + \bar{\chi}\Gamma^M\mathcal{D}_M\chi + \bar{\lambda}\Gamma^M\mathcal{D}_M\lambda \\ &+\frac{1}{4}(\partial_M\phi)(\bar{\psi}_N\Gamma^M\Gamma^N\chi + \bar{\chi}\Gamma^N\Gamma^M\psi_N) \\ &+\frac{1}{24}e^{\frac{1}{2}\phi}\hat{G}_{MNP}(\bar{\psi}^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R\Gamma^{MNP}\Gamma^R\chi \\ &\quad -\bar{\chi}\Gamma^R\Gamma^{MNP}\psi_R - \bar{\chi}\Gamma^{MNP}\chi + \bar{\lambda}\Gamma^{MNP}\lambda) \\ &-\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\hat{F}_{MN}(\bar{\psi}_Q\Gamma^{MN}\Gamma^Q\lambda + \bar{\lambda}\Gamma^Q\Gamma^{MN}\psi_Q + \bar{\chi}\Gamma^{MN}\lambda - \bar{\lambda}\Gamma^{MN}\chi) \\ &+i\sqrt{2}ge^{-\frac{1}{4}\phi}(\bar{\psi}_M\Gamma^M\lambda + \bar{\lambda}\Gamma^M\psi_M - \bar{\chi}\lambda + \bar{\lambda}\chi) \\ &-\frac{e_4}{e_6}T_i\delta^{(2)}(y-y_i),\end{aligned} \tag{66}$$

where the modified gauge field strengths are

$$\hat{F}_{MN} = F_{MN} - \delta^m_M\delta^n_N\epsilon_{mn}\xi_i\frac{\delta^{(2)}(y-y_i)}{e_2}, \tag{67}$$

$$\hat{G}_{MNP} = G_{MNP} - 3\delta^\mu_{[M}\delta^m_N\delta^n_{P]}A_\mu\epsilon_{mn}\xi_i\frac{\delta^{(2)}(y-y_i)}{e_2}, \tag{68}$$

with

$$\xi_i = \frac{T_i}{4g}. \tag{69}$$

All the fermionic SUSY transformations are modified as

$$\delta\chi = -\frac{1}{4}(\partial_M\phi)\Gamma^M\varepsilon + \frac{1}{24}e^{\frac{1}{2}\phi}\hat{G}_{MNP}\Gamma^{MNP}\varepsilon, \tag{70}$$

$$\delta\psi_M = \mathcal{D}_M\varepsilon + \frac{1}{48}e^{\frac{1}{2}\phi}\hat{G}_{PQR}\Gamma^{PQR}\Gamma_M\varepsilon, \tag{71}$$

$$\delta\lambda = \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\hat{F}_{MN}\Gamma^{MN}\varepsilon - i\sqrt{2}g\,e^{-\frac{1}{4}\phi}\varepsilon, \tag{72}$$

but the bosonic SUSY transformations are the same as eqs. (31)-(31) and (32). The important ingredient of the above modifications is that we have a brane term linear in F_{MN}, the brane-localised FI term. In other words, there is a brane coupling to the magnetic flux, which is proportional to the brane tension. Moreover, we get a singular correction to the Chern-Simons term in the field strength for the KR field. We note that the modified field strengths satisfy the Bianchi identities, $\partial_{[Q}\hat{F}_{MN]} = 0$ and $\partial_{[Q}\hat{G}_{MNP]} = \frac{3}{4}\hat{F}_{[MN}\hat{F}_{QP]}$, even with the singular term.

One could be worried by the squared terms of the two-dimensional delta functions appearing in the kinetic term $\hat{F}_{MN}\hat{F}^{MN}$. However, SUSY requires these terms to be present and are a usual ingredient of orbifold supersymmetric theories. The delta squared terms, *i.e.,* $\delta^2(0)$, appear naturally in orbifolds, when bulk and brane fields are coupled supersymmetrically. One can obtain the same form $\hat{F}_{MN}\hat{F}^{MN}$ in a 6D off-shell supersymmetric $U(1)$ theory on T^2/Z_2, after the auxiliary field of the bulk vector multiplet is eliminated. It has been known that the $\delta^2(0)$ term provides counterterms, which are necessary to maintain supersymmetry in explicit calculations on orbifolds, like the scattering amplitude and the self-energy correction for a brane field. In our case, we have not introduced brane multiplets other than the tension. The case with brane multiplets will be studied elsewhere so the usual discussion on the $\delta^2(0)$ term on orbifolds is expected to hold.

There are some known anomaly-free models including the non-abelian gauge fields in 6D gauged supergravity. In these cases, an abelian flux can be also turned on in the direction of the non-abelian gauge fields. For instance, in the model with $E_7 \times E_6 \times U(1)_R$ with hyperino, the $U(1)$ contained in E_6 can also develop a nonzero flux, still maintaining the warped solution that was obtained for the Salam-Sezgin supergravity. As a result, E_6 is broken down to $SO(10)$ in the bulk and the adjoint fermions of E_6 can survive as two chiral **16**'s of $SO(10)$. Even in this more general case, the supersymmetric brane action obtained for the Salam-Sezgin supergravity remains the same.

Furthermore, we can always introduce arbitrary localised FI terms for any abelian factor of the bulk gauge group other than $U(1)_R$ in a supersymmetric way because there is no constraint from the variation of the gravitino kinetic term unlike eq. (60). We only have to modify the field strengths appearing in both the bulk action and the fermionic SUSY transformations like in eqs. (67), (68) and (70)-(72). Thus, it is straightforward to see that the localised FI terms generated in 6D global SUSY case are embedded into a supergravity theory.

5 Brane plus Bulk Supersymmetry in $D = 10$

The standard formulation of $D = 10$ IIA and IIB supergravity has the following field content

$$\begin{array}{lll} \text{IIA} & : & \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, \psi_\mu, \lambda\right\} \,, \\ \text{IIB} & : & \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\nu\rho\sigma}, \psi_\mu, \lambda\right\} \,. \end{array} \quad (73)$$

In the IIA case, the massive theory contains an additional mass parameter $G^{(0)} = m$. In the IIB case, an extra self-duality condition is imposed on the field strength of the four-form. It turns out that one can realize the N=2 supersymmetry on the R-R gauge fields of higher rank as well. These are usually incorporated via duality relations. To treat the R-R potentials democratically we propose a new formulation based upon a pseudo-action. This democratic formulation describes the dynamics of the bulk supergravity in the most elegant way. However, it turns out that this formulation is not well suited for our purposes. For the IIA case, we therefore give a different formulation where the constant mass parameter has been replaced by a field.

To explicitly introduce the democracy among the R-R potentials we propose a pseudo-action whose equations of motion are supplemented by duality constraints. Of course this enlarges the number of degrees of freedom. Since a p- and an $(8-p)$-form potential carry the same number of degrees of freedom, the introduction of the dual potentials doubles the R-R sector. Including the highest potential $C^{(9)}$ in IIA does not alter this, since it carries no degrees of freedom. This 9-form potential can be seen as the potential dual to the constant mass parameter $G^{(0)} = m$. The doubling of number of degrees of freedom will be taken care of by a constraint, relating the lower- and higher-rank potentials. This new formulation of supersymmetry is inspired by the bosonic construction, and, in the case of IIB supergravity, is related to the pseudo-action construction.

A pseudo-action can be used as a mnemonic to derive the equations of motion. It differs from a usual action in the sense that not all equations of motion follow from varying the fields in the pseudo-action. To obtain the complete set of equations of motion, an additional constraint has to be substituted by hand into the set of equations of motion that follow from the pseudo-action. The constraint itself does not follow from the pseudo-action. The construction we present here generalizes the pseudo-action construction in the sense that our construction treats the IIA and IIB case in a unified way, introducing all R-R potentials in the pseudo-action, and (ii) describes also the massive IIA case via a 9-form potential $C^{(9)}$ and a constant mass parameter $G^{(0)} = m$.

Our pseudo-action has the extended field content

$$\begin{array}{lll} \text{IIA} & : & \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, C^{(5)}_{\mu\cdots\rho}, C^{(7)}_{\mu\cdots\rho}, C^{(9)}_{\mu\cdots\rho}, \psi_\mu, \lambda\right\} \,, \\ \text{IIB} & : & \left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\cdots\rho}, C^{(6)}_{\mu\cdots\rho}, C^{(8)}_{\mu\cdots\rho}, \psi_\mu, \lambda\right\} \,. \end{array} \quad (74)$$

It is understood that in the IIA case the fermions contain both chiralities, while in the IIB case they satisfy $\Gamma_{11}\psi_\mu = \psi_\mu \,, \Gamma_{11}\lambda = -\lambda \,, (IIB)$. In that case they are doublets, and we suppress the corresponding index. The explicit form of the pseudo-action is given by

$$S_{Pseudo} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\Big[R(\omega(e)) - 4(\partial\phi)^2 + 12H\cdot H$$
$$-2\partial^\mu\phi\chi_\mu^{(1)} + H\cdot\chi^{(3)} + 2\bar{\psi}_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar{\lambda}\Gamma^\mu\nabla_\mu\lambda + 4\bar{\lambda}\Gamma^{\mu\nu}\nabla_\mu\psi_\nu\Big]$$
$$+\sum_{n=0,1/2}^{5,9/2} 14G^{(2n)}\cdot G^{(2n)} + 12G^{(2n)}\cdot\Psi^{(2n)}\Big\} + \text{ quartic fermionic terms}\,. \qquad (75)$$

It is understood that the summation in the above pseudo-action is over integers ($n = 0, 1, \ldots, 5$) in the IIA case and over half-integers ($n = 1/2, 3/2, \ldots, 9/2$) in the IIB case. In the summation range we will always first indicate the lowest value for the IIA case, before the one for the IIB case. Furthermore,

$$\frac{1}{2\kappa_{10}^2} = \frac{g^2}{2\kappa^2} = \frac{2\pi}{(2\pi\ell_s)^8}\,, \qquad (76)$$

where κ^2 is the physical gravitational coupling, g is the string coupling constant and $\ell_s = \sqrt{\alpha'}$ is the string length. For notational convenience we group all potentials and field strengths in the formal sums

$$\mathbf{G} = \sum_{n=0,1/2}^{5,9/2} G^{(2n)}\,, \qquad \mathbf{C} = \sum_{n=1,1/2}^{5,9/2} C^{(2n-1)}\,. \qquad (77)$$

The bosonic field strengths are given by

$$H = dB\,, \qquad \mathbf{G} = d\mathbf{C} - dB{\scriptstyle\wedge}\mathbf{C} + G^{(0)}\mathbf{e}^B\,, \qquad (78)$$

where it is understood that each equation involves only one term from the formal sums (77). The corresponding Bianchi identities then read

$$dH = 0\,, \qquad d\mathbf{G} - H{\scriptstyle\wedge}\mathbf{G} = 0\,. \qquad (79)$$

In this subsection $G^{(0)} = m$ indicates the constant mass parameter of IIA supergravity. In the IIB theory all equations should be read with vanishing $G^{(0)}$. The spin connection in the covariant derivative ∇_μ is given by its zehnbein part: ${\omega_\mu}^{ab} = {\omega_\mu}^{ab}(e)$. The bosonic fields couple to the fermions via the bilinears $\chi^{(1,3)}$ and $\Psi^{(2n)}$, which read

$$\chi_\mu^{(1)} = -2\bar{\psi}_\nu\Gamma^\nu\psi_\mu - 2\bar{\lambda}\Gamma^\nu\Gamma_\mu\psi_\nu\,, \qquad (80)$$
$$\chi_{\mu\nu\rho}^{(3)} = 12\bar{\psi}_\alpha\Gamma^{[\alpha}\Gamma_{\mu\nu\rho}\Gamma^{\beta]}\mathcal{P}\psi_\beta + \bar{\lambda}{\Gamma_{\mu\nu\rho}}^\beta\mathcal{P}\psi_\beta - 12\bar{\lambda}\mathcal{P}\Gamma_{\mu\nu\rho}\lambda\,, \qquad (81)$$
$$\Psi^{(2n)}_{\mu_1\cdots\mu_{2n}} = \tfrac{1}{2}e^{-\phi}\bar{\psi}_\alpha\Gamma^{[\alpha}\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^{\beta]}\mathcal{P}_n\psi_\beta + \tfrac{1}{2}e^{-\phi}\bar{\lambda}\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^\beta\mathcal{P}_n\psi_\beta + \qquad (82)$$
$$-\tfrac{1}{4}e^{-\phi}\bar{\lambda}\Gamma_{[\mu_1\cdots\mu_{2n-1}}\mathcal{P}_n\Gamma_{\mu_{2n}]}\lambda\,. \qquad (83)$$

We have used the following definitions:

$$
\begin{aligned}
\mathcal{P} &= \Gamma_{11} \quad \text{(IIA)} \quad \text{or} \quad -\sigma^3 \text{ (IIB)}\,, \\
\mathcal{P}_n &= (\Gamma_{11})^n \text{ (IIA)} \quad \text{or} \quad \sigma^1 \text{ (n + 1/2 even)},\ \mathrm{i}\sigma^2 \text{ (n + 1/2 odd) (IIB)}\,.
\end{aligned}
\tag{84}
$$

Note that the fermions satisfy

$$
\Psi^{(2n)} = (-)^{\mathrm{Int}[n]+1} \star \Psi^{(10-2n)}\,. \tag{85}
$$

Due to the appearance of all R-R potentials, the number of degrees of freedom in the R-R sector has been doubled. Each R-R potential leads to a corresponding equation of motion:

$$
d \star (G^{(2n)} + \Psi^{(2n)}) + H_\wedge \star (G^{(2n+2)} + \Psi^{(2n+2)}) = 0\,. \tag{86}
$$

Now, one must relate the different potentials to get the correct number of degrees of freedom. We therefore by hand impose the following duality relations

$$
G^{(2n)} + \Psi^{(2n)} = (-)^{\mathrm{Int}[n]} \star G^{(10-2n)}\,, \tag{87}
$$

in the equations of motion that follow from the pseudo-action (75). It is in this sense that the action (75) cannot be considered as a true action. Instead, it should be considered as a mnemonic to obtain the full equations of motion of the theory. As usual, the Bianchi identities and equations of motions of the dual potentials correspond to each other when employing the duality relation. For the above reason the democratic formulation can be viewed as self-dual, since (87) places constraints relating the field content (74).

The pseudo-action (75) is invariant under supersymmetry provided we impose the duality relations (87) after varying the action. The supersymmetry rules read

$$
\delta_\epsilon e_\mu{}^a = \bar\epsilon \Gamma^a \psi_\mu\,, \tag{88}
$$

$$
\delta_\epsilon \psi_\mu = \Big(\partial_\mu + 14\ \not\omega_\mu + 18\mathcal{P}\ \not H_\mu\Big)\epsilon + 116 e^\phi \sum_{n=0,1/2}^{5,9/2} \frac{1}{(2n)!}\ \not G^{(2n)} \Gamma_\mu \mathcal{P}_n \epsilon\,, \tag{89}
$$

$$
\delta_\epsilon B_{\mu\nu} = -2\,\bar\epsilon \Gamma_{[\mu} \mathcal{P} \psi_{\nu]}\,, \tag{90}
$$

$$
\delta_\epsilon C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} = -e^{-\phi}\,\bar\epsilon\,\Gamma_{[\mu_1\cdots\mu_{2n-2}}\,\mathcal{P}_n \Big((2n-1)\psi_{\mu_{2n-1}]} - 12\Gamma_{\mu_{2n-1}]}\lambda\Big) + \tag{91}
$$

$$
+(n-1)(2n-1)\,C^{(2n-3)}_{[\mu_1\cdots\mu_{2n-3}}\,\delta_\epsilon B_{\mu_{2n-2}\mu_{2n-1}]}\,, \tag{92}
$$

$$
\delta_\epsilon \lambda = \Big(\not\partial\phi + 112\ \not H \mathcal{P}\Big)\epsilon + 18 e^\phi \sum_{n=0,1/2}^{5,9/2} (-)^{2n} \frac{5-2n}{(2n)!}\ \not G^{(2n)} \mathcal{P}_n \epsilon\,, \tag{93}
$$

$$
\delta_\epsilon \phi = 12\,\bar\epsilon\lambda\,, \tag{94}
$$

where ϵ is a spinor similar to ψ_μ, in IIB: $\Gamma_{11}\epsilon = \epsilon$. Note that for n half-integer (the IIB case) these supersymmetry rules exactly reproduce the rules given in another publication.

Secondly, the pseudo-action (75) is also invariant under the usual bosonic NS-NS and R-R gauge symmetries with parameters Λ and $\Lambda^{(2n)}$ respectively:

$$\delta_\Lambda B = d\Lambda\,, \qquad \delta_\Lambda \mathbf{C} = (d\mathbf{L} - G^{(0)}\Lambda)_\wedge \mathbf{e}^B\,, \qquad \textit{with}\;\; \mathbf{L} = \sum_{n=0,1/2}^{4,7/2} \Lambda^{(2n)}\,. \tag{95}$$

Finally, there is a number of $\mathbb{Z}_2$-symmetries. However, in the IIA case these $\mathbb{Z}_2$-symmetries are only valid for $G^{(0)} = m = 0$. Below we show how these symmetries of the action act on supergravity fields. For both massless IIA and IIB there is a fermion number symmetry $(-)^{F_L}$ given by

$$\{\phi, g_{\mu\nu}, B_{\mu\nu}\} \to \{\phi, g_{\mu\nu}, B_{\mu\nu}\}\,, \tag{96}$$
$$\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\} \to -\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\}\,, \tag{97}$$
$$\{\psi_\mu, \lambda, \epsilon\} \to +\mathcal{P}\{\psi_\mu, -\lambda, \epsilon\}\,, \quad (IIA), \tag{98}$$
$$\{\psi_\mu, \lambda, \epsilon\} \to +\mathcal{P}\{\psi_\mu, \lambda, \epsilon\}\,, \qquad (IIB). \tag{99}$$

In the IIB case there is an additional worldsheet parity symmetry Ω given by

$$\{\phi, g_{\mu\nu}, B_{\mu\nu}\} \to \{\phi, g_{\mu\nu}, -B_{\mu\nu}\}\,, \tag{100}$$
$$\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\} \to (-)^{n+1/2}\{C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\}\,, \tag{101}$$
$$\{\psi_\mu, \lambda, \epsilon\} \to \sigma^1\{\psi_\mu, \lambda, \epsilon\}\,, \tag{102}$$

In the massless IIA case there is a similar $I_9\Omega$-symmetry involving an additional parity transformation in the 9-direction. Writing $\mu = (\underline{\mu}, \dot{9})$, the rules are given by

$$\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\} \to \{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\}\,, \tag{103}$$
$$\{C^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\} \to (-)^{n+1}\{C^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\}\,, \tag{104}$$
$$\{\psi_{\underline{\mu}}, \lambda, \epsilon\} \to +\Gamma^9\{\psi_{\underline{\mu}}, -\lambda, \epsilon\}\,. \tag{105}$$

The parity of the fields with one or more indices in the $\dot{9}$-direction is given by the rule that every index in the $\dot{9}$-direction gives an extra minus sign compared to the above rules.

In both IIA and IIB there is also the obvious symmetry of interchanging all fermions by minus the fermions, leaving the bosons invariant.

The $\mathbb{Z}_2$-symmetries are used for the construction of superstring theories with sixteen supercharges. $(-)^{F_L}$ gives a projection to the $E_8 \times E_8$ heterotic superstring (IIA) or the SO(32) heterotic superstring theory (IIB). Ω is used to reduce the IIB theory to the SO(32) Type I superstring, while the $I_9\Omega$-symmetry reduces the IIA theory to the Type I′ $SO(16)\times SO(16)$ superstring theory.

One might wonder at the advantages of the generalized pseudo-action (75) above the standard supergravity formulation. At the cost of an extra duality relation we were able to realize the R-R democracy in the action. Note that only kinetic terms are present; by allowing for a larger field content the Chern–Simons term is eliminated. Under T-duality all kinetic terms are easily seen to transform into each other. The same goes for the duality constraints. This formulation is elegant and comprises all potentials. However, it is impossible to construct a proper action in this formulation due to the doubling of the degrees of freedom. Therefore, to add brane actions to the bulk system, the democratic formulation is not suitable. This is due to two reasons. First, the $I_9\Omega$ symmetry is only valid for $G^{(0)} = 0$, but we will need this symmetry in our construction of the bulk & 8-brane system. Secondly, to describe a charged domain wall, we would like to have opposite values for $G^{(0)}$ at the two sides of the domain wall, i.e. we want to allow for a mass parameter that is only piecewise constant. The R-R democracy has to be broken to accommodate for an action and this will be discussed in the next subsection.

We will present here the new dual formulation with action, available for the IIA case only. A proper action will be constructed in this formulation. It is this formulation that we will apply in our construction of the bulk & brane system. We will call this the dual formulation.

The independent fields in this formulation are

$$\left\{e_\mu^a, B_{\mu\nu}, \phi, G^{(0)}, G^{(2)}_{\mu\nu}, G^{(4)}_{\mu_1\cdots\mu_4}, A^{(5)}_{\mu_1\cdots\mu_5}, A^{(7)}_{\mu_1\cdots\mu_7}, A^{(9)}_{\mu_1\cdots\mu_9}, \psi_\mu, \lambda\right\}. \tag{106}$$

The bulk action reads

$$\begin{aligned} S_{bulk} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{&e^{-2\phi}[R(\omega(e)) - 4(\partial\phi)^2 + 12H\cdot H - 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} \\ &+2\bar\psi_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar\lambda\Gamma^\mu\nabla_\mu\lambda + 4\bar\lambda\Gamma^{\mu\nu}\nabla_\mu\psi_\nu] + \sum_{n=0,1,2} 12G^{(2n)}\cdot G^{(2n)} + G^{(2n)}\cdot\Psi^{(2n)} \\ &-\star\,[12\,G^{(4)}G^{(4)}B - 12\,G^{(2)}G^{(4)}B^2 + 16\,G^{(2)2}B^3 + 16\,G^{(0)}G^{(4)}B^3 - 18\,G^{(0)}G^{(2)}B^4 \\ &+140\,G^{(0)2}B^5 + \mathbf{e}^{-B}\mathbf{G}d(A^{(5)} - A^{(7)} + A^{(9)})]\Big\} + \text{ quartic fermionic terms}, \end{aligned} \tag{107}$$

where all $\wedge$'s have been omitted in the last two lines. In the last term a projection on the 10-form is understood. Here $\mathbf{G}$ is defined as in (77) but where $G^{(0)}$, $G^{(2)}$ and $G^{(4)}$ are now independent fields (which we will call black boxes) and are no longer given by (78). Note that their Bianchi identities are imposed by the Lagrange multipliers $A^{(9)}$, $A^{(7)}$ and $A^{(5)}$. The NS-NS three-form field strength is given by (78). Note that the standard action for IIA supergravity can be obtained by integrating out the dual potentials in (107).

The symmetries of the action are similar to those of the democratic formulation with some small changes. In the supersymmetry transformations of gravitino and gaugino, the sums now extend only over $n = 0, 1, 2$:

$$\delta_\epsilon e_\mu{}^a = \bar\epsilon\Gamma^a\psi_\mu\,, \tag{108}$$

$$\delta_\epsilon\psi_\mu = \Big(\partial_\mu + 14\not\omega_\mu + 18\Gamma_{11}\not H_\mu\Big)\epsilon + 18e^\phi\sum_{n=0,1,2}\frac{1}{(2n)!}\not G^{(2n)}\Gamma_\mu(\Gamma_{11})^n\epsilon\,, \tag{109}$$

$$\delta_\epsilon B_{\mu\nu} = -2\,\bar\epsilon\Gamma_{[\mu}\Gamma_{11}\psi_{\nu]}\,, \tag{110}$$

$$\delta_\epsilon\lambda = \Big(\not\partial\phi - 112\Gamma_{11}\not H\Big)\epsilon + 14e^\phi\sum_{n=0,1,2}\frac{5-2n}{(2n)!}\not G^{(2n)}(\Gamma_{11})^n\epsilon\,,$$

$$
\begin{aligned}
\delta_\epsilon \phi &= 12\, \bar\epsilon \lambda\,, \\
\delta_\epsilon \mathbf{A} &= \mathbf{e}^{-B}{}_\wedge \mathbf{E}\,, \\
\delta_\epsilon \mathbf{G} &= d\mathbf{E} + \mathbf{G}{}_\wedge \delta_\epsilon B - H{}_\wedge \mathbf{E}\,, \qquad (111)
\end{aligned}
$$

$$
\textit{with}\quad E^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} \equiv -e^{-\phi}\, \bar\epsilon\, \Gamma_{[\mu_1\cdots\mu_{2n-2}}\, (\Gamma_{11})^n \Big((2n-1)\psi_{\mu_{2n-1}]} - 12 \Gamma_{\mu_{2n-1}]} \lambda \Big)\,. \qquad (112)
$$

The transformation of the black boxes $\mathbf{G}$ follow from the requirement that $\mathbf{e}^{-B}\mathbf{G}$ transforms in a total derivative. Here the formal sums

$$
\mathbf{A} = \sum_{n=1}^{5} A^{(2n-1)}\,, \qquad \mathbf{E} = \sum_{n=1}^{5} E^{(2n-1)}\,, \qquad \mathbf{G} = \sum_{n=0}^{5} G^{(2n)}\,, \qquad (113)
$$

have been used. Note that the first formal sum in (113) contains fields, $A^{(1)}$ and $A^{(3)}$, that do not occur in the action. The same applies to $\mathbf{G}$, which contains the extra fields $G^{(6)}, G^{(8)}$ and $G^{(10)}$. Although these fields do not occur in the action, one can nevertheless show that the supersymmetry algebra is realized on them. To do so one must use the supersymmetry rules of (112) and the equations of motion that follow from the action (107).

The gauge symmetries with parameters Λ and $\Lambda^{(2n)}$ are

$$
\begin{aligned}
\delta_\Lambda B = \, d\Lambda\,, \qquad \delta_\Lambda \mathbf{A} = \, d\mathbf{L} - G^{(0)}\Lambda - d\Lambda{}_\wedge \mathbf{A}\,, \\
\delta_\Lambda \mathbf{G} = \, d\Lambda{}_\wedge (\mathbf{G} - \mathbf{e}^{B}{}_\wedge (d\mathbf{A} + G^{(0)})) + \mathbf{e}^{B}{}_\wedge \Lambda{}_\wedge dG^{(0)}\,. \qquad (114)
\end{aligned}
$$

Note that, with respect to the R-R gauge symmetry, the $\mathbf{A}$ potentials transform as a total derivative while the black boxes are invariant.

Finally, there are $\mathbb{Z}_2$-symmetries, $(-)^{F_L}$ and $I_9\Omega$, which leave the action invariant. In contrast to the democratic formulation these two $\mathbb{Z}_2$-symmetries are valid symmetries even for $G^{(0)} \neq 0$. The $(-)^{F_L}$-symmetry is given by

$$
\begin{aligned}
\{\phi, g_{\mu\nu}, B_{\mu\nu}\} &\to \{\phi, g_{\mu\nu}, B_{\mu\nu}\}\,, \qquad (115) \\
\{G^{(2n)}_{\mu_1\cdots\mu_{2n}}, A^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\} &\to -\{G^{(2n)}_{\mu_1\cdots\mu_{2n}}, A^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\}\,, \qquad (116) \\
\{\psi_\mu, \lambda, \epsilon\} &\to +\Gamma_{11}\{\psi_\mu, -\lambda, \epsilon\}\,, \qquad (117)
\end{aligned}
$$

while the second $I_9\Omega$-symmetry reads

$$
\begin{aligned}
\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\} &\to \{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\}\,, \qquad (118) \\
\{G^{(2n)}_{\underline{\mu_1}\cdots\underline{\mu_{2n}}}, A^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\} &\to (-)^{n+1}\{G^{(2n)}_{\underline{\mu_1}\cdots\underline{\mu_{2n}}}, A^{(2n-1)}_{\underline{\mu_1}\cdots\underline{\mu_{2n-1}}}\}\,, \qquad (119) \\
\{\psi_{\underline{\mu}}, \lambda, \epsilon\} &\to +\Gamma^9\{\psi_{\underline{\mu}}, -\lambda, \epsilon\}\,. \qquad (120)
\end{aligned}
$$

Having established supersymmetry in the bulk, we now turn to supersymmetry on the brane. As mentioned in the introduction, our main interest is in one-dimensional orbifold constructions with 8-branes at the orbifold points. Using the techniques of the three-brane on the orbifold in five dimensions, we want to construct an orientifold using a $\mathbb{Z}_2$-symmetry of the bulk action. On the fixed points we insert brane actions, which will turn out to be invariant under the reduced ($N = 1$) supersymmetry. For the moment we will not restrict to domain walls (in this case eight-branes) since our brane analysis is similar for orientifolds of lower dimension. In the previous section we have seen that our bulk action possesses a number of symmetries, among which a parity operation. To construct an orientifold, the relevant $\mathbb{Z}_2$-symmetry must contain parity operations in the transverse directions. Furthermore, in order to construct a charged domain wall, we want for a p-brane the $(p+1)$-form R-R potential to be even. For the 8-brane the $I_9\Omega$ symmetry satisfies the desired properties. For the other p-branes, it would seem natural to use the $\mathbb{Z}_2$-symmetry

$$I_{9,8,\ldots,p+1}\Omega \equiv (I_9\Omega)(I_8\Omega)\cdots(I_{p+1}\Omega)\,, \tag{121}$$

where $I_q\Omega$ is the transformation (105) with 9 replaced by q, and I_q and Ω commute. However, for some p-branes ($p = 2, 3, 6, 7$) the corresponding $C^{(p+1)}$ R-R-potential is odd under this $\mathbb{Z}_2$-symmetry. To obtain the correct parity one must include an extra $(-)^{F_L}$ transformation in these cases, which also follows from T-duality.

Thus the correct $\mathbb{Z}_2$-symmetry for a general IIA Op-plane is given by

$$((-)^{F_L})^{p/2} I_{9,8,\ldots,p+1}\Omega\,. \tag{122}$$

The effect of this $\mathbb{Z}_2$-symmetry on the bulk fields reads (the underlined indices refer to the worldvolume directions, $\mu = (\underline{\mu}, p+1, \ldots, 9)$)

$$\{x^{p+1}, \ldots, x^9\} \to -\{x^{p+1}, \ldots, x^9\}\,, \tag{123}$$

$$\{\phi, g_{\underline{\mu\nu}}, B_{\underline{\mu\nu}}\} \to \{\phi, g_{\underline{\mu\nu}}, -B_{\underline{\mu\nu}}\}\,, \tag{124}$$

$$\{A^{(5)}_{\underline{\mu_1}\cdots\underline{\mu_5}}, A^{(9)}_{\underline{\mu_1}\cdots\underline{\mu_9}}, G^{(2)}_{\underline{\mu\nu}}\} \to (-)^{p2}\{A^{(5)}_{\underline{\mu_1}\cdots\underline{\mu_5}}, A^{(9)}_{\underline{\mu_1}\cdots\underline{\mu_9}}, G^{(2)}_{\underline{\mu\nu}}\}\,, \tag{125}$$

$$\{A^{(7)}_{\underline{\mu_1}\cdots\underline{\mu_7}}, G^{(0)}, G^{(4)}_{\underline{\mu_1}\cdots\underline{\mu_4}}\} \to (-)^{p2+1}\{A^{(7)}_{\underline{\mu_1}\cdots\underline{\mu_7}}, G^{(0)}, G^{(4)}_{\underline{\mu_1}\cdots\underline{\mu_4}}\}\,, \tag{126}$$

$$\{\psi_{\underline{\mu}}, \epsilon\} \to -\alpha\Gamma^{p+1\cdots 9}(-\Gamma_{11})^{p2}\{\psi_{\underline{\mu}}, \epsilon\}\,, \tag{127}$$

$$\{\lambda\} \to +\alpha\Gamma^{p+1\cdots 9}(+\Gamma_{11})^{p2}\{\lambda\}\,, \tag{128}$$

and for fields with other indices there is an extra minus sign for each replacement of a worldvolume index $\underline{\mu}$ by an index in a transverse direction. We have left open the possibility of combining the symmetry with the sign change of all fermions. This possibility introduces a number $\alpha = \pm 1$ in the above rules. This symmetry will be used for the orientifold construction.

For this purpose we choose spacetime to be $\mathcal{M}^{p+1} \times T^{9-p}$ with radii $R^{\overline{\mu}}$ of the torus that may depend on the world-volume coordinates. All fields satisfy

$$\Phi(x^{\overline{\mu}}) = \Phi(x^{\overline{\mu}} + 2\pi R^{\overline{\mu}})\,, \tag{129}$$

with $\overline{\mu} = (p+1, \ldots, 9)$. The parity symmetry (122) relates the fields in the bulk at $x^{\overline{\mu}}$ and $-x^{\overline{\mu}}$. At the fixed point of the orientifolds, however, this relation is local and projects out half the fields. This means that we are left with only $N = 1$ supersymmetry on the fixed points, where the branes will be inserted. Consider for example a nine-dimensional orientifold. The projection truncates our bulk $N = 2$ supersymmetry to $N = 1$ on the brane, only half of the 32 components of ϵ are even under (128). The original field content, a $D = 10$, $(128 + 128)$, $N = 2$ supergravity multiplet, gets truncated on the brane to a reducible $D = 9$, $(64 + 64)$, $N = 1$ theory consisting of a supergravity plus a vector multiplet. One may further restrict to a constant torus. This particular choice of spacetime then projects out a $N = 1$ $(8 + 8)$ vector multiplet (containing $e_{\dot{9}}{}^{9}$), leaving us with the irreducible $D = 9$, (56+56), $N = 1$ supergravity multiplet. Similar truncations are possible in lower dimensional orientifolds, on which the $(64 + 64)$ $N = 1$ theory also consists of a number of multiplets.

We propose the p-brane action $(p = 0, 2, 4, 6, 8)$ to be proportional to

$$\mathcal{L}_p = -e^{-\phi}\sqrt{-g_{(p+1)}} - \alpha 1(p+1)!\varepsilon^{(p+1)}C^{(p+1)}\,, \quad \textit{with}\ \varepsilon^{(p+1)}C^{(p+1)} \equiv \varepsilon^{(p+1)}_{\underline{\mu_0}\cdots\underline{\mu_p}}\, C^{(p+1)\underline{\mu_0}\cdots\underline{\mu_p}}\,, \tag{130}$$

with $\varepsilon^{(p+1)\,\underline{\mu_0}\cdots\underline{\mu_p}} = \varepsilon^{(10)\,\underline{\mu_0}\cdots\underline{\mu_p}\dot{p+1}\cdots\dot{9}}$, which follows from $e_{\underline{\mu}}{}^{\overline{a}} = 0$ (being odd). Here the underlined indices are $(p+1)$-dimensional and refer to the world-volume. The parameter α is the same that appears in (128) and takes the values $\alpha = +1$ for branes, which are defined to have tension and charge with the same sign in our conventions, and $\alpha = -1$ for anti-branes, which are defined to have tension and charge of opposite signs. Note that due to the vanishing of B on the brane the potentials $C^{(p+1)}$ and $A^{(p+1)}$ are equal. The p-brane action can easily be shown to be invariant under the appropriate $N = 1$ supersymmetry:

$$\delta_\epsilon \mathcal{L}_p = -e^{-\phi}\sqrt{-g_{(p+1)}}\,\bar{\epsilon}(1 - \alpha\Gamma^{p+1\cdots 9}(\Gamma_{11})^{p2})\,\Gamma^{\underline{\mu}}\,(\psi_{\underline{\mu}} - 118\Gamma_{\underline{\mu}}\lambda)\,. \tag{131}$$

The above variation vanishes due to the projection under (128) that selects branes or anti-branes depending on the sign of α (+1 or −1 respectively). In the following discussions we will assume $\alpha = 1$ but the other case just amounts to replacing branes by anti-branes.

By truncating our theory we are able to construct a brane action that only consists of bosons and yet is separately supersymmetric. Having these at our disposal, we can introduce source terms for the various potentials. In general there are 2^{9-p} fixed points. The compactness of the transverse space implies that the total charge must vanish. Thus the total action will read

$$\mathcal{L} = \mathcal{L}_{bulk} + k_p \mathcal{L}_p \Delta_p\,, \tag{132}$$

$$\Delta_p \equiv (\delta(x^{p+1}) - \delta(x^{p+1} - \pi R^{p+1}))\cdots(\delta(x^9) - \delta(x^9 - \pi R^9)) \tag{133}$$

where the branes at all fixed points have a tension and a charge proportional to $\pm k_p$, a parameter of dimension $1/[length]^{p+1}$. Since anti-branes do not satisfy the supersymmetry condition, we need both positive and negative tension branes to accomplish vanishing total charge. The main new results of this paper of general nature are the new formulations of Type II $D = 10$ supergravity. For both Type IIA and IIB theories, we constructed democratic bulk theories with a unified treatment of all R-R potentials. Due to the doubling of R-R degrees of freedom one had to impose extra duality constraints and thus a proper action was not possible. A so-called pseudo-action, containing kinetic terms for all R-R potentials but without Chern-Simons terms. In general, an elegant solution is difficult to find, but in the eight-brane case the situation simplifies.

6 Conclusion

In the context of $D = 5$, $N = 2$, Yang-Mills Supergavity compactified on S^1/Z_2 we consider the supersymmetric coupling of matter fields propagating on the brane. Working in the on-shell scheme we have derived the terms of the brane action which are relevant for studying the mechanisms of supersymmetry and gauge symmetry breaking. The omitted radion multiplet couplings, as well as other couplings to the brane fields, can be derived, if desired, using the Nöther procedure which we followed in this paper. In order to check if this is indeed the case higher order interactions, in the gravitational constant $k_{(5)}$, of the brane fields with the five-dimensional gravity multiplet have to be derived in the on-shell scheme we have adopted. This rather complicated task, along with the derivation of additional terms coupling the brane fields to the radion multiplet, and other even combination of odd fields, which complete the Lagrangian. The complete brane action including these terms and the mechanisms of supersymmetry and the gauge symmetry breaking in particular unified models, in which both Gravity and Gauge forces propagate in the bulk.

We have constructed a consistent SUSY action for brane matter multiplets in a 6D chiral gauged supergravity. Introducing brane chiral multiplets charged under the $U(1)_R$, we derived the supersymmetric $U(1)_R$ coupling to the brane by modifying both the gauge field strength and the field strength for the KR field together with the necessary modifications of the fermionic SUSY transformations. We also notify that the modified field strength for the KR field is consistent with SUSY and $U(1)_R$ symmetry only at the expense of modifying the SUSY and gauge transformations of the KR field with the singular terms, respectively. In the present paper, we discussed the spectrum of the gravitino of the six-dimensional gauged supergravity model with gauge group $E_7 \times E_6 \times U(1)_R$, where a gauge flux is turned on in the $U(1) \subset E_6$ and the $U(1)_R$ directions. We studied in detail the spectrum in the general warped background where codimension-two branes were supporting the necessary conical singularities. The above property for the massless gravitino and its mass suppression with extra operators, should also hold for the other fermionic states of the spectrum which we did not consider in the present paper. In particular the gauginos which correspond to the directions of isometry of the internal space should have the same feature. This procedure offers an alternative way to obtain light fermions in models with extra dimensions.

We have constructed new formulations of Type II $D = 10$ supergravity. For both Type IIA and IIB theories, we constructed democratic bulk theories with a unified treatment of all R-R potentials. Due to the doubling of R-R degrees of freedom one had to impose extra duality constraints and thus a proper action was not possible. A so-called pseudo-action, containing kinetic terms for all R-R potentials but without Chern-Simons terms, was discussed. Furthermore, we have broken the self-duality explicitly in the IIA case, allowing for a proper action. Instead of all R-R potentials only half of the $C^{(p)}$'s occur in these theories. Both the standard ($p = 1, 3$) as well as the dual ($p = 5, 7, 9$) formulations were discussed. Using these actions all bulk & brane systems can be described. Apart from being a tool to understand the supersymmetric domain walls we were interested, it can be expected that the new effective theories of $D = 10$ supersymmetry will have more general applications in the future.

References

[1] G. Diamandis, B. Georgalas, P. Kouroumalou, A. Lahanas, "On the brane coupling of Unified orbifolds with gauge interactions in the bulk", arXiv:hep-th/0402228.

[2] H. Lee, "Supersymmetric codimension-two branes and $U(1)_R$ mediation in 6D gauged supergravity", arXiv:hep-th/0803.2683.

[3] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest, A. Van Proeyen, "Brane plus Bulk Supersymmetry in Ten Dimensions", arXiv:hep-th/0105061.

[4] J. Bagger, D. Belyaev, "Boundary Conditions in Brane-World Supergravity", arXiv:hep-th/0312072.

[5] S. Parameswaran, S. Randjbar-Daemi, A. Salvio, "Gauge Fields, Fermions and Mass Gaps in 6D Brane Worlds", arXiv:hep-th/0608074.

[6] A. Tolley, C. Burgess, C. de Rham, D. Hoover, "Scaling Solutions to 6D Gauged Chiral Supergravity", arXiv:hep-th/0608083.

[7] H. Lee, A. Papazoglou, "Gravitino in six-dimensional warped supergravity", arXiv:hep-th/0705.1157.

[8] H. Lee, A. Papazoglou, "Supersymmetric codimension-two branes in six-dimensional gauged supergravity", arXiv:hep-th/0710.4319.

[9] H. Lee, "Flux compactifications and supersymmetry breaking in 6D gauged supergravity", arXiv:hep-th/0812.3373.

[10] A. Salvio, "Brane Gravitational Interactions from 6D Supergravity", arXiv:hep-th/0909.0023.

[11] D. Belyaev, T. Pugh, "The supermultiplet of boundary conditions in supergravity", arXiv:hep-th/1008.1574.

[12] S. Parameswaran, J. Schmidt, "Coupling Brane Fields to Bulk Supergravity", arXiv:hep-th/1008.3832.

[13] J. Bagger, F. Feruglio, F. Zwirner, "Brane-induced supersymmetry breaking", arXiv:hep-th/0108010.

[14] T. Fujita, T. Kugo, K. Ohashi, "Off-Shell Formulation of Supergravity on Orbifold", arXiv:hep-th/0106051.

[15] D. Belyaev, "Bulk-brane supergravity", arXiv:hep-th/0710.4540.

[16] J. Bagger, D. Belyaev, "Brane-Localized Goldstone Fermions in Bulk Supergravity", arXiv:hep-th/0406126.