A simple method to obtain the gravitational wave equations

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Abstract

According to general relativity, the cause of gravitational waves is that time and space can be bent. Because the curvature of space-time is closely related to the change of gravitational field energy. Therefore, this article attempts to explore the causes of gravitational waves from the change and propagation process of gravitational field energy. Such an analysis method can make the physical image of gravitational waves clearer, and it is believed that it can also solve more complicated gravitational problems in addition to the gravitational problem of symmetry. The analysis results of this article also show that apart from transverse waves, gravitational waves may also have longitudinal waves. This article gives a method to detect the presence or absence of longitudinal waves of gravitational waves.

1 Elastic space-time

We assume here that space-time is an elastic substance. Then we can use a lot of knowledge of elasticity to deal with the problem of space-time changes.

Now we graphically represent the degree of curvature of space-time, as shown in Figure 1

Figure 1. Flat and curved spacetime

It can be seen from the figure that if it is a flat space-time, it can be expressed as a cylindrical shape. This reflects that space-time have not been bent. And if it is a curved space-time, it will be a twisted shape. Of course, for the sake of simplicity, we still consider the axisymmetric shape here. That is, it is symmetrical along the *z*-axis of cylindrical coordinates.

According to the general theory of relativity, the reason why space-time is bent is because there is energy in it.

Now we assume that there is an energy E in a spacetime, and the energy corresponding to the rest mass is 0. But since there is energy, we can still obtain the dynamic mass m corresponding to the energy through the mass-energy relationship of the theory of relativity. This dynamic mass can be used as an intermediate variable for us to calculate the space-time bending effect. Then

$$m = \frac{E}{c^2}$$

Now we consider the force analysis of Figure 2.

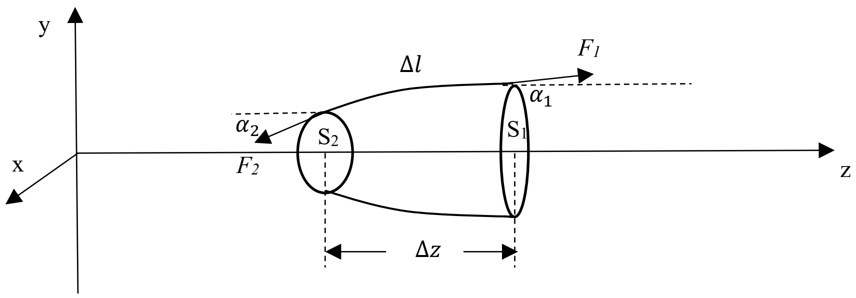

Figure 2. Transverse wave

From the force analysis in Figure 2, due to the action of two forces F_1 and F_2, time and space have been bent. At the same time, considering that this curvature of space-time is very weak, there is actually

$$F_1 \approx F_2 \approx F$$

Two cross-sectional areas

$$S_1 \approx S_2 \approx S$$

Moreover, the two included angles α_1 and α_2 are both very small. So

$$\Delta l \approx \Delta z$$

In addition, considering the symmetry, the force on the lower half of the curved space-time in the figure is also symmetrical. Therefore, we only need to analyze the forces acting on half of the space-time, and then we can extend the conclusion to the other half of space-time. As shown in Figure 3, we consider the force situation in the upper half of space-time

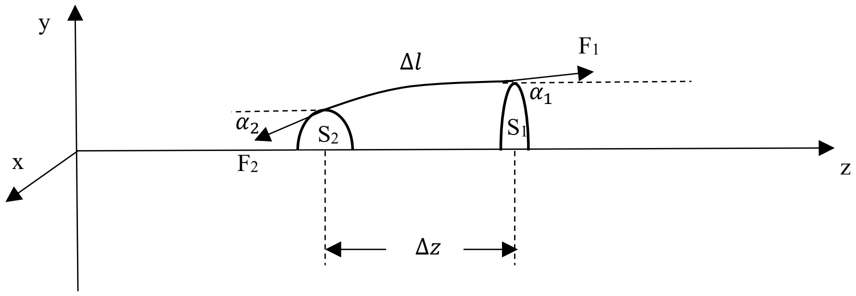

Figure 3. Transverse wave of half spacetime

We first analyze the force along the *z*-axis

$$\Delta F_z = F_1 cos\alpha_1 - F_2 cos\alpha_2 \approx 0$$

The force along the *y*-axis is

$$\Delta F_y = F_1 sin\alpha_1 - F_2 sin\alpha_2 \approx F\left(\frac{dy}{dz}\bigg|_{z+\Delta z} - \frac{dy}{dz}\bigg|_{z}\right)$$

Now consider that in Figure 2, the energy density of the space-time gravitational field is Ɛ, which is equivalent to the mass density

$$\rho = \frac{\mathcal{E}}{c^2}$$

Then we can get from figure 3.

$$\Delta F_y = F\left(\frac{dy}{dz}\bigg|_{z+\Delta z} - \frac{dy}{dz}\bigg|_{z}\right) = \Delta m \frac{d^2y}{dt^2} = \rho \frac{S\Delta z}{2}\frac{d^2y}{dt^2}$$

If $\Delta z \to 0$, then

$$F\frac{d^2y}{dz^2} = \rho S \frac{d^2y}{dt^2} = \frac{\mathcal{E}S}{2}\frac{d^2y}{dt^2 c^2}$$

It can be seen that this is a wave equation.

In addition, if the space-time bending process in Figure 3, the work done by the force *F* is

$$\Delta W = F\Delta l \approx F\Delta z$$

And the energy of the gravitational field in this area is

$$\Delta E = \frac{\mathcal{E}S\Delta z}{2}$$

It can be seen, that if

$$\Delta W = \Delta E$$

Then

$$F = \frac{\varepsilon S}{2}$$

So the above wave equation becomes

$$\frac{d^2 y}{dz^2} = \frac{d^2 y}{dt^2 c^2}$$

Obviously, this is a wave equation that travels at the speed of light. This also means that after the gravitational field causes space-time to bend, it can form a wave form, that is, a gravitational wave, and propagate out at the speed of light.

The wave equation in the x-axis direction can also be obtained

$$\frac{d^2 x}{dz^2} = \frac{d^2 x}{dt^2 c^2}$$

If space-time is an elastic substance, it can be seen from the space-time bending mechanism of Figure 2 or Figure 3 that the vibration waveform in the x-axis direction is exactly 180° out of phase with the y-axis direction.

It can be seen from the above analysis that although conclusions consistent with general relativity can be obtained, the derivation of this article is more concise and the physical picture is clearer. Therefore, if the conclusion of the derivation of this article is valid, the method can be used to solve the more complicated problems of gravitational wave propagation.

2 Gravitational wave longitudinal wave

The wave equation obtained in the previous section is a transverse wave, which is consistent with the result of general relativity. However, it can be seen from the analysis of this article that, in fact, gravitational waves can also be longitudinal waves.

Since the vibration direction of time and space is in the z-axis direction, the displacement generated on the z-axis is represented by $u(z)$ here. In Figure 4, when a force acts on a space-time section, it will cause a slight displacement $du(z)$ in the section.

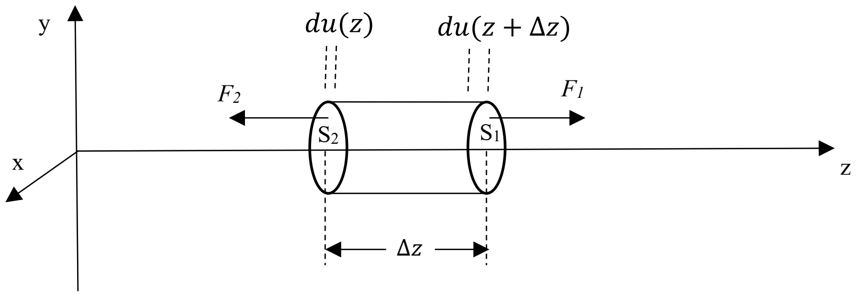

Figure 4. Longitudinal wave

Among them, F_1 in the S_1 position can make the time and space of this position produce a displacement $du(z+\Delta z)$

Then the actual force at the position of section S_1 due to space-time bending is:

$$F_1 \frac{du}{dz}\Big|_{z+\Delta z} \approx F \frac{du}{dz}\Big|_{z+\Delta z}$$

In the same way, the actual force at the S_2 position is:

$$F_2 \frac{du}{dz}\Big|_{z} \approx F \frac{du}{dz}\Big|_{z}$$

In this way, the total space-time force of the length Δz in the figure is

$$\Delta F = F\left(\frac{du}{dz}\Big|_{z+\Delta z} - \frac{du}{dz}\Big|_{z}\right)$$

Consider

$$\Delta F = \rho S \Delta z \frac{d^2u}{dt^2} = \mathcal{E} S \Delta z \frac{d^2u}{dt^2 c^2}$$

Then

$$F\left(\frac{du}{dz}\Big|_{z+\Delta z} - \frac{du}{dz}\Big|_{z}\right) = \rho S \Delta z \frac{d^2u}{dt^2} = \mathcal{E} S \Delta z \frac{d^2u}{dt^2 c^2}$$

If $\Delta z \to 0$, then

$$F \frac{d^2u}{dz^2} = \mathcal{E} S \frac{d^2u}{dt^2 c^2}$$

And if the work done by these two forces is exactly equal to the energy of the gravitational field, then the work done by force F is:

$$\Delta W = F\Delta z$$

And the energy of the gravitational field in this area is:

$$\Delta E = \mathcal{E}S\Delta z$$

Then we can also obtain the wave equation with the speed of light:

$$\frac{d^2u}{dz^2} = \frac{d^2u}{dt^2c^2}$$

But the wave equation reflects the longitudinal wave form of gravitational waves.

3 Detection of gravitational waves and longitudinal waves

At present, the LIGO device has been able to detect gravitational waves predicted by general relativity theory. However, after a slight improvement, it is believed that the device can still be used to detect longitudinal waves of gravitational waves.

It is relatively easy to identify gravitational transverse waves. As shown in Figure 5, the gravitational wave detector has two arms perpendicular to each other. Corresponding to the x-axis and y-axis respectively. If gravitational waves propagate along the z-axis, the detector can detect the alternate length of the two arms. For example, if Arm 1 is elongated, Arm 2 will be shortened at the same time. And vice versa. By analyzing the relative length changes of the two Arms, the incident direction of gravitational waves can also be determined.

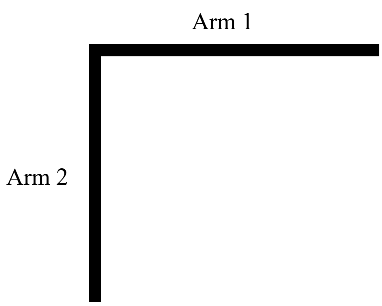

Figure 5. Gravitational wave detection device

If it is a longitudinal wave, you only need to keep the incident direction of the gravitational wave parallel to the plane of the detector. One of the two Arms can be in the z-axis direction of the gravitational wave propagation. In this way, if there is a longitudinal wave, the detector can find that only the length of the Arm in the z-axis direction will change periodically, and the length of the other Arm will not change. Of course, if it is a transverse wave, the length change of Arm is just the opposite.

As for the propagation direction of gravitational waves, it can be determined by other astronomical observation data. The direction adjustment of the detector can be realized by the rotation of the earth. That is, by measuring in different time periods, different propagation directions of gravitational waves can be obtained.

4 Conclusions

The gravitational wave solution is an important achievement of general relativity. Therefore, detecting the presence or absence of gravitational waves has become an important experimental evidence for testing general relativity.

However, the calculation process of the gravitational wave solution of general relativity is more complicated. After using the method of tensor analysis, the complicated mathematical derivation process can easily conceal the physical meaning of it.

In this paper, we try to use the assumption of elastic space-time, combined with the propagation of gravitational field energy, to obtain gravitational wave solutions. The physical meaning is easier to understand. The entire derivation process of gravitational waves is also relatively simple, and I believe this will help solve some more complicated problems of the generation and propagation of gravitational waves.

In addition, the analysis in this article also shows that if the hypothesis of this article is true, gravitational waves should not only have transverse waves, but also longitudinal waves.

This article believes that the existing LIGO gravitational wave measurement device can be used to measure at different time periods to prove whether the longitudinal wave form of gravitational waves exists.

Reference

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