# Heisenberg's uncertainty principle and wave-particle dualism.

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**Abstract:** This work shows that the Heisenberg uncertainty principle is a consequence of the connection between the characteristics of space-time in the microworld with the energy-momentum of elementary particles. The manifestation of such a connection is the wave-particle properties of microparticles, the mathematically rigorous expression of which is the de Broglie interval wave. Moreover, the interval wave is, in fact, an analogue of Einstein's field equations for the microworld, as it expresses the connection between space-time and the energy-momentum of matter at the quantum level.

**Keywords:** Heisenberg's uncertainty principle, de Broglie interval wave, wave-particle duality, space-time, energy-momentum, Einstein's field equations.

**INTRODUCTION.**

Heisenberg's uncertainty principle was discovered in 1927 and is a consequence of the wave-particle duality: an elementary particle can be a wave, or it can be a corpuscle. And this means that an elementary particle is no longer a corpuscle in our usual understanding, and therefore, we will not be able to simultaneously determine the momentum and coordinate of a microparticle, as we usually do this for a corpuscle (material point).

Heisenberg's principle says: "the more precisely the position is determined, the less accurately the impulse is known, and vice versa" [1].

$$\Delta x * \Delta p = \hbar / 2$$

"…There is an exact quantitative analogy between the Heisenberg uncertainty relations and the properties of waves or signals.

Consider a time-varying signal, such as a sound wave. It makes no sense to talk about the frequency spectrum of a signal at any point in time. To accurately determine the frequency, it is necessary to observe the signal for some time, thus losing the accuracy of timing.

In other words, the sound cannot simultaneously have the exact value of the time of its fixation, as a very short impulse has, and the exact value of the frequency, as is the case for a continuous (and, in principle, infinitely long) pure tone (pure sinusoid).

The temporal position and frequency of the wave are mathematically completely analogous to the coordinate and quantum mechanical momentum of the particle. Which is not surprising at all, if we recall de Broglie's formula (λ = h/p), that is, momentum in quantum mechanics, this is the spatial frequency along the corresponding coordinate.

…The uncertainty relation in quantum mechanics in the mathematical sense is a direct consequence of a certain property of the Fourier transform” [2].

**RESULTS AND DISCUSSION.**

The fact that it is impossible to accurately measure both the coordinate and the momentum for an elementary particle can be demonstrated if we take into account, as was shown earlier [3], that the de Broglie wave is an interval wave.

S = h / p

Where S — is interval,

S = ƛc. * (1 – v^2 / c^2)^0.5 * c/v

ƛc. – is the Compton wavelength of the corresponding microparticle.

The formula of Louis de Broglie:

S = h / p

which connects space-time (the interval) and the momentum of the microparticle (and hence its total energy), is an analogue of A. Einstein’s well-known field equations (GR) [4], but only for the microworld.

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

This means that an elementary particle in the space-time continuum occupies a certain area, which is determined by the interval. That is, a microparticle has a certain extent in time and space (Δt, Δx). Therefore, the simultaneous determination of the coordinate and momentum is impossible by definition, which was well demonstrated above with the example of a sound wave.

Let's write the de Broglie interval wave in extended form:

ƛc. * (1 - v^2 / c^2)^0.5 * c/v = h / Δp

The left side of the equation, or rather, ƛc. * (1 - v^2 / c^2)^0.5 will determine the wavelength, that is, the spatial uncertainty of the coordinate, which will also lead to the uncertainty of the momentum.

If we want to strictly determine the coordinate, then we must reduce this "extent" of the particle as much as possible, that is,

$$\Delta x = \bar{\lambda}c. * (1 - v^2 / c^2)^{0.5} = \min.$$

Then, the impulse uncertainty will be:

$$\Delta p = (h * v) / (\bar{\lambda}c. * c * (1 - v^2 / c^2)^{0.5})$$

From the last formula it is clearly seen that the narrower the "length" of the microparticle, the greater the impulse uncertainty. And with the exact determination of the coordinate ($\Delta x = 0$, $v = c$), the momentum becomes absolutely undefined (since there is no division by zero).

$$\Delta x \to \min, \quad v \to c, \quad \Delta p \to \infty$$

If the coordinate is precisely defined, that is, $x = a$ ($\Delta x = 0$), then the impulse uncertainty is equal to infinity.

$$\Delta x = 0, \quad x = a, \quad v = c, \quad \Delta p = \infty$$

When determining the exact momentum ($v = const$), the "length" of the microparticle will always be non-zero:

$$L = \bar{\lambda}c. * (1 - v^2 / c^2)^{0.5}$$

And therefore, the uncertainty of the coordinate of the microparticle $\Delta x$ will be equal to the reduced value of the "extension".

$$\Delta x = L = \bar{\lambda}c. * (1 - v^2 / c^2)^{0.5}$$

Note that the momentum and energy of a microparticle are related by the Einstein equation:

$$E^2 = (p * c)^2 + (m * c^2)^2$$

where E - is the energy of a microparticle,

p - is the impulse of the microparticle,

m - is the mass of the system, that is, in our case, the mass of a microparticle.

It strictly follows from this equation that if the rest mass of a microparticle is zero, then it can only move at the speed of light, since the rest of the speeds are forbidden for it.

If a microparticle is composite and not elementary, for example, like nucleons, then the mass of such a system will no longer be equal to the sum of the masses of its constituent particles. Since the law of conservation of mass does not exist: in an isolated physical system, only energy and momentum are conserved in accordance with Noether's theorem [5].

The mass conservation law used in chemistry is an approximation. No more. Theoretically, strictly, the sum of the masses of the objects making up the system is not equal to the mass of the system. In chemistry, this is not noticeable, since the effect is very weak. Even when determining the mass defect of the nucleus, it is believed that the mass of the nucleus should be equal to the sum of the masses of its constituent nucleons. But, this is not so, the mass of the nucleus should not be equal to the sum of the masses of the nucleons, since the mass of any system is strictly determined by the Einstein formula:

E^2 = (p * c)^2 + (m * c^2)^2

And therefore, the mass of a composite system cannot be equal to the sum of the masses of its components. This is very well demonstrated by the example of the proton and neutron.

A proton is made up of three quarks, and the sum of the masses of these quarks is only 1 percent of the actual mass of the proton. In a neutron, the sum of the masses of its quarks is only 1.3 % of the real mass of the neutron. The sum of the quark masses in π-mesons is 3 - 5 % of the meson mass.

The reason is that the law of conservation of mass does not exist, but there is only the law of conservation of energy and momentum, in accordance with Einstein's formula. You can see for yourself the given figures if you visit the Particle Data Group website [6] and look at the masses of quarks and nucleons.

Body mass is simply a measure of its energy content, as A. Einstein pointed out in his work [7]: Einstein, A. Does the inertia of a body depend upon its energy-content? Annalen der Physik 18 (1905).

The energy of a microparticle moving with a speed v = const will be equal (if we make elementary substitutions: the de Broglie interval wave and the reduced Einstein formula):

E = ( ((h * v) / ƛc.)^2 + (m0 * c^2)^2 )^0.5 / (1 - v^2 / c^2)^0.5

It is clearly seen from the formula that at speeds that approach the speed of light, the energy of a microparticle rushes to infinity.

Finally, note that Einstein's formula:

E^2 = (p * c)^2 + (m * c^2)^2

directly confirms what we got from quantization: the world of elementary particles is a two-dimensional world [8].

## CONCLUSION.

Thus, the Heisenberg uncertainty principle is a consequence of the particle-wave properties of elementary particles, which are most fully expressed in the de Broglie interval wave. Since the interval wave connects space-time (interval) and the total energy of a microparticle (impulse of a microparticle), just like A. Einstein's field equations connect the curvature of space-time with the energy-momentum of matter. In fact, the de Broglie interval wave is an analogue of Einstein's equations for the microworld, since both the equations and the interval wave connect the characteristics of space-time with the energy-momentum of matter. Consequently, both quantum mechanics and general relativity describe the connection between space-time and energy-momentum (elementary particles, matter).

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