

Study on the Fundamental Principle of Quantum mechanics[†]

Chenggang Zhang(张成刚^{*})

*School of Nuclear Science and Technology (2009-2013) , University of South
China, Hengyang 421001, China*

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Abstract: This paper investigates Born's statistical interpretation and the processes in which the Schrodinger equation are constructed, and delves into the fundamental principles of quantum mechanics. In addition, several examples and the foundation of quantum mechanics are discussed from a new perspective.

Introduction:

As is known to all, quantization problems in the microscopic domain can be accurately solved by calculating the Schrodinger equation, a fact that has verified the accuracy and rationality of the equation. Nevertheless, the Schrodinger equation was not strictly derived in a mathematical or physical level, which results in its unclear physical essence. Moreover, although in quantum mechanics, the statistical interpretation of the wave function, on which the construction of the Schrodinger equation must be based, is made possible in line with the facts produced by microscopic particle diffraction experiments, it is still impossible to fundamentally understand the probabilistic property of microscopic particles.

Two problem-solving logics can be applied to such a difficult issue. The first is that quantum mechanics is theoretically based on the most essential law of nature, thus making it impossible to be derived from other theories. In other words, the probabilistic property of microscopic particles is their intrinsic property, a fact that we can only accept but can't truly perceive. The second is that the probabilistic property of microscopic particles is not the inherent property or the real property, but an indirect manifestation of a more essential law of nature. For example, the gravity is reflected by the phenomenon that an object thrown into the air always falls back to the ground, but we can't simply assume that it is an inherent property of the object. Taking the second logic may be more of practical significance and in the spirits of scientific exploration for interpreting the theoretical basis of quantum mechanics as well as enhancing the theories of physics (the unity and harmony between quantum mechanics and classical physics).

Therefore, this paper follows the second logic to explore a principle more fundamental than the theoretical basis of quantum mechanics. The results are hypothesized as follows: quantum mechanics will be based on rigorous mathematical physics derivation; the foundation of quantum mechanics is only a necessary inference of the more fundamental principles; thus, the basic problem of quantum mechanics is solved and quantum mechanics and classical physics are harmonized and unified.

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^{*} E-mail: zhangchenggang@163.com;SXSLZA,GDSWCQ;202106021534163

§1. Implications of Born's statistical interpretation [1] and the processes of constructing stationary Schrodinger equation [2]

In my opinion, The best way to explore the fundamental principles of quantum mechanics is to trace back to the creation of quantum mechanics for more inspiration and breakthroughs, that is, the initial construction process of the Schrodinger equation. According to Schrodinger, the Schrodinger equation can be constructed from the Hamilton-Jacobi equation in classical physics, which is also how Schrodinger initially proposed the Schrodinger equation. For the problems of general conservative system in classical mechanics, the classical Hamilton-Jacobi equation is as follows:

$$-E + \frac{1}{2m}(\nabla S)^2 + V = 0 \quad (1)$$

The construction process is as follows: assuming a new unknown function ψ represents the action S in the Hamilton-Jacobi equation, there is:

$$\psi = e^{S/\hbar}$$

where $\hbar = h/2\pi$, and h is the Planck constant (note that mathematically, ψ must be always greater than 0); According to Born's statistical interpretation to wave function, which says that $|\psi|^2$ gives the relative probability density of finding the particle at point (x, y, z) (relative probability density is the probability that is not normalized, and it is meaningless when S is a virtual function, so we think S is a real function in here), relative probability density is written as ω , Therefore, this paper obtains a relation between relative probability density ω and action S :

$$\omega = e^{2S/\hbar}, \text{ has: } \omega|_{S=0} = 1$$

Consequently, this is strongly reminiscent of the Boltzmann probability distribution of classical physics in the form of mathematics, the Boltzmann probability distribution is:

$$\omega = e^{-\varepsilon_p/KT}, \text{ has: } \omega|_{\varepsilon_p=0} = 1$$

Where ε_p represents potential function; So we have sufficient reason to associate the action S with the potential function ε_p , that is to say, we found that: the action S which in Hamilton-Jacobi differential equation is closely related to the potential function ε_p , has:

$$S = (-\hbar/2KT)\varepsilon_p \quad (2)$$

As the paper discusses quantum mechanics, ε_p herein should be associated with the microscopic domain, which is regarded as a microscopic potential function. The differences between ε_p and the potential function V in the Hamilton-Jacobi equation as well as their associations will be elaborated on in the following sections.

§2. Mathematical physics derivation of the stationary Schrodinger equation

Based on the findings in §1, the Hamilton-Jacobi equation is given a clearer physical definition, which is associated with the microscopic domain. The Hamilton-Jacobi equation can be a constraint equation regarding two functions, macroscopic potential function V and microscopic potential function ε_p . The

original classical Hamilton-Jacobi equation can be rewritten as:

$$-\frac{1}{2m} \left(-\frac{\hbar}{2KT} \nabla \varepsilon_p \right)^2 + (E - V) = 0 \quad (3)$$

This equation is more basic than the Schrodinger equation and is associated with classical physics as well as the microscopic domain. Moreover, all quantities in the equation have clear physical definitions. Compared with the classic Hamiltonian principle and Jacobi principle of least action, the equation above reveals another principle of least action, i.e.: For microscopic systems, the real action (potential function) is extremum in comparison with all the other actions (potential function), has: $\delta \varepsilon_p = 0$. It is applied to the derivation of the stationary Schrodinger equation.

Regarding the mathematical physics derivation of the stationary Schrodinger equation, the exploration process in §1 is an attempt to obtain more inspiration from Schrodinger's initial process of constructing the Schrodinger equation by the Hamilton-Jacobi equation, rather than the strict mathematical physics derivation of the Schrodinger equation, even though such an attempt has shown that the Schrodinger equation can be obtained from the Hamilton-Jacobi equation. Given the confusions about the theoretical basis of quantum mechanics, it is necessary to present the complete process of mathematical physics derivation of the Schrodinger equation. When a lot of particles in the microscopic potential field obey Boltzmann statistics, the function of the relative probability density ω with respect to the microscopic potential field ε_p under the condition $\omega|_{\varepsilon_p=0} = 1$ is:

$$\varepsilon_p = -KT \ln \omega = -KT \ln \psi^2, \text{ has: } \omega|_{\varepsilon_p=0} = 1$$

Let's plug this relation into Hamilton-Jacobi equation that is rewritten in the above, Unlike what Schrodinger initially did to substitute the action S for $\hbar \ln \psi$ mathematically, or substituting the action S by analogy, the substitution operation herein has sufficiently clear physical processes and definitions, from which the following can be obtained:

$$-\frac{\hbar^2}{2m} (\nabla \psi / \psi)^2 + (E - V) = 0$$

Let the left-hand side of this equation be M , which is taken as the Lagrangian of generalized coordinates ψ , $\partial \psi / \partial r$; the following can be acquired by solving the integration equation:

$$I = \int M dr = \int \left[-E + \frac{\hbar^2}{2m} (\nabla \psi / \psi)^2 + V \right] dr$$

Based on the variational problem, the extremum of I is solved (Euler-Lagrange equation), from which we have the following equation:

$$\frac{\partial I}{\partial \psi} - \frac{\partial}{\partial r} \left[\frac{\partial I}{\partial (\partial \psi / \partial r)} \right] = 0$$

The equation for microscopic particles regarding ψ is thus obtained:

$$\frac{\hbar^2}{2m} \nabla^2 \psi + (E - V) \psi = 0$$

This is the stationary Schrodinger equation in quantum mechanics.

In view of this paper's point, has: $\varepsilon_p = -KT \ln \psi^2$, thus, the stationary Schrodinger

equation can also be rewritten in the following form only with ε_p and V :

$$\frac{\hbar^2}{2m} \nabla^2 \left\{ e^{-\varepsilon_p/2KT} \right\} + (E - V) \left\{ e^{-\varepsilon_p/2KT} \right\} = 0 \quad (4)$$

According to the physical properties of Equation (4), it is hereby named as a potential function for easy reference in the following sections.

§3. Discussion on the physics principles revealed by the Schrodinger equation

It can be noted that the microscopic potential function ε_p is derived from the macroscopic potential function V in the potential equation. Since the potential function manifests the interaction between objects and the potential functional equation contains the energy term E of the system, the potential functional equation explicitly reveals that: (1) microscopic action ε_p must be simultaneously associated with the energy E and space r of the system, and thus $\varepsilon_p = \varepsilon_p(E, r)$. (2) The probabilistic property exhibited by microscopic particles is not an inherent property of microscopic particles, but an indirect reflection of microscopic action, has : $\omega = \psi^2 = e^{-\varepsilon_p/KT}$. (3) The microscopic interaction is different from the macroscopic one. In other words, the proved classical mechanical laws used to describe the macroscopic natural phenomena are just the limit result or approximate result of quantum mechanics.

Several common forms of microscopic action corresponding to macroscopic action will be obtained by solving the potential function equation, in order to prove that the classical mechanical laws used to describe the macroscopic natural phenomena are just the limit result or approximate result of quantum mechanics; that is, the macroscopic action V is the limit or approximate result of the microscopic action ε_p .

(1): For a one-dimensional macroscopic simple harmonic motion (SHM), its macroscopic action is $V(x) = m\omega^2 x^2 / 2$, with the potential function equation as follows:

$$\frac{\hbar^2}{2m} \nabla^2 \left\{ e^{-\varepsilon_p/2KT} \right\} + \left(E - m\omega^2 x^2 / 2 \right) \left\{ e^{-\varepsilon_p/2KT} \right\} = 0$$

Desired ε_p can be obtained by solving the potential function equation:

$$\begin{cases} E_n = \hbar\omega \left(n + \frac{1}{2} \right), \text{ has: } n = 0, 1, 2, 3, \dots \\ \varepsilon_p(n, x) = -KT \left\{ \ln N_n^2 - a^2 x^2 + \ln [H_n^2(ax)] \right\} \end{cases}$$

Where, $a = \sqrt{\frac{mE_n}{\hbar^2(n+1/2)}}$, $N_n = \left(\frac{a}{\pi^{1/2} 2^n n!} \right)^{1/2}$, $H_n(ax)$ are Hermite polynomials.

When $n=0$ or $n \rightarrow 0$, the microscopic potential function is approximately the potential function form of the macroscopic SHM.

(2): For the macroscopic central gravitational field between m_1 and m_2 , its macroscopic action is $V(r) = -B/r$, with the potential function equation as follows:

$$\frac{\hbar^2}{2\mu} \nabla^2 \left\{ e^{-\varepsilon_p/2KT} \right\} + (E + B/r) \left\{ e^{-\varepsilon_p/2KT} \right\} = 0$$

Where, μ is the reduced mass, which can be expressed as $\mu = \frac{m_1 m_2}{m_1 + m_2}$; desired ε_p can be obtained by solving the potential function equation:

$$\left\{ \begin{array}{l} E_n = -\frac{\mu B}{2n^2 \hbar^2}, \text{ has: } n = 1, 2, 3, \dots \\ \varepsilon_p(n, r) = -KT \left\{ \ln \left(\frac{na_0}{2} N_{n,n-1} \right)^2 + \ln [L_{2n-1}^{2n-1}(\rho)]^2 + \ln \rho^{2n} - \rho \right\} \end{array} \right.$$

Where, $\rho = \frac{2r}{na_0}$, $a_0 = \frac{2\hbar^2}{\mu B}$, and $N_{n,n-1}$ are normalization factors and $L_{n+1}^{2l+1}(\rho)$ is the associated Laguerre polynomial. According to mathematical calculation, $N_{n,n-1}$ and $L_{2n-1}^{2n-1}(\rho)$ are the constants only associated with n and are irrelevant to ρ . It can be clearly concluded from the mathematical form of the microscopic potential function that, for m_1 and m_2 , when the energy of the system is E_n , the minimum of the corresponding microscopic potential function is found at $r = n^2 a_0$. In other words, when the system energy is E_n , the corresponding well bottom of the microscopic potential function is at $r = n^2 a_0$; or when the distance between m_1 and m_2 is $r = n^2 a_0$, they are immune from each other's force, which is exactly the case where the system can maintain stability; meanwhile, the relationship between quantum energy E_n and well bottom $n^2 a_0$ is that:

$$E_n = -\frac{\mu B^2}{2n^2 \hbar^2} = -\frac{B}{n^2 a_0}$$

When $a_0 \rightarrow 0$, the well bottom $n^2 a_0$ of a series of potential wells is approximately continuous, which is the macroscopic potential function formula as follows:

$$E_n = -\frac{B}{n^2 a_0} \approx -\frac{B}{r} = V(r)$$

For hydrogen-like atoms, $B = Ze^2 / 4\pi\epsilon_0$, when the energy of the electrons outside the nucleus is E_n , the minimum of the corresponding microscopic potential function is at $r = n^2 a_0$. The extranuclear electrons at $r = n^2 a_0$ will not be affected by the nucleus. This is where extranuclear electrons with the energy of E_n should exist stably. It can be judged from this that extranuclear electrons in atoms do not move like celestial bodies that stay in an orbit by high-velocity motion. Instead, extranuclear electrons are trapped by microscopic action in the potential wells around the nucleus. That's why classical physics cannot explain the fact of the stable existence of atoms. Therefore, according to the discussion herein, the internal scene of an atom can be described as follows: all positive charges are concentrated in the nucleus; extranuclear electrons are trapped in the potential wells corresponding to their energy by microscopic action; and the energy of extranuclear electrons can only be quantized; the most probable condition (stable condition) is that extranuclear electrons are at the

bottom of the potential wells corresponding to their energy; when there is a transition in the energy of extranuclear electrons, the action from the nucleus changes accordingly and then extranuclear electrons move to the bottom of the new potential wells.

Conclusions: The familiar macroscopic action is only a limit or approximate result of microcosmic action. They are constrained by the following potential function equation:

$$\frac{\hbar^2}{2\mu} \nabla^2 \left\{ e^{-\varepsilon_p/2KT} \right\} + (E - V) \left\{ e^{-\varepsilon_p/2KT} \right\} = 0$$

Where, ε_p is microcosmic action; V is macroscopic action; and E is system energy. The solution of the potential function equation shows that the energy of any system is quantized, or microscopic action is quantized; and the continuous changes in macroscopic energy is the limit case and approximate form of quantization. The probabilistic property of the microscopic system is not its inherent property, but an indirect manifestation of the microscopic action.

Comments: in the course of physics, what classical physics has achieved in handling macroscopic field is visible to all, but its limitations when being applied to the microscopic one is also prominent. According to the spirit of scientific exploration, we can't simply accept the fact that classical physics is inapplicable to microscopic field, because microscopic and macroscopic field could have been unified in theory as microscopic and macroscopic are only relative concepts in scales and have no strict demarcation. Therefore, the limited application of classical physics to microscopic field proves only its laws are not sufficiently accurate or even partial in some case. We also can't simply accept that microscopic field is probabilistic, either. Inspired by spirit of scientific exploration, we can keep asking why microscopic field is probabilistic and what's the inherent principle. The most scientific practice here is to unveil the inherent nature of quantum mechanics. In this process, limitation of classical physics may be solved and understood so that certain correction may be done in this respect, quantum mechanics may be understood, quantum mechanics and classical physics may be unified and coordinated. This is exactly what this paper aspires to do.

References:

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