New notation in series of functions II.

Juan Elias Millas Vera

juanmillaszgz@gmail.com

Zaragoza (Spain)

June 2021

0- Abstract:

In this paper we will see the next part of my theory of notation in series. Focusing on summation and productory we will do a defined explanation of an iterated serial operators.

1- Introduction:

We are going to remember the polynomial expressions of series in summation and productory with an interval which were introduced in the first paper:

$$(1)\quad \sum_{n=a(c)}^{b} f(x) = \mathrm{A}_{1}{}_{n=a(c)}^{b} f(x) = +f(a)+f(a+c)+f(a+2c)+\ldots+f(b-2c)+f(b-c)+f(b)$$

$$(2)\quad \prod_{n=a(c)}^{b} f(x) = \mathrm{A}_{2}{}_{n=a(c)}^{b} f(x) = f(a)\cdot f(a+c)\cdot f(a+2c)\cdot\ldots\cdot f(b-2c)\cdot f(b-c)\cdot f(b)$$

2- Operators with a sequence of operators:

Now we are going to present the development in two steps of the serial operator defined between two states of other operator.

- In the first part we see the summation of summations and we can visualize the development equaling f(x) to x:

$$(3)\quad \sum_{n=a=\lambda\sum\limits_{m=a'(c')}^{b'} g(x)\; c=\theta\sum\limits_{m=a'(c')}^{b'} g(x)}^{b=\lambda'\sum\limits_{m=a'(c')}^{b'} g(x)} f(x) = \lambda\sum_{m=a'(c')}^{b'} g(x) + (\lambda+\theta)\sum_{m=a'(c')}^{b'} g(x) + (\lambda+2\theta)\sum_{m=a'(c')}^{b'} g(x) + \ldots$$

$$+(\lambda'-2\theta)\sum_{m=a'(c')}^{b'} g(x) + (\lambda'-\theta)\sum_{m=a'(c')}^{b'} g(x) + \lambda'\sum_{m=a'(c')}^{b'} g(x)$$

Next, we are going to see the polynomial evolution of this expression equaling g(x) to x:

$$(4)\quad \lambda(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')+(\lambda+\theta)(a'+(a'+c')+(a'+2c'))$$
$$(+\ldots+(b'-2c')+(b'-c')+b')+(\lambda+2\theta)(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')$$
$$+\ldots+(\lambda'-2\theta)(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')+(\lambda'-\theta)(a'+(a'+c')+(a'+2c'))$$
$$(+\ldots+(b'-2c')+(b'-c')+b')+\lambda'(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')$$

- Secondly, we are going to see a productory of summations:

$$(5)\quad \prod_{n=a=\lambda\sum_{m=a'(c')}^{b'} g(x)\; c=\theta\sum_{m=a'(c')}^{b'} g(x)}^{b=\lambda'\sum_{m=a'(c')}^{b'} g(x)} f(x) = \lambda\sum_{m=a'(c')}^{b'} g(x)\cdot(\lambda+\theta)\sum_{m=a'(c')}^{b'} g(x)\cdot(\lambda+2\theta)\sum_{m=a'(c')}^{b'} g(x)\cdot\ldots$$
$$\ldots\cdot(\lambda'-2\theta)\sum_{m=a'(c')}^{b'} g(x)\cdot(\lambda'-\theta)\sum_{m=a'(c')}^{b'} g(x)\cdot\lambda'\sum_{m=a'(c')}^{b'} g(x)$$

Of course we can also do the polynomial:

$$(6)\quad \lambda(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')\cdot(\lambda+\theta)(a'+(a'+c')+(a'+2c'))$$
$$(+\ldots+(b'-2c')+(b'-c')+b')\cdot(\lambda+2\theta)(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')\cdot\ldots$$
$$\ldots\cdot(\lambda'-2\theta)(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')\cdot(\lambda'-\theta)(a'+(a'+c')+(a'+2c'))$$
$$(+\ldots+(b'-2c')+(b'-c')+b')\cdot\lambda'(a'+(a'+c')+(a'+2c')+\ldots+(b'-2c')+(b'-c')+b')$$

- Third part, the productory of products:

$$(7)\quad \prod_{n=a=\lambda\prod_{m=a'(c')}^{b'} g(x)\; c=\theta\prod_{m=a'(c')}^{b'} g(x)}^{b=\lambda'\prod_{m=a'(c')}^{b'} g(x)} f(x) = \lambda\prod_{m=a'(c')}^{b'} g(x)\cdot(\lambda+\theta)\prod_{m=a'(c')}^{b'} g(x)\cdot(\lambda+2\theta)\prod_{m=a'(c')}^{b'} g(x)\cdot\ldots$$
$$\ldots\cdot(\lambda'-2\theta)\prod_{m=a'(c')}^{b'} g(x)\cdot(\lambda'-\theta)\prod_{m=a'(c')}^{b'} g(x)\cdot\lambda'\prod_{m=a'(c')}^{b'} g(x)$$

Now we can develop it as polynomial too:

$$(8)\quad \lambda(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')+b')\cdot(\lambda+\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots)$$
$$(\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')\cdot(\lambda+2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b'\cdot\ldots)$$
$$\ldots\cdot(\lambda'-2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')\cdot(\lambda'-\theta)(a'\cdot(a'+c')\cdot(a'+2c'))\cdot\ldots$$
$$(\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')\cdot\lambda'(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')$$

- Fourth, we can do the final combination which is the summation of products:

$$(9)\quad \sum_{n=a=\lambda\prod_{m=a'(c')}^{b'}g(x)\;c=\theta\prod_{m=a'(c')}^{b'}g(x)}^{b=\lambda'\prod_{m=a'(c')}^{b'}g(x)} f(x) = \lambda\prod_{m=a'(c')}^{b'}g(x)+(\lambda+\theta)\prod_{m=a'(c')}^{b'}g(x)+(\lambda+2\theta)\prod_{m=a'(c')}^{b'}g(x)+\ldots$$

$$\ldots+(\lambda'-2\theta)\prod_{m=a'(c')}^{b'}g(x)+(\lambda'-\theta)\prod_{m=a'(c')}^{b'}g(x)+\lambda'\prod_{m=a'(c')}^{b'}g(x)$$

And finally we can do in this last combination the polynomial:

$$(10)\quad \lambda(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')+b')+(\lambda+\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots)$$
$$(\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')+(\lambda+2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')$$
$$+\ldots+(\lambda'-2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')+(\lambda'-\theta)(a'\cdot(a'+c')\cdot(a'+2c'))\cdot\ldots$$
$$(\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')+\lambda'(a'\cdot(a'+c')\cdot(a'+2c')\cdot\ldots\cdot(b'-2c')\cdot(b'-c')\cdot b')$$

We assume for all options this should be true:

$$(11)\quad \frac{\lambda'-\lambda}{\theta}=s \qquad \forall s\in\mathbb{N}$$

3- Other functions.

If we want we can also equal the functions inside the operator to a different types of functions.
For summation of summations and equaling f(x) to x^2 we can see this result:

$$(12)\quad \sum_{n=a=\lambda\sum_{m=a'(c')}^{b'}g(x)\;c=\theta\sum_{m=a'(c')}^{b'}g(x)}^{b=\lambda'\sum_{m=a'(c')}^{b'}g(x)} f(x)=x^2 =\left(\lambda\sum_{m=a'(c')}^{b'}g(x)\right)^2+\left((\lambda+\theta)\sum_{m=a'(c')}^{b'}g(x)\right)^2+\left((\lambda+2\theta)\sum_{m=a'(c')}^{b'}g(x)\right)^2+\ldots$$

$$+\left((\lambda'-2\theta)\sum_{m=a'(c')}^{b'}g(x)\right)^2+\left((\lambda'-\theta)\sum_{m=a'(c')}^{b'}g(x)\right)^2+\left(\lambda'\sum_{m=a'(c')}^{b'}g(x)\right)^2$$

And the polynomial growth will be the next one if g(x) is x:

(13)

$$(\lambda(a'+(a'+c')+(a'+2c')+...+(b'-2c')+(b'-c')+b'))^2+((\lambda+\theta)(a'+(a'+c')+(a'+2c')))$$
$$((+...+(b'-2c')+(b'-c')+b'))^2+((\lambda+2\theta)(a'+(a'+c')+(a'+2c')+...+(b'-2c')+(b'-c')+b'))^2$$
$$+...+((\lambda'-2\theta)(a'+(a'+c')+(a'+2c')+...+(b'-2c')+(b'-c')+b'))^2+((\lambda'-\theta)(a'+(a'+c')+(a'+2c')))$$
$$((+...+(b'-2c')+(b'-c')+b'))^2+(\lambda'(a'+(a'+c')+(a'+2c')+...+(b'-2c')+(b'-c')+b'))^2$$

To finish we are going to present a final example, a productory of products where f(x) is equal to 3x and g(x) is x:

(14)

$$\prod_{n=a=\lambda\prod_{m=a'(c')}^{b'} g(x)\;c=\theta\prod_{m=a'(c')}^{b'} g(x)}^{b=\lambda'\prod_{m=a'(c')}^{b'} g(x)} f(x)=3x \quad =3\left(\lambda\prod_{m=a'(c')}^{b'} g(x)\right)\cdot 3\left((\lambda+\theta)\prod_{m=a'(c')}^{b'} g(x)\right)\cdot 3\left((\lambda+2\theta)\prod_{m=a'(c')}^{b'} g(x)\right)\cdot ...$$
$$...\cdot 3\left((\lambda'-2\theta)\prod_{m=a'(c')}^{b'} g(x)\right)\cdot 3\left((\lambda'-\theta)\prod_{m=a'(c')}^{b'} g(x)\right)\cdot 3\left(\lambda'\prod_{m=a'(c')}^{b'} g(x)\right)$$

Polynomial:

(15)

$$3(\lambda(a'\cdot(a'+c')\cdot(a'+2c')\cdot...\cdot(b'-2c')\cdot(b'-c')+b'))\cdot 3((\lambda+\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot...))$$
$$((...\cdot(b'-2c')\cdot(b'-c')\cdot b'))\cdot 3((\lambda+2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot...\cdot(b'-2c')\cdot(b'-c')\cdot b'\cdot...))$$
$$...\cdot 3((\lambda'-2\theta)(a'\cdot(a'+c')\cdot(a'+2c')\cdot...\cdot(b'-2c')\cdot(b'-c')\cdot b'))\cdot 3((\lambda'-\theta)(a'\cdot(a'+c')\cdot(a'+2c')))\cdot...$$
$$((...\cdot(b'-2c')\cdot(b'-c')\cdot b'))\cdot 3(\lambda'(a'\cdot(a'+c')\cdot(a'+2c')\cdot...\cdot(b'-2c')\cdot(b'-c')\cdot b'))$$

4- Conclusion.

As we can see we could combine summation and productory forms to obtain different polynomial results. In my opinion this is a success of the notation in series theory. Besides we could do a combination of the main and secondary term with also restory, divisory, exponentory and rootory (the others operators introduced previously in my other papers).