Prime Generation and primality test using 2x+1 and Summation of a constant

by: Mar Detic

Abstract:

We introduce another way to enumerate primes up to N using 2x+1 and the summation of a constant. By which can also be used for primality test of a given integer.

Definition:

For all x integer, where x is equal to $2a+1$ and a is equal to $(\frac{x-1}{2})$. And the multiples of a prime can be written as $(\frac{x-1}{2})$ adding to the summation of a constant c multiplied by 2 adding 1; where c is equal to x; thus $((\frac{x-1}{2})+\sum_{i=1}^{n} x)\times 2+1$, which means the congruence $((\frac{x-1}{2})+\sum_{i=1}^{n} x)\times 2+1\equiv 0 \mod x$.

By definition above we're gonna use the formulae:

Let a ={a_1,a_2,…a_{k-1}} a set of integers

n is a tuple (sequence)where:

$$(((a_k+\sum_{i=1}^{n} c)\times 2)+1)\leq a_k$$

On prime generation(prime sieve**)**

Let say we are given an integer **a**:

a= 10

since we need to find all primes less than a well use the list **{a_1,a_2,…a_{k-1}}**; thus {1,2,3,4,5,6,7,8,9}

<table>
<tr><td colspan="2">x</td><td>3</td><td>5</td><td>7</td><td>---------</td><td>---------</td><td>---------</td><td>---------</td><td>---------</td><td>---------</td><td>---------</td></tr>
<tr><td colspan="2">a_k</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td rowspan="2">$a_k+\sum_{i=1}^{n} x$</td><td>n=1</td><td>4</td><td>Here we skip since the functio n will give 7 7(2)+1=15 which is > a</td><td>Here we skip since the functio n will give10 10(2)+1=2 1 which is > a</td><td>Here we skip since 4 is on the a_1</td><td colspan="6">Here we gonna skip since the function will add up if multiplied by 2 then added 1 is greater than a
meaning instead of listing all {$a_{1,\ldots}a_{n-1}$} we can list only up $\frac{a_k}{2}$ since $(((\frac{a_k}{2})\times 2)+1)>a_k$</td></tr>
<tr><td>n=2 so on</td><td>Here we skip since the functio n will give 7 7(2)+1=15 which is > a</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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Checking if $a_k \neq (a_{k-(k-1)} + \sum_{i=1}^{n} x)$; if true x is prime

On primality test (trial division using the prime generation above**)**

example:
Given integer a, we check first if even or not.
a=100

we'll gonna use the method from prime generation above but we'll gonna use the limit since we know that the largest factor of a number is the squareroot; so $(a=\sqrt{a}) \Rightarrow (a_k=10)$

Checking if $a \, mod ((a_k \times 2)+1)$; if true **a** not is prime

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As you can see above we started generating primes from 3 because:

if we consider 1 as prime:

$1 \Rightarrow \frac{1-1}{2} \Rightarrow \frac{0}{2}$	thus the x above will start at 0 then if we feed 0 to the $a_k + \sum_{i=1}^{n} x$ where a_k is 0 and x is 1.
$0 + \sum_{i=1}^{n} 1$	this set will produce an integer a where 2a+1 will produce all odd and even integer. So we can say this function is the prime of primes for all odd integer.

if we consider 2 as prime:

$2 \Rightarrow \frac{2-1}{2} \Rightarrow \frac{1}{2}$	thus the x above will start at $\frac{1}{2}$ then if we feed $\frac{1}{2}$ to the w $a_k + \sum_{i=1}^{n} x$ here x is $\frac{1}{2}$ and p is 2.
$\frac{1}{2} + \sum_{i=1}^{n} 2$	this set will produce a where 2a+1,we'll produce all even integer that if divide by 2 is equal to all odd integer. Which be written as $2 \times (0 + \sum_{i=1}^{n} 1)$,where n is only odd integer(including primes). So we can say this function is the prime of primes for all even integer. So if we don't consider 1 as prime then so is 2 we can't consider as prime.

note:

and the gaps of primes is bounded by how many multiple of primes between 2 given primes
example:
89,97 gap is 8
(89-1)/2=44
(97-1)/2=48 44,{45,46,47}48 ; thus 3 is the gap

Now to calculate the gaps; the formula is:

2x+2

Where x is equals $\left(\frac{a-1}{2}-\frac{b-1}{2}\right)-1$ and a is the bigger prime and b is the smaller prime.