Lorentz transformation of gravitational field

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Abstract

The transformation relationship between different reference systems follows the principle of Lorentz transformation. In general relativity, the curvature of space-time caused by mass is regarded as a non-inertial frame of reference, so there is also a problem of frame of reference transformation. If the concept of virtual space-time is introduced, we can see that the existence of gravity can also introduce the Lorentz transformation relationship, so that we can deal with the problem of gravity in a simpler way. This article analyzes the static gravitational field, and obtains a result consistent with the Schwarzschild solution of relativity in a more concise way.

1 Introduction

The Lorentz transformation relationship can be used to transform time and space between different reference systems. In this way, the physical laws in one frame of reference can be transformed into another frame of reference. This has been supported by a large number of physical facts.

For space-time with mass, general relativity regards it as a non-inertial frame of reference. The characteristic of the non-inertial reference frame is the bending phenomenon of time and space.

Since the existence of gravity leads to changes in the frame of reference, the Lorentz transformation relationship should also exist in the gravitational field. This article points out through analysis that a non-inertial system with mass can also have its own set of Lorentz transformation relations. Through the Lorentz transformation of this gravitational field, some seemingly complex gravitational field problems can be analyzed relatively simply.

2 The essence of Lorentz transformation

We start with the analysis from the mass-energy relationship in the theory of relativity:

$$\mathcal{E}^2 = m_0^2 c^4 + p^2 c^2 \tag{1}$$

Where, the $\mathcal{E}$ is the total energy; m_0 is the rest mass; p is the momentum; c is the light speed.

This reflects the relationship between mass, momentum and energy.

The above formula can also be written as

$$m^2c^4 = m_0^2c^4 + m^2v^2c^2 \tag{2}$$

Where, the m is the movement mass. Then the result will be

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 \tag{3}$$

In fact, it can also be

$$\gamma = \frac{mc^2}{m_0c^2} = \frac{\mathcal{E}}{\mathcal{E}_0} \tag{4}$$

Where the $\mathcal{E}_0 = m_0c^2$

In other words, the Lorentz transformation coefficient is the ratio of the energies of the two reference systems. This may better reflect the nature of Lorentz transformation.

The transformation relationship of the time interval can be expressed as

$$dt = \gamma dt_0 \tag{5}$$

That is

$$\frac{\mathcal{E}_0}{\mathcal{E}} = \frac{dt_0}{dt} \tag{6}$$

Where dt_0 is the time interval of the rest reference system and *dt* is the time interval of the motion reference system.

It can be seen from this transformation relationship that the time interval between the two reference systems is consistent with the ratio of the energy of the particles in the two reference systems. This reflects that the time of the moving frame of reference is slower than the time of the stationary frame of reference.

For the transformation relationship of length. If the frame of reference moves along the *x*-axis, then

$$dx = \gamma^{-1}dx_0 \tag{7}$$

It can also be represented as

$$\frac{\mathcal{E}_0}{\mathcal{E}} = \frac{dx}{dx_0} \tag{8}$$

3 Lorentz transformation of gravitational field

If there is a gravitational field, two frames of reference can also be used for comparison.

Think of infinity as a frame of reference $\mathcal{R}_\infty$. There is no gravitational field in this frame of reference.

Consider the distance from the mass M is *r*. We can use this position as another frame of reference $\mathcal{R}_r$

Then in the frame of reference $\mathcal{R}_\infty$, a particle has energy $\mathcal{E}_\infty = m_0c^2$

Considering that the frame of reference has changed, the mass of particles in the frame of reference $\mathcal{R}_r$ may change. This is the same as in different inertial systems. Suppose the mass of the particle in the frame of reference $\mathcal{R}_r$ is *m*. In addition, the particle is still in the gravitational field generated by a large mass with a mass of *M*. Adding the potential energy V, the total energy of the particle is

$$\mathcal{E}_R = mc^2 + V \tag{9}$$

This is a well-known formula for calculating total energy in gravitational potential, except that mass is not equal to rest mass.

Below we consider the existence of a virtual space-time. Then the above-mentioned energy is the energy measured from real space-time. From the perspective of virtual space-time, according to the viewpoint of virtual space-time physics [1], mass corresponds to the energy of virtual space-time. However, the potential energy here is the energy of the real space-time. Therefore, if the energies of two different time and space are to be added, the sum of squares relationship should be used. That is, if you observe this particle from virtual space-time, its total energy should be

$$\mathcal{E}_V = \sqrt{(m_0c^2)^2 + V^2} \tag{10}$$

Of course, it is also possible to do not rely on the physics of virtual space-time, considering that the potential energy and kinetic energy in the gravitational field can be converted into each other, so the potential energy of an object has the same properties as kinetic energy. In this way, we can directly replace the kinetic energy term *pc* in the relativistic energy formula with potential energy, so that we can also get the above formula. After all, in classical physics or general relativity, we can directly add kinetic energy and potential energy together to calculate. This also proves that the two energies, kinetic energy and potential energy, are energy in the same dimension.

Then consider that although the energy is observed in different time and space, the two should be equal. therefore

$$\mathcal{E}_R = \mathcal{E}_V \tag{11}$$

Then

$$mc^2 + V = \sqrt{(m_0c^2)^2 + V^2} \tag{12}$$

Then we can get

$$(mc^2)^2 + 2Vmc^2 = (m_0c^2)^2 \tag{13}$$

The formula of potential energy is

$$V = -\frac{GMm}{r} \tag{14}$$

Then we can get

$$\gamma^2 = \frac{\mathcal{E}^2}{{\mathcal{E}_0}^2} = \frac{(mc^2)^2}{(m_0c^2)^2} = \frac{1}{1 - \frac{2GM}{rc^2}} \tag{15}$$

Therefore, according to formula (5), we can get

$$\frac{{dt_0}^2}{dt^2} = 1 - \frac{2GM}{rc^2} \tag{16}$$

Or

$${dt_0}^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 \tag{17}$$

It can be seen from this formula that the time period observed at the position *r* away from the gravitational source is longer than the time period observed when there is no gravitational field (equivalent to infinity). This also means that the existence of a gravitational field causes time to become slower.

And from formula (7) we can get

$$\frac{dr^2}{{dr_0}^2} = 1 - \frac{2GM}{rc^2} \tag{18}$$

Or

$${dr_0}^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 \tag{19}$$

This shows that the existence of the gravitational field causes the radial length to become shorter.

From the perspective of the Euclidean distance change, the Euclidean distance without a gravitational field is

$$ds^2 = -dt^2c^2 + dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2 \tag{20}$$

If the gravitational field exists, dr^2 and dt^2 will change. which is

$$dt^2 \rightarrow \left(1 - \frac{2GM}{rc^2}\right)dt^2 \tag{21}$$

$$dr^2 \rightarrow \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 \tag{22}$$

And $r, \theta, d\theta, d\varphi$ will not change. At this time, the Euclidean distance will become

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)dt^2c^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2 \tag{23}$$

It can be seen that this is completely consistent with Schwarzschild's solution.

Through the above analysis, we can also see that the mass of the object has changed in the gravitational field. The mass m_0 at infinity has become the mass m in the gravitational field. This may be used as important evidence to test the conclusion of this article.

4 Effect of mass change in gravitational field on gravitational constant measurements

Although the mass of the object increases in the gravitational field. However, this increase is actually very small, and it is difficult to have an observable effect. Here's a rough estimate.

As can be seen from equations (13) and (15), the formula for the mass of an object in a gravitational field is

$$m = \frac{m_0}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{24}$$

Hence the mass of the object in the gravitational field is increased.

For the Earth, which is subjected to the gravitational field of the Sun, its mass increases by approximately

$$m_{earth} = \frac{m_0}{\sqrt{1 - \frac{2GM}{rc^2}}} \approx m_0 \left(1 + \frac{GM}{rc^2}\right) \tag{25}$$

$$\Delta m \approx \frac{GM}{rc^2} m_0 \approx 10^{-8} m_0 \tag{26}$$

The current error of gravitational measurements is about 10^{-5}. It can be seen that changes in the Sun's gravitational field do not affect the measurement of the gravitational constant on Earth.

However, if we take into account the influence of the gravitational field generated by all the matter in the Milky Way, and then consider that dark matter accounts for 80% of the mass of the Milky Way's matter, then the gravitational field of the Milky Way matter can cause a change in the mass of objects on Earth by approximately

$$\Delta m \approx 2.35 \times 10^{-5} m_0 \tag{27}$$

This change in mass is an order of magnitude consistent with the current accuracy of gravitational constant measurements. Therefore, it can be considered that the current accuracy of gravitational constant measurement cannot be improved, which may be related to the change of gravitational field caused by the fluctuation of the mass of matter in the Milky Way, which in turn leads to the inability to improve the measurement accuracy of gravitational constant.

5 Conclusions

This article obtains Schwarzschild's solution through another simpler way. Of course, this is not to overturn the conclusion of the theory of relativity, but to think that besides the theory of relativity, there may be another way to understand the laws of nature.

Judging from the analysis results of this article, the existence of gravity actually changes the energy of the object. This energy change will also produce another form of reference frame transformation. If the energy change of an object caused by the existence of gravity is regarded as the same parameter as the energy change caused by the change of speed, then the Lorentz transformation of gravity can also be introduced to obtain some meaningful results. Of course, gravity is different from electromagnetic force. There are no two fields, electric and magnetic fields, that can exchange energy with each other. Therefore, it may be necessary to introduce virtual space-time when conducting similar analysis. This can also prove the existence of virtual space-time on another level. Of course I have also noticed that many authors have tried to perform the Lorentz transformation of the gravitational field a long time ago [2]. However, since the existence of virtual space-time is not considered, the Lorentz transformation process of this gravitational field will be more complicated.

Of course, the analysis in this article must have conclusions that are not consistent with general relativity. For example, the analysis in this article shows that in the gravitational field, the mass of particles in gravitational field will change. This is the same as the change in the speed of the object's movement causes a change in the mass of the object. The analysis in this paper also suggests that the current inability to measure gravitational constants on Earth may be due to the influence of fluctuations in the gravitational field within the Milky Way.

Reference

[1] Cheng, Z. Foundations of Virtual Spacetime Physics. LAP LAMBERT Academic Publishing. Brivibas gatve 197, LV-1039, Riga. Latvia, European Union.

[2] Kibble, T. . (1961). Lorentz invariance and the gravitational field. *Journal of Mathematical Physics, 2*(2), 212-221.