The hidden constants

Les constantes cachées

الثوابت الكامنة

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Abstract

In this article we have unify the electromagnetic-field and the gravitational field by introducing a new universal constant. We define also the absolute degree of temperature as $1K = 0.627\,\frac{c}{k}\sqrt{ha}$ where "c" is the celerity of light, "k" the Boltzmann constant , "h" the Planck constant and "a" the mechanical impedance of vacuum .

Résumé

Dans cet article on a unifié le champ électromagnétique et le champ gravitationnel en un seul champ par l'introduction d'une nouvelle constante universelle. On a aussi définit le degré absolu de température comme $1K = 0{,}627\,\frac{c}{k}\sqrt{ha}$ où "c" est la célérité de la lumière, "k" la constante de Boltzmann, "h" la constante de Planck et "a" l'impédance mécanique du vide.

ملخص

في هذا المقال قمنا بتوحيد المجال الكهرومغناطيسي و مجال الجاذبية و ذالك عبر اعتماد ثابت كوني جديد . لقد عرفنا

أيضا بوحدة درجة الحرارة في المطلق $1K = 0.627\,\frac{c}{k}\sqrt{ha}$ أين

"c" هي سرعة الضوء

"k" هي ثابت بولتزمان

"h" هي ثابت بلانك

"a" هي المقاومة الميكانيكية للفراغ (الثابت الكوني الجديد)

Keywords :

Mechanical impedance of vacuum, Newton gauge, scale invariance gauge, unity-multiplicity duality, viscosity-dispersion duality, Maxwell charge , De Broglie quantum of energy, vacuum energy levels, negative mass, dark energy, dark matter, unification of fields, vacuum density current , second law of radiation displacement, black hole radiation envelop.

1)Introduction:

"It offers the possibility of establishing units for length, mass, time and temperature which are independent of specific bodies or materials and which necessarily maintain their meaning for all time and for all civilizations, even those which are extraterrestrial and nonhuman, constants which therefore can be called fundamental physical units of measurement" ***M. Planck (1900)***

From 1899 , Planck had established an absolute system of unities as follows[1]:

$$M_P = \sqrt{\frac{\hbar.c}{G}} = 2.18\ \ 10^{-8} kg \qquad (1\text{-}1)\ ,$$

$$L_P = \sqrt{\frac{\hbar.G}{c^3}} = 1.6\ \ 10^{-35} m \qquad (1\text{-}2)\ ,$$

$$T_P = \sqrt{\frac{\hbar.G}{c^5}} = 5.39\ \ 10^{-44} s \qquad (1\text{-}3).$$

The Planck system signify that extension in space-temps is equivalent to energy. So the equation of motion of a corpuscle should be written in a full space-time which act on the corpuscle by a friction force in the opposite direction of its speed. The equation of motion of such a corpuscle is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (1\text{-}4)$$

Where: $\boldsymbol{p}$**:** the moment of the corpuscle;

$\boldsymbol{f}$: All unknown forces which act on the corpuscle;

$-a.\boldsymbol{v}$: An universal friction force due to the energy of the space-time. This force is really cancelled by the negative pressure of space-time otherwise we are in contradiction with the principle of inertia. In other terms the space-time is a super fluid without friction;

a : Friction coefficient of the space-time;

The friction coefficient $''a''$of the space-time is declared as a new universal constant. Space-time is vacuum and vacuum is space-time.

Of course equation (1-4) is not invariant by transformations of space and time but we will see how to change our view in exchanging energy. Let's take it as the first idea which comes to us as thinking in classical manner.

The MKS system (or cgs system) is a natural system of measuring any physical quantity. Those unities of measure are as follows:

$M = 1\ kg$: The unit of mass;

$L = 1\ m$: The unit of length;

$T = 1\ s$: The unit of time.

But here there is not any relationship between those unities. Universal constants are constraints to respect in any theory of motion. When Michelson tries to measure the speed of light in 1891 in order to detect any motion of the "ether" he was surprised that the speed of light is constant in any direction: the speed of light $''c''$ was declared an universal constant and it is independent from the choice of the referential of motion and from any corpuscle. With the constant $''G''$ the gravitational constant we can establish another relationship between mass and length for example and with Planck constant $''\hbar''$ we can resolve a system of three equations with three parameters and so Planck get the solutions (1-1), (1-2) & (1-3): there is a general equivalence between mass, length and time. Space-time can't be "inert" and should act on corpuscles. Space-time can't be only a theatre of interactions between corpuscles but it participates and interacts with them.

The Planck system should conduct us to the minimum energy in the minimum volume of the space-time. If we calculate it i.e. we have a mass M_P in a volume L_P^3 we get an enormous value which is in flagrant contradiction with observations which speaks about a value of $10^{-9}\ joule.m^{-3}$ .Nothing can't move in this media: we conclude that Planck system is not the good choice, another system should replace it and so one of the constants which form the Planck system is a derived constant or is a coupling force constant from a certain scale.

It is evident that we have particles which have a mass less than Planck mass (for example electrons). The constant which should be removed is the gravitational constant: gravitation strength is neglected in the subatomic particles.

We propose to built the new system of unities with three constants $''\hbar''$, $''c''$&$''a''$. The result is:

$M = \frac{1}{c}.\sqrt{\hbar.a}$ (1-5) ,

$L = \sqrt{\frac{\hbar}{a}}$ (1-6) ,

$T = \frac{1}{c}.\sqrt{\frac{\hbar}{a}}$ (1-7) .

We can expect that in the MKS system the constant $''a''$ should have a very low value: if we neglect it in the equation (1-4) we get the classical dynamic law.

The most evident experience to have an idea about the value of the constant $''a''$ is to determine the density of energy of vacuum in the Universe by observations and to identify it to the theoretical vacuum energy [2]. There is also others experiences which allow us to determine this constant such as the photo-electric experience and the black body radiation experience [3]. We will try in the following to determine this constant referring to recent cosmology observations and old German scientists' radiation experiences. But if we want to continue, we should refer to a consistent theory of motion. Equation (1-4) is not invariant by Lorentz transformations of space & time; we can accept it only in the particular case that it is defined locally.

Let's have a corpuscle of a mass m in motion in an inertial referential$R(O, x, y, z, t)$. Let's have another inertial referential $R'(O', x', y', z', t')$ in motion with a speed V along the axis (O, x) and that origins are coincident in the beginning of motion. Axis (O, x)&(O', x') are co-linear.

The Lorentz transformations of space and time between the two referential are [4]:

$x' = \frac{x - V.t}{\sqrt{1 - \frac{V^2}{c^2}}}$ (1-8)

$t' = \frac{t - x.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}$ (1-9) or $c.t' = \frac{c.t - \frac{V}{c}.x}{\sqrt{1-\frac{V^2}{c^2}}}$ (1-10)

For monochromatic plane waves the transformations of wave-vector and frequency are the same of space and time:

$k' = \frac{k - \omega.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}$ (1-11) or $c.k' = \frac{c.k - \frac{V}{c}.\omega}{\sqrt{1-\frac{V^2}{c^2}}}$ (1-12)

$\omega' = \frac{\omega - k.V}{\sqrt{1-\frac{V^2}{c^2}}}$ (1-13)

Where: k & k' are respectively the wave-vector in the referential R& R';

ω & ω' are respectively the frequency in the referential R & R'.

The principle of relativity is that the equations of Nature are invariant by Lorentz transformations i.e. there is invariance of the action of the corpuscle and also there is invariance of phase for plane waves.

Every theory takes its validity by respecting the following conditions:

1st condition–The respect of the principle of least action i.e. that every physical phenomenon is described by a principle of action. The principle of conservation of energy and conservation of momentum comes from the least action principle. We should search an action that the equations of motion which comes from its minimisation describe the phenomenon in the laboratory and Nature. The action of a corpuscle is:

$$S_{corpuscle} = \int L(X, \dot{X}, t).dt \quad (1\text{-}14)$$

Where $L(X, \dot{X}, t)$ its Lagrange function.

2nd condition-The respect of the principle of locality i.e. the phenomenon that happen in a region of space and time affect directly only their nearest environment. If we act on a system in the position (X, t) in space-time, the only direct effect is on the nearest infinitely close neighbourhood. How to guaranty that a theory respect the principle of locality?: it is done by the principle of least action.

Let's take the action of a corpuscle as:

$$S_{corpuscle} = \int L(X, \dot{X}, t).dt \quad (1\text{-}15)$$

To guaranty the locality, the Lagrange function in equation (1-15) should depend only of the spatial coordinates of the system i.e. for a corpuscle it should depend only from its position $X(t)$ and its first derivative $\dot{X}(t)$. The neighbouring points does not intervene only via its time derivative i.e. at the limit when $\Delta t \to zero$ by $[X(t+\Delta t) - X(t)]/\Delta t$.

In other terms in a referential, the effect is detected at a distance x only after a time t and that $\frac{dx}{dt}$ has a finite value $v(x,t)$ which is less or equal to the speed of information c.

3rd condition-The respect of the principle of relativity i.e. the equations of motion should be invariants by Lorentz transformations. The equations of motion should be invariants in inertial referential i.e. the same equations in different referential.

4th condition-The respect of the gauge invariance i.e. a change on the system which does not affect the action or the equations of motion: it is called a symmetry. Let's take an example in the classical fundamental law of dynamics:

$$\boldsymbol{F} = m.\frac{d^2\boldsymbol{X}}{dt^2} \quad (1\text{-}16)$$

This equation remains the same if we translate the origin of coordinates of a fix value or we rotate the axles of coordinates of fixed angles. Which is conserved in classical dynamics is the total energy of the corpuscle: When there is a symmetry there is something which is conserved.

The first idea to come for anyone in order to have an idea about the constant "a" is to suppose that the vacuum density is one mass M given by (1-5) in a volume L^3 given by (1-6) and to equalize to the admitted recent cosmology observation as $10^{-9}\ joule.m^{-3}$. We get $a \approx 1.8\ 10^{-26}\ kg.s^{-1}$: it is a very low value and it is logic to accept it.

The most non evident idea for all even thought for me is to wonder: what we have as absolute unity in the MKS system?. There is only one response which is the Kelvin degree of temperature and so we have:

$$k.1K = M.c^2 = c.\sqrt{\hbar.a} \quad (1\text{-}17)$$

Where: k :is the Boltzmann constant.

After calculation we get $a = 3.2\ 10^{-30}\ kg.s^{-1}$: it also a very low value but it should be confirmed by experience and theory.

After getting results from old experiments we will be surprised how it is worse for science home-made constants.

In fact the introduction of the new constant "a" in physics is totally different from which I wrote in this article. I had introduce this constant in august 1986 when I had try to resolve the problem of the trajectory of the electron around the nucleus by means of only classical mechanics because I had not accept in my baccalaureate year when our professor Mr Ahmed present the first lesson of chemistry "the structure of matter" that there is three points of views or hypothesis about the motion of the electron (Schrödinger view , De Broglie view & Bohr view) and he write on the board the fourth point of view will be one of you (our classroom) . This article is written in this manner only to conserve a certain earlier demarche & a chronological evolution of ideas. If Planck theory of radiation had influence the mechanics the inverse sense is also true: mechanics will influence the Planck theory.

2) The principle of least action for a non charged corpuscle:

2-1) The Euler-Lagrange equations:

A corpuscle which have the generalised coordinates $\{q_i, i = 1,2,3\}$ follow a trajectory developed in time and which have the equation [5]:

$$q = q_i(t), \ \ i = 1,2,3 \qquad (2\text{-}1)$$

Here referring to the referential R we have $(q_1 = x, q_2 = y, q_3 = z)$.

The components of generalised speed are defined as:

$$\dot{q}_i = \frac{dq_i(t)}{dt}\ , i = 1,2,3 \qquad (2\text{-}2)$$

The action S associated to the corpuscle is defined as:

$$S = \int L(q_i, \dot{q}_i, t).dt \qquad (2\text{-}3)$$

Where L is a function of q_i, $\dot{q}_i$ and possible of t.

The quantity S is extreme for the real trajectory of the corpuscle. We have:

$dS = 0$ (2-4)

If we put that the q_i are independents from each else and that the variation of the function $L(q_i, \dot{q}_i, t)$is happened at constant time we get the Euler-Lagrange equations as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad (2\text{-}5)$$

The solutions of equations (2-5) define the trajectory of the corpuscle.

The quantity L is linked to the energy of the corpuscle and it is called almost the kinetic potential. It is the difference of kinetic energy and the potential energy of the corpuscle in the case that the forces which act on the corpuscle are derived from a potential i.e. they are conservatives forces.

In case that those forces are non conservatives the Euler-Lagrange are :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad (2\text{-}6)$$

Where Q_i :generalized forces.

The importance of Lagrange equations is that instead we treat with vectors quantity such as forces and accelerations in the classical dynamics, we have scalar quantities where appear only positions and speeds .

2-2) The Hamilton equations:

We define the generalised momentum or conjugate momentum as:

$$p_\alpha = \frac{\partial L}{\partial \dot{q_\alpha}} \quad (\alpha = 1, \dots \dots, n), n\colon number\ of\ freedom\ degrees \quad (2\text{-}7)$$

We define the Hamilton function or Legendre transformation of Lagrange function $L(q, \dot{q}, t)$ referring to the generalised speed as:

$$H(q, p, t) = \sum \dot{q_\alpha}\,(q, p). p_\alpha - L(q, \dot{q}(q, p), t) \qquad (2\text{-}8)$$

From that the $q_\alpha \& p_\alpha$ are independents we get the equations of Hamilton as the following:

$$\dot{q_\alpha} = \frac{\partial H}{\partial p_\alpha} \qquad (2\text{-}9)$$

$$\dot{p}_\alpha = -\frac{\partial H}{\partial q_\alpha} \qquad (2\text{-}10)$$

$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$ (2-11)

With the condition that $H(q_i, \dot{q}_i, t)$ is independent from time we get also [6]:

$\frac{dH}{dt} = \frac{\partial H}{\partial t} = 0$ (2-12)

The Hamiltonian of a corpuscle is the total energy of the corpuscle i.e. the sum of its kinetic energy and its potential energy. The advantage of the equations of Hamilton is that they are equations of the first order but Lagrange equations are of the second order. Also with the equations of Hamilton we deal only with positions and moments, the notion of inertia doesn't appear explicitly.

2-3) Canonical formalism:

Let's have a dynamical system with only one degree of freedom [7].Its configuration is completely determined by giving two dynamical variables q & p called conjugate variables. Its evolution is determined by the Hamiltonian function $H(q,p)$ which characterise the system and from it we get the equations of motion:

$\frac{dq}{dt} = \frac{\partial H}{\partial p} = [q, H]$ (2-13)

$\frac{dp}{dt} = -\frac{\partial H}{\partial q} = [p, H]$ (2-14)

Equations (2-13) &(2-14) are the canonical equations or Hamilton equations of the system.

The symbol [,] signify the Poisson crochet defined as:

$[f, g] = \frac{\partial f}{\partial q}.\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}.\frac{\partial g}{\partial q}$ (2-15)

Where f & g are any dynamical function of the variables q& p .

In particular we have:

$[q, p] = 1$ (2-16)

A replacement of reversible variables $Q(q,p)$& $P(q,p)$ are canonicals when the new variables Q & P satisfy equation (2-16):

$[Q,P]=1$ (2-17)

It is the only condition required for the conservation of the Poisson crochet by this variables replacement i.e.:

$$[f,g]_{q,p}=\frac{\partial f}{\partial q}.\frac{\partial g}{\partial p}-\frac{\partial f}{\partial p}.\frac{\partial g}{\partial q}=\frac{\partial f}{\partial Q}.\frac{\partial g}{\partial P}-\frac{\partial f}{\partial P}.\frac{\partial g}{\partial Q}=[f,g]_{Q,P} \quad (2\text{-}18)$$

And also the only condition required to conserve the canonical form of the equations of motion:

$$\frac{dQ}{dt}=\frac{\partial H}{\partial P}=[Q,H] \quad (2\text{-}19)$$

$$\frac{dP}{dt}=-\frac{\partial H}{\partial Q}=[P,H] \quad (2\text{-}20)$$

Let's consider this case: let's have a given $P(q,p)$ can we associate to it a function $Q(q,p)$ in order to get a replacement of dynamical variables which is defined will be canonical?. As it is indicated, it is sufficient to insure that the transformation is reversible i.e. to insure that:

$\frac{\partial P}{\partial q}$ & $\frac{\partial P}{\partial p}$ don't vanishes simultaneously: (2-21)

So the condition (2-17) becomes a partial derivation which determine (q,p) .

Let's remark that the condition (2-21) is only locally condition, it doesn't insure the reversibility of the transformation only in a certain neighbours of the plan (q,p) .

2-4) Equation of motion:

Let's suppose that the corpuscle is in rest in the referential R'. The action S' of the corpuscle in this referential is as:

$dS'=L'.dt'$ (2-22)

Where L' is a constant that we search. In principle this constant is the kinetic potential of the corpuscle in the referential R'. It is different from the conception of the classical mechanics which consider the mass is ''inert'' and has no role only to resist to the variation to the speed of the corpuscle.

The principle of relativity requires that:

$dS'=dS$ (2-23)

So we get:

$L.dt = L'.dt'$ (2-24)

The position of the corpuscle in the referential R is $x = V.t$ and so from equation (1-9) we get:

$dt' = dt.\sqrt{1 - \frac{V^2}{c^2}}$ (2-25)

Replace (2-25) in (2-24) we get :

$L = L'.\sqrt{1 - \frac{V^2}{c^2}}$ (2-26)

For low speeds we should found the expression of the kinetic energy in the classical mechanics. So when $V \ll c$ we get from equation (2-26):

$L \approx L' - \frac{1}{2}.L'.\frac{V^2}{c^2}$ (2-27)

It is evident from equation (2-27) that:

$L' = -m.c^2$ (2-28)

And so from (2-26) and (2-28) we have :

$L = -m.c^2.\sqrt{1 - \frac{V^2}{c^2}}$ (2-29)

We generalise the equation (2-29) for every speed of the corpuscle :

$L = -m.c^2.\sqrt{1 - \frac{v^2}{c^2}}$ (2-30)

And that from (2-30) the moment of the corpuscle is:

$\boldsymbol{p} = \frac{\partial L}{\partial \boldsymbol{v}} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ (2-31)

We define the inertia of the corpuscle as the ratio of its moment to its speed:

$\xi = \frac{p}{v} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ (2-32)

The energy of the corpuscle is equal to its Hamiltonian in (2-8) i.e.:

$$E = H = \boldsymbol{p}.\boldsymbol{v} - L = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \xi.c^2 \quad \text{(2-33)}$$

If this corpuscle is in motion in the referential R it signify that it is under a friction force due to vacuum which is due to the extension of the space-time. Remainder that space-time according to Planck system of unities is like a ‘’foam’’ of energy. The same idea of continuous media for vacuum (or space-time) is presented by Lev Landau &E.Lifchitz in their theory of fields [8]: to determine the position of a corpuscle we should have a referential filled with an infinite number of bodies everywhere in space and it behave like a ‘’medium’’. Every body had its own clock which it function in an arbitrary manner. This system of bodies is the referential of the theory of the gravitation i.e. the theory of general relativity as presented by Landau & E.Lifchitz. If we choose an arbitrary referential in the theory of general relativity than the laws of Nature should be available in any system of coordinates: we will conclude that the motion of a corpuscle can’t never be in a straight line, space-time or vacuum act on this corpuscle and curve its direction. The gravitational interaction is a limit interaction and it is universal. Yes constant $''G''$ is an universal constant and it is also predictable.

The action of the ‘’foam’’ on the corpuscle is a friction force which act in the opposite direction of motion. This force is a serial coefficients of the exponents of the speed of the corpuscle. We take only the first exponent i.e. the friction force is as:

$$\boldsymbol{f} = -a.\boldsymbol{v} \quad \text{(2-34)}$$

This friction is of course independent from the choice of the corpuscle i.e. it is universal. The coefficient of friction $''a''$ is declared as a new universal constant.

We associate to the corpuscle an inertial time as:

$$\xi = a.\tau \quad \text{(2-35)}$$

Its inertial time in rest is as :

$$m = a.\tau_0 \quad \text{(2-36)}$$

Idem for inertial length as :

$$l = c.\tau \qquad (2\text{-}37)$$

And inertial length in rest as :

$$l_0 = c.\tau_0 = \frac{m.c}{a} \qquad (2\text{-}38)$$

We have always this relation ship for inertial time :

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2\text{-}39)$$

It is very easy to verify that equation (2-39) is invariant by Lorentz transformations i.e. we have in referential R' the inertial time of the corpuscle is:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{v\prime^2}{c^2}}} \qquad (2\text{-}40)$$

Where :

$$v' = \frac{dx\prime}{dt\prime} \qquad (2\text{-}41)$$

Proof:

We can consider that the speed of the corpuscle is constant between the instant t and $t + dt$ in the referential R which corresponds to the instants t' and $t' + dt'$ in the referential $'$. From equation (1-9) we get:

$$dt' = \frac{1-v.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}.dt \qquad (2\text{-}42)$$

If we indicate by τ' the inertial time of the corpuscle in the referential R' and by τ its inertial time in the referential R than from (2-42) we deduce that:

$$\tau' = \frac{1-v.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}.\tau \qquad (2\text{-}43)$$

Replace (2-39) the expression of τ in (2-43) than we have:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{(v-V)^2}{(1-\frac{v.V}{c^2})^2.c^2}}} \quad (2\text{-}44)$$

Let’s determinate the speed v' of the corpuscle. From (1-8) and (1-9) we have:

$$v' = \frac{dx\prime}{dt\prime} = \frac{v-V}{1-\frac{v.V}{c^2}} \quad (2\text{-}45)$$

Replace (2-45) in (2-44) we get:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{v\prime^2}{c^2}}} \quad (2\text{-}46)$$

And that’s CQFD.

The constancy of the speed $''c''$ implies the constancy of the energy and momentum in inertial referential as the following:

$$E^2 - p^2.c^2 = m^2.c^4 \quad (2\text{-}47)$$

And also the constancy of the pseudo-module :

$$ds^2 = c^2.dt^2 - dx^2 - dy^2 - dz^2 \quad (2\text{-}48)$$

The moment can be written as the following:

$$\boldsymbol{p} = \xi.\boldsymbol{v} = \frac{E}{c^2}.\boldsymbol{v} \quad (2\text{-}49)$$

It is evident that a corpuscle with a speed c has a moment $\frac{E}{c}$ according to (2-49).

The Hamiltonian is as:

$$H = a\tau.c^2 \quad (2\text{-}50)$$

The energy of the corpuscle is a time component of its action. Its inertial position along the time component is $''c\tau''$ and its speed along this dimension is from (2-9):

$$\frac{\partial H}{\partial(a\tau.c)} = c \quad (2\text{-}51)$$

Where $''a\tau.c''$ is the moment of the corpuscle along its inertial dimension.

The friction force along this dimension is from (2-10) as:

$\frac{-\partial H}{\partial(c\tau)} = -a.c$ (2-52)

It is like that the corpuscle had a speed of $''c''$ along its time component dimension and there is always a friction force equal to $'' - a.c''$ which act on along this dimension.

It is evident that the equation of motion of the corpuscle in the three dimensional space is as :

$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v}$ (2-53) or $\frac{d}{dt}(\boldsymbol{p} + a.\boldsymbol{X}) = \boldsymbol{f}$ (2-54)

Where $\boldsymbol{f}$ **:** all unknown forces which act on the corpuscle ;

$-a.\boldsymbol{v}$: an universal friction force which act always in the opposite side of the direction of motion due to vacuum which the same the space-time.

$\boldsymbol{X}$**:** the position of the corpuscle.

But equation (2-54) is non relativist invariance we can't take it and we will see how to replace it to get the kinetic energy of the corpuscle as in classical mechanics.

Let's remark that when we write the position $\boldsymbol{X}$of the corpuscle in fourth space dimensions as:

$\boldsymbol{X} = (c\tau, x, y, z)$ (2-55)

The speed of the corpuscle in fourth dimensions is :

$\boldsymbol{V} = (c\frac{d\tau}{dt}, \dot{x}, \dot{y}, \dot{z})$ (2-56)

This speed should coincide to $''c''$ along the inertial dimension when the energy of the corpuscle is varying. So we have:

$\frac{d\tau}{dt} = 1$ (2-57) :when the energy of the corpuscle is varying;

$d\tau = 0$ (2-58) : when the energy of the corpuscle is constant (i.e. its three dimensional speed is constant in module) .

The equation of motion can be written as the following:

$\boldsymbol{f} = \frac{d^2(\xi.\boldsymbol{X})}{dt^2}$ (2-59)

With : $\xi = a.\tau$ the inertia of the corpuscle

$\boldsymbol{X} = (x, y, z)$ the three dimensional position of the corpuscle.

So the speed in fourth dimensions is :

$\boldsymbol{V}=(c, \dot{x}, \dot{y}, \dot{z}) = (c, \boldsymbol{v})$ (2-60)

The moment in fourth dimensions is :

$$\boldsymbol{P} = a.\tau.\boldsymbol{V} = (a.\tau.c, \boldsymbol{p}) = m.c.\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{\boldsymbol{v}}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) = m.c.u^i = p^i \qquad (2\text{-}61)$$

Where u^i is the quadric-dimensional speed in the forth dimensions space-time (ct, x, y, z).

$$u^i = \frac{dx^i}{ds} \qquad (2\text{-}62)$$

With $x^0 = c.t, x^1 = x, x^2 = y, x^3 = z$ (2-63)

$$ds = c.dt.\sqrt{1 - \frac{v^2}{c^2}}$$ (2-64): deduced from (2-48).

And:

$$p^i = m.c.u^i = (\frac{E}{c}, \boldsymbol{p}) \qquad (2\text{-}65)$$

The square of the vector x^i is the following pseudo-module:

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \qquad (2\text{-}66)$$

This square doesn't change for every fixed rotations of the fourth coordinates system where the Lorentz transformations are a particular case (We follow the same analysis done by Lev Landau & E.Lifchitz in their book "theory of fields").

In general we call a quadric-vector A^i the fourth quantities A^0, A^1, A^2, A^3 when in fourth transformations of coordinates system they are transformed as the x^i. In Lorentz transformations we have:

$$A^{0\prime} = \frac{A^0 - \frac{V}{c}.A^1}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A^{1\prime} = \frac{A^1 - \frac{V}{c}.A^0}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A^{2'} = A^2 \ , \quad A^{3\prime} = A^3 \qquad (2\text{-}67)$$

Here V is the three dimensional speed of R' $(O', c.t', x', y', z')$.

The square of every quadric-vector is the pseudo-scalar as defined in (2-48).

In order to simplify writing equations, we introduce another kind of quadric-vector as the following:

$$A_0 = A^0, \quad A_1 = -A^1, \quad A_2 = -A^2, \qquad A_3 = -A^3 \qquad \text{(2-68)}$$

The square of the quadric-vector is now as the following:

$$A^iA_i = A^0A_0 + A^1A_1 + A^2A_2 + A^3A_3 \qquad \text{(2-69)}$$

The quantities A^i are called the contra-variant corposants and the quantities A_i are called the covariant corposants of the quadri-vector.

The scalar product of two quadric-vectors is as the following:

$$A^iB_i = A^0B_0 + A^1B_1 + A^2B_2 + A^3B_3 = A_iB^i \qquad \text{(2-70)}$$

The product A^iB_i is an invariant scalar for every fixed rotations of coordinates system.

So for a corpuscle its four speed is :

$$u_0 = u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ,$$

$$u_1 = -u^1 = \frac{-\dot{x}}{c.\sqrt{1-\frac{v^2}{c^2}}} \, ,$$

$$u_2 = -u^2 = \frac{-\dot{y}}{c.\sqrt{1-\frac{v^2}{c^2}}} \quad ,$$

$$u_3 = -u^3 = \frac{-\dot{z}}{c.\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{(2-71)}$$

We remark that:

$$ds^2 = dx_i dx^i \qquad \text{(2-72)}$$

So we get that:

$$u^iu_i = 1 \qquad , \; p^ip_i = m^2.c^2 \qquad \text{(2-73)}$$

The fourth acceleration of the corpuscle is defined as:

$$w^i = \frac{du^i}{ds} = \frac{d^2x^i}{ds^2} \qquad (2\text{-}74)$$

Deriving (2-72) we get:

$$u_i w^i = u^i w_i = 0 \qquad (2\text{-}75)$$

We define the quadric- force vector as :

$$g_i = \frac{dp_i}{ds} = m.c.\frac{du_i}{ds} \qquad (2\text{-}76)$$

Its elements verify the identity:

$$g_i u^i = 0$$ (2-77) :deduced from (2-73)

We have:

$$g^i = \frac{dp^i}{ds} = m.c.\frac{du^i}{ds} = \left(\frac{\boldsymbol{F.v}}{c^2.\sqrt{1-\frac{v^2}{c^2}}}, \frac{\boldsymbol{F}}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (2\text{-}78)$$

With:

$$\boldsymbol{F} = \frac{\boldsymbol{dp}}{dt} = \frac{d}{dt}\left(\frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$ (2-79) :the ordinary three-dimensional force

Lets develop the expression of g_i :

$$g_0 = g^0 = \frac{1}{c^2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{dE}{dt} \quad \text{with} \quad E = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad \& \quad \boldsymbol{F.v} = \frac{dE}{dt}$$

$$g_1 = -g^1 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{x}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$g_2 = -g^2 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{y}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$g_3 = -g^3 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{z}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (2\text{-}80)$$

Replace (2-71) & (2-80) in (2-77) we get:

$$0 = g_i u^i = \frac{1}{c^2.(1-\frac{v^2}{c^2})}.\frac{dE}{dt} - \frac{\dot{x}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{x}}{\sqrt{1-\frac{v^2}{c^2}}}\right) - \frac{\dot{y}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{y}}{\sqrt{1-\frac{v^2}{c^2}}}\right) -$$

$$\frac{\dot{z}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{z}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{dE}{dt} - \dot{x}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{x}\right) - \dot{y}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{y}\right) - \dot{z}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{z}\right) = 0 \qquad (2\text{-}81)$$

Suppose that $v \neq c$ than (2-81) becomes:

$$\frac{dE}{dt} - \frac{(\dot{x}^2+\dot{y}^2+\dot{z}^2)}{c^2}.\frac{dE}{dt} - \frac{\dot{x}}{c^2}.E.\frac{d\dot{x}}{dt} - \frac{\dot{y}}{c^2}.E.\frac{d\dot{y}}{dt} - \frac{\dot{z}}{c^2}.E.\frac{d\dot{z}}{dt} = 0 \qquad (2\text{-}82)$$

So:

$$\frac{dE}{dt}\left(1-\frac{v^2}{c^2}\right) - \frac{E}{2.c^2}.\frac{dv^2}{dt} = 0 \qquad (2\text{-}83)$$

Let’s do the following balance in equation (2-83):

*If $v = constant$ than we get from (2-83) that $0 = 0$ (nothing) but we are in contradiction to our hypothesis that the energy is varying so we should exclude this case.

*The energy is varying so we have $\frac{dE}{dt} = \frac{d}{d\tau}(a.c^2.\tau) = a.c^2$ than we get from (2-83) & (2-39):

$$a.c^2.\left(1-\frac{v^2}{c^2}\right) - \frac{m}{2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{dv^2}{dt} = a.c^2.\left(1-\frac{v^2}{c^2}\right) - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\varepsilon}{dt} = 0 \qquad (2\text{-}84)$$

With:

$$\varepsilon = \frac{1}{2}.m.v^2 \qquad (2\text{-}85)$$

And :

$$dt = \frac{1}{a.c^2}.\frac{d\varepsilon}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2\text{-}86)$$

So we get from (2-84), (2-85) & (2-86) that:

$$1 - \frac{v^2}{c^2} = 1 \qquad (2\text{-}87)$$

It comes that the corpuscle should be in rest $(v = 0)$ but we have exclude the constant speed. From (2-84) we have:

$$\frac{d\varepsilon}{dt} = a.c^2.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} \qquad (2\text{-}88)$$

To resolve the dilemma of equation (2-88) that the energy is varying and the speed of the corpuscle is approximately constant is that the speed of the corpuscle vary slowly close to the speed of light $''c''$:it is the balance which we search. I think that this dilemma is similar to the dilemma of wave-corpuscle duality: how to have a corpuscle present in a position $(t, \boldsymbol{x})$ and the same time it is a plane wave present everywhere, the solution was that the speed of the corpuscle should be identified to the group speed of a packet of plane waves.

Another solution is that the speed of the corpuscle is varying slowly nearly zero to be approximately conform to equation (2-88). In this case we have:

$$\varepsilon \approx \frac{1}{2}.m.v^2 - \frac{3}{4}.m.\frac{v^4}{c^2} \qquad (2\text{-}89)$$

Let's remark that when the corpuscle has a speed closely to $''c''$ it signify that a corpuscle is like light and it can have a wave behaviour. It signify also that it should be maintained in motion by an external force equal approximately to $''a.c''$.

Let's define the proper time $''\zeta''$of the corpuscle as the following:

$$d\zeta = \frac{ds}{c} = dt.\sqrt{1-\frac{v^2}{c^2}} \qquad (2\text{-}90)$$

The time $''\zeta''$ is the time indicated by a clock moving with the corpuscle at the same speed: it is like it is attached to the corpuscle. Between two positions A & B in space-time we have:

$$\zeta_B - \zeta_A = \frac{1}{c}.\int_{t_A}^{t_B} ds = \int_{t_A}^{t_B} \sqrt{1-\frac{v^2}{c^2}}.dt \qquad (2\text{-}91)$$

We remark that the proper time is always less the time of the referential of motion :we conclude that a mobile clock function slowly than a fixed clock.

The laws of Nature are invariants in inertial referential. The referential of the fixed clock is an inertial referential but the referential of the mobile clock is not an inertial referential. If the motion of the corpuscle is happened approximately in constant speed than its *universe line* is a straight line parallel to the axle of time. A *universe line* of a corpuscle is its trajectory in four-dimensional space-time constructed by the *universe points* from which the corpuscle passes. A *universe point* is the three dimensional space coordinates x, y, z and the time t when the corpuscle passes.

The interval of time of any clock is of course $\frac{1}{c}. \int ds$ among its universe line (Suppose that this clock is attached to a corpuscle). The universe line of a fixed clock in an inertial referential is a straight line parallel to the axle of time. In another hand we have that the fixed clock indicates always a time interval superior than the interval time indicated by the mobile clock. It comes that the integral $\int ds$ between two universe points presents its maximum value if those points are linked by a universe straight line. We suppose that those points and lines which links them are that the elementary intervals ds along those lines are *times genre.*

An interval is a *time genre* when the module of the line which link two universe points $(c.t_1, x_1, y_1, z_1)$ and $(c.t_2, x_2, y_2, z_2)$ is positive.

This module is the following square:

$$s_{12}^2 = c^2.t_{12}^2 - l_{12}^2 \quad \text{(2-92)}$$

With : $t_{12} = t_2 - t_1$ and $l_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

If the square (2-92) is negative than the interval is a *space genre.*

Two events had a causal relation only if the interval which separate them is time genre. This result comes from that any interaction can't spread with a speed more than the light speed. The notions of "before'' and " after" had an absolute sense.

In our case the corpuscle can't be in motion in constant speed because its energy is varying continuously. Than to be conform to the image of constant speed the motion of the corpuscle should be in curved trajectory to get the same length interval as in straight motion in constant speed. An important conclusion that the motion of free massive corpuscle never can't be in straight line: it is always curved. This phenomenon is called universal gravitation: the vacuum curve the motion of every massive corpuscle.

The same analysis can be done referring to the proper time of the corpuscle. From (2-90) we have:

$$\int ds = \int c.d\zeta = \int \sqrt{1 - \frac{v^2}{c^2}}c.dt < \int c.dt \quad \text{(2-93)}$$

If the speed is augmenting continuously than from (2-93) we get an interval less than the case of constant speed and to be in coherence with the image of constant speed than an observer attached to the corpuscle see the light in curved motion.

To be coherent from the beginning, let's search the action of the corpuscle in the quadric-dimensional space-time. The first idea which comes is that this action is proportional to the integral of the interval ds. We should at first verify that $ds' = ds$ to be conform with (2-14). We can take it also as a demonstration of equation (2-15) by following the same path of Lev Landau & E.Lifchitz in their book "Theory of fields".

In the referential R' we have also from equation (2-92):

$s'^2_{12} = c^2.t'^2_{12} - l'^2_{12}$ (2-94)

With : $t'_{12} = t'_2 - t'_1$ and $l'^2_{12} = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2$

For light which move with constant speed c in the two inertial referentials R & R' we have of course that:

$s'^2_{12} = s^2_{12} = 0$ (2-95)

We can consider that equation (2-92) or equation (2-48) are the intervals between two points in the quadric-dimensional space-time (ct, x, y, z) but with a special form geometry: The Minkowski geometry.

From (2-95) it is evident that if $ds = 0$ in an inertial referential that $ds' = 0$ in any other inertial referential. In another hand ds & ds' are infinitely little in the same order that we can consider ds^2 & ds'^2 are mutually proportional:

$ds^2 = \gamma.ds'^2$ (2-96)

The factor of proportionality γ is only a function of the absolute relative speed of the two referentials and is not a function of space coordinates and time coordinate otherwise the points of space-time will not be equivalents as implies the homogeneity of space-time. This factor is also independent from the sense of the relative speed because this makes default of the isotropy of space-time .

Let's have three inertial referential R, R_1 & R_2 and let's take V_1 & V_2 the relatives speeds of respectively R_1 & R_2 in the referential R .So we have:

$ds^2 = \gamma(V_1).ds_1^2$ & $ds^2 = \gamma(V_2).ds_2^2$ (2-97)

We have also:

$$ds_1^2 = \gamma(V_{12}).ds_2^2 \qquad (2\text{-}98)$$

Where V_{12} the absolute speed of K_2 referring to K_1. So from (2-97) & (2-98) we get:

$$\frac{\gamma(V_2)}{\gamma(V_1)} = \gamma(V_{12}) \qquad (2\text{-}99)$$

But V_{12} depends not only of the modules of $\boldsymbol{V_1}$& $\boldsymbol{V_2}$ but also of the angle between the two vectors. As this angle doesn't exist in the first member of equation (2-99), so equation (2-99) can't be verified if only the function $\gamma(V)$ is equal to one. So we have:

$$ds^2 = ds'^2 \qquad (2\text{-}100)$$

We return now to search the action of the corpuscle in four dimensions. For a free corpuscle i.e. a corpuscle which is not under any force, the action is an integral of a scalar. The unique convenient scalar is the interval ds .So the action should be as:

$$S = \alpha.\int_A^B ds \qquad (2\text{-}101)$$

With : α a constant which characterise the corpuscle.

The integral $\int_A^B ds$ has its great value along a universe straight line. Along a curved line this integral will be more great. For the principle of least action of a mechanical system we can define an integral S called action which presents a minimum for real motion i.e. its variations δS are equal to zero. It comes that the constant α should be negative.

We have:

$$\delta S = \alpha.\int_A^B \delta.ds = 0 \qquad (2\text{-}102)$$

Or from (2-62) we have:

$ds = \frac{dx_i dx^i}{ds}$ so $\delta ds = \frac{dx_i \delta dx^i}{ds} = u_i d\delta x^i$

Then:

$$\delta S = \alpha.\int_A^B u_i d\delta x^i = \alpha.\int_A^B [d(u_i\delta x^i) - \delta x^i du_i] \qquad (2\text{-}103)$$

So:

$$\delta S = \alpha.\,[u_i \delta x^i]_A^B - \alpha.\int_A^B \delta x^i \frac{du_i}{ds} ds \qquad (2\text{-}104)$$

As in classical mechanics to find the equations of motions of a corpuscle we should compare many trajectories of the corpuscle which all pass from two given points i.e. satisfying the limit conditions that $(\delta x^i)_A = (\delta x^i)_B = 0$. The real trajectory of the corpuscle is deduced from the condition that $\delta S = 0$. Than we conclude from (2-94) that $\frac{du_i}{ds} = 0$ expressing the constancy of the speed of a free corpuscle in the quadric-dimensional form of coordinates.

To determinate the variation of the action as a function of coordinates , we put that the point A is given as $(\delta x^i)_A = 0$ and that the point B is any point of space-time which satisfy the equation of motion i.e. its belong to the real trajectory of the corpuscle. In consequence the integral in (2-94) is equal to zero and by putting that $(\delta x^i)_B = \delta x^i$ it comes:

$$\delta S = -\alpha.\,u_i.\,\delta x^i \qquad (2\text{-}105)$$

Referring to classical mechanics the partials derivations $\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z}$ are the components of the moment vector of the corpuscle and $-\frac{\partial S}{\partial t}$ is its energy. So it comes that the covariant components of the four moment of the corpuscle is:

$$p_i = -\frac{\partial S}{\partial x^i} = -\alpha.\,u_i = (\frac{E}{c}, -\boldsymbol{p}) \qquad (2\text{-}106)$$

So we deduce from (2-106) and (2-71) that:

$$\alpha = -m.\,c \qquad (2\text{-}107)$$

The transformations of energy and moment are as follows:

$$E' = \frac{E - \boldsymbol{p}.\boldsymbol{V}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (2\text{-}108)$$

$$\boldsymbol{p}' = \frac{\boldsymbol{p} - \frac{E}{c^2}.\boldsymbol{V}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (2\text{-}109)$$

Equations (2-108), (2-109) are what implies the relativist invariance of equation of motion deduced from the principle of least action in four dimensions.

To find the equation of motion it comes from (2-63) that:

$$p^i p_i = \frac{\partial S}{\partial x_i}\frac{\partial S}{\partial x^i} = \frac{1}{c^2}(\frac{\partial S}{\partial t})^2 - (\frac{\partial S}{\partial x})^2 - (\frac{\partial S}{\partial y})^2 - (\frac{\partial S}{\partial z})^2 = m^2.c^2 \qquad (2\text{-}110)$$

Equation (2-110) is the Hamilton-Jacobi relativist equation of motion of the corpuscle. .

We have obtain the equation of motion (2-110) with the hypothesis that the quadric-dimensional speed of the corpuscle $\frac{du_i}{ds} = 0$. But we had seen that a massive corpuscle can't never move in a straight line with constant speed otherwise it should be like light or it is in rest.

Let's develop equation (2-59) and see what does it mean:

$$\boldsymbol{f} = \frac{d^2(a.\tau.\boldsymbol{X})}{d\tau^2} = a.\tau.\frac{d^2\boldsymbol{X}}{d\tau^2} + 2.a.\boldsymbol{v} = a.\tau.\frac{d\boldsymbol{v}}{d\tau} + 2.a.\boldsymbol{v} = \frac{\boldsymbol{d^2}(\xi.\boldsymbol{X})}{dt^2} = \frac{d\boldsymbol{p}}{dt} - a.\boldsymbol{v} \qquad (2\text{-}111)$$

Let's note that equation (2-111) is independent from the choice of the origin of the referential.

Let's develop the conventional definition of the force:

$$\frac{d\boldsymbol{p}}{dt} = \frac{d(a.\tau.\boldsymbol{v})}{dt} = a.\boldsymbol{v} + a.\tau.\frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}\left(\frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\boldsymbol{v}}{dt} + \frac{m.\boldsymbol{v}}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.dt} \qquad (2\text{-}112)$$

It comes that:

$$a.\boldsymbol{v} = \frac{m.\boldsymbol{v}}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.dt} \qquad (2\text{-}113)$$

So:

$$a = \frac{m}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.d\tau} \qquad (2\text{-}114)$$

We have:

$$d\tau = d\left(\frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\tau_0}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2} \quad (2\text{-}115)$$

Replace (2-115) in (2-114) we have:

$$a = \frac{m}{\tau_0} \qquad (2\text{-}116)$$

We get nothing special.

Let's define the following force as:

$$\boldsymbol{G} = \frac{d\boldsymbol{P}}{ds} = \left(\frac{1}{c^2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right), \frac{1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\boldsymbol{p}}{dt}\right) = \left(\frac{\frac{dW}{dt}}{c^2.\sqrt{1-\frac{v^2}{c^2}}}, \frac{\boldsymbol{F}}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) \quad (2\text{-}117)$$

With that:

$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$ (2-118): the classical definition of the force.

$$\frac{dW}{dt} = \frac{d}{dt}\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{dH}{dt} \quad (2\text{-}119)$$

If we adopt the same definition in classical mechanics that W is the work of the classical force $\boldsymbol{F}$ than we have:

$$\boldsymbol{F}.\boldsymbol{v} = \frac{dW}{dt} = \frac{dE}{dt} \quad (2\text{-}120)$$

We have also this relation:

$$\boldsymbol{p} = \frac{E}{c^2}.\boldsymbol{v} \quad (2\text{-}121)$$

We have also the following invariant as a consequence of the invariance of the speed of light:

$$\frac{E^2}{c^2} - p^2 = m^2.c^2 \quad (2\text{-}122)$$

So from (2-122) and (2-121) we get:

$$v^2 = c^2 - \frac{m^2.c^6}{E^2} \quad (2\text{-}123)$$

From (2-117) we have:

$$\boldsymbol{f}.\boldsymbol{v} = \boldsymbol{F}.\boldsymbol{v} - a.v^2 = \frac{dE}{dt} - a.c^2 - \frac{a.m^2.c^6}{E^2} = \frac{dE}{dt} - a.c^2.\left(2 - \frac{v^2}{c^2}\right) = \frac{dE}{dt} - a.v^2 \quad (2\text{-}124)$$

It comes that:

$$v = c \quad (2\text{-}125)$$

The speed of the corpuscle is equal to $''c''$ only referring to its proper time but the referential which is attached to the corpuscle is not an inertial referential. The only way to get out from this contradiction is to accept that the corpuscle can have wave behaviour like light in a curved line motion. Along its inertial time coordinate the corpuscle is like light and had the speed $''c''$. It is like maintained in motion with a force:

$f = a.c$ (2-126)

The work of this force along the inertial length of the corpuscle is:

$\Delta E_{kinetic} = f.\Delta l$ (2-127)

With:

$\Delta l = l - l_0 = c(\tau - \tau_0) \approx \frac{1}{2}.\frac{\tau_0.v^2}{c}$ if $v \ll c$ (2-128)

Of course the force $f = a.c$ respect relativist invariance. This force ''maintain'' the corpuscle in motion with a speed c along its inertial coordinate. In the four dimensions space-time the motion of the corpuscle is in a curved universe line but we can consider this motion is constant between the time t and $t + dt$ and the corpuscle is maintained in motion by the force $\boldsymbol{f} = a.\boldsymbol{v}$ just in this laps of time. At the same time the corpuscle can have a waving behaviour.

Let's suppose that the corpuscle is in constant speed. The characteristics of a plane wave is its frequency ω and wave-vector **.** We can form a four-vector k^i as the following:

$k^0 = \frac{\omega}{c}$, $k^1 = k_x$, $k^2 = k_y$, $k^3 = k_z$ (2-129)

Of course the four components should have the same dimension.

If the corpuscle can have a waving behaviour necessary there is a relation ship between its four dimension moment and its four dimension wave-vector:

$p^i = \beta.k^i$ (2-130)

Where : β: a new universal constant .

So we have:

$\frac{E}{c} = \beta.\frac{\omega}{c}$ (2-131)

$\boldsymbol{p} = \beta.\boldsymbol{k}$ (2-132)

It comes that from equation (2-38) :

$(\beta.\frac{\omega}{c})^2 - (\beta.k)^2 = (m.c)^2$ (2-133)

In other terms equation (2-123) becomes:

$\frac{\omega^2}{c^2} - k^2 = (\frac{m.c}{\beta})^2$ (2-134)

The wave-vector associated to the corpuscle is not a linear function of the frequency i.e. the medium in which the corpuscle move is a dispersive medium. A dispersive medium for waves correspond for the corpuscle to a viscous medium : there is friction in space-time and this doesn't surprise us.

The group speed of the wave is as defined :

$v_g = \frac{d\omega}{dk} = \frac{d\omega}{dE}.\frac{dE}{dk} = v$ (2-135)

The phase speed is as defined:

$v_f = \frac{\omega}{k} = \frac{\beta}{p}.c.\sqrt{\frac{p^2}{\beta^2} + \frac{m^2.c^2}{\beta^2}} = c.\sqrt{1 + \frac{m^2.c^2}{p^2}} = c.\sqrt{1 + \frac{c^2}{v^2}.(1 - \frac{v^2}{c^2})} = \frac{c^2}{v}$ (2-136)

The corpuscle is like a packet of waves which are reinforced around its position and

annihilate themselves above. This condition requires that:

$\Delta k_x.\Delta x \geq 1$, $\Delta k_y.\Delta y \geq 1$, $\Delta k_z.\Delta z \geq 1$ (2-137)

$\Delta\omega.\Delta t \geq 1$ (2-138)

Where Δk: The uncertainty about the wave-vector;

ΔX: The uncertainty about the position of the corpuscle;

$\Delta\omega$: The uncertainty about the frequency (i.e. about the energy of the corpuscle)

Δt: The uncertainty about the time.

The equation of motion is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (2\text{-}139)$$

Which is only valid locally in the position $\boldsymbol{X} \pm \Delta\boldsymbol{X}$ at the time $\boldsymbol{t} \pm \Delta\boldsymbol{t}$ .So this equation can't be the solution to found the real trajectory of the corpuscle. The only way to found the trajectory of the corpuscle is to determine its action i.e. to apply the principle of least action in a curved space-time and respecting the principle of uncertainty (2-137) & (2-138).

3) The principle of least action for a charged corpuscle in motion in an electromagnetic field:

"Nature presents us with 3 such units. (build from G, c, e)"

G. Johnstone-Stoney (1881)

In the interaction of a charged corpuscle with an electromagnetic field we consider only its electric charge e which can be positive, negative or equal to zero and we neglect its spin the intrinsic momentum of the corpuscle.

The electromagnetic field is characterised by the quadric-potential A_i which its components are a function of coordinates and time. The action of the corpuscle is the sum of the action (2-91) for a free corpuscle and the action of the electromagnetic field:

$$S = \int_A^B (-mc.ds - \gamma.\frac{e}{c}A_i dx^i) \qquad (3\text{-}1)$$

The factor $\frac{1}{c}$ is chosen for commodity.

The coefficient γ is a conversion factor since there is not any known relationship between the potential vector or the electric charge with the MKS system or cgs system. The only fact which we know is that gravitational field is acting in great scale and electromagnetic field is acting in microscopic scale.

The time component of the quadric-potential is the *scalar potential* of the field and it is noted $A^0 = \varphi$ and the three space components of the field are the *vector potential* $\boldsymbol{A}$ of the field.

We have:

$$A^i = (\varphi, \boldsymbol{A}) \qquad (3\text{-}2)$$

We can write the integral (3-1) as:

$$S = \int_A^B (-mc.ds + \gamma.\frac{e}{c}.\boldsymbol{A}.d\boldsymbol{X} - \gamma e.\varphi dt) \qquad (3\text{-}3)$$

Introduce the speed $\boldsymbol{v} = \frac{d\boldsymbol{X}}{dt}$ of the corpuscle , equation (3-3) becomes:

$$S = \int_{t_1}^{t_2} \left(-m.c^2.\sqrt{1-\frac{v^2}{c^2}} + \gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi\right).dt \qquad (3\text{-}4)$$

So the Lagrange function of the corpuscle is:

$$L = -m.c^2.\sqrt{1-\frac{v^2}{c^2}} + \gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi \qquad (3\text{-}5)$$

The equation (3-5) is different from equation (2-21) for a free corpuscle by the term $\gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi$ which describe the interaction of the corpuscle with the field.

The derivative $\frac{\partial L}{\partial \boldsymbol{v}}$ is the generalised momentum of the corpuscle noted as. We found that:

$$\boldsymbol{P} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} + \gamma\frac{e}{c}.\boldsymbol{A} = \boldsymbol{p} + \gamma\frac{e}{c}.\boldsymbol{A} \qquad (3\text{-}6)$$

Where $\boldsymbol{p} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ is the ordinary momentum of the corpuscle.

The Hamilton function of the corpuscle in the field is:

$$H = \boldsymbol{v}.\frac{\partial L}{\partial \boldsymbol{v}} - L = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \gamma e.\varphi \qquad (3\text{-}7)$$

The Hamilton function should be written as a function of generalised momentum and as a function of speed. From (3-6) & (3-7) we have:

$$(\frac{H-\gamma e.\varphi}{c})^2 - (\boldsymbol{P} - \gamma\frac{e}{c}.\boldsymbol{A})^2 = m^2.c^2 \qquad (3\text{-}8)$$

Than we have:

$$H = \sqrt{m^2.c^4 + c^2.(\boldsymbol{P} - \gamma \frac{e}{c}.\boldsymbol{A})^2} + \gamma e.\varphi \qquad (3\text{-}9)$$

Let's write the Hamilton-Jacobi equation for a corpuscle placed in an electro-magnetic field. This equation is obtained by replacing in the Hamilton function the generalised momentum by $\frac{\partial S}{\partial \boldsymbol{X}}$ and H by $-\frac{\partial S}{\partial t}$. We get from (3-8):

$$(grad\, S - \gamma \frac{e}{c}.\boldsymbol{A})^2 - \frac{1}{c^2}.(\frac{\partial S}{\partial t} + \gamma e.\varphi)^2 + m^2.c^2 = 0 \qquad (3\text{-}10)$$

3-1)Locally equation of motion of an electric charge in a field:

We suppose that the electric charge is small and can't affect the electromagnetic field. We get the equation of motion by varying the action so we can use the Lagrange equations (2-5) as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{v}}\right) - \frac{\partial L}{\partial \boldsymbol{X}} = \boldsymbol{0} \qquad (3\text{-}11)$$

We neglect any non conservative force in equation (3-11). The Lagrange function L is given by equation (3-5).

The derivative $\frac{\partial L}{\partial \boldsymbol{v}}$ is given by equation (3-6). Also we have:

$$\frac{\partial L}{\partial \boldsymbol{X}} \equiv \nabla L = \gamma \frac{e}{c}.grad(\boldsymbol{A}.\boldsymbol{v}) - \gamma e.grad(\varphi) \qquad (3\text{-}12)$$

As we know in mathematics that for any two vectors $\boldsymbol{A}$ & $\boldsymbol{v}$ we have:

$$grad(\boldsymbol{A}.\boldsymbol{v}) = (\boldsymbol{A}\nabla).\boldsymbol{v} + (\boldsymbol{v}\nabla).\boldsymbol{A} + \boldsymbol{v} \times rot\boldsymbol{A} + \boldsymbol{A} \times rot\boldsymbol{v} \qquad (3\text{-}13)$$

Locally the speed $\boldsymbol{v}$ of the corpuscle is approximately constant so we take it constant . We have:

$$\frac{\partial L}{\partial \boldsymbol{X}} = \gamma \frac{e}{c}(\boldsymbol{v}\nabla).\boldsymbol{A} + \gamma \frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} - \gamma e.grad(\varphi) \qquad (3\text{-}14)$$

And :

$$(\boldsymbol{A}\nabla)\boldsymbol{v} + \boldsymbol{A} \times rot\boldsymbol{v} \approx \boldsymbol{0} \qquad (3\text{-}15)$$

So locally the Lagrange equations are as the following:

$$\frac{d}{dt}\left(\boldsymbol{p} + \gamma \frac{e}{c}\boldsymbol{A}\right) = \gamma \frac{e}{c}(\boldsymbol{v}\nabla).\boldsymbol{A} + \gamma \frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} - \gamma e.grad(\varphi) \qquad (3\text{-}16)$$

Or the total differential $\frac{d\boldsymbol{A}}{dt}.dt$ include two terms the local variation $\frac{\partial \boldsymbol{A}}{\partial t}.dt$ of the potential vector as a function of time in a given point of space and its variation when we translate of a distance $d\boldsymbol{X}$ to another point. This second term is equal to $(d\boldsymbol{X}\nabla)\boldsymbol{A}$. So we have:

$$\frac{d\boldsymbol{A}}{dt} = \frac{\partial \boldsymbol{A}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{A} \qquad (3\text{-}17)$$

Introduce equation (3-17) in equation (3-16) we get:

$$\frac{d\boldsymbol{p}}{dt} = -\gamma\frac{e}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma e.grad(\varphi) + \gamma\frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} \qquad (3\text{-}18)$$

Equation (3-18) is the locally equation of motion of a charged corpuscle in an electromagnetic field. In the first member of this equation we found the derivative of the momentum of the corpuscle to time so the second member of this equation represent the force exerted by the electromagnetic field on the electric charge. This force is composed of two terms. The first term is independent from the speed of the corpuscle but the second term it depends and it is proportional to the module of the speed and perpendicular to it.

The first term is called the *intensity of electric field,* it is noted **E** and is equal for one unit charge:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma.grad(\varphi) \qquad (3\text{-}19)$$

The factor after the speed $\boldsymbol{v}$ is the second term of the force applied on a unit charge and it is called *magnetic field vector* which is noted $\boldsymbol{B}$ and so we have:

$$\boldsymbol{B} = \frac{\gamma}{c}.rot\boldsymbol{A} \qquad (3\text{-}20)$$

So the locally equation of motion of an electric charge in an electromagnetic field is:

$$\frac{d\boldsymbol{p}}{dt} = e.\boldsymbol{E} + e.\boldsymbol{v} \times \boldsymbol{B} \qquad (3\text{-}21)$$

Let's establish the variation of the total energy of the corpuscle locally:

$$\frac{dE_{total}}{dt} = \boldsymbol{v}.\frac{d\boldsymbol{p}}{dt} = e\boldsymbol{E}.\boldsymbol{v} \qquad (3\text{-}22)$$

We remark that the work furnished to the corpuscle is only due to electric field, the magnetic field doesn't do any work for electric charges in motion.

Mechanical motion are locally invariants to the change of the direction of time which we can deduce it from $\frac{d\boldsymbol{p}}{dt} = \frac{d}{dt}\left(\frac{m.\frac{dX}{dt}}{\sqrt{1-\frac{\left(\frac{dX}{dt}\right)^2}{c^2}}}\right)$ in other terms the motion in inverse direction of a mechanical system is possible and product the same affects as in the first direction.

For electromagnetic field let's remark that when we do the following substitutions $t \to -t$, $\boldsymbol{E} \to \boldsymbol{E}$, $\boldsymbol{B} \to -\boldsymbol{B}$ the equation (3-21) doesn't change but regarding to equations (3-19) & (3-20) the scalar potential doesn't change and the potential vector change its sign: $\varphi \to \varphi$, $\boldsymbol{A} \to -\boldsymbol{A}$. So when a certain local motion is possible in the electromagnetic field, the motion in the inverse direction is also possible with the condition to inverse the direction of magnetic field.

3-2) Gauge invariance:

Let's remark that when we add a constant to potential vector and/or another constant to the scalar potential in the equations (3-20) and (3-19) than the electric field and the magnetic field doesn't change: the question is that are the potentials of the field determined in only one manner.

The electromagnetic field is characterised by its action on the charges in which they move and in the equation (3-21) there is only the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ so we can conclude that two fields are physically the same only if they are characterised by the same vectors $\boldsymbol{E}$ & $\boldsymbol{B}$.

For a given potentials $\boldsymbol{A}$ & φ we can determine the field by equations (3-19) & (3-20) but as we had seen that to a unique and the same field can correspond many different potentials. In the general case let's add to the components of the potential A_k the quantity $-\frac{\partial f}{\partial x^k}$ where f is an arbitrary function of coordinates and time. We get the new potential:

$$A'_k = A_k - \frac{\partial f}{\partial x^k} \qquad (3\text{-}23)$$

This substitution engenders in the integral of the action (3-4) a supplement term which is a total differential:

$$\gamma \frac{e}{c} . \frac{\partial f}{\partial x^k} . dx^k = d(\gamma \frac{e}{c} . f) \qquad (3\text{-}24)$$

Which doesn't affect the equations of motion.

If we introduce instead the quadric-dimensional potential another vector potential and another scalar potential and instead the coordinates x^i the coordinates ct, x , y, z we can write the fourth equalities (3-23) as:

$$\boldsymbol{A'} = \boldsymbol{A} + grad(f) \quad , \; \varphi' = \varphi - \frac{1}{c}.\frac{\partial f}{\partial t} \qquad (3\text{-}25)$$

It is very easy to verify that the electric field and the magnetic field given by equations (3-19) & (3-20) don't vary if we replace $\boldsymbol{A}$ & φ by the potentials $\boldsymbol{A'}$& φ' given by equations (3-25). So the transformation of potentials (3-23) doesn't affect the field. So the potentials are not defined in a unique manner, the vector potential is defined with a gradient function nearly and the scalar potential is also defined to time derivative of the same function nearly.

Only the values invariants referring to the transformations of potentials (3-25) have a physical signification. So all the equations should be invariants referring to this transformation: this invariance is called *gauge invariance.*

3-3) Unification of fields:

Let's take the locally equation of motion (2-54). It can be written as:

$$\frac{d}{dt}(\boldsymbol{p} + a.\boldsymbol{X}) = \boldsymbol{f} \qquad (3\text{-}26)$$

Where:

- $\boldsymbol{p}$: the momentum of the corpuscle;
- $\boldsymbol{X}$: the position of the corpuscle.

We can write the equation (3-26) as the following:

$$\frac{d\boldsymbol{P}}{dt} = \boldsymbol{f} \qquad (3\text{-}27)$$

With :

$$\boldsymbol{P} = \boldsymbol{p} + a.\boldsymbol{X} \qquad (3\text{-}28)$$

Equation (3-27) is like equation (3-16) but here the corpuscle is not charged. The only potential which can curve the motion of the corpuscle is the field of gravitation.

Let's define the following generalised momentum (canonical momentum) as the following:

$\boldsymbol{P} = \boldsymbol{p} + \mu.\boldsymbol{U}$ (3-29)

With:

$\boldsymbol{p}$: The momentum of the corpuscle;

$\mu = \gamma.\frac{e}{c}$ (3-30):if we are dealing with a charged corpuscle

$\mu = a$ (3-31): if we are dealing with a non charged corpuscle

$\boldsymbol{U} = \boldsymbol{A}$ (3-32): if we are dealing with a charged corpuscle in motion in an electromagnetic field;

$\boldsymbol{U} = \boldsymbol{X}$ (3-33): if we are dealing with a non charged corpuscle in motion in a gravitational field.

We should verify that the generalised momentum (3-29) is canonical.

We have for one dimension:

$P_x = p_x + \mu.U_x$ (3-34)

$[U_x, P_x] = \frac{\partial U_x}{\partial U_x}.\frac{\partial P_x}{\partial p_x} - \frac{\partial U_x}{\partial p_x}.\frac{\partial P_x}{\partial U_x} = 1 \times 1 - 0 \times \mu = 1$ (3-35)

For a given $P = P(U_x, p_x) = p_x + \mu.U_x$ we can associate to it a function $Q(U_x, p_x) = U_x$ with the objective that the dynamical variables change as defined is canonical ?. It signify that the transformation is reversible →necessary condition: $\frac{\partial P}{\partial p_x}$ & $\frac{\partial P}{\partial U_x}$ don't vanishes simultaneously. We have:

$\frac{\partial P}{\partial p_x} = 1$ & $\frac{\partial P}{\partial U_x} = \mu$ (3-36)

We generalise (3-34) & (3-35) for the other dimensions. So our transformation of dynamical variables is canonical. Than we conclude that the equation of motion (3-26) is defined only locally.

So we can write the Hamilton-Jacobi equation of motion for any corpuscle charged or not charged in any field by going back to the equation (3-10) as:

$$(grad\ S - \mu.\boldsymbol{U})^2 - \frac{1}{c^2}.(\frac{\partial S}{\partial t} + c.\mu.\varphi)^2 + m^2.c^2 = 0 \quad (3\text{-}37)$$

Let’s take a non charged corpuscle. The force of first specie which is applied on the corpuscle as referring to equation (3-19) is:

$$\boldsymbol{G} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{X}}{\partial t} - \gamma.grad(\varphi) = -grad(\varphi') \qquad (3\text{-}38)$$

With:

$\frac{\partial \boldsymbol{X}}{\partial t} = \boldsymbol{0}$: We accept that this force vary slowly with time or independent from time.

$\varphi' = \gamma.\varphi$: is the gravitational field

We choose the coefficient γ as $\boldsymbol{G}$ becomes acceleration. The corpuscle will be under a force of a first specie as in classical mechanics:

$$\boldsymbol{f} = m.\boldsymbol{G} \qquad (3\text{-}39)$$

The force of the second specie is as referring to equation (3-20):

$$\boldsymbol{F} = \frac{\gamma}{c}.rot\boldsymbol{X} = \boldsymbol{0} \qquad (3\text{-}40)$$

The total energy of the corpuscle as referring to equation (3-7) is:

$$E_{total} = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{a.c}{\gamma}.\varphi' \qquad (3\text{-}41)$$

If we have $v \ll c$ than the total energy is :

$$E_{total} \approx m.c^2 + \frac{1}{2}.m.v^2 + \varphi'' \quad (3\text{-}42)$$

With:

$$\varphi'' = \frac{a.c}{\gamma}.\varphi' \qquad (3\text{-}43)$$

3-3-1) Newton gauge:

Let’s take a corpuscle in rest. Which gravitational field it create?.

From equation (3-38) we have:

$div(\boldsymbol{G}) = -\nabla^2\varphi' = -\Delta\varphi'$ (3-44)

The *Newton gauge* is when we have:

$div(\boldsymbol{G}) = -4.\pi.G.\rho$ (3-45)

With:

G: Gravitationnel constant (*Newton constant*)

ρ: Density of masses

The general solution for the equation (3-45) is as:

$\varphi' = -G\int\frac{\rho dV}{R}$ (3-46)

Where:

$dV = dx.dy.dz$: Volume element;

R: The distance between the corpuscle in the center and the volume element.

Of course we suppose that $\boldsymbol{G}$ has a spherical symmetry.

For low speed corpuscles the equation (3-46) determines the gravitational field for any masses distribution. For one corpuscle we have:

$\varphi' = -\frac{G.m}{R}$ (3-47)

The force which is exerted on a corpuscle of a mass m' in this field is:

$f = -m'.\frac{\partial\varphi'}{\partial R} = -\frac{G.m.m'}{R^2}$ (3-48)

We found the *second law of Newton.*

3-3-2) Coulomb gauge :

Let's have a charge in rest .Which electrical field create?

From equation (3-19) & (3-20) we have in general:

$rot\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial}{\partial t}(rot\boldsymbol{A}) - \gamma.rotgrad(\varphi) = -\frac{\partial\boldsymbol{B}}{\partial t}$ (3-49)

$div\boldsymbol{B} = \boldsymbol{0}$ (3-50)

Equations (3-49) and (3-50) are called the *first group of Maxwell equations without sources* .

In electrostatic the *Coulomb gauge* is as:

$div\boldsymbol{A} = \nabla\boldsymbol{A} = \boldsymbol{0}$ (3-51)

So we have in electrostatic:

$div\boldsymbol{E} = \nabla\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial\nabla\boldsymbol{A}}{\partial t} - \gamma.\nabla^2\varphi = -\gamma.\nabla^2\varphi = -\nabla^2\varphi'$ (3-52)

$rot\boldsymbol{B} = \frac{\gamma}{c}\nabla(\nabla\boldsymbol{A}) - \nabla^2\boldsymbol{A} = -\nabla^2\boldsymbol{A}$ (3-53)

With:

$\varphi' = \gamma.\varphi$

Like the *Newton gauge* we suppose that the field $\boldsymbol{E}$ is spherical and radial and we impose that:

$div\boldsymbol{E} = 4.\pi.K.\rho$ (3-54)

With:

K: Electrical constant (*Coulomb constant)*;

ρ: Density of charges;

Equation (3-54) is called *First Maxwell equation with source.*

The general solution of equation (3-54) is as:

$\varphi' = K.\int\frac{\rho dV}{R}$ (3-55)

Where :

$dV = dx.dy.dz$: Volume element;

R: The distance between the charge in the center and the volume element.

For low speed charges the equation (3-55) determines the scalar potential for any charges distribution. For one charge we have:

$$\varphi' = \frac{K.e}{R} \quad (3\text{-}56)$$

The force which is exerted on a charge e' by the charge e is :

$$F = -e'.\frac{\partial \varphi\prime}{\partial R} = \frac{K.e.e\prime}{R^2} \quad (3\text{-}57)$$

So we found the *law of Coulomb.*

The electric field is as:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - grad(\varphi') = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} + \frac{\sigma.e}{R^3}.\boldsymbol{R} \quad (3\text{-}58)$$

If we add another condition which is that the electrical field vary slowly or constant in time , we get:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{0} \quad (3\text{-}59)$$

$$\boldsymbol{E} = \frac{K.e}{R^3}.\boldsymbol{R} = \frac{\boldsymbol{F}}{e\prime} \quad (3\text{-}60)$$

3-3-3) Continuity equation for masses:

The variation of density of masses as a function of time is:

$$\frac{\partial}{\partial t}\int \rho.dV \quad (3\text{-}61)$$

The variation of quantity of masses per unit time depends on the quantity of masses getting out or in the element volume. The quantity of masses going in this volume is equal to $\rho\boldsymbol{v}.d\boldsymbol{f}$ where $\boldsymbol{v}$ is the speed of a corpuscle in the point of space where exist the element. The total mass going out the volume is as:

$$\oint \rho\boldsymbol{v}.d\boldsymbol{f} \quad (3\text{-}62)$$

Where the integral (3-61) is extended to the total closed surface bordering the volume. So we have:

$$\frac{\partial}{\partial t}\int \rho.dV = -\oint \rho\boldsymbol{v}.d\boldsymbol{f} = -\oint \boldsymbol{j}.d\boldsymbol{f} \quad (3\text{-}63)$$

The negative sign forward the second member of equation (3-63) is that the first member should be positive when the total mass in the volume augment. The element of surface $d\boldsymbol{f}$ is oriented to the exterior of the volume.

Apply Gauss theorem to the second member of equation (3-63) we get:

$$\oint \boldsymbol{j}.d\boldsymbol{f} = \int div\boldsymbol{j}\, dV \qquad (3\text{-}64)$$

Replace (3-64) in equation (3-63) we have:

$$\int (div\boldsymbol{j} + \frac{\partial \rho}{\partial t})\, dV = 0 \qquad (3\text{-}65)$$

The equation (3-65) is valid for any volume so we should have:

$$div\boldsymbol{j} + \frac{\partial \rho}{\partial t} = 0 \qquad (3\text{-}66)$$

Equation (3-66) is *the continuity equation*.

3-3-4) Quadric-vector current of mass or flux of mass:

For a corpuscle an element of its mass is as:

$$dm = \rho.dV \qquad (3\text{-}67)$$

Multiplying the two terms by dx^i we get:

$$dm.dx^i = \rho.dV.dx^i = \rho.dV.\frac{dx^i}{dt}.dt \qquad (3\text{-}68)$$

In the left dm is a scalar and dx^i is a quadric-vector so the product is a quadric-vector. In the right $dV.dt$ is a scalar so $\rho.\frac{dx^i}{dt}$ is a quadric-vector noted as j^i and called *quadric-vector of density of current of mass or flux of mass.* We have:

$$j^i = \rho.\frac{dx^i}{dt} \qquad (3\text{-}69)$$

The three space components of this quadric-vector define the three dimensional flux of mass:

$$\boldsymbol{j} = \rho.\boldsymbol{v} \qquad (3\text{-}70)$$

The time component of this quadric-vector is $\rho.c$ so we have:

$$j^i = (\rho.c, \boldsymbol{j}) \qquad (3\text{-}71)$$

The total mass in a volume V is the mass of the corpuscle so we have:

$$m = \int \rho . dV = \frac{1}{c} . \int j^0 dV \qquad (3\text{-}72)$$

From equation (3-66) we deduce that:

$$div\boldsymbol{j} + \frac{\partial \rho}{\partial t} = \frac{\partial j^1}{\partial x^1} + \frac{\partial j^2}{\partial x^2} + \frac{\partial j^3}{\partial x^3} + \frac{1}{c} . \frac{\partial (\rho . c)}{\partial t} = \frac{\partial j^i}{\partial x^i} = 0 \qquad (3\text{-}73)$$

For vacuum where absence of mass we have from (3-72):

$$m_0 = \rho_0 . \frac{4}{3} . \pi . R^3 \qquad (3\text{-}74)$$

With:

m_0 : The mass of vacuum contained in a sphere of radius R

ρ_0 : The density of vacuum

$\boldsymbol{j_0} = \boldsymbol{0}$: Density of current of vacuum.

Normally we should take in consideration the action of vacuum on the corpuscle when establishing the equation of motion of the corpuscle referring to the principle of least action .

3-3-5) Applications in classic physics:

Let's have a classic corpuscle in motion nearly the border of the Universe. We draw a sphere tangent to its motion. The gravitational action on this corpuscle referring to (3-48) is:

$$F_0 = \frac{-G . m_0 . m}{R^2} \qquad (3\text{-}75)$$

Replace m_0 by its expression in (3-74) so:

$$F_0 = -G . \frac{4}{3} . \rho_0 . \pi . m . R \qquad (3\text{-}76)$$

The corpuscle interacts with every point of space-time as like an harmonic oscillator.

The density of vacuum referring to equation (1-5) & (1-6) is:

$$\rho_0 = \frac{M}{L^3} = \frac{a^2}{\hbar . c} \qquad (3\text{-}77)$$

So:

$F_0 = -\frac{4}{3}.\pi.G.\frac{a^2}{\hbar.c}.m.R$ (3-78)

For fine structure the electromagnetic force is the most important.

Let's have two charges e in interaction .From equation (3-57) we have:

$F = \frac{K.e^2}{R^2}$ (3-79)

To compare the generalized momentums of classic non charged corpuscle and a classic charged corpuscle, they differ by terms $a.\boldsymbol{X}$ and $.\frac{e}{c}.\boldsymbol{A}$. Let's choose the coefficient γ as a product of two coefficients $\gamma' and\ \varepsilon$ in order to get:

$a = \gamma'.\frac{e}{c}$ (3-80)

$\boldsymbol{X} = \varepsilon.\boldsymbol{A}$ (3-81)

If we replace (3-80) in (3-78) we get for fine structure:

$F = -\frac{4}{3}.\pi.G.\frac{\gamma'^2.e^2}{\hbar.c^3}.m.R$ (3-82)

The same conclusion, in fine structure charges interact with others like harmonic oscillator.

In equation (3-80) , a & c are universal constants , γ' is a conversion factor so we can deduce that there is an universal constant e_0 which has a dimension of electric charge. This constant is called *Maxwell constant*.

In fine structure the electrical force is so great compared to the gravitational force. For two charged corpuscles with the same mass and the same charge in absolute value we have:

$\frac{K.e^2}{R^2} \gg \frac{G.m^2}{R^2}$ (3-83)

So we get:

$m \ll e.\sqrt{\frac{K}{G}}$ (3-84)

The gravitational interaction is negligible if the mass of the corpuscles are under the following constant :

$$M_M = M_{JS} = e_0.\sqrt{\frac{K}{G}} = \frac{a.c}{\gamma'}.\sqrt{\frac{K}{G}} = \sqrt{\frac{K.e_0^2}{G}} \qquad (3\text{-}85)$$

For: $e_0 = e = 1.6\ 10^{-19}\ Coulomb\ , K = 9.10^9\ unities\ of\ MKSA\ system,$

and : $G = 6.67\ 10^{-11}\ N.m^2.kg^{-2}$,

We have:

$$M_M = 1.858\ \ 10^{-9}\ kg \qquad (3\text{-}86)$$

The constant M_M is called *Maxwell mass or G. Johnstone-Stoney mass*

We deduce two others constants build with $e_0\ , c\ \ and\ G$ as:

$$T_{JS} = \sqrt{\frac{K.e_0^2.G}{c^6}} = 4.59\ 10^{-45}\ s \qquad (3\text{-}87)$$

$$L_{JS} = \sqrt{\frac{K.e_0^2.G}{c^4}} = 1.37\ 10^{-36}\ m \qquad (3\text{-}88)$$

Which are the *G. Johnstone-Stoney time and length in the MKSA system.*

In the cgs-ues system take $K = 1$ in equations (3-85), (3-87) & (3-88).

The radius of interaction for microscopic sizes is as:

$$M_M = \frac{4}{3}.\pi.R_M^3.\rho_0 \qquad (3\text{-}89)$$

So:

$$R_M = (\frac{3}{4.\pi}.\frac{\hbar.c}{a^2}.M_M)^{\frac{1}{3}} \qquad (3\text{-}90)$$

The constant R_M is called *Maxwell radius.*

We define also the *Maxwell force:*

$$f_M = a.c \qquad (3\text{-}91)$$

Also the *Maxwell pressure:*

$w_M = \frac{f_0}{4.\pi.R_0^2}$ (3-92)

Maxwell period is as per definition:

$T_M = \frac{R_0}{c}$ (3-93)

With M_M, T_M, R_M we can define a new system of unities which match well with the microscopic scale. For great scale we take the system of unities as defined by equations (1-5), (1-6) & (1-7).

For microscopic scale the motion of the corpuscle is like nearly the center of the Universe. The vacuum around the corpuscle create an attractive/repulsive force because of the dissymmetry of the position of the two half of the Universe referring to the position of the corpuscle: one half is always more near to the corpuscle than the other half referring to the center: this force maintain the orbital speed of the corpuscle as constant in module after a certain distance of the corpuscle from the center :we say that it is due to *Dark Matter*.

For great scale the motion of the corpuscle is like nearly the border of the Universe. The vacuum in the sphere tangent to the motion of the corpuscle create a repulsive/attractive force in the direction of the center. The corpuscle becomes accelerated in the direction of the position of the corpuscle- center of the Universe: we say that it is due *Dark Energy*.

This phenomenon in the Universe is called *scale invariance gauge* [9].

Of course equation of continuity is also valid for charges: instead we speak about flux of masses, we say flux of charges…etc.

Negative charge correspond to negative pressure of vacuum in microscopic scale and vice versa.

3-3-6) Lorentz gauge:

The *second Maxwell equation with sources* is as the following [10]:

$rot\boldsymbol{B} = \frac{4.\pi.K}{c^2}.\boldsymbol{j} + \frac{1}{c^2}.\frac{\partial \boldsymbol{E}}{\partial t}$ (3-91)

It is possible to get the scalar potential φ and the vector potential $\boldsymbol{A}$ not coupled.

From equation (3-54) &(3-53)we have:

$$div\boldsymbol{E} = \nabla.\boldsymbol{E} = -\gamma.\nabla^2\varphi - \gamma.\frac{e}{c}.\frac{\partial\nabla.\boldsymbol{A}}{\partial t} = 4.\pi.K.\rho \qquad (3\text{-}92)$$

$$\nabla\times\boldsymbol{B} = \frac{\gamma}{c}\nabla\times(\nabla\times\boldsymbol{A}) = \frac{\gamma}{c}\nabla(\nabla.\boldsymbol{A}) - \frac{\gamma}{c}\nabla^2\boldsymbol{A} = \frac{4.\pi.K}{c^2}.\boldsymbol{j} + \frac{1}{c^2}.\frac{\partial\boldsymbol{E}}{\partial t} \qquad (3\text{-}93)$$

With the condition (*Lorentz gauge*):

$$\frac{\gamma}{c^2}.\frac{\partial}{\partial t}\varphi + \gamma.\frac{e}{c}.\nabla.\boldsymbol{A} = 0 \qquad (3\text{-}94)$$

We get:

$$(3\text{-}92)\rightarrow\frac{\gamma}{c^2}.\frac{\partial^2\varphi}{\partial t^2} - \gamma.\nabla^2\varphi - \frac{\partial}{\partial t}\left(\frac{\gamma}{c^2}.\frac{\partial\varphi}{\partial t} + \gamma.\frac{e}{c}\nabla\boldsymbol{A}\right) = 4.\pi.K.\rho \qquad (3\text{-}95)$$

$$(3\text{-}93)\rightarrow\frac{\gamma e}{c}.\nabla^2\boldsymbol{A} = -\frac{4.\pi.K.e}{c^2}.\boldsymbol{j} - \frac{e}{c^2}.\left(-\frac{\gamma}{c}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \gamma\nabla\frac{\partial\varphi}{\partial t}\right) + \nabla\left(\frac{\gamma}{c^2}.\frac{\partial}{\partial t}\varphi\right) = -\frac{4.\pi.K.e}{c^2}.\boldsymbol{j} + \frac{\boldsymbol{\gamma.e}}{\boldsymbol{c^3}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} + \nabla[\left(\frac{\gamma}{c^2} + \frac{\gamma.e}{c^2}\right)\frac{\partial\varphi}{\partial t}] \qquad (3\text{-}96)$$

Let’s add another condition:

$$\frac{\partial\varphi}{\partial t} = 0 \qquad (3\text{-}97)$$

(3-96) becomes:

$$\gamma.\nabla^2\boldsymbol{A} = -\frac{4.\pi.K}{c}.\boldsymbol{j} + \frac{\gamma}{c^2}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} \qquad (3\text{-}98)$$

Finally we have:

$$(3\text{-}95)\rightarrow\frac{1}{c^2}.\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = -\nabla^2\varphi = \frac{4.\pi.K.\rho}{\gamma} \qquad (3\text{-}99)$$

$$(3\text{-}96)\rightarrow\frac{\boldsymbol{1}}{\boldsymbol{c^2}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \nabla^2\boldsymbol{A} = \frac{\boldsymbol{1}}{\boldsymbol{c^2}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} = \frac{4.\pi.K}{c.\gamma}.\boldsymbol{j} \qquad (3\text{-}100)$$

If vacuum, equations (3-99) and (3-100) had solutions in microscopic scale & also in great scale.

3-3-7) Equation of motion of a charged corpuscle:

Equations (3-49) and (3-50) are the *two first Maxwell equations without sources.* Those equations don't characterize completely the electromagnetic field because if we want to determine for example $\frac{\partial \boldsymbol{B}}{\partial t}$ we haven't another equation for $\frac{\partial \boldsymbol{E}}{\partial t}$.

Equation (3-50) can be written as referring to Gauss theorem:

$$\int div\boldsymbol{B}\, dV = \oint \boldsymbol{B}.d\boldsymbol{f} = 0 = \Phi \qquad (3\text{-}101)$$

The integral of the second term in (3-101) is for all the surface bordering the volume on which done the integral of the first term.

The integral of a vector taken on a surface is called *flux of this vector* (Φ) through the surface. So the flux of an electromagnetic field through a closed surface is equal to zero.

Equation (3-49) can be written as referring to Stokes theorem:

$$\int rot\boldsymbol{E}.d\boldsymbol{f} = \oint \boldsymbol{E}.d\boldsymbol{l} = -\frac{\partial}{\partial t}\int \boldsymbol{B}.d\boldsymbol{f} = -\frac{\partial \Phi}{\partial t} = f.e.m \qquad (3\text{-}102)$$

The integral of a vector along a closed contour is called *circulation* of this vector along this contour. The circulation of electric field is called *force electromotive (fem.)* in the considered contour.

Let's write the equation of motion of a charged corpuscle in motion in en electromagnetic field using quadric-coordinates.

From equation (3-1) we have:

$$\delta S = \delta \int \left(-m.c.ds - \gamma.\frac{e}{c}.A_i dx^i\right) = 0 \qquad (3\text{-}103)$$

Or $ds = \sqrt{dx_i dx^i}$ so we get:

$$\delta S = -\int (m.c.\frac{dx_i d\delta x^i}{ds} + \gamma.\frac{e}{c}.A_i d\delta x^i + \gamma.\frac{e}{c}.\delta A_i dx^i)$$

$$= -\int \left(m.c.u_i d\delta x^i + \gamma.\frac{e}{c}.A_i d\delta x^i + \gamma.\frac{e}{c}.\delta A_i dx^i\right) = 0 \quad (3\text{-}104)$$

With:

$u_i = \frac{dx_i}{ds}$: The quadric-vector speed.

Integrate by party the first two terms in (3-104):

$\int (m.c.u_i d\delta x^i + \gamma.\frac{e}{c}.A_i d\delta x^i) = \int (m.c.u_i + \gamma.\frac{e}{c}.A_i) d\delta x^i - \int (m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i =$

$0 - \int (m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i$ (3-105)

Replace (3-105) in (3-104) we get:

$$\delta S = -\int \{(m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i - \gamma.\frac{e}{c}.\delta A_i dx^i\}$$

$$= -\int \{(m.c.du_i + \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k} dx^k)\delta x^i - \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k}\delta x^k dx^i\} \text{(3-106)}$$

Equation (3-106) because we have:

$dA_i = \frac{\partial A_i}{\partial x^k} dx^k$, $\delta A_i = \frac{\partial A_i}{\partial x^k}\delta x^k$

Replace in (3-106) $du_i = \frac{du_i}{ds}.ds$ and $dx^i = u^i ds$ and permit indices i & k in the third term (which doesn't change the result):

$\delta S = -\int \{(m.c.\frac{du_i}{ds} ds\delta x^i + \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k} u^k ds\delta x^i - \gamma.\frac{e}{c}.\frac{\partial A_k}{\partial x^i} u^k ds\delta x^i =$

$-\int \{m.c.\frac{du_i}{ds} - \gamma.\frac{e}{c}.(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k})u^k\} ds\delta x^i = 0$ (3-107)

The variations δx^i are arbitrary so we should have:

$m.c.\frac{du_i}{ds} - \gamma.\frac{e}{c}.\left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right) u^k = 0$ (3-108)

Equation (3-108) is the equation of motion of the charge written in quadric coordinates.

Let's introduce the notation:

$F_{ik} = \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right)$ (3-109)

This anti-symmetric tensor is called *tensor of electromagnetic field.* The equation of motion of the corpuscle becomes as:

$$m.c.\frac{du_i}{ds} = \gamma.\frac{e}{c}.F^{ik}u_k \qquad (3\text{-}110)$$

With the notation $A_i = (\varphi, -\boldsymbol{A})$ we have from (3-109) & (3-49) & (3-50):

$$i = 0, k = 0 \rightarrow F_{00} = \frac{\partial A_0}{\partial x^0} - \frac{\partial A_0}{\partial x^0} = 0$$

$$i = 0, k = 1 \rightarrow F_{01} = \frac{\partial A_1}{\partial x^0} - \frac{\partial A_0}{\partial x^1} = -\frac{\partial A_x}{\partial (c.t)} - \frac{\partial \varphi}{\partial x} = \frac{1}{\gamma}.E_x$$

$$i = 0, k = 2 \rightarrow F_{02} = \frac{1}{\gamma}.E_y$$

$$i = 0, k = 3 \rightarrow F_{03} = \frac{1}{\gamma}.E_z$$

$$i = 1, k = 0 \rightarrow F_{10} = \frac{\partial A_0}{\partial x^1} - \frac{\partial A_1}{\partial x^0} = \frac{\partial \varphi}{\partial x} + \frac{\partial A_x}{\partial (c.t)} = \frac{-1}{\gamma}.E_x$$

$$i = 1, k = 1 \rightarrow F_{11} = 0$$

$$i = 1, k = 2 \rightarrow F_{12} = \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} = -\frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} = -\frac{c}{\gamma}B_z$$

$$i = 1, k = 3 \rightarrow F_{13} = \frac{\partial A_3}{\partial x^1} - \frac{\partial A_1}{\partial x^3} = -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} = \frac{c}{\gamma}B_y$$

$$i = 2, k = 3 \rightarrow F_{23} = \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} = -\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} = -\frac{c}{\gamma}B_x$$

……(etc.)

So:

$$F_{ik} = \frac{1}{\gamma}.\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -c.B_z & c.B_y \\ -E_y & c.B_z & 0 & -c.B_x \\ -E_z & -c.B_y & c.B_x & 0 \end{pmatrix} \qquad (3\text{-}111)$$

$$F^{ik} = \frac{1}{\gamma}.\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -c.B_z & c.B_y \\ E_y & c.B_z & 0 & -c.B_x \\ E_z & -c.B_y & c.B_x & 0 \end{pmatrix} \qquad (3\text{-}112)$$

For space components $(i = 1,2,3)$ the equation (3-110) is exactly the vector equation of motion (3-21).

For time component $(i = 0)$ the equation (3-110) is exactly the work equation (3-22) which becomes from the locally equation of motion.

We can verify that only three equations from the four equations (3-110) are independents: we can multiply the two members of equation (3-110) by u^i and as we know that u^i and $\frac{du_i}{ds}$ are orthogonal and the second member is :

$$F^{ik}u_k u^i = F_{ik}u^k u^i = 0 \qquad (3\text{-}113)$$

From equation (3-104) we have:

$$\delta S = -(mcu_i + \gamma\frac{e}{c}.A_i)\delta x^i \qquad (3\text{-}114)$$

So we have:

$$-\frac{\partial S}{\partial x^i} = m.c.u_i + \gamma\frac{e}{c}.A_i = p_i + \gamma\frac{e}{c}A_i = P_i \qquad (3\text{-}115)$$

So:

$$P^i = \left(\frac{\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}+\gamma.e.\varphi}{c}, \boldsymbol{p} + \gamma\frac{e}{c}.\boldsymbol{A}\right) \qquad (3\text{-}116)$$

4) Lorentz transformations of the field:

4-1) Tensor algebra:

4-1-1) four-vector position of a universe point :

Let's have an inertial reference $R(O, ct, x, y, z)$ .A universe point $\boldsymbol{X}$ have the coordinates in this space-time (*Minkovski vectorial space*) as the following [10] & [11]:

$$\boldsymbol{X} = \sum_i x^i.e_i \qquad (4\text{-}1)$$

Where:

$\boldsymbol{e}_i$: Base of the Minkovski vectorial space M.

$x^0 = c.t,\ x^1 = x\ , x^2 = y\ , x^3 = z$: *The contra-variant coordinates of the Universe point.*

$i = 0,1,2,3$: Variable indices.

With *Einstein convention for repetitive indices* we write (4-1) as the following:

$$\boldsymbol{X} = x^i . e_i \qquad (4\text{-}2)$$

We do implicitly summation if only the same indices are viewed *one time in the top and another time in the down.* For example T_{ii} represent a diagonal element of a tensor (matrices) and not a summation. The trace of the matrices is T_i^i so the convention summation is applied for repetitive indices.

We call *free indices* an indices on which the summation rule is not applied and so it remains as it is in the final expression and we call *mute indices* an indices which is the subject of an implicitly summation and don't appear as it is in the final expression. For free indices we respect the *rule of "balance"*. In an equation the free indices which appears in the two members should corresponds one to one and appears in the same position (up or down).

We can associate for our space-time a scalar product , which is of course commutative. Let's have two four-vectors $\boldsymbol{X} = x^i . \boldsymbol{e_i}$ & $\boldsymbol{Y} = y^j . \boldsymbol{e_j}$, the scalar product is as follows:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i y^j \boldsymbol{e_i}.\boldsymbol{e_j} \qquad (4\text{-}3)$$

We pose a table of numbers of two indices as:

$$g_{ij} = \boldsymbol{e_i}.\boldsymbol{e_j} \qquad (4\text{-}4)$$

The scalar product is as:

$$\boldsymbol{X}.\boldsymbol{Y} = g_{ij} x^i y^j \qquad (4\text{-}5)$$

We hope of course that the scalar product have an expression which compatible with the notion of interval in four dimension space-time. For this we should have $\boldsymbol{X}.\boldsymbol{X} = c^2 t^2 - x^2 - y^2 - z^2$. We get a convenient scalar product is the g_{ij} which we call *metric tensor* is as follows:

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (4\text{-}6)$$

In this table i are the line indices and j are the column indices.

4-2) Covariant coordinates:

We pose:

$$y_i = g_{ij} y^j \qquad (4\text{-}7)$$

The repetitive indices (up and down) in the right member of (4-7) are j .We should made summation on all values of this indices. The indices i are a free index which appears with the same name with the same position in the two members of the equation. We call *covariant coordinates* of the Universe point the components y_i.

With this notation we have $y_0 = y^0$ & $y_i = -y^i \ for \ i = 1,2,3$. The metric tensor permit to download or to upload the indices as an escalator with a general rule: the downloading or the uploading of a space indices changes its sign and the downloading and uploading of a time indices doesn't change the sign.

With those notations the scalar product of two four-vectors is as:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i y_i \qquad (4\text{-}8)$$

And also:

$$\boldsymbol{X}.\boldsymbol{Y} = x_i y^i \quad with \ x_i = g_{ij} x^j \qquad (4\text{-}9)$$

And:

$$\boldsymbol{X}.\boldsymbol{e_i} = x^j \boldsymbol{e_j}.\boldsymbol{e_i} = g_{ji} x^j = x_i \qquad (4\text{-}10)$$

We can also write the inverse transformation which gives us the contra-covariant coordinates as a function of covariant coordinates by defining a new *table of numbers* g^{ij} as:

$$y^i = g^{ij} y_j \qquad (4\text{-}11)$$

We can write:

$$y^i = g^{ij} y_j = g^{ij} g_{jk} y^k = \delta^i_k y^k \qquad (4\text{-}12)$$

With:

$g^{ij}g_{jk} = \delta_k^i$ (4-13)

With:

$\delta_k^i = 0\ if\ i \neq j\ and\ 1\ if\ i = j$: Kronecker symbols.

As matrices the g^{ij} is the inverse of the matrices g_{ij}. We have:

$$g^{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (4\text{-}14)$$

4-3) Duality:

For a space vector M we can define linear forms. A linear form associate for every vector a real number (or complex). We note $\tilde{R}$ a linear form and $\tilde{R}(\boldsymbol{X})$ the real number associated to the vector . A linear form is a linear function of its vector argument. So we have such relations as $\tilde{R}(\boldsymbol{X}+\boldsymbol{Y}) = \tilde{R}(\boldsymbol{X}) + \tilde{R}(\boldsymbol{Y})$ etc…

We can define on the ensemble of linear forms an addition and a multiplication with a real scalar. This two operations confer to the ensemble of linear forms a structure of space vector: it is called the *dual* of our initial space vector M and noted as M^*.

Also if M have a finite dimension, its dual have the same dimension. Also if it is defined a scalar product in the space vector M we can define a bijection between the space and its dual. We associate to every vector $\boldsymbol{Y}$ a linear form $\tilde{Y}$ defined as $\tilde{Y}(\boldsymbol{X}) = \boldsymbol{Y}.\boldsymbol{X}$ .A Universe point in four dimensions space-time can be considered as a vector or a linear form. In fact the two representations are the same one subject.

In the dual space we choose the base:

$\tilde{\epsilon}^i(\boldsymbol{e}_j) = \delta_j^i$ (4-15)

We have:

$\boldsymbol{e}_i.\boldsymbol{Y} = \boldsymbol{e}_i.e_j y^j = g_{ij}y^j = y_i$ (4-16)

$\tilde{\epsilon}^i(\boldsymbol{Y}) = \tilde{\epsilon}^i(y^j e_j) = y^j\tilde{\epsilon}^i(\boldsymbol{e_j}) = y^j\delta_j^i = y^i \neq y_i\ \ for\ i = 1,2,3.$ (4-17)

So we have in general:

$\boldsymbol{e}_i.\boldsymbol{Y} = y_i \qquad \& \qquad \tilde{\epsilon}^i(\boldsymbol{Y}) = y^i \qquad\qquad (4\text{-}18)$

So we can form from a four-vector $\boldsymbol{Y} = y^j e_j$ a linear form $y_j\tilde{\epsilon}^j$. The action of this linear form on a vector $\boldsymbol{X} = x^i e_i$ is as $y_j\tilde{\epsilon}^j(x^i e_i) = y_j x^j = \boldsymbol{Y}.\boldsymbol{X}$. The form constructed coincide with the linear form $\tilde{Y}$ associated to the vector $\boldsymbol{Y}$. If the components contra-variants are the components of the four-vector, the covariant components are the components of the linear form associated to this vector on the dual base. As we can confound vector and linear form in one physical subject, the writing of contra-variants components and covariant components are different writing of the same quantity.

4-4) Change of referential, change of base:

We can write the contra-variant coordinates by Lorentz transformations as:

$x'^i = \mathcal{L}^i{}_j x^j \qquad\qquad (4\text{-}19)$

Where x^j are the contra-variant components in the referential R and x'^i are the contra-variant components of the Universe point in the referential R'. We associated the line indices i for the new referential and the column indices j for the old referential. In the Lorentz transformations the table $\mathcal{L}^i{}_j$ is the following matrices:

$$\mathcal{L}^i{}_j = \begin{pmatrix} \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} & \frac{-\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}} & 0 & 0 \\ \frac{-\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}} & \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (4\text{-}20)$$

The inverse transformation is as follows:

$x^i = (\mathcal{L}^{-1})^i{}_j x'^j \qquad\qquad (4\text{-}21)$

The inverse matrices $(\mathcal{L}^{-1})^i{}_j$ is obtained by change in the matrices $\mathcal{L}^i_j$, $V\ by - V$.

The transformations of covariant coordinates in the dual space are defined as:

$x'_i = \mathcal{L}_i{}^j x_j \qquad\qquad (4\text{-}22)$

Where $\mathcal{L}_i^{\ j}$ is a table of numbers. Note that $\mathcal{L}_i^{\ j}$ is different from $\mathcal{L}^j_{\ \ i}$.We can deduce the link between $\mathcal{L}_j^i$ and $\mathcal{L}_i^j$ by the invariance of the scalar product which is a consequence of the invariance of the interval. We have in this case:

$$x'^i y'_i = x^i y_i \quad with \quad x'^i = \mathcal{L}^i_{\ j} x^j \ \ and \ \ y'_i = \mathcal{L}_i^{\ k} y_k \qquad (4\text{-}23)$$

So:

$$\mathcal{L}^i_{\ j} x^j \mathcal{L}_i^{\ k} y_k = x^i y_i = x^j y_j = x^j y_k \delta_j^k \qquad (4\text{-}24)$$

The relation (4-24) should be verified for every couple of vectors so:

$$\mathcal{L}^i_{\ j} \mathcal{L}_i^{\ k} = \delta_j^k \qquad (4\text{-}26)$$

Let’s note that the left term in (4-26) is not a product of two matrices. It is a summation on two lines indices. In Lorentz transformations the matrices are symmetric and the matrices of transformations of covariant coordinates is the inverse matrices of transformations of contra-variant coordinates and we get this matrices by changing the speed V in contra-variant matrices by $-V$.

The link between the two transformations is:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i g_{ij} y^j = x'^k g_{kl} y'^l \qquad (4\text{-}27)$$

The metric tensor which expressing an orthogonal base is the same in all bases and so it is an invariant by Lorentz transformation. From (4-27)and (4-23) we get:

$$x^i g_{ij} y^j = \mathcal{L}^k_{\ j} x^j g_{kl} \mathcal{L}^l_{\ m} y^m \qquad (4\text{-}28)$$

This relation (4-28) is always verified , so we deduce that:

$$g_{jm} = \mathcal{L}^k_{\ j} g_{kl} \mathcal{L}^l_{\ m} \qquad (4\text{-}29)$$

Multiply (4-29) by g^{nj} we get:

$$g^{nj} g_{jm} = \delta_m^n = g^{nj} \mathcal{L}^k_{\ j} g_{kl} \mathcal{L}^l_{\ m} \qquad (4\text{-}30)$$

So:

$$\left(g^{nj} g_{kl} \mathcal{L}^k_{\ j}\right) \mathcal{L}^l_{\ m} = \delta_m^n \qquad (4\text{-}31)$$

(4-26)→ $\left(g^{nj}g_{kl}\mathcal{L}^{k}{}_{j}\right)\mathcal{L}^{l}{}_{m} = \mathcal{L}_{m}^{i}\mathcal{L}_{i}^{n} = \mathcal{L}_{m}^{l}\mathcal{L}_{l}^{n}$ (4-32)

So:

$$\mathcal{L}_{l}{}^{n} = g^{nj}g_{kl}\mathcal{L}^{k}{}_{j} \quad (4\text{-}33)$$

The inverse relation of (4-33) is:

$$\mathcal{L}^{n}{}_{l} = g_{il}g^{nj}\mathcal{L}_{j}{}^{i} \quad (4\text{-}34)$$

Let's remark that for the coordinates the change of a space coordinate indices change the sign and the change of the time coordinate indices doesn't change the sign . In the passage from a transformation to another only changes in the sign the coefficients indexed space & time. The coefficients only space or only time are unchanged. This is what we observe in Lorentz transformation.

We can choose that acting on Lorentz transformations is only by the metric tensor and define new quantities as the following:

$$\mathcal{L}^{ij} = g^{jk}\mathcal{L}^{i}{}_{k} \quad (4\text{-}35)$$

$$\mathcal{L}_{ij} = g_{ik}\mathcal{L}^{k}{}_{j} \quad (4\text{-}36)$$

We have with those tensors:

$$x'^{i} = \mathcal{L}^{i}{}_{j}x^{j} = \mathcal{L}^{i}{}_{j}g^{jk}x_{k} = \mathcal{L}^{ik}x_{k} \quad (4\text{-}37)$$

Where the line indices is assigned to the new referential and the column indices is assigned to the old referential.

Also we have:

$$x'_{i} = \mathcal{L}_{ik}x^{k} \quad (4\text{-}38)$$

For the inverse change of the referential we can use the relation (4-26) without using the transformation $\mathcal{L}^{-1}$.We have:

$$\mathcal{L}^{i}{}_{m}x'_{i} = \mathcal{L}^{i}{}_{m}\mathcal{L}_{i}{}^{k}x_{k} = \delta_{m}^{k}x_{k} = x_{m} \quad (4\text{-}39)$$

So:

$x_i = \mathcal{L}^k{}_i x'_k$ (4-40)

This transformation is of course different from the direct transformation:

$x'_i = \mathcal{L}_i{}^k x_k$ (4-41)

The indices relative to the new referential i and the other relative to the old referential k change the position (up/down) between the two expressions. In terms of matrices in case of Lorentz transformation it correspond to a change of sign in the components space & time and so taking the inverse of the matrices.

We can do the same for the contra-variant components of coordinates or any combination of mixture components as the following:

$x'_i = \mathcal{L}_i{}^j x_j$ (4-42)

$x'^i = \mathcal{L}^i{}_j x^j$ (4-43)

$x_i = \mathcal{L}^j{}_i x'_j$ (4-44)

$x^i = \mathcal{L}_j{}^i x'^j$ (4-45)

Those four combinations are obtained by respecting the balance rule by assigning the first indices to the new referential, assigning the second indices to the old referential and summing on the indices of the coordinate which to be transformed.

We finish this paragraph by examining the transformations of the base vectors of our space-time. We remark that:

$y_i = \boldsymbol{Y}. e_i = \tilde{Y}(e_i)$ & $y'_i = \boldsymbol{Y}. e'_i = \tilde{Y}(e'_i)$ (4-46)

Where the e'_i are the transformations of the base vectors. We can write:

$\boldsymbol{Y}. e'_i = \mathcal{L}_i{}^j \boldsymbol{Y}. e_j$ (4-47)

In other terms the law of vector transformations is the same law of the transformations of covariant coordinates which the inverse of the one of contra-variant components transformations.

For the dual base we have:

$y'^i = \widetilde{e'}^i(\boldsymbol{Y})$ & $y^i = \tilde{e}^i(\boldsymbol{Y})$ (4-48)

So we deduce:

$$\widetilde{e'}^i = \mathcal{L}^i{}_j \tilde{e}^j \qquad (4\text{-}49)$$

The vectors of the dual base are transformed like the contra-variant components.

4-5) Tensors:

4-5-1) Contra-variant tensors :

The operation of tensors product permits to associate to a vector space M a space $M \otimes M$ more great. For every couple of vectors $\boldsymbol{X}$ & $\boldsymbol{Y}$ of M we associate a vector $\boldsymbol{X} \otimes \boldsymbol{Y}$ of $M \otimes M$.A base of $M \otimes M$ is formed of 16 tensor products obtained with the four-vectors base of M , $e_i \otimes e_j$. The components of $\boldsymbol{X} \otimes \boldsymbol{Y}$ in this base are the components of $\boldsymbol{X}$ & $\boldsymbol{Y}$ as the following:

$$\boldsymbol{X} \otimes \boldsymbol{Y} = x^i y^j e_i \otimes e_j \qquad (4\text{-}50)$$

The dimension of tensor product space is 16. We define a tensor of order 2 completely contra-variant T^{ij} which components are defined on the base $e_i \otimes e_j$. In a base change with Lorentz transformation the new components of the tensor are as:

$$T'^{ij} = \mathcal{L}^i{}_k \mathcal{L}^j{}_m T^{km} \qquad (4\text{-}51)$$

The inverse transformation is as:

$$T^{ij} = \mathcal{L}_k{}^i \mathcal{L}_m^{\ j} T'^{km} \qquad (4\text{-}52)$$

We can also consider that the tensor T^{ik} is the image of its dual M^* in M. The image $\boldsymbol{W}$ of a vector $\boldsymbol{V}$ is :

$$W^i = T^{ij} V_j \qquad (4\text{-}53)$$

Its transformation as a four vector is:

$$W'^i = \mathcal{L}^i{}_k W^k = \mathcal{L}^i{}_k T^{kj} V_k \qquad (4\text{-}54)$$

But:

$$V_k = \mathcal{L}^m{}_k V'_m \qquad (4\text{-}56)$$

So:

$W'^i = \mathcal{L}^i{}_k \mathcal{L}^j{}_m T^{km} V'_j = T'^{ij} V'_j$ (4-57)

The operation of tensor product can be generalised for any number of terms. We can define the space $M^{\otimes k}$ tensor product of M , k manner its self. The elements of this space have a dimension 4^k are the tensors completely contra-variant of order k and their elements are written as $T^{ijkl\dots p}$.Those components are transformed as k Lorentz transformations.

4-5-2) Covariant tensors, Mix tensors :

Which was done for the space M can be done for its dual M^* .We can define tensors of an order 2 completely covariant where the components are written on the dual base product tensor $\tilde{e}^i \otimes \tilde{e}^j$ as T_{ij} . The Lorentz transformations of those quantities are:

$T'_{ij} = \mathcal{L}_i{}^k \mathcal{L}_j{}^l T_{kl}$ (4-58)

We can do the tensor product of any number of dual spaces. We can also define subjects as the tensor product of the space M with its dual M^*.We obtain mix tensors of an order 2 (or more if we use many times M & $M*$) which the components are written as $T^i{}_j$ for $M \otimes M^*$ and $T_i{}^j$ for $M^* \otimes M$. The transformation rule of such mix tensor is :

$T'^i{}_j = \mathcal{L}^i{}_l \mathcal{L}_j{}^k T^l{}_k$ (4-59)

And it can be generalised for every mix tensor of any order.

Covariant components and contra-variants components describe the same physical subject. The same thing is for tensors: a physical quantity represented as a tensor can be also written as a tensor completely contra-variant, completely covariant, or mix in arbitrary manner. Like for four-vectors the metric tensor g^{ij} or g_{ij} can be used to upload or download indices, so we can write:

$T^{ij} = g^{ik} g^{jl} T_{kl}$ (4-60)

$T^i{}_j = g^{ik} g_{jl} T_k{}^l$ (4-61)

$T^i{}_j = g_{jl} T^{il}$ (4-62)

In terms of linear applications, all those forms are different manners to write the image $\boldsymbol{Y}$ of a four-vector:

$$Y^i = T^{ik}X_k = T^i{}_k X^k \qquad (4\text{-}63)$$

And:

$$Y_i = T_{ik}X^k = T_i{}^k X_k \qquad (4\text{-}64)$$

4-5-3) Terminologies:

A tensor of an order 2 is symmetric if:

$$T^{ij} = T^{ji} \qquad (4\text{-}65)$$

We deduce immediately $T_{ij} = T_{ji}$ & $T^i{}_j = T_j{}^i$. So for a symmetric mix tensor we can write it as T_j^i without order of indices. Note that in this case it doesn't implies that T_j^i is the same T_i^j.

A tensor of an order2 is anti-symmetric if:

$$T^{ij} = -T^{ji} \qquad (4\text{-}66)$$

A symmetric tensor can be written as:

$$T^{ij} = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix} = (\boldsymbol{a}, \boldsymbol{b}) \qquad (4\text{-}67)$$

Where $\boldsymbol{a}$ is a vector and $\boldsymbol{b}$ is a pseudo-vector (which is transformed to its symmetric opposite in a base change included a space reflexion). The couple electric field/magnetic field obey to those conditions.

We call *trace* of a tensor of an order 2 the quantity $T^i{}_i = T_i{}^i$.

We call *contraction* of a tensor the expression like $T^i{}_i{}^j$. The contraction of a tensor order k is a tensor order $k-2$. The contraction of a tensor order 3 for example gives a tensor order 1 i.e. a four-vector. The trace is a contraction of a tensor order 2 and it gives a tensor order 0 i.e. a four-scalar.

Example for contraction of a tensor order3:

$T'^{i}{}_{i}{}^{j} = \mathcal{L}^{i}{}_{l}\mathcal{L}_{i}{}^{m}\mathcal{L}^{j}{}_{n}T^{l}{}_{m}{}^{n} = \delta_{l}^{m}\mathcal{L}^{j}{}_{n}T^{l}{}_{m}{}^{n} = \mathcal{L}^{j}{}_{n}T^{l}{}_{l}{}^{n}$ (4-68)

So it is a four-vector.

As a tensor we have the metric tensor which is invariant by Lorentz transformation. It is a symmetric tensor. Its mix form $g_j^i = g^{ik}g_{kj} = \delta_j^i$. The Kronecker symbol is the mix form of the metric tensor. The relation between the contra-variant form and the covariant form $g^{ij}g_{jk} = \delta_k^i$ is only a simple downloading of indices.

Finally we define a tensor order 4 completely anti-symmetric (Levi-Civita tensor) ϵ^{ijkl}.By the 256 elements of this tensor only are not equal to zero whose indices correspond to one permutation of (0,1,2,3). If the permutation is pair the correspondent element is equal to +1. It is equal to -1 if the permutation is impaired. So there is 24 elements of the tensor not equal to zero, 12 equal to +1 and 12 equal to -1. We have $\epsilon^{ijkl} = -\epsilon_{ijkl}$. Finally we have: $\epsilon^{ijkl}\epsilon_{ijkl} = -24$.

4-5-4) Derivation & vector analysis:

4-5-4-1) Derivation:

We can define for a four-vector which is a Universe point , the derivation by the contra-variant coordinate as:

$\partial_i = \frac{\partial}{\partial x^i}$ (4-69)

For a scalar function its variation is:

$df = \partial_i f(x^i).dx^i = \frac{\partial f}{\partial x^i}.dx^i$ (4-70)

df is a scalar, and dx^i is a contra-variant vector, ∂_i is a covariant *vector*. It is transformed as it is in a Lorentz transformation:

$\partial'_i = \mathcal{L}_i{}^j\partial_i$ (4-71)

Where ∂' represents the derivatives according to the new contra-variant coordinates.

The derivative according to the covariant coordinates is:

$\partial^i = g^{ij}\partial_j$ (4-72)

4-5-4-2) Vector analysis:

If f is a scalar function, $\partial_i f$ generalise the gradient and we have:

$\partial_i f = (\frac{\partial f}{c\partial t}, \nabla f)$ (4-73)

And:

$\partial^i f = (\frac{\partial f}{c\partial t}, -\nabla f)$ (4-74)

If we have a four-vector $A^i(x^j) = (a^0, \boldsymbol{a})$ its divergence is defined as:

$\partial^i A_i = \partial_i A^i = \frac{\partial a^0}{c\partial t} + \nabla.\boldsymbol{a}$ (4-75)

The analogue of the rotational is a tensor of order 2 completely anti-symmetric :

$\partial^i A^j - \partial^j A^i$ (4-76)

In its covariant form the rotational is:

$\partial_i A_j - \partial_j A_i$ (4-77)

The *Laplace operator* of the space-time is the norm of the vector ∂^i :

$\partial_i \partial^i = \frac{\partial^2}{c^2\partial t^2} - \Delta = \Box$ (4-78)

Which is the dalembertian $\Box$.

4-5-4-3) Integration:

We define a volume integral of space-time for any types of quantity as :

$\int d\Omega$ (4-79)

Where $d\Omega = cdtdxdydz$ the integral element in space-time.

A surface in space in three dimensions is a variety in three dimensions. We can define an integral on those surfaces (a flux) with the condition to define a four-vector element surface dS^i . A surface element is a little subject of three dimensions. It is defined by three four-vector dx^i, dy^i, dz^i . dS^i should be orthogonal to any vector of the element and its length should be a measure of the *volume* of the surface element. To define dS^i we form at first a tensor of an order 3, dS^{ijk} as:

$$dS^{ijk} = \begin{vmatrix} dx^i & dy^i & dz^i \\ dx^j & dy^j & dz^j \\ dx^k & dy^k & dz^k \end{vmatrix} \qquad (4\text{-}80)$$

The surface element is obtained by contracting this tensor with the tensor of order 4 completely anti-symmetric:

$$dS^i = -\frac{1}{6}\epsilon^{ijkl}\, dS_{jkl} \qquad (4\text{-}81)$$

We establish for surface integrals a theorem which generalise the Gauss theorem as:

$$\int_S A^i dS_i = \int_V \partial_i A^i d\Omega \qquad (4\text{-}82)$$

Where V is a volume in space-time and S is its surface border.

So the integral of the divergence extended to all the space is equal to the flux on the *sphere at infinite*. This is in general equal to zero for physical fields.

We can also define an integral on two dimensions varieties . The element of the integral is a tensor anti-symmetric of an order 2 madden on the vectors dx^i& dy^j delimitate the integral element:

$$df^{ij} = dx^i dy^j - dx^j dy^i \qquad (4\text{-}83)$$

Finally we can define a curvilinear integral on a universe line. The theorem of Stokes link the integral on a variety in two dimensions to the integral on its contour:

$$\int A_i dx^i = \int df^{ij}(\partial_i A_j - \partial_j A_i) \qquad (4\text{-}85)$$

5)Generalised equation of motion:

5-1) A corpuscle in a field:

Let's have a system of many corpuscles in a free field. The total action of this system is as:

$$S = S_{free\ corpuscles} + S_{free\ \ fields} + S_{Interaction} \qquad (5\text{-}1)$$

We consider here charged corpuscles to facilitate writing the equation of motion. For non charged corpuscles do the conversion factors in unified field as defined in (3-30), (3-31), (3-32) & (3-33).

In a first step we consider only one charged corpuscle in interaction with a free field. In a second step we consider many charged corpuscles in interaction between each other and the free field.

The action of a free corpuscle is as:

$$S_{free\ corpuscles} = -mc \int ds \qquad (5\text{-}2)$$

The field can be represented by a unique potential four-vector as $A^i = (\varphi, \boldsymbol{A})$ and its action is as:

$$S_{Intercation} = -\int \gamma.\frac{e}{c} A_i dx^i = -q \int A_i dx^i \qquad (5\text{-}3)$$

With:

$q = \gamma.\frac{e}{c}$ is a constant.

5-2) Electromagnetic field tensor :

F_{ik} is per definition an anti-symmetric tensor of an order 2, the four-rotational of the potential $(\varphi, \boldsymbol{A})$. It depends only of six independent coordinates. The three space-time coordinates are the components of a space vector, and the three only space coordinates are the components of a pseudo-vector.

We can write the space-time components as the following:

$$F_{0i} = \partial_0 A_i - \partial_i A_0 = -\frac{\partial A_i}{c\partial t} - \frac{\partial \varphi}{\partial x^i} = \frac{E_i}{\gamma} \quad \text{for } i = 1,2,3 \qquad (5\text{-}4)$$

We pose:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma.grad(\varphi) \qquad (5\text{-}5)$$

This is called electric field the real space vector defined.

The space coordinates of the field tensor are:

$$F_{12} = -\frac{c.B_z}{\gamma} = -\frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} \qquad (5\text{-}6)$$

$$F_{13} = \frac{c.B_y}{\gamma} = -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} \qquad (5\text{-}7)$$

$$F_{23} = -\frac{c.B_x}{\gamma} = -\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} \qquad (5\text{-}8)$$

If we introduce the pseudo-vector called *magnetic field*:

$$\boldsymbol{B} = \frac{\gamma}{c}.rot\boldsymbol{A} \qquad (5\text{-}9)$$

The electromagnetic tensor describes well the Maxwell equations of electromagnetism.

We have:

$$F_{ik} = (\boldsymbol{E}, \boldsymbol{B}) \quad \& \quad F^{ik} = (-\boldsymbol{E}, \boldsymbol{B}) \qquad (5\text{-}10)$$

5-2-1) Change of referential for the field:

We have in an inertial referential:

$$F'^{ik}\left(x'^i = \mathcal{L}^i{}_j x^j\right) = \mathcal{L}^i{}_l \mathcal{L}^k{}_m F^{lm}(x^j) \qquad (5\text{-}11)$$

Where the quantities F'^{ik} are relative to the new referential R' .

The transformations of fields are as the following:

$$E'_x = E_x \qquad (5\text{-}12)$$

$$E'_y = \frac{E_y - V.B_z}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}13)$$

$$E'_z = \frac{E_z + V.B_y}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}14)$$

$$B'_x = B_x \qquad (5\text{-}15)$$

$$B'_y = \frac{B_y + \frac{V}{c^2}.E_z}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}16)$$

$$B'_z = \frac{B_z - \frac{V}{c^2}.E_y}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}17)$$

For the inverse transformations change $V\ by - V$.

The transformations of potentials are as the following [12]:

$$A^0 = \frac{A'^0 + \frac{V}{c}.A'^1}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}18)$$

$$A^1 = \frac{A'^1 + \frac{V}{c}A'^0}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (5\text{-}19)$$

$$A^2 = A'^2 \qquad (5\text{-}20)$$

$$A^3 = A'^3 \qquad (5\text{-}21)$$

For covariant components of the potentials we have:

$$A_0 = \frac{A'_0 - \frac{V}{c}.A'_1}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_1 = \frac{A'_1 - \frac{V}{c}.A'_0}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_2 = A'_2 \quad , \quad A_3 = A'_3 \qquad (5\text{-}22)$$

With the four-vector $A^i = (\varphi, \boldsymbol{A})$ we have:

$$\varphi = \frac{\varphi' + \frac{V}{c}.A'_x}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_x = \frac{A'_x + \frac{V}{c}.\varphi'}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_y = A'_y \quad , \quad A_z = A'_z \qquad (5\text{-}23)$$

5-2-2) Invariants of the field:

There is two invariants which have physical interest. They are:

$$F_{ik}F^{ik} = inv. \qquad (5\text{-}24)$$

$$\epsilon^{iklm}F_{ik}F_{lm} = inv. \qquad (5\text{-}25)$$

This is due to the power of mathematics.

It comes that:

$$c^2B^2 - E^2 = inv. \qquad (5\text{-}26)$$

$$\boldsymbol{E}.\boldsymbol{B} = inv \qquad (5\text{-}26)$$

Another approach be described for the invariants of the field represented by anti-symmetric four-tensor.

Let's consider the complex vector:

$$\boldsymbol{F} = \boldsymbol{E} + ic\boldsymbol{B} \qquad (5\text{-}27)$$

The Lorentz transformation of this vector along the axle (O, x) according to (5-12)...(5-17) is as:

$$F_x = F'_x \quad , \quad F_y = F'_y ch\theta - iF'_z sh\theta = F'_y \cos(i\theta) - F'_z \sin(i\theta) \qquad (5\text{-}28)$$

$$F_z = F'_z \cos(i\theta) + F'_y \sin(i\theta) \qquad , \qquad th(\theta) = \frac{V}{c} \qquad (5\text{-}29)$$

The rotation of the vector $\boldsymbol{F}$ in the plan (O, x, t) of the four-dimensional space (it is the Lorentz transformation which we search here) is equivalent of a rotation of an imaginary angle in the plan (O, y, z) of the three dimensional space. The ensemble of all possible rotations in the four-dimensional space (included the simples rotations of axles $(x, y$ & $z)$ is equivalent to the ensemble of all possible rotations of complexes angles in the three dimensional space (for the six rotation angles in the four-dimensional space correspond three complexes rotation angles of the three dimensional referential).

The unique invariant of the vector according to those rotations is its square $F^2 = E^2 - c^2B^2 + 2ic\boldsymbol{E}.\boldsymbol{B}$ .So the real quantities $E^2 - c^2B^2$ and $\boldsymbol{E}.\boldsymbol{B}$ are the unique invariants of the tensor F_{ik}.

5-2-3) First group of Maxwell equations:

Equation (3-109) signifies the electromagnetic tensor is the rotational of the potential. In three dimensions this propriety implies the nullity of its divergence. Let's establish this propriety in four dimensions. We have:

$$F_{ik} = \partial_i A_k - \partial_k A_i \qquad (5\text{-}30)$$

We deduce that:

$$\partial_j F_{ik} = \partial_j \partial_i A_k - \partial_j \partial_k A_i \qquad (5\text{-}31)$$

$$\partial_k F_{ji} = \partial_k \partial_j A_i - \partial_k \partial_i A_j \qquad (5\text{-}32)$$

$$\partial_i F_{kj} = \partial_i \partial_k A_j - \partial_i \partial_j A_k \qquad (5\text{-}33)$$

The sum of (5-31), (5-32) & (5-33) gives us:

$$\partial_j F_{ik} + \partial_k F_{ji} + \partial_i F_{kj} = 0 \qquad (5\text{-}34)$$

There is only four independents equations of (5-34) where $i \neq j \neq k$. Otherwise the components of (5-34) are equal to zero.

The first one is for indices 1,2,3:

$$\partial_1 F_{23} + \partial_3 F_{12} + \partial_2 F_{31} = 0 \qquad (5\text{-}35)$$

i.e.:

$$\nabla.\boldsymbol{B} = 0 \qquad (5\text{-}36)$$

For the other three equations we have also that:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad (5\text{-}37)$$

So we found the first pair of Maxwell equations (homogeny Maxwell equations)which are the existence of a scalar potential and a vector potential.

5-2-4) Fields as a function of sources:

We will establish the equations which links the field to its sources i.e. to the motion of charged corpuscles. In the following we suppose that is imposed the dynamics of corpuscles and we are interested only to the dynamics of the field. The dynamic variables are the values of potentials or fields in every space-time point.

5-2-4-1) Interaction field-current:

We consider an ensemble of punctual charged corpuscles whom motion is imposed and they are indexed with indices (α) .

Instead to pose that the charges are punctual we consider that the charge is repatriated in a continuous form. This allows as defining the *density of charge* ρ and to pose that ρdV is the charge contained in the volume dV.The density of charge is a function of coordinates and time. The integral of ρ represents for a given volume the charge contained in this volume. We shouldn't forget that the charges are punctual and that the density ρ is equal to zero everywhere except in the points where punctual charges localised; the integral $\int \rho dV$ should be equal to the sum of the charges contained in this volume. This permits us to represent the density ρ as:

$\rho = \sum_\alpha e_{(\alpha)}\delta(\boldsymbol{r} - \boldsymbol{r}(\alpha))$ (5-38)

Where:

$e_{(\alpha)}, \boldsymbol{r}_{(\alpha)}$ are respectively the charge and the position of the corpuscle α.

$\delta(x)$ is the function defined as $\delta(x) = 0$ for every $x \neq 0$, for $x = 0, \delta(0) = \infty$ but $\int_{-\infty}^{+\infty} \delta(x)dx = 1$.

If $f(x)$ is an arbitrary continuous function than:

$$\int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$$

In consequence we have $\int_{-\infty}^{+\infty} f(x)\delta(x)dx = f(0)$.

The limit of integration can be different than $\pm\infty$ and the domain of integration can be anyone but should contain the point where δ exist.

The signification of those equalities is that their members furnish the same result when used as a factors under the sign of integration:

$$\delta(-x) = \delta(x)\, , \delta(ax) = \frac{1}{|a|}\delta(x)\ .$$

And in general we have:

$$\delta[\varphi(x)] = \sum_i \frac{1}{|\varphi'(a_i)|}\delta(x - a_i)$$

Where the a_i are solution of $\varphi(x) = 0$ and $\varphi'(a_i)$ the derivative of $\varphi(x)$ at the point a_i.

For three dimensional space we can define a function $\delta(\boldsymbol{r})$ which equal to zero everywhere except in the origin the three-dimensional coordinates system and also its integral extended to the total space is equal to one. This function can be represented in the form of a product of $(x)\delta(y)\delta(z)$.

Per definition the charge of a corpuscle is an invariant i.e. it is independent from the choice of the referential. The density ρ is not an invariant but the product ρdV is an invariant.

Let's multiply by dx^i the two terms of the equality $= \rho dV$:

$dedx^i = \rho dV dx^i = \rho dV dt \frac{dx^i}{dt}$ (5-39)

In the left we found a four-vector (because de is a scalar and dx^i is a four-vector).In consequence we should found in the right a four-vector. As $dVdt$ is a scalar ,so $\rho\frac{dx^i}{dt}$ is a four-vector. This one noted j^i is called four-vector of *current density*:

$$j^i = \rho\frac{dx^i}{dt} \qquad (5\text{-}40)$$

The three space components of this four-vector define the three-dimensional density of current:

$$\boldsymbol{j} = \rho\boldsymbol{v} \qquad (5\text{-}41)$$

Where $\boldsymbol{v}$ is the speed of the considered charge. The time component of this four-vector is ρc . So we have:

$$j^i = (\rho c, \boldsymbol{j}) \qquad (5\text{-}41)$$

The total charge contained in all space is equal to the integral $\int \rho dV$ extended for all space. We can represent this integral in a four-dimensional form:

$$\int \rho dV = \frac{1}{c}\int j^0 dV = \frac{1}{c}\int j^i\, dS_i \qquad (5\text{-}42)$$

Where the integral is extended to a four-dimensional hyper-plan orthogonal to the axle x^0 .In a general manner the integral $\frac{1}{c}\int j^i dS_i$ extended to an arbitrary hyper-plan represents the sum of charges whom the universe lines cut this hyper-surface.

Instead of punctual charges e we introduce a continuous repartition of density ρ and so the action due to interaction charge-charge and charge-current is:

$$S_{interaction} = -\frac{1}{c}\int \rho\, A_i dx^i dV \qquad (5\text{-}43)$$

If we write it in this form:

$$S_{interaction} = -\frac{1}{c}\int \rho\frac{dx^i}{dt}A_i dVdt = -\frac{1}{c^2}\int A_i j^i\, d\Omega \qquad (5\text{-}44)$$

Where :

$$d\Omega = dVcdt = cdtdxdydz \qquad (5\text{-}45) \text{ (four-volume)}$$

5-2-4-2) Interaction charge-field:

To find the form of the action $S_{free\ field}$ we refer to an important propriety of electromagnetic fields which is that the experience shows that the electromagnetic fields satisfy the *principle of superposition*: the field generated by a system of charges result only in a simple addition of fields due to every charge taken separately. In other terms the field vector resultant is equal to the sum of all vectors values in the point of every fields considered separately. Every solution of field equations is a field which can be realised in the Nature. According to the principle of superposition the sum of two fields should be a field which can exist in the Nature and should verify the equations of field.

It is known that the linear differentials equations had the propriety that the sum of their solutions is also a solution. In consequence the equations of the electromagnetic field should be linear differentials equations.

So in the action $S_{free\ field}$ we should have a quadratic expression under the integral referring to the field.

The potentials of the field can't be used in the expression of the action $S_{free\ field}$ because they are not defined in one manner (univocal manner and this univocal manner have no importance in the definition of $S_{interaction}$. We conclude that $S_{free\ field}$ is an integral of the tensor F_{ik} of the electromagnetic field. But because the action should be a scalar , it should be the integral of a scalar. The unique scalar existent in this case is the product $F_{ik}F^{ik}$. The function under the sign of the integral in the expression of the action $S_{free\ field}$ shouldn't contain any derivative of F_{ik} because that the Lagrange function can't contain except the coordinates of the system ,only the firsts derivatives according to time. The role of *coordinates* (i.e. the variables according to them we execute the variations of principle of least action) is assumed here by the potentials A_k of the field. Reminder that in classical mechanics the Lagrange function of a mechanical system contain only the coordinates of corpuscles and their firsts derivatives according to time.

Concerning the quantity $\epsilon^{iklm}F_{ik}F_{lm}$ it represents the total four-dimensional divergence and its insertion in the expression of the $S_{free\ field}$ doesn't affect the equations of motion. This quantity is excluded from the expression of the action independently of the fact that is a pseudo-scalar. This pseudo scalar can be represented as a form of four-divergence $\epsilon^{iklm}F_{ik}F_{lm} = 4\frac{\partial}{\partial x^i}(\epsilon^{iklm}A_k\frac{\partial}{\partial x^l}A_m)$ which can be easily verified because ϵ^{iklm} is anti-symmetric.

So the action of fields is as:

$$S_{free\ field} = -\sqrt{\frac{\kappa}{2048.\pi^3.G}}\int F_{ik}F^{ik}d\Omega \qquad (5\text{-}46)$$

Where κ & G are positive constants to choose one and determine the other.

The total action is:

$$S = -\{\int mcds + \frac{1}{c^2}\int A_i j^i\, d\Omega + \sqrt{\frac{\kappa}{2048.\pi^3.G}}\int F_{ik}F^{ik}d\Omega\} \qquad (5\text{-}47)$$

For many charges the action is the sum of equation (5-47).

6) The second pair of Maxwell equations:

When we search to establish the equation of the field from the principle of least action , we are obligated to pose that the motion of the charges are given and to vary only the potentials of the field (which play in this case the role of *coordinates* of the system).In the inverse sense to establish the equations of motion, we had pose that the field is given and we vary only the trajectory of the corpuscle.

In consequence the variation of the first term of equation (5-47) is maintained equal to zero but in the second term we should only vary the current j^i.So:

$$\delta S = -\frac{1}{c}\int[\frac{1}{c}j^i\delta A_i + \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}\int F_{ik}\delta F^{ik}]d\Omega \qquad (6\text{-}1)$$

For equation (6-1) take in consideration that $F_{ik}\delta F^{ik} \equiv F^{ik}\delta F_{ik}$.

Substitute in (6-1) $F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}$ we get:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i\delta A_i + \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^i}\delta A_k - \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^k}\delta A_i\}d\Omega \ (6\text{-}2)$$

Permute in the second term of (6-2) the indices i & k on which we do the summation and replace F_{ki} by $-F_{ik}$:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i\delta A_i - \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^k}\delta A_i\}d\Omega \qquad (6\text{-}3)$$

Integrate by party the second integral which means apply the theorem of Gauss:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i + \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\frac{\partial F^{ik}}{\partial x^k}\}\delta A_i d\Omega - \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\int F^{ik}\delta A_i dS_k \qquad (6\text{-}4)$$

In the second term we should take its value in the limits of integration. The limits of integration on the coordinates are extended to the infinite because the field disappear in the infinite. In the limits of integration on time i.e. in the initial and final instants given the variation of the potentials is equal to zero because according to the principle of least action those potentials are known in those instants. In consequence the second term of (328) is equal to zero and thus we get:

$$\int\{\frac{1}{c}j^i + \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\frac{\partial F^{ik}}{\partial x^k}\}\delta A_i d\Omega = 0 \qquad (6\text{-}5)$$

As the principle of least action implies that the variations δA_i are arbitrary, the coefficient of δA_i in (6-5) should be equal to zero:

$$\frac{\partial F^{ik}}{\partial x^k} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}j^i \qquad (6\text{-}6)$$

Rewrite those equations $(i = 0,1,2,3)$ in three-dimensional form.

For $i = 1$ we have:

$$\frac{1}{c}\frac{\partial F^{10}}{\partial t} + \frac{\partial F^{11}}{\partial x} + \frac{\partial F^{12}}{\partial y} + \frac{\partial F^{13}}{\partial z} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}j^1 \qquad (6\text{-}7)$$

By substituting the values of the components of the tensor F^{ik} we get:

$$\frac{1}{c}\frac{\partial E_x}{\partial t} - c.\frac{\partial B_z}{\partial y} + c.\frac{\partial B_y}{\partial z} = -\frac{\gamma}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}j_x \qquad (6\text{-}8)$$

The equation (6-8) and the succeeded equations for $(i = 2,3)$ can be written as a unique vector form:

$$rot\boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{\gamma}{c^3}\sqrt{\frac{128.\pi^3.G}{\kappa}}\boldsymbol{j} \qquad (6\text{-}9)$$

Finally for $i = 0$ we have:

$$\frac{\partial F^{00}}{\partial x^0} + \frac{\partial F^{01}}{\partial x^1} + \frac{\partial F^{02}}{\partial x^2} + \frac{\partial F^{03}}{\partial x^3} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}j^0 \qquad (6\text{-}10)$$

By substituting the values of the tensor F^{ik} and the current j^0 we get:

$$divE = \gamma \frac{\rho}{c}\sqrt{\frac{128.\pi^3.G}{\kappa}} \qquad (6\text{-}11)$$

The equations (6-9) and (6-11) are the second pair of Maxwell equations as formulated by H.A.Lorentz for the electromagnetic field in vacuum contained punctual charges.

We can write equation (6-11) as:

$$divE = 4\pi K\rho \qquad (6\text{-}12)$$

With K constant. So we have:

$$\gamma = Kc\sqrt{\frac{\kappa}{8.\pi.G}} \qquad (6\text{-}13)$$

In fact don't forget that conversion coefficient γ is a product of two conversion coefficients γ'& ε . If we resolve the problem for one coefficient, it remains unsolved for the other so the second group of Maxwell equations remains inhomogeneous equations.

To resolve the problem for one coefficient we had build a theory so there is experiences to do & so there are new technologies to rise.

Resolving the problem for the second conversion coefficient needs a new theory to build, new experiences to do and new technologies will rise but we will notice that this coefficient is in fact a product of two coefficients....etc. So the science will be never had an end: welcome to the city of science.

7) Continuity equation for charges:

The variation of the charge contained in a given volume is represented by the derivative $\frac{\partial}{\partial t}\int \rho dV$.

In other hand the variation per unit of time depends on the quantity of charge going out or in this volume. The quantity of charges going in this volume per unit time among the element $\boldsymbol{df}$ of the surface bordering this volume is equal to $\rho\boldsymbol{v}\boldsymbol{df}$ where $\boldsymbol{v}$ is the speed of displacement of the charge in the point of space where exist the element $\boldsymbol{df}$. As it is useful in usage , the vector $\boldsymbol{df}$ is directed in the same sense of the extern vector orthogonal to this surface i.e. in the same sense of the orthogonal vector directed out the considered volume. So the quantity

$\rho\boldsymbol{v}d\boldsymbol{f}$ is positive if the charge go out the volume and negative if the charge go in the volume. The total charge going out per unit time of a given volume is equal to $\oint \rho\boldsymbol{v}d\boldsymbol{f}$ where the integral is extended for all the closed surface bordering this volume. So we have:

$$\frac{\partial}{\partial t}\int \rho dV = -\oint \rho\boldsymbol{v}d\boldsymbol{f} \qquad (7\text{-}1)$$

The minus sign before the second member of (7-1) is introduced to consider the first member positive when the total charge in the volume is augmenting. The equation (7-1) is the conservation law of continuous charge called also *continuity equation* written in an integral form.

We remark that $\rho\boldsymbol{v}$ is the density of current, we can write the equation (7-1) as:

$$\frac{\partial}{\partial t}\int \rho dV = -\oint \boldsymbol{j}d\boldsymbol{f} \qquad (7\text{-}2)$$

In a differential form we apply Gauss theorem for the second member of (339) :

$$\oint \boldsymbol{j}d\boldsymbol{f} = \int div\boldsymbol{j}dV \qquad (7\text{-}3)$$

We get:

$$\int \left(div\boldsymbol{j} + \frac{\partial\rho}{\partial t}\right) dV = 0 \qquad (7\text{-}4)$$

Equation (7-4) should be verified by integrate in any volume, so we should have:

$$div\boldsymbol{j} + \frac{\partial\rho}{\partial t} = 0 \qquad (7\text{-}5)$$

(7-5) is the differential form of the continuity equation.

We can insure that the expression (5-38) which give ρ as a function of δ verify automatically the equation (7-5).

Let's suppose that it exist only one charge as:

$$\rho = e\delta(\boldsymbol{r} - \boldsymbol{r_0}) \qquad (7\text{-}6)$$

The current is :

$$\boldsymbol{j} = e\boldsymbol{v}\delta(\boldsymbol{r} - \boldsymbol{r_0}) \qquad (7\text{-}7)$$

Where $\boldsymbol{v}$ **:** is the speed of the charge.

Calculate the derivative $\frac{\partial \rho}{\partial t}$. When the charge move its coordinates varies so $\boldsymbol{r_0}$ vary.

We have:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial \boldsymbol{r_0}} \frac{\partial \boldsymbol{r_0}}{\partial t}$$

Or $\frac{\partial \boldsymbol{r_0}}{\partial t}$ is the speed $\boldsymbol{v}$ of the charge. In the other hand as ρ is a function of $\boldsymbol{r} - \boldsymbol{r_0}$ we have $\frac{\partial \rho}{\partial \boldsymbol{r_0}} = -\frac{\partial \rho}{\partial \boldsymbol{r}}$ and so in consequence:

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{v} grad\rho = -div(\rho \boldsymbol{v}) = -div\boldsymbol{j}$$ considering that $\boldsymbol{v}$ as independent from $\boldsymbol{r}$.

So we get the equation (7-5).

In a four-dimensional form the equation (7-5) is obtained by putting equal to zero the four-divergence of the four-current:

$$\frac{\partial j^i}{\partial x^i} = 0 \qquad (7\text{-}8)$$

In equation (5-42) we had established that the total charge can represented as $\frac{1}{c} \int j^i \, dS_i$ where the integration is extended to the hyper-plan $x^0 = const.$ In another instant the total charge can be represented by a similar integral extended to another hyper-plan orthogonal to the axle x^0 . We can easily verify that the law of charge conservation is coming from the equation (7-8) i.e. the integral $\int j^i \, dS_i$ is the same for any hyper-plan of integration $x^0 = const.$

The difference between the integrals $\int j^i \, dS_i$ taken on two hyper-surfaces $x^0 = const$ can be written as $\oint j^i dS_i$ where the integral is extended to the closed hyper-surface bordering the four-volume existing between the considered hyper-plans (this integral is different from the difference obtained by an integral extended to the *lateral* hyper-surface localised at the infinite which is eliminated because there is no charge in the infinite). With Gauss theorem we can transform this one on an integral in the four-volume existing between the two hyper-plans and be insured that :

$$\oint j^i dS_i = \int \frac{\partial j^i}{\partial x^i} d\Omega = 0 \qquad (7\text{-}9).$$

CQFD.

This demonstration remains available for two integrals $\int j^i dS_i$ extended to two hyper-surfaces infinites chosen arbitrary (and not only for hyper-plans $x^0 = const$ which is bordering all the three-dimensional space. Those considerations shows that the integral $\frac{1}{c}\int j^i \, dS_i$ only a unique value (equal to the total charge contained in the space for any integration hyper-surface.

We had seen in equation (3-94) that the gauge invariance of equations implies the conservation of charge . In the equation of motion (5-47) let's replace A_i by $A_i \frac{\partial f}{\partial x^i}$, the integral $\frac{1}{c^2}\int j^i \frac{\partial f}{\partial x^i} d\Omega$ will be added to the second term of (5-47): this is the conservation of charge given by the continuity equation (7-5) which allow us to write the expression under the symbol of integration as a four-divergence $\frac{\partial}{\partial x^i}(f j^i)$ and with Gauss theorem this integral will be transformed as an integral on the hyper-surfaces bordering the four-volume .Those integrals will be eliminated when we vary the action and thus will not affect the equations of motion.

8) Density & flux of energy:

Let's multiply equation (6-11) by $\boldsymbol{E}$ and equation (6-9) by $\boldsymbol{B}$ we get :

$$\frac{1}{c^2}\boldsymbol{E}.\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{B}.\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\gamma}{c^3}\sqrt{\frac{128.\pi^3.G}{\kappa}}\boldsymbol{E}.\boldsymbol{j} + \boldsymbol{E}.rot\boldsymbol{B} - \boldsymbol{B}.rot\boldsymbol{E} = -\frac{\gamma}{c^3}\sqrt{\frac{128.\pi^3.G}{\kappa}}\boldsymbol{E}.\boldsymbol{j} - div(\boldsymbol{E}\times\boldsymbol{B})$$

(8-1)

With the notation $\boldsymbol{H} = \boldsymbol{B}.c$ equation (8-1) becomes:

$$\frac{\partial}{\partial t}\left[\frac{c}{\gamma}.\sqrt{\frac{\kappa}{512.\pi^3.G}}(E^2 + H^2)\right] = -\boldsymbol{j}.\boldsymbol{E} - div\boldsymbol{S} \qquad (8\text{-}2)$$

With:

$$\boldsymbol{S} = \frac{c^2}{\gamma}.\sqrt{\frac{\kappa}{128.\pi^3.G}}\boldsymbol{E}\times\boldsymbol{H} = \lambda.\frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{H} \qquad (8\text{-}3)$$

$\boldsymbol{S}$: is called *Pointing vector*.

With:

$$\lambda = \frac{c}{\gamma}.\sqrt{\frac{\kappa}{8.\pi.G}} \qquad (8\text{-}4)$$

Equation (8-2) becomes:

$$\frac{\partial}{\partial t}\left[\frac{\lambda}{64\pi}.(E^2+H^2)\right] = -\boldsymbol{j}.\boldsymbol{E} - div\boldsymbol{S} \qquad (8\text{-}5)$$

Integrate (8-5) in a certain volume V and apply Gauss theorem for the second member we get:

$$\frac{\partial}{\partial t}\int \frac{\lambda.(E^2+H^2)}{64\pi}dV = -\int \boldsymbol{j}.\boldsymbol{E}\ dV - \oint \boldsymbol{S}.d\boldsymbol{f} \qquad (8\text{-}6)$$

If the integration is extended for all the space, the surface integral disappears because at the infinite the field is equal to zero. Referring to equations (3-67) & (3-38) or (7-9) &(3-38) for punctual charge instead of density of charges we can write:

$$\int \boldsymbol{j}.\boldsymbol{E}\ dV = \int \rho v.E\, dV = \frac{d}{dt}[\sum\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}+\varphi''\right)] \qquad (8\text{-}7)$$

So (8-6) becomes:

$$\frac{d}{dt}\left[\int \frac{\lambda.(E^2+H^2)}{64\pi}dV + \sum\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}+\varphi''\right)\right] = 0 \qquad (8\text{-}8)$$

For a closed system which composed of charged corpuscles and electromagnetic field the quantity between parentheses in (8-8) is conserved. The second term of this expression is the total energy of all the corpuscles i.e. it evolves their kinetic energy and their potential energy. So the first term represent the energy of the electromagnetic field itself. The quantity:

$$W = \frac{\lambda.(E^2+H^2)}{64\pi} \qquad (8\text{-}9)$$

Is called *density of energy* of the electromagnetic field.

For vacuum when there is no charges & no masses and if we suppose that the average density of the energy of vacuum tends to zero, we get from (8-9):

$$E_0 = \pm iH_0 \qquad (8\text{-}10)$$

There is an electromagnetic field associate to vacuum. If the magnetic field of vacuum H_0 is imaginary than the electric field of vacuum E_0 is real and so it can interact with charged corpuscle. The other way is also correct. We have take only corpuscles without spin, if they are with spin they can also interact with the magnetic field of vacuum. We can take equation

(8-9) as an average of electric field and magnetic field which the sum given by (8-9) is equal to zero.

For a finite volume (8-6) becomes:

$$\frac{\partial}{\partial t}\left[\int \frac{\lambda.(E^2+H^2)}{64\pi} dV + \sum\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \varphi''\right)\right] = -\oint \boldsymbol{S}.d\boldsymbol{f} = -\int div\boldsymbol{S}\, dV \quad (8\text{-}11)$$

For vacuum (8-11) becomes:

$$\frac{\partial}{\partial t}\left[\int \frac{\lambda.(E^2+H^2)}{64\pi} dV\right] + \int div\boldsymbol{S}\, dV = 0 \qquad (8\text{-}12)$$

Equation (8-12) is valid for any volume so:

$$\frac{\lambda}{64\pi}.\frac{\partial}{\partial t}(E_0^2 + H_0^2) + div\boldsymbol{S_0} = 0 \qquad (8\text{-}13)$$

We can deduce (8-13) directly from (8-2).

9) The meaning of the constant $''\boldsymbol{a}''$:

From equation (2-39) we have in Cartesian coordinates:

$$\tau^2.\left(1 - \frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2}\right) = \tau_0^2 \qquad (9\text{-}1)$$

Differentiate (9-1) we will get :

$$2.\tau.\left(1 - \frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2}\right) + \tau^2.\left(\frac{-2}{c^2}.[\dot{x}.\ddot{x} + \dot{y}.\ddot{y} + \dot{z}.\ddot{z}]\right) = 0 \quad (9\text{-}2)$$

Let’s consider one dimension the abscissa coordinate:

$$\left(1 - \frac{\dot{x}^2}{c^2}\right) - \frac{\tau_0}{c^2.\sqrt{1-\frac{\dot{x}^2}{c^2}}}(\dot{x}.\ddot{x}) = 0 \qquad (9\text{-}3)$$

So we get :

$$\left(1 - \frac{\dot{x}^2}{c^2}\right)^{\frac{3}{2}} - \frac{\tau_0}{c^2}.\dot{x}.\ddot{x} = 0 \qquad (9\text{-}4)$$

Let’s suppose that the speed of the corpuscle tends to $''c''$ so from equation (9-4) we deduce that the acceleration tends to zero: we can’t apply on the corpuscle any force, there is a maximum force to apply . Also the power to transmit to the corpuscle had a superior limit: we can't exchange any amount of energy with the corpuscle instantaneously, there is always a delay time to exchange energy and that's which translate this new constant.

10) Wave-corpuscle duality:

For $m = 0$ it correspond to light which have a peed equal to $''c''$. It is possible that we can treat corpuscles and waves as the same thing.

If we can treat a corpuscle as a wave, we can represent it by a plane wave as the following:

$$\psi(\boldsymbol{x}, t) = A.\exp(i\boldsymbol{k}.\boldsymbol{x} - i\omega.t) \qquad (10\text{-}1)$$

Where :

A: The amplitude of the wave function;

$\boldsymbol{k}$: The wave-vector;

ω: The frequency of the wave.

The principle of relativity implies the invariance of phase of the wave i.e. we have:

$$x.k - \omega.t = x'.k' - \omega'.t' \qquad (10\text{-}2)$$

Let's suppose that the corpuscle is in rest in the referential R', so we consider that it's wave-vector in this referential is equal to zero (nothing is in propagation):

$$k' = 0 \qquad (10\text{-}3)$$

And so we deduce that:

$$\boldsymbol{k} = \frac{\omega'.\boldsymbol{V}/c^2}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (10\text{-}4)$$

$$\omega = \frac{\omega'}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (10\text{-}5)$$

So:

$$\boldsymbol{k} = \frac{\omega'}{m.c^2}.\boldsymbol{p} \qquad (10\text{-}6)$$

$$\omega = \frac{\omega'}{m.c^2}.E \qquad (10\text{-}7)$$

$\boldsymbol{k} = \frac{\omega}{c^2}.\boldsymbol{V}$ (10-8)

We generalise those equations for every speed of the corpuscle.

But in this conception there is a major contradiction: how do we accept that a corpuscle which limited in space-time extension can be represented by a plane wave which is present everywhere. In 1927 de Broglie had found the solution by applying the principle of non-contradiction: the corpuscle is a packet of waves which reinforce each other in a limited region of space-time and annihilate each other above. The group speed of waves is identified to the corpuscle speed.

11) Unity-multiplicity duality:

As a consequence of de Broglie conception the corpuscle is considered as a unique object and also multiple of superposing waves. The wave function associated to the corpuscle has a quasi-monochromatic frequency ω and a wave vector k verifying the principle of uncertainty as follows:

$\Delta k.\Delta x \geq 1$ (11-1)

$\Delta\omega.\Delta t \geq 1$ (11-2)

The group speed v_g of the packet of waves is the speed with which the energy is transmitted .Its definition is as follows:

$\frac{1}{v_g} = \frac{dk}{d\omega}$ (11-3)

From (10-6) and (10-7) the uncertainties are as the following:

$\Delta k = \frac{\omega'}{m.c^2}.\Delta p$ (11-4)

$\Delta\omega = \frac{\omega'}{m.c^2}.\Delta E$ (11-5)

We have:

$\Delta k.\Delta x = \frac{\omega'}{m.c^2}.\Delta p.\Delta x$ (11-6)

$\Delta\omega.\Delta t = \frac{\omega'}{m.c^2}.\Delta E.\Delta t$ (11-7)

From equations (11-1) & (11-2) we deduce that:

$\Delta p.\Delta x \geq \frac{m.c^2}{\omega'}$ (11-8)

$\Delta E.\Delta t \geq \frac{m.c^2}{\omega'}$ (11-9)

The constant $\frac{m.c^2}{\omega'}$ should be independent from any corpuscle. We declare it an universal constant.

We put:

$\frac{m.c^2}{\omega'} = \hbar$ (11-10)

The constant $\hbar$ should have a very low value in the MKS system.

Equations (10-6) & (10-7) becomes:

$\hbar.\boldsymbol{k} = \boldsymbol{p}$ (11-11)

$\hbar.\omega = E$ (11-12)

It is very easy to verify that:

$\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{1}{v}$ (11-13)

And that what is CQFD.

12)Viscosity-dispersion duality:

The equation of propagation of the wave function (Klein-Gordon equation) is:

$\frac{1}{c^2}.\frac{\partial^2}{\partial t^2}\psi(\boldsymbol{x},t) - \nabla^2\psi(\boldsymbol{x},t) = -\frac{m^2 c^2}{\hbar^2}\psi(\boldsymbol{x},t)$ (12-1)

We define the following operator called also d'Alembertian:

$\Box \equiv \frac{1}{c^2}.\frac{\partial^2}{\partial t^2} - \nabla^2$ (12-2)

Equation (12-1) can be written also as the following:

$\Box\psi(\boldsymbol{x},t) = -\frac{m^2\,c^2}{\hbar^2}\psi(\boldsymbol{x},t)$ (12-3)

The d'Alembertian of the wave function is not equal to zero so there is dispersion of the wave function. The medium in which the packet of waves is in propagation is a dispersive medium: there is attenuation of the packet of waves with absorption of energy.

A dispersive medium for waves correspond for corpuscles to a viscous medium. The equation of motion of the corpuscle is:

$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v}$ (12-4)

Where:

$\boldsymbol{f}$: All unknown forces which act on the corpuscle;

$-a.\boldsymbol{v}$: An universal friction force which act on the opposite side of motion of the corpuscle;

a : Friction coefficient of the space-time (or mechanical impedance of vacuum).

So we can conclude that for wave-corpuscle duality there is another duality which is viscosity-dispersion duality of space-time.

Equation (12-5) is the same equation (1-4).

We define the momentum of the corpuscle as:

$\boldsymbol{p} = \xi.\boldsymbol{v}$ (12-5)

Where:

ξ : The inertia of the corpuscle;

$\boldsymbol{v}$: The speed of the corpuscle.

From equation (12-4) we deduce that:

$\dot{\xi}.\boldsymbol{v} + \xi.\frac{d\boldsymbol{v}}{dt} = \boldsymbol{f} - a.\boldsymbol{v}$ (12-6)

We put that:

$\dot{\xi} = a$ (12-7): if the energy of the corpuscle is varying;

$\dot{\xi} = 0$ (12-8): if the energy of the corpuscle is constant.

So we deduce from (12-7) & (12-8) that :

$\xi = a.\tau$ (12-9)

Where:

τ: is the inertial time of the corpuscle;

With:

$d\tau = dt$ (12-10): if the energy of the corpuscle is varying;

$d\tau = 0$ (12-11): if the energy of the corpuscle is constant.

If the corpuscle is in rest that we associate to it an inertial time in rest as:

$m = a.\tau_0$ (12-12)

So we get from (12-6):

$lim(\frac{d\boldsymbol{v}}{dt})_{t\to+\infty} = lim(\frac{\boldsymbol{f}-2.a.\boldsymbol{v}}{a.\tau})_{\tau\to+\infty} = \boldsymbol{o}$ (12-13)

So the speed of the corpuscle tends to a constant $''c''$ and this constant is declared as an universal constant: we know that it is the speed of light in vacuum.

We get the transformations of space and time (1-8) & (1-9). Also we have the transformations of momentum and inertia as the following:

$\xi' = \frac{\xi-\boldsymbol{p}.\boldsymbol{V}/c^2}{\sqrt{1-\frac{V^2}{c^2}}}$ (12-14)

$\boldsymbol{p}' = \frac{\boldsymbol{p}-\xi.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}}$ (12-15)

$\boldsymbol{p}'$and ξ' are respectively the momentum and the inertia of the corpuscle according to the reference R'. There are always invariants in inertial references to conserve the same speed of light. We have here an invariant which is:

$\xi^2 - (\frac{p}{c})^2 = m^2$ (12-16)

Now we can write equation (12-4) as:

$$\boldsymbol{f} = \frac{d^2(\xi.\boldsymbol{X})}{dt^2} \qquad (12\text{-}17)$$

Where:

$\boldsymbol{X}$**:** the position of the corpuscle.

We can take this formulae as a beginning for the equation of motion of a corpuscle and we get the same results. Let's develop equation (12-17):

$$\boldsymbol{f} = \ddot{\xi}.\boldsymbol{X} + 2.\dot{\xi}.\boldsymbol{v} + \xi.\ddot{\boldsymbol{X}} \qquad (12\text{-}18)$$

In a referential inertia equation (12-18) should be independent from the choice of its origin so we get from (12-18):

$$\ddot{\xi} = 0 \qquad (12\text{-}19)$$

So:

$$\dot{\xi} = a \qquad (12\text{-}20)$$

And so on.

13)How to determine the constant "$\boldsymbol{a}$" :

"If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or motion or the mass of our planet, but in the wavelength, the period of vibration, and absolute mass of these imperishable and unalterable and perfectly similar molecules. " **J.C. Maxwell (1870)**

13-1) Black body radiation:

This experiment is the black body radiation [13] .Let's have a cavity in a temperature T with a little hole from which we measure the energy and power of the emergent radiation.

13-1-1) Black body thermal equilibrium:

The average of number of photons of the frequency ν at the thermal equilibrium is :

$$n = \frac{1}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (13\text{-}1)$$

Where: $h = 6.626\ 10^{-34}\ j.s$: The constant of Planck

$k = 1.380\ 10^{-23} j.°K^{-1}$: The constant of Boltzmann

T: The temperature of the black body

The average of the energy of photons at the frequency ν is:

$$E_\nu = n.h.\nu = \frac{h.\nu}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (13\text{-}2)$$

The average of the power of photons at the frequency ν is:

$$P_\nu = n.a.c^2 = \frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (13\text{-}3)$$

13-1-2) Number of modes contained in the interval of frequency :

For a length L there is a stationary polarized wave if we have:

$$L = j.\frac{\lambda}{2} = j.\frac{\pi}{k_j} \qquad (13\text{-}4)$$

So:

$$k_j = j.\frac{\pi}{L} \qquad (13\text{-}5)$$

j: Integer , λ: Wave length , k_j: Wave vector.

The interval between two successive waves numbers is:

$$\delta k = \frac{\pi}{L} \qquad (13\text{-}6)$$

The number of values of k included in an interval δk is very high than δk. This number is:

$$\frac{\delta k}{(\frac{\pi}{L})} = \delta k.\frac{L}{\pi} \qquad (13\text{-}7)$$

A stationary wave contains two waves. The number of modes δM is the half of the number of values of k so:

$$\delta M = \frac{\delta k.L}{2.\pi} \qquad (13\text{-}8)$$

In three dimensions we get:

$$\delta M = \delta M_x.\delta M_y.\delta M_z = \frac{L_x.L_y.L_z.\delta k_x.\delta k_y.\delta k_z}{(2\pi)^3} \qquad (13\text{-}9)$$

i.e.:

$$\delta M = \frac{V.\delta k^3}{(2\pi)^3} \qquad (13\text{-}10)$$

With: $V = L_x.L_y.L_z$

The photon had two states of possible polarisation so:

$$\delta M = \frac{2.V.\delta k^3}{(2\pi)^3} \qquad (13\text{-}11)$$

δk^3 is the spherical volume interval in the space of k and it is equal to :

$$4\pi.k^2.\delta k = \delta k^3 \qquad (13\text{-}12)$$

So:

$$\delta M = \frac{8\pi.V.k^2.\delta k}{(2\pi)^3} = \frac{V}{\pi^2}.k^2.\delta k \qquad (13\text{-}13)$$

With $k = \frac{2\pi\nu}{c}$: the wave vector.

So we get:

$$\delta M = 8\pi.V.\frac{\nu^2}{c^3}.\delta\nu \qquad (13\text{-}14)$$

13-1-3) Black body volume power density:

The cavity of the black body encloses δM modes which everyone contains the power given in (13-3).

So the power which is contained in the interval of frequency $\delta\nu$ is:

$$\delta P = P_\nu.\delta M = 8\pi.V.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.\delta\nu \qquad (13\text{-}15)$$

Here k is the Boltzmann constant.

The volume power per frequency interval $\delta\nu$ is:

$$dP = \frac{\delta P}{V} = 8\pi.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu = p_\nu.d\nu \qquad (13\text{-}16)$$

Integrate (13-16) for all frequencies:

$$P = \int_0^\infty 8\pi.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu \qquad (13\text{-}17)$$

Replace $x = \frac{h.\nu}{k.T}$ in (13-17) we get:

$$P = \int_0^\infty 8\pi.\frac{1}{c^3}.\frac{k^2.T^2}{h^2}.x^2\frac{a.c^2}{\exp(x)-1}.\frac{k.T}{h}dx \qquad (13\text{-}18)$$

So:

$$P = \int_0^\infty \frac{8\pi.a.k^3.T^3}{c.h^3}.\frac{x^2}{\exp(x)-1}.dx = \frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.T^3 \qquad (13\text{-}19)$$

With: $\zeta(3) = 1.202056 \ldots$, $\zeta(x) = \sum_{n=1}^{\infty}\frac{1}{n^x}$ $(x > 1)$: Riemann function (or Zeta function).

$$\sigma = \frac{8.\pi^5.k^4}{15.h^3.c^3} = 7.56\ 10^{-16}\ j.m^{-3}.°K^{-4}$$

$$P\ have\ a\ dimension\ of [W.m^{-3}]$$

Than P is a linear function of T^3 where the linearity coefficient contains $''a''$ so we can determinate it.

13-1-4) Volume energy of the black body (Planck law):

The energy contained in the interval of frequency $\delta\nu$ is:

$$\delta U = U_\nu.\delta M = \frac{8\pi.V.\nu^2}{c^3}.\frac{h.\nu}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.\delta\nu \qquad (13\text{-}20)$$

The volume energy per interval of frequency $d\nu$ is:

$$dU = \frac{\delta E}{V} = \frac{8\pi.h.\nu^3}{c^3}.\frac{1}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu = u_\nu.d\nu \qquad (13\text{-}21)$$

By integrate (13-21) we found:

$$U = \frac{8\pi.k^4}{c^3h^3}.T^4.\int_0^{\infty}\frac{\left(\frac{h\nu}{kT}\right)^3}{\exp\left(\frac{h\nu}{kT}\right)-1}.d(\frac{h\nu}{kT}) = \sigma.T^4 \qquad (13\text{-}22)$$

With:

$$\sigma = \frac{8.\pi^5.k^4}{15.h^3.c^3} = 7.56\ 10^{-16}\ j.m^{-3}.°K^{-4} \qquad (13\text{-}23)$$

Because we have:

$$\int_0^{\infty}\frac{x^3}{\exp(x)-1}.dx = \frac{\pi^4}{15} \qquad (13\text{-}24)$$

$$U\ have\ a\ dimension\ of\ [j.m^{-3}]$$

13-2) Intensity of radiation (Stefan-Boltzmann law):

The law of Stefan-Boltzmann gives the power of radiation going out from the black body per unit surface i.e. the radiated energy dE per unit time dt going out through a little hole dS i.e. its intensity $R(T) = \frac{dW}{dS.dt}$ or its spectral density of intensity per unit of frequency [14]:

$$r_\nu(T) = \frac{dE}{dS.dt.d\nu} \qquad (13\text{-}25)$$

This energy is equal to the irradiance of the hole by the electromagnetic field in the black body. The radiation is isotropic and independent from the characteristics of the black body. Its density per solid angle is $\frac{1}{\frac{4\pi}{2}} = \frac{1}{2\pi}$ (we take the half of the total space).The incident energy for one mode which have an angle θ with the perpendicular line to the element dS is the one in the volume $\frac{1}{2}\ c.dt.dS.cos\theta$. The factor $\frac{1}{2}$ is due to that there is in stationary mode two waves an incident & a reflected wave: we take only the incident one on the surface dS. So we have:

$$r_\nu(T) = \frac{dE}{dS.dt.d\nu} = \frac{u_\nu}{dS.dt.d\nu}.\left(\frac{1}{2}\ c.dt.dS.d\nu.\int_0^{\frac{\pi}{2}}cos\theta\frac{d\Omega}{2\pi}\right) = \frac{c}{2}.u_\nu.\int_0^{\frac{\pi}{2}}cos\theta\,.\frac{2\pi sin\theta.d\theta}{2\pi} = \frac{c}{4}.u_\nu$$

(13-26)

Than we get that:

$$R = \int_0^\infty r_\nu . d\nu = \frac{c}{4}\int_0^\infty u_\nu . d\nu = \frac{c}{4} . U = \frac{2.\pi^5.k^4}{15.c^2h^3} . T^4 = \sigma' . T^4 \qquad (13\text{-}27)$$

With:

$$\sigma' = \frac{2.\pi^5.k^4}{15.c^2h^3} = 5.67040 \times 10^{-8}\, W.m^{-2}.K^{-4} \qquad (13\text{-}28)$$

$R\ have\ a\ dimension\ of\ [W.m^{-2}]$

It is like the quarter of energy going out through 1 m^2 to a distance of 3 $10^8\ m$ in 1 $second$.

The coefficient $\frac{c}{4}$ is the same coefficient which use Planck in the F.Kurlbaum experimental formulae to cancel time and prove that the constants h & k have the same values as in its earlier communication [15].

13-3) Disposal power per unit surface & per unit time:

The same analysis in §13-2 than we get:

$$w_\nu(T) = \frac{c}{4} . p_\nu(T) \qquad (13\text{-}29)$$

Than we have:

$$W(T) = \frac{c}{4} . P(T) \qquad (13\text{-}30)$$

For a short time Δt at a distance d of a black body we have:

$$R.4\pi.d^2 = W.4\pi.d^2.\Delta t \qquad (13\text{-}31)$$

So:

$$U(T) = P(T).\Delta t \qquad (13\text{-}32)$$

13-3) Limits of the constant $''a''$:

The total power is (the variation of the energy per unit time in the volume) at any temperature is the same power that will go out through the surface delimitate this volume:

$$P_{tot} = P.V = R.S \qquad (13\text{-}33)$$

With: $V = \frac{4}{3}.\pi.D^3$: the volume of a sphere of a radius D .

$S = 4\pi.D^2$: the surface delimitate the sphere

So we have:

$$\frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.T^3.\frac{4}{3}\pi.D^3 = \frac{c}{4}.\sigma.T^4.4\pi.D^2 \qquad (13\text{-}34)$$

Than:

$$\frac{30.\zeta(3).a.c^2}{\pi^4.k}.\frac{D}{3} = \frac{c}{4}.T \qquad (13\text{-}35)$$

So:

$$40.\zeta(3).a.c.D = k.T.\pi^4 \qquad (13\text{-}36)$$

Let's design by D a characteristic dimension of the Universe, we have:

$$40.\zeta(3).a.c = k.T.\pi^4.\frac{1}{D} \qquad (13\text{-}37)$$

Then we will have that:

$$a = \frac{k.T.\pi^4}{40.\zeta(3).c.D} \qquad (13\text{-}38)$$

We can think in a classical manner: A mole of perfect gas occupy a volume of $22.4\ liters$ at the normal conditions of pressure and temperature ($1bar$ & $20\ °C$). We suppose that photons doesn't interact with each other.

So the distance $D = (22.4\ liters)^{\frac{1}{3}} = 0.282\ m$ and we have $kT = 404.34\ \ 10^{-23} joule$ than:

$$a \approx\ \ 10^{-28}\ kg.s^{-1} \qquad (13\text{-}39)$$

We should expect that the number of corpuscles in this volume should be near the Avogadro number ($N_A = 6.02\ 10^{23}$) if of course Boltzman statistics still valid for photons (but it is wrong because photons are relativist corpuscles and there is another statistics to follow).

From equation (13-19) we deduce the density of photons:

$$n = \frac{P}{a.c^2} = \frac{30.\zeta(3).\sigma}{\pi^4.k}.T^3 \qquad (13\text{-}40)$$

So for $T = 20\ °C$ & $V = 22.4\ liters$ the number of photons is:

$$N = n.V = 1.14\ \ 10^{13} \neq 6.02\ 10^{23} \qquad (13\text{-}41)$$

This signifies that we can't treat a gas of photons like a classical gas.

13-4) The orbital speed of a galaxy:

Let's have a galaxy with a mass m which gravitate around another galaxy with a mass. The motion is happened in a plan and we use polar coordinates (R, θ). It is very easy to get the equation of motion of the galaxy m as:

$$m.\left(\ddot{R} - R.\dot{\theta}^2\right) = -\frac{G.m.M}{R^2} - a.\dot{R} \qquad (13\text{-}42)$$

$$m.\left(2\dot{R}.\dot{\theta} + R.\ddot{\theta}\right) = -a.R.\dot{\theta} \qquad (13\text{-}43)$$

Integrate equation (13-45) we get:

$$R^2.\dot{\theta} = 2.K.\exp(-\frac{m}{a}t) \qquad (13\text{-}44)$$

Where K is Kepler constant of area.

The Hubble law of an expanding Universe is:

$$\dot{R} = H.R \qquad (13\text{-}45)$$

So:

$$\ddot{R} = H^2.R \qquad (13\text{-}46)$$

With $H = 50\ \ km.s^{-1}.Mpc^{-1} = 1.62\ \ 10^{-18}\ s^{-1}$: is the Hubble constant.

The Universe is expanding, so constant "a" can take negative value otherwise referring to equation (13-44) it will collapse.

Replace $\dot{R}$ & $\ddot{R}$ in the equation (13-42) as given in (13-45) & (13-4) we get:

$$\dot{\theta}^2 = H^2 + \frac{G.M}{R^3} + \frac{a.H}{m} \qquad (13\text{-}47)$$

From (13-44) we deduce that:

$$4.\frac{K^2}{R^4}.\exp\left(-2.\frac{a}{m}.t\right) = H^2 + \frac{G.M}{R^3} + \frac{a.H}{m} \quad (13\text{-}48)$$

The orbital speed of the galaxy can be constant from a certain distance R_0 if we can resolve the following equation:

$$R^2(H^2 + \frac{a.H}{m}) + \frac{G.M}{R} = U_0^2 \qquad (13\text{-}49)$$

Where U_0 is the orbital speed of the galaxy.

Let's remark that when only we take Newton gravitational law without caring about Hubble law we get the following equation:

$$\ddot{R} + \frac{G.M}{R^2} - \frac{4.K^2}{R^3}.\exp\left(-\frac{2.a}{m}t\right) = -\frac{a.\dot{R}}{m} \qquad (13\text{-}50)$$

If $\frac{a}{m} \to 0$ than we found Newton in (13-50).

13-5) The photo-electric experiment:

13-5-1) Definitions and vocabulary:

The quantum of energy of a photon as given by Planck formulae is [19]:

$$E = \hbar.\omega = \frac{\hbar.\omega}{u_0}.u_0 = N.u_0 \qquad (13\text{-}51)$$

With:

$$u_0 = \hbar.\omega_0 = M.c^2 = c.\sqrt{\hbar.a} \qquad (13\text{-}52)$$

u_0 : *De Broglie quantum of energy or the elementary quantum of energy or the grain of energy or the hidden constant of energy or the individual substance of energy which can occupy any volume of space.*

$$N = \frac{\hbar.\omega}{u_0} \qquad (13\text{-}53)$$

$N: Number\ of\ DeBroglie\ quanta$

We define also the *De Broglie Quantum of Power or the elementary quantum of power or the grain of power or the hidden constant of power or the individual substance of power as:*

$$s_0 = a.c^2 \qquad (13\text{-}54)$$

The *De Broglie quantum of momentum or the elementary quantum of momentum or the grain of momentum or the hidden constant of momentum or the individual substance of momentum is defined as:*

$$p_0 = M.c = \frac{u_0}{c} = \sqrt{\hbar.a} \qquad (13\text{-}55)$$

The *Planck time variable or the elementary constant of time or the grain of time or Einstein hidden constant or the hidden constant of time or one oscillation or the individual substance of time* is defined as:

$$h_0 = T = \frac{1}{c}.\sqrt{\frac{\hbar}{a}} \qquad (13\text{-}56)$$

The De Broglie quantum of space or the elementary quantum of space or the grain of space or the hidden constant of space or the individual substance of space is defined as:

$$l_0 = L = \sqrt{\frac{\hbar}{a}} \qquad (13\text{-}57)$$

The De Broglie quantum of force or the elementary quantum of force or the grain of force or the Maxwell force or the hidden constant of force or the individual substance of force is defined as:

$$f_0 = a.c \qquad (13\text{-}58)$$

We have always:

$$\hbar = u_0.h_0 = f_0.l_0.h_0 = p_0.l_0 \quad \text{and} \quad \left(\frac{u_0}{c}\right)^2 - p_0^2 = 0 \qquad (13\text{-}59)$$

13-5-2)The photo-electric effect experiment:

Let's take a photo emissive cell in short circuit with a resistance and which is lightened with a monochromatic light of a frequency ν at a power to have a measurable current and tension in the circuit. The internal resistance of the cell is negligible compared to the external resistance or be considered as equal to zero.

The photo cell is supposed equivalent to a capacitor of a C_0 capacity. We suppose that the incident light interact with the cell anode in batch and extract a number N photons at a time τ

and this time is very near to the life-time of a surface electron of the atom anode. We suppose also that the inertial time of light is very near to the life-time of a surface electron.

The power in the circuit is:

$$P = U.I = U.\frac{N.e}{\tau} = U.\frac{N.e}{h\nu}.\alpha_0 \quad (13\text{-}60)$$

With: $\alpha_0 = a.c^2$: the power of a photon.

The tension at the ends of the cell is:

$$U = \frac{Q}{C_0} = \frac{N.e}{C_0} \quad (13\text{-}61)$$

Than we have:

$$I = \frac{\alpha_0.C_0}{h\nu}.U \quad (13\text{-}62)$$

If we know the ratio $\frac{U}{I}$ than we can determine the constant α_0 .

We can deduce the value of the constant α_0 directly from the ordinary photoelectric experiment with a generator to stop the current in the circuit (Fig2):

The stopping potential for it the current in the circuit fall to zero is:

$$V_0 = \frac{h}{e}.\nu - \frac{W}{e} \quad (13\text{-}63)$$

Where $\frac{W}{e}$: a constant of the material of the photo-cathode.

$e = 1.6\ 10^{-19}\ Colomb$: the electron charge.

We know that in general the constant of the material of the photo-cathode is in the order of one volt and that the phenomena of the photoelectric effect happen with the visible light i.e. with frequencies of light which belongs to the interval $[4.3 - 7.5]10^{14}\ Hz$. So:

$$e * 1volt \approx \alpha_0.\tau \quad (13\text{-}64)$$

The life-time of a surface electron is about $10^{-7}\ second$ and so we deduce from (13-64):

$$\alpha_0 \approx 1.6\ 10^{-12}\ Watt \quad (13\text{-}65)$$

13-6) choosing a characteristic dimension of the Universe as an universal constant:

Let's have a part of the Universe as a sphere of radius D. The energy contained in this sphere is :

$$E = U.\frac{4}{3}.\pi.D^3 = \sigma.T^4.\frac{4}{3}.\pi.D^3 \qquad (13\text{-}66)$$

The same energy is the one which go out through the sphere during the time $\frac{D}{c}$:

$$E = P.\frac{D}{c}.\frac{4}{3}.\pi.D^3 \qquad (13\text{-}67)$$

It comes that:

$$\frac{k.T}{D} = \frac{30.\zeta(3).a.c}{\pi^4} \qquad (13\text{-}68)$$

For $D = \sqrt{\frac{\hbar}{a}}$ and $T = 3\ °K$ we get:

$$a = \frac{k^2.T^2.\pi^8}{900.\hbar.\zeta(3)^2.c^2} \approx 13\ \ 10^{-28}\ \ kg.s^{-1} \quad \& \quad D = 0.3\ mm \qquad (13\text{-}69)$$

Distance D is very near to the wave-length of the cosmic radiation ($\lambda_{max} \approx 1\ mm$) .

13-7) The second form of Wien displacement law :

Planck had deduced the values of its constant h & Boltzmann constant k from the values of the experiments done by F.Kurmbaum & Wien [15]:

"§11. The values of both universal constants h and $k$may be calculated rather precisely with the aid of available measurements. F. Kurlbaum, designating the total energy radiating into air from 1 sq cm of a black body at temperature t ○ C in 1 sec by S_t, found that:

$S_{100}-S_0=0.0731\ \frac{Watt}{cm^2}=7.31\ 10^5\ \frac{erg}{cm^2.sec}$"

The second form of Wien displacement law is [16]:

$$\nu_{maxenergy}.T^{-1} = C_{2w} = 5.879\ 10^{10}\ \ Hz.°K^{-1} \qquad (13\text{-}70)$$

The number of photons having a frequency between ν & $\nu + d\nu$ going through out an elementary surface dS in the time dt and the solid angle $d\Omega$ around the direction of propagation orthogonal to the surface element [21] is:

$$dN = \frac{\delta M}{V}.n.c.dt.dS.d\Omega = \frac{8\pi.\nu^2}{c^2}.\frac{1}{exp\left(\frac{h\nu}{kT}\right)-1}.dt.dS.d\nu.d\Omega \qquad (13\text{-}71)$$

The maximum of power which go out through $\Delta S = 1\ cm^2$ of the black body in $\Delta t = 1\ sec$ is:

$$\Delta p = a.c^2.\Delta N \qquad (13\text{-}72)$$

And:

$$\Delta N = \left(\frac{8\pi.\nu_{100}^2}{c^2}.\frac{1}{exp\left(\frac{h.\nu_{100}}{k.373}\right)-1} - \frac{8\pi.\nu_0^2}{c^2}.\frac{1}{exp\left(\frac{h.\nu_0}{k.273}\right)-1}\right).\Delta t.\Delta S.\Delta\Omega.\Delta\nu \qquad (13\text{-}73)$$

With:

$$\nu_{100} = C_{2w}.373 = 2192.867\ 10^{10}\ Hz$$

$$\nu_0 = C_{2w}.273 = 1604.967\ 10^{10}\ Hz$$

$\Delta\Omega \approx 2\pi\ maximum$

So from F.Kurlbaum experiment we deduce that:

$a.c^2\Delta N = 0.0731\ Watt$

And so we get:

$$a = 5.38\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}74)$$

13-8The second law of radiation displacement:

The density of power in a black body for the range of frequency ν & $\nu + d\nu$ is:

$$p_\nu(T) = 8\pi.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1} = 8\pi.\frac{h^2\nu^2}{c.h^2.k^2.T^2}.\frac{a.k^2.T^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (13\text{-}75)$$

Replace $u = \frac{h\nu}{kT}$ in (13-75) we get:

$$p_\nu(T) = 8\pi.\frac{a.k^2.T^2}{c.h^2}.\frac{u^2}{\exp(u)-1} \qquad (13\text{-}76)$$

The maximum of power radiation density is given when $\frac{dp_\nu(T)}{d\nu} = \frac{dp_\nu(T)}{du} = 0$ so:

$$2u.\frac{1}{\exp(u)-1} - u^2.\frac{\exp(u)}{(\exp(u)-1)^2} = 0 \qquad (13\text{-}77)$$

Than we have:

$-2.\exp(-2) = (u-2)\exp(u-2)$ (13-78)

This equation can be resolved graphically by Lambert's W-functions as:

$u - 2 = W(-2.e^{-2}) = W(-0.27)$ (13-79)

The solution of equation (13-79):

$u - 2 = -0.406$ (13-80)

It comes that for a temperature T there is a maximum of *power density* of a black body as:

$\nu_{maxpower}.T^{-1} = 3.32\ \ 10^{10}\ Hz.°K^{-1}$ (13-81)

If somebody verify experimentally the relation (13-81) it signify that our first assumptions are good.

However let's suppose that the value of constant "a" given in (13-74) is good. We deduce like Planck do for the Boltzmann constant that we can associate to a perfect gas a coefficient of inner impedance as:

$\kappa = N_A.a = 6.02\ 10^{23}x5.38\ 10^{-29} = 32.3876\ \ 10^{-6}\ kg.s^{-1}$ (13-82)

If somebody find the theory which include the *inner impedance of a perfect gas* than we can verify again our assumptions.

Let's have a mole of a perfect gas in a constant volume . Its classical equation of state is the Boyle-Mariotte law:

$P.V = R.T$ (13-83)

Where : $P = pressure\ of\ the\ gas$

$R = 8.314\ j.mol^{-1}.°K^{-1}$

$T = Temperature\ \ of\ the\ gas$

If we want to augment the temperature of the gas of only *one degree Kelvin* than we have:

$\Delta P.V = R.\Delta T = N_A.k.\Delta T$ (13-84)

With:

$N_A = 6.02\ 10^{23}$: the Avogadro number (the number of corpuscles in the volume V corresponding to one mole;

$\Delta T = 1K$

From (13-84) we conclude that every corpuscle will absorb an energy of $.\Delta T$. The energy is taken from the radiation inner the volume. The power transmitted by a photon is $a.c^2$ and it takes at minimum a time of $\frac{1}{c}.\sqrt{\frac{\hbar}{a}}$ to be absorbed than we have:

$$k.\Delta T = (a.c^2).\left(\frac{1}{c}.\sqrt{\frac{\hbar}{a}}\right) = c.\sqrt{\hbar.a} \qquad (13\text{-}85)$$

Of course this description is qualitative description (we had neglect any interaction between the corpuscles of the gas and normally we should suppose that there is only a few corpuscles in the volume and not a mole).

We get from (13-86):

$$a = \frac{k^2.\Delta T^2}{c^2.\hbar} = \frac{1.38^2\ 10^{-46}.1^2}{9\ 10^{16}.1{,}054\ 10^{-34}} = 2.0076\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}86)$$

Note that equation (13-86) can serve to define what an absolute degree of temperature as:

$$1K = \frac{c}{k}.\sqrt{\hbar.a} \qquad (13\text{-}87)$$

However the only unit in the MKSA system defined as absolute is temperature in degree Kelvin. The degree Kelvin hide four universal constants.

13-9)The radiated power as a function of power density:

Lets' take an isotherm cavity at a temperature T [22].Let's drill a little hole in this cavity. The energy ΔE going out the hole at an angle θ is:

$$\Delta E = \frac{U}{4\pi}.c\Delta t.\Delta S_{hole}.cos\theta.\Delta\Omega_{detector} \qquad (13\text{-}88)$$

With :

$$\Delta\Omega_{detector} = \frac{\Delta S_{detector}}{\lambda^2} \qquad (13\text{-}89)$$

$$\Delta S_{detector} = \Delta S_{hole}.\cos\left(\frac{\pi}{2} - \theta\right).\Delta\theta\Delta\varphi = \Delta S_{hole}.sin\theta.\Delta\theta\Delta\varphi \quad (13\text{-}90)$$

$$\Delta S_{hole} = \lambda^2 = \frac{c^2}{\nu^2}$$

$$\lambda = c.\Delta t = \frac{c}{\nu} \quad \text{so} \quad \Delta t = \frac{1}{\nu} \quad (13\text{-}91)$$

After doing calculation we will fix the base of time at a value as:

$$T = 1K \ \ and \ \Delta t = h_0 = \frac{1}{\nu_0} = \frac{1}{c}.\sqrt{\frac{\hbar}{a}} \quad (13\text{-}92)$$

Because the little hole should have constant dimensions.

From (13-88) we have:

$$\frac{\frac{\Delta E}{\Delta t}}{\lambda^3.\Delta\nu}.\Delta\nu = \frac{1}{\lambda^3}.\frac{U}{4\pi}.c.\lambda^2.cos\theta.\frac{\lambda^2.sin\theta}{\lambda^2}.\Delta\theta\Delta\varphi = p_\nu.\Delta\nu \quad (13\text{-}93)$$

Integrate (13-93) in all space and all frequency:

$$\int_0^\infty \frac{p_\nu}{\nu}.d\nu = \frac{U}{4\pi}.\int_0^{2\pi} d\varphi.\int_0^{\frac{\pi}{2}} cos\theta.sin\theta.d\theta = \frac{U}{4} \quad (13\text{-}94)$$

We have from (13-93):

$$\int_0^\infty \frac{p_\nu}{\nu}.d\nu = \int_0^\infty 8\pi.\frac{\nu}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu \quad (13\text{-}95)$$

Put $x = \frac{h.\nu}{k.T}$ than $dx = \frac{h}{kT}d\nu$ so:

$$\int_0^\infty \frac{p_\nu}{\nu}.d\nu = \int_0^\infty \frac{8\pi}{c^3}.\left(\frac{kT}{h}\right)^2.x.\frac{a.c^2}{\exp(x)-1}dx = \frac{8\pi a.k^2.T^2}{c.h^2}.\int_0^\infty \frac{x}{\exp(x)-1}.dx \quad (13\text{-}96)$$

We have:

$$\int_0^\infty \frac{x}{\exp(x)-1}.dx = \Gamma(2).\zeta(2) = \frac{\pi^2}{6} \quad \text{so we get:}$$

$$\int_0^\infty \frac{p_\nu}{\nu}.d\nu = \frac{4\pi^3 a.k^2.T^2}{3.c.h^2} = \frac{\sigma.T^4}{4} \quad (13\text{-}97)$$

For $T = 1K$ we have:

$$a = \frac{3.c.h^2.\sigma}{16.\pi^3.k^2} = \frac{3x3\ 10^8 x6.62^2\ 10^{-68} x7.56\ 10^{-16}}{16.\pi^3 x1.38^2\ 10^{-46}} = 3.156\ 10^{-30}\ kg.s^{-1} \quad (13\text{-}98)$$

It is like we have do the following:

$$P(1K) \approx \frac{U(1K)}{h_0} \quad (13\text{-}99)$$

So:

$$\frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.1^3 = \frac{\sigma.1^4}{\frac{1}{c}.\sqrt{\frac{\hbar}{a}}}$$

It comes that:

$$a = \frac{\pi^8.k^2}{900.\zeta(3)^2.c^2.\hbar} = \frac{\pi^8 x 1.38^2 x 10^{-46}}{900 x 1.2^2 x 9.10^{16} x 1.054\ 10^{-34}} = 1.47\ 10^{-28}\ kg.s^{-1} \quad (13\text{-}100)$$

Let’s verify our assumptions for (13-99):

$$P = \frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.T^3 = \frac{U}{h_0} = \frac{\sigma.T^4}{h_0} \quad \text{so:}$$

$$T = \frac{30.\zeta(3).h_0.a.c^2}{\pi^4.k} = \frac{30.\zeta(3).c.\sqrt{\hbar.a}}{\pi^4.k} \quad (13\text{-}101)$$

Let’s take for $a \approx 2\ 10^{-29}\ kg.s^{-1}$ as a good experimental value given by (13-86):

$$T = \frac{30 x 1.2 x 3\ 10^8 x \sqrt{1.054\ 10^{-34} x 2\ 10^{-29}}}{\pi^4 x 1.38\ 10^{-23}} = 0.37\ °K \quad (13\text{-}102)$$

It is a value very near to $1K$.

The same analyses can be taken for instantaneously energy & power radiation. We should have:

$$p_\nu(T) \approx \frac{U_\nu(T)}{h_0} \quad (13\text{-}103)$$

It comes that:

$$h_0 = \frac{h.\nu}{a.c^2} \quad (13\text{-}104)$$

So:

$$\nu = \frac{a.c^2}{h}.h_0 \quad (13\text{-}105)$$

If we choose for $h_0 = \frac{1}{c}.\sqrt{\frac{\hbar}{a}}$ than (13-104) becomes:

$$\nu = \nu_0 = c.\sqrt{\frac{a}{\hbar}} = \frac{1}{h_0} \qquad (13\text{-}106)$$

And if we take $\nu = \frac{1}{h_0}$ than (13-104) gives us:

$$h_0 = \frac{1}{c}.\sqrt{\frac{h}{a}} \qquad (13\text{-}107)$$

The question is what is the good choice for the definition of the base of time?.

The answer is that one replace $\hbar$ by h in the equation (13-99) and we get:

$$T \approx 0.9\ K \qquad (13\text{-}108)$$

It is more accurate value (the nearest to one Kelvin).

So we can began from the first and redefine the natural absolute system of unities as $h = c = a = 1$. I can't do this because it is hard and takes time to do again. But this invite us to think about the Planck system: if this system have any physical sense it should be in a space-time where the common laws of nature are not available . To be in continued coherence with our natural absolute system the Planck system should be better defined as $h = c = G = 1$.

Let's do the following summary for the energy radiation of a black body (Planck law)[20]:

In order to resolve the theoretical problems founded by Rayleigh-Jeans , Planck propose to limit the energy of an oscillator for every mode of vibration as $h\nu$ .He add also those constraints:

1-To excite a mode of vibration with a high probability , the thermal energy should be greater than the energy of excitation of the vibration mode $(kT > h\nu)$.

2-A mode of vibration should leave all its energy at the same time.

The loss of energy is called loss by *quanta* because that every de-excited mode of vibration product a unique radiated energy equal to $h\nu$. The conclusion of Planck is that it should be accepted that a *black body loss* energy by *radiation* only by *quanta*. The following is a summary how to interpret the distribution of energy loss as a function of the frequency ν of every mode of vibration:

Frequency of the mode $(\boldsymbol{\nu})$	$\boldsymbol{\nu}$ **is great**	$\boldsymbol{\nu \approx \nu_{max}}$	$\boldsymbol{\nu}$ **is low**
Mode frequency (ν)	Very high	High	Low
Energy of the mode $(h\nu)$	Very high	High	Low
Excitation / des-excitation probability	Low	High	High
Energy loss proportion	Low	High	Low

It is logic to think to choose the base of time when $\nu_0 = \nu_{max}\ and\ T = 1K$. From Wien displacement law we deduce that:

$$\nu_0 = 5.879\ \ 10^{10}\ \ Hz = c.\sqrt{\frac{a}{h}} \qquad (13\text{-}109)$$

It comes that:

$$a = h.\frac{\nu_0^2}{c^2} = 6.62\ 10^{-34}\ \frac{5.879^2\ 10^{20}}{9\ 10^{16}} = 2.542\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}110)$$

Note that the maximum of energy emission is not the maximum of power emission.

From the second law of radiation displacement (13-81) (I suppose that experience will confirm this law) we have for $T = 1K$:

$$\nu_0 = 3.32\ \ 10^{10}\ Hz = c.\sqrt{\frac{a}{h}} \qquad (13\text{-}111)$$

It comes that:

$$a = h.\frac{\nu_0^2}{c^2} = 6.62\ 10^{-34}\ \frac{3.32^2\ 10^{20}}{9\ 10^{16}} = 8.11\ 10^{-30}\ \ kg.s^{-1} \qquad (13\text{-}112)$$

The good value should be calculated as :

$$1K = \frac{c}{k}.\sqrt{h.a} \qquad (13\text{-}113)$$

So:

$a = \frac{k^2}{c^2}.\frac{1^2}{h} = \frac{1.38^2\ 10^{-46}}{9\ 10^{16}\ x\ 6.62\ 10^{-34}} = 3.2\ \ 10^{-30}\ kg.s^{-1}$ (13-114)

However this value should be confirmed by another experiment such as the photo-electric experiments (independents from temperature) which I had describe the manner how to do. The value in (13-114) is very near to (13-112) but not exactly the same.

Let's return to the perfect gas. The interaction of the gas with radiation is that some corpuscles absorb radiation and others radiate energy but not all. The definition of temperature is that [29]:

$\frac{1}{2}m.v^2 = \frac{3}{2}k.T$ (13-115)

Where m: mass of a corpuscle of the gas;

v^2: the average of the quadratic speed of the corpuscles of the gas;

If we augment the temperature of the gas of $1K$ than the quadratic speed will change as:

$\frac{1}{2}m.(v+\Delta v)^2 = \frac{1}{2}m.v^2 + mv.\Delta v + \frac{1}{2}m.(\Delta v)^2 = \frac{3}{2}k.(T+1) = \frac{1}{2}m.v^2 + \frac{3}{2}k.1K$ (13-116)

So:

$mv.\Delta v + \frac{1}{2}m.(\Delta v)^2 = \frac{3}{2}k.1K$ (13-117)

Equation (13-117) can be resolved as an equation of the second order and we will find Δv as a function of temperature.

The essential is that a certain corpuscles absorb the energy $\frac{3}{2}k.1K$ to augment their kinetic energy. This energy comes from the radiation into the volume V by heating. So a population of corpuscles will take the power from radiation $a.c^2$at a time in minimum as $\frac{1}{c}\sqrt{\frac{h}{a}}$. So we have:

$\frac{3}{2}k.1K = (a.c^2).\left(\frac{1}{c}.\sqrt{\frac{h}{a}}\right) = c.\sqrt{h.a}$ (13-118)

It comes that:

$a = \frac{9}{4}.\frac{k^2}{c^2.h} = \frac{9x1.38^2\ 10^{-46}}{4x9\ 10^{16}x6.62\ 10^{-34}} = 0.72\ \ 10^{-29}\ \ kg.s^{-1}$ (13-119)

(13-119) fit exactly with equation (13-112) deduced from the second law of radiation displacement (values are very near each either).We declare that equation(13-111) is the good equation theoretically and experimentally. However it will be necessary to confront this value with another value deduced from another experiment independent from temperature. But our assumptions are based on the mono-atomic gas which internal energy for one mole is

$U = \frac{3}{2}R.T = \frac{3}{2}\ N_A.k.T$ and than to augment the temperature of the gas of $1K$ it signify that every corpuscle of the gas absorb the energy of $\frac{3}{2}k.1K$.

For one mole of a diatomic gas we have another equation for the internal energy which is $U = \frac{5}{2}kT$ than we should stop the discussion and declare that:

-The Wien displacement law (the first law of radiation displacement) serve to determine the Planck constant h and the Boltzmann constant k.

-The second law of radiation displacement (13-82) serve to determine what does 'it mean one degree Kelvin in the MKS system and so the final value of constant "a" is given by equation (13-112). The hazard makes that this value coincide exactly with the value calculated from one mole of a mono-atomic perfect gas.

DEFITION:

$$\frac{h.\nu_0}{k.1K} = \frac{M.c^2}{k.1K} = \frac{c.\sqrt{h.a}}{k.1K} = 2 + W(-2.e^{-2}) = 1.594 \qquad \text{(13-120)}$$

So:

$$1K = 0.627\ \frac{c}{k}.\sqrt{h.a} \qquad \text{(13-121)}$$

Note that $0.627 \approx \frac{2}{3}$ which justify the coincidence with the mono-atomic perfect gas.

Note that we can deduce the formulae (13-121) in a very easier manner.

From equation (13-32) we deduce that:

$$u_\nu = p_\nu.\Delta t \qquad \text{(13-122)}$$

With Δt : a very short time to measure power. It is our base of time.

It comes that:

$\Delta t = \frac{h.\nu}{a.c^2}$ (13-123)

There is $\nu = \nu_0$ when the power per unit volume & per unit of frequency is in its maximum in which we choose the base of time. So:

$\Delta t \approx h_0 = \frac{h.\nu_0}{a.c^2}$ (13-125)

The first idea which comes is to equal h_0 to the absolute unity of time (the reference) to $\frac{1}{c}.\sqrt{\frac{\hbar}{a}}$ but we will rapidly conclude that we should choose $\frac{1}{c}.\sqrt{\frac{h}{a}}$ and get that:

$1K = \frac{1}{2+W(-2.e^{-2})}.\frac{c}{k}.\sqrt{h.a}$ (13-126)

13-10)Other manners to determine approximately the constant "a" :

Let's take one mole of iron and elevate its temperature of $1K$. Every atom of the iron will absorb the power "$a.c^2$" given by heating at a minimum of time of "$\frac{1}{c}\sqrt{\frac{h}{a}}$" so we have for the energy absorbed by on atom:

$Q = \frac{m_{mole}.c_{cal}.\Delta T}{N_A} = c.\sqrt{h.a} = Constant$ (13-126)

With : $m_{mole} = 56\ g\ ;\ c_{cal} = 460\ j.kg^{-1}.K^{-1};\ \Delta T = 1K$

It comes that:

$a = \frac{m_{mole}^2.c_{cal}^2.\Delta T^2}{N_A^2.c^2.h} \approx 3.7\ 10^{-29}\ kg.s^{-1}$ (13-127)

Let's see for copper:

$m_{mole} = 63.546\ g\ ;\ c_{cal} = 385\ j.kg^{-1}.°K^{-1};\ \Delta T = 1K$

So:

$a \approx 0.268\ 10^{-28}\ kg.s^{-1}$ (13-128)

Lets' see for liquids such as water:

$m_{mole} = 18\ g\ ;\ c_{cal} = 4180\ j.kg^{-1}.°K^{-1};\ \Delta T = 1K$

So:

$a \approx 2.62\ 10^{-28}\ kg.s^{-1}$ (13-129)

Another liquid mercury:

$m_{mole} = 200.6\ g\ ;\ c_{cal} = 139\ j.kg^{-1}.K^{-1};\ \Delta T = 1K$

So :

$a \approx 0.35\ 10^{-28}\ \ kg.s^{-1}$ (13-130)

Let's see for gases such as hydrogen:

$m_{mole} = 1\ g\ ;\ c_{Pcal} = 14266\ j.kg^{-1}.K^{-1};\ c_{Vcal} = 10300\ j.kg^{-1}.K^{-1}; \Delta T = 1K$

We don't take care about c_{Pcal} because it concern the enthalpy of the gas when there is variation of the volume so there is mechanical work. We take only c_{Vcal} the heat capacity in constant volume so the energy of heat is totally transmitted to the gas. We get by (13-126):

$a \approx 4.9\ 10^{-30}\ kg.s^{-1}$ (13-131)

For Helium we have:

$m_{mole} = 4\ g\ ;\ c_{Vcal} = 5193\ j.kg^{-1}.K^{-1}; \Delta T = 1K$

So:

$a \approx 2\ 10^{-29}\ kg.s^{-1}$ (13-132)

For chloride:

$m_{mole} = 71\ g\ ;\ c_{Pcal} = 33\ j.kg^{-1}.K^{-1};\ c_{Vcal} = 480\ j.kg^{-1}.K^{-1}; \Delta T = 1K$

So: $a \approx 5\ 10^{-29}\ kg.s^{-1}$ (13-133)

For the most heaviest mono-atomic gas the Xenon:

$m_{mole} = 130\ g\ ;\ c_{Vcal} = 158\ j.kg^{-1}.K^{-1}; \Delta T = 1K$

So:

$a \approx 1.95\ 10^{-29}\ kg.s^{-1}$ (13-134)

For Air considered as a perfect gas:

$m_{mole} = 29\ g\ ;\ c_{Vcal} = 1005\ j.kg^{-1}.K^{-1}; \Delta T = 1K$

So:

$$a \approx 3.93\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}135)$$

In general for a mono-atomic gas or a dia-atomic gas at low temperature (neglecting vibration energy and rotational energy) we have always:

$$c_V = \frac{3}{2}.\frac{R}{m_{mole}} \qquad (13\text{-}136)$$

With: $R = 8.314\ joule.mole^{-1}.°K^{-1}$

It comes that for $\Delta T = 1K$:

$$\frac{m_{mole}}{N_A}.c_V.\Delta T = c.\sqrt{h.a} \qquad (13\text{-}137)$$

So:

$$a = \frac{9.R^2}{4.c^2.h.N_A^2} = \frac{9x8.314^2}{4x9\ 10^{16}x6.62\ 10^{-34}x6.02^2\ 10^{46}} = 0.72\ 10^{-29}\ kg.s^{-1} \quad (13\text{-}138)$$

In fact for solids the heat capacity depends on temperature as Debye formulae [23]:

$$c_V = \frac{12}{5}.\pi^4.R.\left(\frac{T}{\theta}\right)^3\ \ if\ T \ll \theta\ \ \&\ c_V = 3R\ if\ T \gg \theta \qquad (13\text{-}139)$$

For Iron for example $\theta = 420\ K$ and so at for example $T = 27\ C$ we have
$c_V = 707\ j.kg^{-1}.K^{-1}$ so with the formulae:

$$\frac{m_{mole}}{N_A}.c_V.1K = c.\sqrt{h.a} \qquad (13\text{-}140)$$

We get:

$$a = 7.2\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}141)$$

For Copper $\theta = 315\ K$ and at $T = 27\ C$ we get $c_V = 1676\ joule.kg^{-1}.K^{-1}$ we deduce from (13-139) that:

$$a \approx 0.5\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}142)$$

For solids at high temperature we deduce the maximum molar mass :

$$M_{mole} = \frac{c.\sqrt{\hbar.a}\,.N_A}{3R.1°K} \approx 527.3\ g \qquad (13\text{-}143)$$

But the exchanging energy happen at maximalist situation so we should have:

$$M_{mole} \leq \frac{c.\sqrt{\hbar.a}\,.N_A}{3R.1K}.\frac{1}{2+W(-2.e^{-2})} \approx 330.4\ g \qquad (13\text{-}144)$$

Equation (13-144) gives us the maximum molar mass of a pure chemical element which can exist in nature. The maximum molar mass of a chemical element which had been discovered until this day is the Oganesson with a molar mass equal to $294\ g$: it seems that there is many other discoveries to do.

13-11)About the Planck system:

From Planck system we can define the following entities:

-The Planck elementary energy:

$$U_0 = M_P.c^2 = \sqrt{\frac{\hbar.c^5}{G}} \qquad (13\text{-}145)$$

-The Planck mechanical impedance:

$$A_0 = \frac{M_P}{T_P} = \frac{c^3}{G} \qquad (13\text{-}146)$$

-The Planck force:

$$F_0 = A_0.c = \frac{c^4}{G} \qquad (13\text{-}147)$$

-The Planck time:

$$H_0 = T_P = \sqrt{\frac{\hbar.G}{c^5}} \qquad (13\text{-}148)$$

-The Planck power:

$$S_0 = A_0.c^2 \qquad (13\text{-}149)$$

-The Planck momentum:

$$P_0 = \frac{U_0}{c} = \sqrt{\frac{\hbar . c^3}{G}} \qquad (13\text{-}150)$$

-The Planck length:

$$L_0 = L_P = c . T_P = \sqrt{\frac{\hbar . G}{c^3}} \qquad (13\text{-}151)$$

We have always:

$$\hbar = U_0 . H_0 = F_0 . L_0 . H_0 = P_0 . L_0 \qquad \& \left(\frac{U_0}{c}\right)^2 - P_0^2 = 0 \qquad (13\text{-}152)$$

Are there any physical significance of *Planck thing?*.It seems from its mechanical impedance nothing can't move: we should apologize to use this system. All scientists should not deal in the future with the Planck system: it doesn't mean nothing. For the constant G it can be deduced from the new system of unities as a coupling constant for the gravitational force. The new system of unities is a system for life.

As we had seen the gravitational force becomes important when the masses used are superior to $m_0 = 1.86\ 10^{-9}\ kg$. Planck mass respond to this condition. Let's take the new mass and so it should verify that:

$$\frac{1}{c} . \sqrt{\hbar . a} > e . \sqrt{\frac{K}{G}} \qquad (13\text{-}153)$$

It comes that:

$$a > \frac{e^2 . c^2 . K}{G . \hbar} = \frac{1.6^2\ 10^{-38} . 9.10^{16} . 9.10^9}{6.67\ 10^{-11} . 1{,}054\ 10^{-34}} = 29.5\ 10^{21}\ kg . s^{-1} \qquad (13\text{-}154)$$

It is a very great value: nothing can't move that's why the Planck system hasn't any physical significance. All scientists should renounce to built theories with this system. The Planck system hidden the natural system and induct us in error. Nevertheless the Planck system can push us to suppose that it exist a mechanical impedance for vacuum "a" as for every corpuscle to get $\hbar . \omega = a . \tau . c^2$ ….etc and build a new theory when we can predict the existence of an universal constant like "G" . The common point for the two systems is equation (13-152) that's why one hid the other. If we replace in the new system constant "a" by $\frac{c^3}{G}$ we get the Planck system. Nevertheless if Planck system has any physical sense it will be for very special situations where the known natural physical laws are not applicable.

Let’s take for Planck system the following definitions:

$$M_P = \sqrt{\frac{h.c}{G}} \quad , L_P = \sqrt{\frac{h.G}{c^3}} \quad , T_P = \sqrt{\frac{h.G}{c^5}} \qquad (13\text{-}155)$$

Suppose that we will determine the constants h , k & a in other manner than the Planck one.

The first law of radiation displacement is:

$$\frac{h.\nu_{max-energy}}{k.T} = 2.8214 \qquad (13\text{-}156)$$

The second law of radiation displacement is:

$$\frac{h.\nu_{max-power}}{k.T} = 2 + W(-2.e^{-2}) \qquad (13\text{-}157)$$

The definition of Planck temperature is as:

$$k.T = M_P.c^2 = \sqrt{\frac{h.c^5}{G}} \qquad (13\text{-}158)$$

We do that:

$$P(T) \approx \frac{U(T)}{H_0} \qquad (13\text{-}159)$$

With : $H_0 = T_P = \sqrt{\frac{h.G}{c^5}}$ the base of time for Planck.

We get:

$$2.\zeta(3).a = \frac{\pi^4.c^3}{15.G} \qquad (13\text{-}160)$$

It is an horrible result or impossible result if we take the experimental values of constants in this equation .

Let’s take for the base of time $h_0 = \frac{1}{c}.\sqrt{\frac{h}{a}}$ so we get:

$$4.\zeta(3)^2.a = \frac{\pi^8.c^3}{15^2.G} \qquad (13\text{-}161)$$

It is another horrible result or impossible result if we take the experimental values of constants in this equation .

We should correct the Planck temperature to avoid which is horrible or impossible as :

$$k.T = \alpha.M_P.c^2 \qquad (13\text{-}162)$$

Replacing (13-162) in Planck law of black body and Wien law we get the ratios $\frac{\alpha^4}{h^3}$ & $\frac{h}{\alpha}$ referring to F.Kurlbaum experiment. But we can deduce $\frac{k^4}{h^3}$ & $\frac{h}{k}$ without needing equation (13-162) as Planck had did it. So we can deduce temperature T to compare it with $1K$. If it is different than there is not any relation ship between a & G.

We get using Planck base of time:

$$2.\zeta(3).a = \alpha.\frac{\pi^4.c^3}{15.G} \qquad (13\text{-}163)$$

So taking the experimental values of constants we get:

$$\alpha = 7.3\ 10^{-66} \quad \& \quad T = 256.84\ 10^{-35}\ K \approx zero\ K \qquad (13\text{-}164)$$

This temperature correspond to the classic definition vacuum where nothing move or we take it as a black hole temperature where the physical common laws are not available.

Using the new base of time we get:

$$4.\zeta(3)^2.a = \alpha.\frac{\pi^8.c^3}{15^2.G} \qquad (13\text{-}165)$$

$$\alpha = 2.7\ 10^{-66} \quad \& \quad T = 95\ 10^{-35}\ K \approx zero\ K \qquad (13\text{-}166)$$

It is not horrible: it is a classic result or consider it as a temperature of a black hole (we disconnect constants).

However we can establish relation ship between the two systems using ratios as the following:

$$\frac{1}{c}.\sqrt{h.a} = \alpha.\sqrt{\frac{h.c}{G}} \Rightarrow \alpha = \sqrt{\frac{a.G}{c^3}} = \sqrt{\frac{0{,}8\ 10^{-29}x6.67\ 10^{-11}}{27\ 10^{24}}} = 0.44\ 10^{-32} \qquad (13\text{-}167)$$

$$\sqrt{\frac{h}{a}} = \beta.\sqrt{\frac{h.G}{c^3}} \Rightarrow \beta = \frac{1}{\alpha} = 2.27\ 10^{32} \qquad (13\text{-}168)$$

$$\frac{1}{c}.\sqrt{\frac{h}{a}} = \gamma.\sqrt{\frac{h.G}{c^5}} \Rightarrow \gamma = \beta \qquad (13\text{-}169)$$

We can do the same analyses for the Stoney-Jones system compared to the absolute natural system.

13-12) Black holes:

The temperature of a black hole considered as a black body is according to Hawking point of view is as [24]:

$$T = \frac{\hbar.\gamma}{2\pi ck} \qquad (13\text{-}170)$$

Where γ: is the surface gravity of the black hole [25].

Probably we need corrective coefficients[26].

According to the point of view of Unruh, an accelerated observer in a Minkowski referential see its environment as a black body with a temperature as given by equation (13-170) where an other observer linked to the Minkowski referential can't detect anything.

Let's have a black hole with an horizon radius $D_0 = \frac{2.G.M}{c^2}$ where M is the mass of the black hole. Near this radius the emitted radiation by the black hole (X-ray or gamma-ray) is enveloped by a wave as given by (13-68):

$$\frac{kT}{\lambda} = \frac{15.\zeta(3).a.c}{\pi^4} \qquad (13\text{-}171)$$

i.e. the horizon of the black hole is equal to: $D(t) = D_0 \pm \lambda(t)$.

Replace kT by its expression in (13-170) we get:

$$\frac{-\hbar.\ddot{\lambda}}{2\pi c.\lambda} = \frac{15.\zeta(3).a.c}{\pi^4} \qquad (13\text{-}172)$$

It comes that:

$$\ddot{\lambda} + \frac{30.\zeta(3).a.c^2}{\pi^3.\hbar}.\lambda = 0 \qquad (13\text{-}173)$$

So the envelop of the radiation has an angular frequency as:

$$\omega = \sqrt{\frac{30.\zeta(3).a.c^2}{\pi^3.\hbar}} = \sqrt{\frac{30x1.2x9\ 10^{16}x0.8\ 10^{-29}}{\pi^3x1.054\ 10^{-34}}} = 0.89\ 10^{10}\ Hz = 2\pi\nu \quad (13\text{-}174)$$

Than the frequency is:

$\nu = 0.14\ 10^{10}\ Hz$ (13-175)

We deduce the wavelength of the envelop as:

$\lambda_0 = \frac{c}{\nu} = \frac{3\ 10^8}{0.14\ 10^{10}} \approx 21.43\ 10^{-2} m = 21.43\ cm$ (13-176)

If cosmologists prove the existence of such envelop so the existence of black holes is true and Hawking-Unruh assumptions are good.

13-13)From the mechanical properties of metals:

Let's have a beam of metal under pressure. The change of the length of this beam obey to Hooke law:

$\sigma = B.\varepsilon$ (13-177)

Where σ: the strength under the beam;

B : Elasticity module ;

ε : relative augmentation of the length of the beam;

For a beam with pressure under all sides we have:

$\Delta P = -B.\frac{\Delta V}{V_0}$ (13-178)

With: V_0: the initial volume of the beam

ΔV: the variation of the volume due to pressure ΔP.

The beam is like a spring so it can magazine the energy :

$\Delta U = \Delta P. V_0 = -B.\Delta V$ (13-179)

The same phenomenon (variation of the volume) can be observed if we heat the beam. Let's take one mole of the metal and we elevate its temperature to one degree Kelvin.

The variation of the internal energy of the beam is :

$\Delta U = B.\Delta V$ (13-180)

Where $\Delta V = \alpha. V_0. \Delta T$: the variation of the volume of the metal due to heat;

$\Delta T = 1K$

α : coefficient of thermal dilatation of the metal ;

V_0: volume of one mole of the metal.

From (13-180) we have:

$$\Delta U = B.\Delta V = B.\alpha.V_0.\Delta T = N_A.c.\sqrt{h.a} \qquad (13\text{-}181)$$

To augment the temperature of the beam of one degree Kelvin, every atom of the beam will absorb at least the energy "$M.c^2 = c.\sqrt{h.a}$" given by heating.

From (13-181) we deduce that:

$$a = \frac{B^2.\alpha^2.V_0^2.\Delta T^2}{N_A^2.c^2.h} \qquad (13\text{-}182)$$

*For steel (Iron with 1% carbon) we have:

$B = 200\ GPa\ ,\ \ \alpha = 12\ 10^{-6}\ K^{-1}\ \ ,V_{mole} = 7.09\ \ 10^{-6}\ m^3$

So:

$$a = \frac{4\ 10^{22} x\ 144\ 10^{-12} x 7.09^2\ 10^{-12}}{6.02^2\ 10^{46} x 9\ 10^{16} x 6.62\ 10^{-34}} = 1.34\ 10^{-29}\ \ kg.s^{-1}$$

*For copper we have:

$B = 124\ GPa\ ,\ \ \alpha = 17\ 10^{-6}\ K^{-1}\ \ ,V_0 = 7.11\ \ 10^{-6}\ m^3$

So:

$$a = \frac{124^2\ 10^{18} x\ 17^2\ 10^{-12} x 7.11^2\ 10^{-12}}{6.02^2\ 10^{46} x 9\ 10^{16} x 6.62\ 10^{-34}} = 1.04\ 10^{-29}\ \ kg.s^{-1}$$

*For zinc we have:

$B = 78\ GPa\ ,\ \ \alpha = 35\ 10^{-6}\ K^{-1}\ \ ,V_0 = 9.16\ \ 10^{-6}\ m^3$

So:

$$a = \frac{78^2\ 10^{18} x\ 35^2\ 10^{-12} x 9.16^2\ 10^{-12}}{6.02^2\ 10^{46} x 9\ 10^{16} x 6.62\ 10^{-34}} = 2.89\ 10^{-29}\ \ kg.s^{-1}$$

*For aluminum we have:

$B = 69\ GPa\ ,\ \ \alpha = 26\ 10^{-6}\ K^{-1}\ \ ,V_0 = 10\ \ 10^{-6}\ m^3$

So:

$a = \frac{69^2\ 10^{18}x\ 26^2\ 10^{-12}x10^2\ 10^{-12}}{6.02^2\ 10^{46}x9\ 10^{16}x6.62\ 10^{-34}} = 1.49\ 10^{-29}\ \ kg.s^{-1}$

We get approximately the same values.

13-14)From cosmology:

Let's have an inertial referential $R(O, x, y, z, t)$ .Let's have another inertial referential $R'(O', x', y', z', t')$ in motion with a speed V along the axle (O, x). At time $t = t' = 0$ the origins coincide and all axles are co-linear.

Let's have a corpuscle of a mass m in motion in those referentials from a universe point A to a universe point B.

The Lorentz transformations of space and time between the referentials are:

$$x' = \frac{x-V.t}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad t' = \frac{t-x.\frac{V}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (13\text{-}183)$$

The transformations of inertia and momentum are as the following:

$$\xi' = \frac{\xi-\boldsymbol{p}.\frac{V}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad \boldsymbol{p}' = \frac{\boldsymbol{p}-\xi.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (13\text{-}184)$$

Where $\xi = \frac{E}{c^2} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$: the inertia of the corpuscle;

$\boldsymbol{p} = \xi.\boldsymbol{v}$: the momentum of the corpuscle .

From (13-208) we have:

$$\frac{x\prime}{t\prime} = \frac{x}{t}.\frac{1-\frac{V.t}{x}}{1-\frac{x.V}{t.c^2}} \qquad (13\text{-}185)$$

If $x = ct$ in equation (13-210) than $x' = ct'$.

The action of a free corpuscle is defied as [30]:

$S = -\alpha . \int_A^B ds$ (13-186)

With $\alpha = m.c$: a characteristic of the corpuscle ;

$ds = c.dt.\sqrt{1 - \frac{v^2}{c^2}}$:the interval in a Minkowski space-time.

According to the principle of least action the action of the corpuscle is minimum for a real motion of this one. It is clear that from equation (13-186) the action of a free corpuscle can't be minimum along a straight line. The motion of the corpuscle is always along a curved line : vacuum influence the motion of the corpuscle and curve its trajectory. The only way to move in a straight line is to be in rest or to have the speed of light "c" :there is a problem with Restraint Relativity.

Locally we can consider that the speed of the corpuscle is approximately constant between the instants t & $t + dt$.

From equation (13-183) we have:

$dt' = dt.\frac{1-\boldsymbol{v}.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}}$ (13-187)

From equation (13-184) we have:

$d\xi' = d\xi.\frac{1-\frac{\boldsymbol{v}.\boldsymbol{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}$ (13-188)

Because $\boldsymbol{v} \approx \boldsymbol{constant}$ between t & $t + dt$.

It comes that:

$\frac{d\xi\prime}{dt\prime} = \frac{d\xi}{dt}$ (13-189)

The most simple solution for a curved motion of the corpuscle is that:

$\frac{d\xi}{dt} = a$ (13-190)

Where "a" : is a constant considered as the mechanical impedance of vacuum.

We associate to the corpuscle the inertial time "τ" as:

$\xi = a.\tau$ (13-191)

With : $d\tau = dt$ when the energy of the corpuscle is varying

$d\tau = 0$ if the energy of the corpuscle is constant.

It is clear that:

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (13\text{-}192)$$

With $\tau_0 = \frac{m}{a}$: is the inertial time of the corpuscle in rest.

Constant "a" is declared as an universal constant .

The locally equation of motion of the corpuscle is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \quad (13\text{-}193)$$

Where: all unknown forces which act on the corpuscle;

$-a.\boldsymbol{v}$: the action of space-time on the corpuscle.

If the corpuscle can move with a speed of light than it can have a wave behavior. We define the quadric impetus of the corpuscle in a Minkowski space-time as:

$$p^i = (\xi c, \boldsymbol{p}) \quad (13\text{-}194)$$

If the corpuscle can have a wave behavior we associate to it a four-wave vector as:

$$k^i = \left(\frac{\omega}{c}, \boldsymbol{k}\right) \quad (13\text{-}195)$$

The duality wave-corpuscle implies that there is a relationship between the corpuscle characteristics and the wave characteristics so we have:

$$p^i = \beta.k^i \quad (13\text{-}196)$$

Where β: is a new universal constant;

It is clear that we have:

$$\xi.c^2 = \beta.\omega \quad (13\text{-}197)$$

$\boldsymbol{p} = \beta.\boldsymbol{k}$ (13-198)

The wave function of the corpuscle is a plane wave as:

$\psi(x,t) = A \exp(ikx - i\omega t)$ (13-199)

Where A: the amplitude of the wave;

But here there is a contradiction: in a hand the corpuscle is a point of space-time & in the other hand it is a plane wave present in all space-time. To resolve this problem we apply the principle of non-contradiction: the corpuscle is a pocket of waves that its group speed is equal to the speed of the corpuscle (De Broglie postulate or unity-multiplicity duality) with the conditions that:

$\Delta k.\Delta x \geq 1$ & $\Delta\omega.\Delta t \geq 1$ (13-200)

Equation (13-200) is Heisenberg principle of uncertainty.

From equation (13-201) we deduce that:

$\Delta E.\Delta t \geq \beta$ (13-201)

But we have $\Delta E = a.c^2\Delta t$, it comes that:

$\Delta t \geq \frac{1}{c}.\sqrt{\frac{\beta}{a}}$ (13-202)

We define the absolute system as the following:

$T = \frac{1}{c}.\sqrt{\frac{\beta}{a}}$, $M = a.T = \frac{1}{c}.\sqrt{\beta.a}$, $L = c.T = \sqrt{\frac{\beta}{a}}$ (13-203)

It is clear that if we choose $\beta = \hbar$ & $a = \frac{c^3}{G}$ we get from (13-203) the Planck system, but this choice is horrible at least for the constant "a" because it implies that nothing can't move in such medium. So the constant $a \neq \frac{c^3}{G}$.

The reference units of measure are now:

$T = \frac{1}{c}.\sqrt{\frac{\hbar}{a}}$, $M = a.T = \frac{1}{c}.\sqrt{\hbar.a}$, $L = c.T = \sqrt{\frac{\hbar}{a}}$ (13-204)

The wave function of the corpuscle obey to Klein-Gordon equation: it signify that the medium is dispersive for the pocket of waves. A dispersive medium for waves correspond for corpuscle to a viscous medium. The wave-corpuscle duality hide two dualities: unity-multiplicity duality & viscous-dispersive duality.

In 1965 , Panzias & Wilson had discovered the CMB radiation at $3\ K$. Universe is considered as a perfect black body with its maximum radiation of energy is at $T_0 = 3\ K$: there is no vacuum in space-time.

From Wien displacement law we deduce that:

$$\lambda_0 . T_0 = C_0 = 3\ 10^{-3}\ m.K \quad so \ \ \lambda_0 = \frac{C_0}{T_0} = 10^{-3} m = \frac{c}{\nu_0}$$

Than : $\nu_0 = 3.\ 10^{11}\ Hz$

The base of energy is:

$$E_0 = h.\nu_0 = 19.89\ 10^{-23}\ joule$$

And we have that:

$$a = \frac{E_0^2}{\hbar.c} \approx 0.95\ 10^{-29}\ kg.s^{-1} \qquad (13\text{-}205)$$

13-15)From natural width radiation of atoms:

Let's have a Bohr atom . The intensity of radiation of an electron in interaction with an external radiation of frequency ω_0 is given by Weisskopf & Wigner formulae is [31]:

$$I(\omega) = \frac{\Gamma}{2\pi} . \frac{\hbar.\omega_0}{(\omega-\omega_0)^2 + \frac{1}{4}.\Gamma^2} \qquad (13\text{-}206)$$

With $\Gamma = \frac{1}{\tau}$ & $\tau : life\ time\ of\ the\ excited\ level$.

And by definition [31]:

$$I(\omega). d\omega = \hbar\omega. dP_r \qquad (13\text{-}207)$$

P_r : the probability to excite the level of energy.

Or:

$$\frac{\Delta E}{\tau} = a.c^2 = \frac{\hbar.(\omega - \omega_0)}{\tau} \qquad (13\text{-}208)$$

So:

$$\tau = \frac{\hbar.(\omega - \omega_0)}{a.c^2} \qquad (13\text{-}209)$$

It comes that:

$$I(\omega) = \frac{1}{\pi a.c^2}.\frac{2\hbar^2.(\omega-\omega_0).\omega_0}{1+\frac{4\hbar^2}{a^2c^4}(\omega-\omega_0)^3} \approx \frac{2\hbar^2.(\omega-\omega_0).\omega_0}{\pi a.c^2}\left[1 - \frac{4\hbar^2}{a^2c^4}(\omega-\omega_0)^3\right] \approx \frac{2\hbar^2.(\omega-\omega_0).\omega_0}{\pi a.c^2}$$

(13-210)

I suppose that the value of $a = 0.8\ 10^{-29}\ kg.s^{-1}$ will be confirmed by the photo-electric experiment.

From equation (13-207) & (13-210) we deduce that:

$$dP_r = \frac{2.\hbar\omega_0}{\pi ac^2}\left(1 - \frac{\omega_0}{\omega}\right).d\omega \qquad (13\text{-}211)$$

The maximum of the probability (13-211) will be one so:

$$1 = \frac{2.\hbar\omega_0}{\pi ac^2}.\left[\omega - \omega_0 - \omega_0.Ln\left(\frac{\omega}{\omega_0}\right)\right] \qquad (13\text{-}212)$$

Neglecting the fine structure in (13-212) so $\omega \approx \omega_0$ i.e. the energy of the excited level is exactly $E = E_0 + \hbar.\omega_0$ where E_0 is the energy of the fundamental state .So it comes that:

$$\hbar(\omega - \omega_0) = \frac{\pi a.c^2.\hbar}{2\hbar.\omega_0} \qquad (13\text{-}213)$$

For optic transition the energy $\hbar.\omega_0 \approx 10\ eV$ for all atoms (practically the incident energy needed to ionize the atom) so we get:

$$\hbar.\Delta\omega \approx \frac{\pi x 0.8\ 10^{-29} x 9\ 10^{16} x 1.054\ 10^{-34}}{2x10x1.6\ 10^{-19}} \approx 0.745\ 10^{-28}\ joule \approx 0.46\ 10^{-9}\ eV \quad (13\text{-}214)$$

So:

$$\tau \approx 14\ \ 10^{-7}\ s \qquad (13\text{-}215)$$

Which is a value confirmed in general by the experience (in all books this time is about $10^{-8}\ second$).

Without neglecting the fine structure equation (13-212) becomes:

$$1 = \frac{2.\hbar\omega_0}{\pi a c^2}.[\omega - \omega_0 - \omega_0.Ln\left(1 + \frac{\Delta\omega}{\omega_0}\right)]$$

With the condition that:

$Ln(1 + x) \approx x - \frac{x^2}{2}\ when\ x \to 0$ we get :

$$\tau \approx \frac{1}{c}\sqrt{\frac{\hbar}{\pi.a}} \approx 0.682\ \ 10^{-11}\ s \qquad (13\text{-}216)$$

In the inverse sense we have [32]:

$$\Delta E = \frac{m.e^8}{32.\hbar^4.c^2} = a.c^2.\tau = \frac{\alpha^2}{16}.R_\infty = c.\sqrt{\frac{\hbar.a}{\pi}} \qquad (13\text{-}217)$$

Equation (13-216) is an important equation: it signify that the constant "$m.e^8$" is an universal constant where "m" is the mass of the electron and "e" is its electric charge. Constant "$\alpha = \frac{1}{137}$" is the fine structure constant and "$R_\infty = 13.6\ eV$" is the non relativist ionization potential of the hydrogen atom .

It is very easy to deduce from (13-216) that:

$$a = \frac{\pi.\alpha^4.R_\infty^2}{256.c^2.\hbar} = \frac{\pi x 13.6^2 x 1.6^2\ 10^{-38}}{256 x 9\ 10^{16} x 1.054\ 10^{-34} x 137^4} = 10^{-29}\ kg.s^{-1} \quad (13\text{-}218)$$

Our predictions are very good.

If we want that equation (13-218) fit exactly the value given by the second law of radiation displacement we should choose that the integration of the probability (13-211) is exactly equal to $\sqrt{0.8} = 0.89$ and than we have:

$$a = 0.8\ \frac{\pi.\alpha^4.R_\infty^2}{256.c^2.\hbar} = \frac{\pi.\alpha^4.R_\infty^2}{320.c^2.\hbar} \qquad (13\text{-}219)$$

In fact it is not a choice , it is an obligation to be coherent.

To ionize a gas the maximum of probability to find N_∞ atoms in the ionized state is at maximum equal to 89% of the fundamental state N_0. We have [34]:

$$\frac{N_\infty}{N_0} = \exp\left(-\frac{E_\infty - E_0}{k.T}\right) = 0.89\ max \qquad (13\text{-}220)$$

With $E_\infty = 0$ the energy of the ionized state

E_0: the energy of the fundamental state

So:

$$T = \frac{E_0}{k.Ln(0.89)} \qquad (13\text{-}221)$$

14)Vacuum energy levels:

14-1) The energy of the corpuscle as an exchange energy with vacuum:

The work of the friction force between two points A & B of the trajectory of the corpuscle is as follows (there is sign minus which we omit for commodity):

$$\varepsilon_{AB} = \int_A^B a.\boldsymbol{v}.d\boldsymbol{x} = \int_A^B a.v^2.d\tau = a.c^2.(\tau_B - \tau_A) + a.c^2.\tau_0^2.\left(\frac{1}{\tau_B} - \frac{1}{\tau_A}\right)$$

$$= a.c^2.(\tau_B - \tau_A).(1 - \frac{\tau_0^2}{\tau_A.\tau_B}) \qquad (14\text{-}1)$$

We take the origin of the energy as the rest state of the corpuscle so:

$$\varepsilon_{AB} = \varepsilon_B - \varepsilon_A \qquad (14\text{-}2)$$

With:

$$\varepsilon_B = a.c^2.(\tau_B - \tau_0).\left(1 - \frac{\tau_0}{\tau_B}\right) = a.c^2.\tau_B.\left(1 - \frac{\tau_0}{\tau_B}\right)^2 \ (14\text{-}3)$$

Idem for ε_A.

We define the energy exchanged by the corpuscle with vacuum as:

$$\varepsilon = a.c^2.\tau.(1 - \frac{\tau_0}{\tau})^2 = \xi.c^2.(1 - \frac{m}{\xi})^2 \qquad (14\text{-}4)$$

In general this energy can be positive of negative.

If the speed of the corpuscle tends to the celerity of light than we have from (14-4):

$$\varepsilon \approx \xi.c^2 \qquad (14\text{-}5)$$

The energy exchanged with vacuum (14-1) corresponds exactly to the energy exchanged with vacuum by light i.e. by a corpuscle which has a mass equal to zero. We can deduce that the energy of a corpuscle of a mass m is approximately as follows:

$$E \approx \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (14\text{-}6)$$

Its moment is as follows :

$$\boldsymbol{p} = \frac{E}{c^2}.\boldsymbol{v} \approx \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (14\text{-}7)$$

Its Lagrangian is as follows :

$$L = \boldsymbol{p}.\boldsymbol{v} - E \approx -m.c^2.\sqrt{1-\frac{v^2}{c^2}} \qquad (14\text{-}8)$$

14-2) Vacuum energy levels:

In the interval $(x \pm \Delta x, t \pm \Delta t)$the corpuscle is a superposition of many monochromatic waves at every point of this interval and also above. If we choose a certain discernible number of positions in this interval we can accept that the exchanging energy with vacuum is approximately:

$$\varepsilon_n = n.\varepsilon = n.\xi.c^2.(1-\frac{m}{\xi})^2 \qquad (14\text{-}9)$$

Where : is an integer which can be positive or negative (the corpuscle can take energy from vacuum or loose energy for vacuum).

With the condition that $\varepsilon_n \to 0$ when $n \to \pm\infty$: the total energy of the corpuscle should be finite.

The total energy of the corpuscle is:

$$E = \xi.c^2 + \varepsilon_n = \xi.c^2 + n.\xi.c^2.(1-\frac{m}{\xi})^2 \qquad (14\text{-}10)$$

With the condition that $\xi \to m$ when $n \to \pm\infty$ (the momentum is very well defined and it tends to zero but the position is bad defined).

This image is that the corpuscle is like an harmonic oscillator maintained in oscillation by a force $\boldsymbol{f} = a.\boldsymbol{v}$ where $''a''$ is a coefficient of a mechanical impedance.

The ‘’stability’’ for any mechanical system is in general defined as when its energy is an extremum i.e. in our case that :

$$\frac{dE}{d\xi} = 0 \qquad (14\text{-}11)$$

So we get:

$$\xi = \pm m.\sqrt{\frac{n}{n+1}} \qquad (14\text{-}12)$$

Replace (14-12) in (14-10) we get:

$$E_{(+)} = 2.m.c^2.\sqrt{n}.(\sqrt{n+1} - \sqrt{n}) \qquad (14\text{-}13)$$

$$E_{(-)} = -2.m.c^2.\sqrt{n}.(\sqrt{n+1} - \sqrt{n}) \qquad (14\text{-}14)$$

With :n positive integer

We can draw the curve $E = function(\xi)$ and we find that it had a minimum given by equation (14-12) for positive ξ and that there is no inflexion point because $\frac{d^2E}{d\xi^2}$ doesn’t change in sign.

If we consider the rest state as the origin of energy, the exchanged energy is:

$$\Delta E = E - m.c^2 \qquad (14\text{-}15)$$

For vacuum we take the mass m as equal in equation (1-5) and by equations (14-12) , (14-13) & (14-14) we get many levels of vacuum energy and exchanged energy with vacuum.

For every level of vacuum energy we can define a certain mass as for example $\frac{E_{(+)}}{c^2}$ and from this origin we get infinite other levels and so on.

14-3) Classical wave-corpuscle duality:

If the speed of the corpuscle is very low than we deduce from (14-9) the exchanged energy with vacuum:

$$\varepsilon \approx n.m.c^2.\frac{v^4}{4.c^4} \qquad (14\text{-}16)$$

It is evident that from (14-16) we have $n \rightarrow +\infty$ when $\rightarrow zero$: the classical case.

The energy of the corpuscle is:

$$E \approx m.c^2 + \frac{1}{2}m.v^2 = \hbar.\omega \qquad (14\text{-}17)$$

Wave-corpuscle duality:

$$\frac{1}{v} = \frac{dk}{d\omega} = \frac{dk}{dv}.\frac{dv}{d\omega} \qquad (14\text{-}18)$$

So we get from (14-17) & (14-18):

$$\hbar.\boldsymbol{k} = m.\boldsymbol{v} = \boldsymbol{p} \qquad (14\text{-}19)$$

14-4) Relativist wave-corpuscle duality:

The relativist case is when $v \to c$ so we have for energy:

$$E \approx (n+1)\xi.c^2 = \hbar.\omega \qquad (14\text{-}20)$$

From equation (14-18) and (14-20) it is easy to get:

$$\hbar.\boldsymbol{k} = (n+1).\frac{m.v}{\sqrt{1-\frac{v^2}{c^2}}} = (n+1).\boldsymbol{p} \qquad (14\text{-}21)$$

14-5)Physical mass:

The definition of the physical mass is when we need a cutting energy to determine a dynamic parameter of a corpuscle.

From (14-20) we have:

$$E^2 - p^2c^2 = m_\varphi^2.c^4 \qquad (14\text{-}22)$$

With: $p \approx m.c$ & $E \approx (n+1)m.c^2$

m_φ : is the physical mass (by definition)

It comes that:

$$m_\varphi = m.\sqrt{n^2 + 2.n} \qquad (14\text{-}23)$$

14-6)Classical quantum gravity:

In general relativity a gravitational field is constant when we can find a referential in which all the components of the metric tensor are independent from time component x^0 . The time component is called in this case *universe time.*

The physical significance of the universe time is that in a constant gravitational field the interval of this time between two events in a point of the space coincide with the time interval between any two other events in another point of space which happen simultaneously with the first couple of events.

In non-relativist mechanics the motion of a corpuscle in a gravitational field is determined by the Lagrange function as:

$$L = -m.c^2 + \frac{1}{2}m.v^2 - m.\varphi \quad \text{(14-24)}$$

With $\varphi = \frac{-GM}{R}$:the potential of the field

In (14-24) is added the constant " $-m.c^2$" in order to get that the Lagrange function without gravitational field "$L = -m.c^2 + \frac{1}{2}m.v^2$" is the same obtained from the relativist function "$L = -m.c^2\sqrt{1 - \frac{v^2}{c^2}}$" when "$\frac{v}{c} \rightarrow 0$".

The non relativist action S of the corpuscle is as per definition:

$$S = \int L\,.dt = -m.c\int\left(c - \frac{v^2}{2c} + \frac{\varphi}{c}\right)dt = -m.c\int ds \quad \text{(14-25)}$$

So by identification we get that:

$$ds = \left(c - \frac{v^2}{2c} + \frac{\varphi}{c}\right)dt \quad \text{(14-26)}$$

Than:

$$ds^2 = \left(c^2 + \frac{v^4}{4c^2} - v^2 + \frac{\varphi^2}{c^2} + 2\varphi - \varphi.\frac{v^2}{c^2}\right)dt^2 \approx (c^2 + 2\varphi).dt^2 - d\boldsymbol{R}^2 \quad \text{(14-27)}$$

With $d\boldsymbol{R} = v.dt$

So the time component of the metric tensor is :

$g_{00} = 1 + \frac{2\varphi}{c^2}$ (14-27)

In the theory of General Relativity, there is no restriction of the choice of the referential: any quantities which characterise the disposition of bodies in space can play the role of the space components x^1, x^2, x^3 and the time component x^0 can be determined by a clock which indicate its proper time.

Let's search the link between the real time noted as τ and the coordinate x^0. Let's consider two events which happen infinitely reproached which happen in the same point of space. In those conditions the interval ds^2 between the two events is equal to "$c.d\tau$" where "$d\tau$" is the real time between the two events.

Putting $dx^1 = dx^2 = dx^3 = 0$ we get for the interval:

$ds^2 = g_{ik}dx^i dx^k = g_{00}(dx^0)^2$ (14-28)

Than:

$d\tau = \frac{1}{c}.\sqrt{g_{00}}.dx^0$ (14-29)

So the time which flow between any two events happened in the same space point is:

$\tau = \frac{1}{c}\int \sqrt{g_{00}}.dx^0$ (14-30)

In a constant gravitational field we have from (14-30):

$\tau = \frac{1}{c}.\sqrt{g_{00}}.x^0$ (14-31)

Let's remark that we should have always $g_{00} > 0$. A tensor which don't satisfy this condition don't correspond at any real gravitational field i.e. to a real space-time metric.

In a weak gravitational field we have from (14-31):

$\tau \approx \frac{x^0}{c}.(1 + \frac{\varphi}{c^2})$ (14-32)

So the proper time is flowing more slowly when the gravitational field in a space point is more weak.

When the corpuscle is in motion in a constant field, its energy as the derivate of the action $\left(-c.\frac{\partial S}{\partial x^0}\right)$ by the universe time is conserved because x^0 doesn't exist explicitly in the Hamilton-Jacobi equation. The energy is than the time component of the covariant four-vector of the moment $p_k = m.c.u_k = m.c.g_{ki}u^i$.

In a constant field we have:

$$ds^2 = g_{00}(dx^0)^2 - dl^2 \qquad (14\text{-}33)$$

With dl the elementary space distance as:

$$dl^2 = -g_{\alpha\beta}.dx^\alpha.dx^\beta \qquad (14\text{-}34)$$

$\alpha = 1,2,3$ & $\beta = 1,2,3$

The energy of the corpuscle is:

$$p_0 = m.c.u_0 = m.c.g_{00}u^0 = \frac{E_0}{c} \qquad (14\text{-}35)$$

So:

$$E_0 = m.c^2.g_{00}.\frac{dx^0}{ds} = m.c^2.g_{00}.\frac{dx_0}{\sqrt{g_{00}(dx^0)^2 - dl^2}} \qquad (14\text{-}36)$$

Introduce the speed of the corpuscle as:

$$v = \frac{dl}{d\tau} = \frac{c.dl}{\sqrt{g_{00}}.dx^0}$$

measured in proper time with an observer placed in a given place.

It comes that:

$$E_0 = \frac{m.c^2.\sqrt{g_{00}}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (14\text{-}37)$$

Than for a weak gravitational field (14-37) becomes:

$$E_0 \approx m.c^2 + \frac{1}{2}m.v^2 + \frac{3}{8}m.\frac{v^4}{c^2} + m.\varphi + \frac{1}{2}m.\varphi.\frac{v^2}{c^2} + \frac{3}{8}m.\varphi.\frac{v^4}{c^4} \qquad (14\text{-}38)$$

This energy measured in real time for a free corpuscle interacting with vacuum is:

$$E_0 = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + n.\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}.\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2 \approx m.c^2 + \frac{1}{2}m.v^2 + \frac{3}{8}m.\frac{v^4}{c^2} + n.\left(m.c^2 + \frac{1}{2}m.v^2 + \frac{3}{8}m.\frac{v^4}{c^2}\right).\frac{1}{4}.\frac{v^4}{c^4} \qquad (14\text{-}39)$$

It comes that:

$$m.\varphi + \frac{1}{2}m.\varphi.\frac{v^2}{c^2} + \frac{3}{8}m.\varphi.\frac{v^4}{c^4} = n.\left(m.c^2 + \frac{1}{2}m.v^2 + \frac{3}{8}m.\frac{v^4}{c^2}\right).\frac{1}{4}.\frac{v^4}{c^4}$$

Than we have:

$$\varphi.(1 + \frac{1}{2}.\frac{v^2}{c^2} + \frac{3}{8}.\frac{v^4}{c^4}) \approx \frac{n}{4}.\frac{v^4}{c^2} \qquad (14\text{-}40)$$

So:

$$\varphi \approx \frac{n}{4}.\frac{v^4}{c^2} = -\frac{G.M}{R} \qquad (14\text{-}41)$$

With n: integer which characterise the interaction of the corpuscle m with vacuum. It is so great because we have the classic condition that $v \ll c$.

With the condition that φ is constant i.e. the distance R between the body of a mass M and the corpuscle is practically constant . We can resolve the problem of the trajectory of the corpuscle under the classical gravitational force with the condition (14-41) & we will find quantified trajectories.

For circular motion of the corpuscle we have:

$$v = R.\omega \qquad (14\text{-}42)$$

With ω: angular speed of the corpuscle chosen as constant;

R: its position ;

From (14-41) we deduce the possible solutions for the trajectory;

$$R = R_n = \left(-\frac{4GM.c^2}{n.\omega^4}\right)^{0.2} \qquad (14\text{-}43)$$

With Kepler law dependent of time the system (M, m) will collapse. The only way to avoid this collapse is to take in consideration the principle of uncertainty :

$$\Delta p.\Delta R \approx \hbar \qquad (14\text{-}44)$$

$$\Delta E.\Delta t \approx \hbar \qquad (14\text{-}45)$$

It comes that from (14-42) & (14-44):

$$\Delta R = \sqrt{\frac{\hbar}{m.\omega}} \qquad (14\text{-}46)$$

From (14-39) we deduce that:

$$\Delta E = \frac{\Delta n}{4}.m.\frac{v^4}{c^2} \approx \frac{m.R^4.\omega^4}{4.c^2} \qquad (14\text{-}47)$$

So from (14-45) we get:

$$\Delta t = \frac{4.\hbar.c^2}{m.R^4.\omega^4} \qquad (14\text{-}48)$$

It comes that the radial speed is :

$$V = \frac{\Delta R}{\Delta t} = \frac{R^4}{4.c^2}.\sqrt{\frac{m.\omega^7}{\hbar}} \qquad (14\text{-}49)$$

So:

$$\frac{dR}{R^4} = \frac{1}{4.c^2}.\sqrt{\frac{m.\omega^7}{\hbar}}\ dt = K.dt \qquad (14\text{-}50)$$

With $K = \frac{1}{4.c^2}.\sqrt{\frac{m.\omega^7}{\hbar}} \approx \omega^3.\sqrt{m.\omega}$;

Than:

$$R^3 = \frac{R_0^3}{1-3K.R_0^3.t} \approx R_0^3.(1+3K.R_0^3.t) \qquad (14\text{-}51)$$

So:

$$R \approx R_0(1+K.R_0^3.t) \qquad (14\text{-}52)$$

The radial speed is:

$$V = K.R_0^4 = H.R_0 \qquad (14\text{-}53)$$

With :

$H = K.R_0^3$ (14-54)

H: is Hubble constant .

15)The magnetic field of a non-charged corpuscle:

As we had associate to a corpuscle (Energy, momentum) a wave function (frequency ,wave-vector) we can also do the same thing for the quadric vector (scalar potential, vector potential).

From equation (10-1) we deduce that:

$i\hbar \frac{\partial\psi(x,t)}{\partial t} = \frac{E}{c}\psi(\boldsymbol{x},t)$ (15-1)

$i\hbar\nabla\psi(\boldsymbol{x},t) = -\boldsymbol{p}\psi(\boldsymbol{x},t)$ (15-2)

It comes that :

$i\hbar(\frac{\partial\psi(x,t)}{\partial x^0}, \nabla\psi(x,t)) = \hbar k_i\psi(x,t)$ (15-3)

$\partial^i\psi(x,t) = -i\psi(x,t)k_i$ (15-4)

We can write the wave function as with two components:

-The time component:

$\psi_0(x,t) = \Theta_0.exp(ik.x - i\omega t)$ (15-5)

-The space component:

$\Gamma(x,t) = \kappa_0.exp(ik.x - i\omega t)$ (15-6)

And so we associate to the corpuscle a four-vector of the wave-function as:

$\psi_i = (\frac{E}{c}.\Theta_0, -\boldsymbol{k}.\kappa_0)$ (15-7)

And in general we have :

$i\hbar\partial^i\psi(x,t) = \hbar\psi_i\exp(ikx - i\omega t)$ (15-8)

The corpuscle is represented by its wave function or by its four vector potential so there is a constant χ that correspond as the following:

$$\psi_i = \chi . A_i \qquad (15\text{-}9)$$

We declare constant χ as a universal constant.

So we have from (15-9):

$$-\boldsymbol{k}.\kappa_0 = -\chi.\boldsymbol{A} \qquad (15\text{-}11)$$

We deduce that:

$$\varphi = \frac{E}{c.\chi}.\Theta_0 \qquad (15\text{-}12)$$

$$\boldsymbol{A} = \frac{\boldsymbol{p}}{\hbar.\chi}.\kappa_0 \qquad (15\text{-}13)$$

To simplify calculations we suppose that we have a non relativist corpuscle . This corpuscle is in constant circular motion with an angular speed Ω around the axle (O, z). We use polar coordinates. The position of the corpuscle is:

$$\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} R.cos\theta \\ R.sin\theta \\ 0 \end{pmatrix} \qquad (15\text{-}14)$$

With $\theta = \Omega.t$

The speed of the corpuscle is:

$$\boldsymbol{v} = \begin{pmatrix} -R.\dot{\theta}sin\theta \\ R.\dot{\theta}cos\theta \\ 0 \end{pmatrix} = \begin{pmatrix} R.\Omega sin\theta \\ -R.\Omega cos\theta \\ 0 \end{pmatrix} \qquad (15\text{-}15)$$

From (15-13) we have:

$$\boldsymbol{A} = \frac{m.\boldsymbol{v}}{\hbar.\chi}.\kappa_0 = \frac{m.R.\Omega}{\hbar.\chi}.\kappa_0 \begin{pmatrix} sin\Omega t \\ -cos\Omega t \\ 0 \end{pmatrix} = \frac{m.\Omega}{\hbar.\chi}.\kappa_0 \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} \qquad (15\text{-}16)$$

So:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \frac{m.R.\Omega^2}{\hbar.\chi}.\kappa_0 \begin{pmatrix} cos\Omega t \\ sin\Omega t \\ 0 \end{pmatrix} = \frac{m.\Omega^2}{\hbar.\chi}.\kappa_0.\boldsymbol{X} \qquad (15\text{-}17)$$

$grad\varphi = \mathbf{0}$ (15-18)

So the electric field of the corpuscle is:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma.grad\varphi = -\frac{\gamma}{c}.\frac{m.\Omega^2}{\hbar.\chi}.\kappa_0.\boldsymbol{X} = \frac{Q_e}{4\pi\varepsilon_0.R^2}.\boldsymbol{u_r} \qquad (15\text{-}19)$$

With:

Q_e: the electric charge associated to the corpuscle

ε_0: electric permittivity of vacuum

$\boldsymbol{u}_r = \frac{\boldsymbol{X}}{R}$: radial vector for polar coordinates.

We deduce from (15-19) that :

$$Q_e = -4\pi\varepsilon_0.R^3.\frac{\gamma}{c}.\frac{m.\Omega^2}{\hbar.\chi}.\kappa_0 \qquad (15\text{-}20)$$

We have also:

$$\nabla \times \boldsymbol{A} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}.\frac{m.\Omega}{\hbar.\chi}.\kappa_0 = \frac{m.\Omega}{\hbar.\chi}.\kappa_0 \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} \qquad (15\text{-}21)$$

So the magnetic field associated to the corpuscle is:

$$\boldsymbol{B} = -\frac{2.m.\Omega}{\hbar.\chi}.\kappa_0.\boldsymbol{e}_z \qquad (15\text{-}22)$$

We tends $R \to 0$ so from (15-20) we get:

$$Q_e = 0 \qquad (15\text{-}23)$$

We conclude that a non charged corpuscle which spin around its proper center have a charge equal to zero and a magnetic field not equal to zero and that’s what it should be expected: an experimental proof is our Earth.

For a relativist corpuscle we have from (15-13):

$$A = \frac{p}{\hbar.\chi}.\sqrt{\kappa_0\kappa_0^*} = \frac{\omega}{c.\chi}.\sqrt{\kappa_0\kappa_0^*}.\sqrt{1 - \frac{m^2.c^4}{\hbar^2.\omega^2}} \qquad (15\text{-}24)$$

Where A & p are respectively the module of vector-potential and the module of the momentum.

For photons and ultra-relativist corpuscles it comes that:

$$A \approx \frac{\omega}{c.\chi}.\sqrt{\kappa_0 \kappa_0^*} \qquad (15\text{-}25)$$

Where κ_0^* is the conjugate of κ_0 .

There is a linear relationship between the module of the amplitude of the potential-vector and the frequency of an ultra-relativist corpuscle.

$\sqrt{\kappa_0 \kappa_0^*}$: is the confinement of the corpuscle.

16)Conclusion:

In equation (2-131) we had concluded that the energy of corpuscle is as:

$$E = \beta.\omega \qquad (16\text{-}1)$$

If we take two electric charge separated by a distance R the Coulomb force is:

$$f = K.\frac{e^2}{R^2} \qquad (16\text{-}2)$$

The question is what is the value of constant β. We can take the way of Planck which is that:

$$\beta = \hbar \qquad (16\text{-}3)$$

Where the Planck constant $\hbar$ is determined by thermodynamics experiment (black body radiation).

There is another way which to have a force $f = ac$ acting between charges separated with the distance $R = \sqrt{\frac{\beta}{a}}$ so we have :

$$\beta = K.\frac{e^2}{c} \qquad (16\text{-}4)$$

The two constants should be equal to avoid any contradiction, so there is an universal constant e as :

$$e = \sqrt{\frac{\hbar.c}{K}} \qquad (16\text{-}5)$$

Which is equivalent to:

$$K.\frac{e^2}{\hbar c} = 1 \qquad (16\text{-}6)$$

From equation (3-77) & (16-5) we have:

$$\gamma' = a.\sqrt{\frac{K.c}{\hbar}} \qquad (16\text{-}7)$$

If we take the value of constant "a" as given by (13-100) than we get:

$$\gamma' = 8.611\ \ 10^{-3}\ unities\ of\ MKSA\ system \qquad (16\text{-}8)$$

Don’t forget that conversion factor γ is a product of two conversion factors.

It is clear that conversion factors open us for more theories so more experiments & so more technologies. The same problem will be found in thermodynamics because we have the conversion relationship $\hbar\omega \ll kT$ or $\hbar\omega \gg kT$ and we see that in relationship (16-7) there is a conversion constant in relation with constant $\hbar$.

The most important thing done here is the unification of fields in a Minkowski space-time i.e. in inertial referential where Lorentz transformations are available. But physics experiments should be independent from the choice of the referential. The reader is invited to rewrite this paper in a Riemann space-time (any transformations of space & time between referentials). The reader can take the document 49089264 of Pierre Paillere [27]available on the internet and rewrite the 115 pages available with the same spirit of this paper. Join to this document the paper of C.LANZANOS [28].

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