New Roots Algorithm for Any Index Using the Same Method

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Abstract
The proposal of this document was to find an alternative to the traditional forms of calculating radicals using an algorithm that works for any index, as the only method. This algorithm is designed for the set of integers, so I use the rest as the basis for the development of this document.

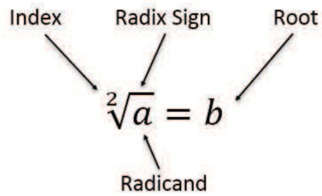
Calculating a square root by hand is not a great difficulty knowing the standard algorithm, but if we must calculate one cubed, fourth, fifth, sixth, etc. Things change remarkably and it gets complicated. Since each root has its own algorithm or procedure. So we usually turn to the calculator. Since the larger the radical index, the more complicated it is.

While a fourth root can be solved by taking the square root of the square root, I am conceptually looking for something different.
The proposal of this document was to find an alternative to the traditional forms of calculating radicals using an algorithm that works for any index, as the only method.
This algorithm is designed for the set of integers, so I use the rest as the basis for the development of this document.

A proposal that is not taught in school and about which I did not find information on the internet or in any book.
It is highly probable that the remains algorithm may become unknown to hundreds of mathematicians and students.

Square Root

Traditional Standard Algorithm: This is the method of calculating a traditional square root



Example 1: $\sqrt[2]{3.250}=$

Algorithm Standard

$\sqrt[2]{3}$	2	5	0	**=57**	
-2	5	↓		5^2	$\mathbf{a^2}$
0	7	**5**	**0**		
-	7	4	9	2*5=10__*__	**=107*7**
0	0	0	1	Resto	

Therefore : $57^2 + r1 = 3.250$

This is solved in two steps, one easy and the other complex.

Step 1 is the same in both algorithms
We are looking for a square number that is close to 32.

Step 2 The empty boxes are completed with a number that by joining 10 forms a new 3-digit number, then we multiply this new number by the same one that we added before, the result should be close to 750.

This is the step that generates some complexity in the student, since he has to create a number and multiply it to solve a 4-digit subtraction.

If you want to see how the standard algorithm works in detail, enter: https://en.wikipedia.org/wiki/Square_root

To calculate the cube root of a number I have a much more complex procedure than in the previous example. And as the index gets higher the level of complexity increases. Therefore, this calculation method ends up being very difficult to apply.

Another traditional way to solve it

ROOTS BY DECOMPOSITION OF A NUMBER IN PRIME FACTORS.

Example 2) $\sqrt[2]{3.250}$=

Decomposition	
3.250/2 1.625/5 325/5 65/5 13/13 1	$3.250 = 2^1 * 5^3 * 13^1$ $\sqrt[2]{3.250}$= $5\sqrt[2]{2 * 5 * 13}$ $5\sqrt[2]{130}$

New root algorithm
For composite numbers

A composite number is one that is not a prime number.
The method consists of decomposing it into prime factors to form two smaller numbers that multiply with each other. Therefore, applying the property of radical multiplication, I can calculate separately and apply the property of remainders.
This is another way of breaking down the number to achieve the goal of finding a result.

$$(K, A, B, r, n, a, b) \in N$$
$$ra = rest\ of\ a$$
$$rb = rest\ of\ b$$

$$\sqrt[n]{A * B} = \sqrt[n]{A} * \sqrt[n]{B} = \boldsymbol{k} + \boldsymbol{r}$$

First Step

$$\sqrt[n]{A} * \sqrt[n]{B}$$
$$= \sqrt[n]{a^n + ra} * \sqrt[n]{b^n + rb}$$
$$= (a + \boldsymbol{ra}) * (b + \boldsymbol{rb})$$

Second Step

Property 1

$$(a + ra) * (b + rb)$$
$$\boldsymbol{a} * \boldsymbol{b} = \boldsymbol{k}$$
$$\boldsymbol{r} = ((ra * b^n) + (rb * a^n) + (ra * rb))$$

$$\boldsymbol{k} + \boldsymbol{r}$$

<u>Example 1</u>: $\sqrt[2]{30}$

$$\sqrt[2]{5} * \sqrt[2]{6} = \sqrt[2]{5*6} = \sqrt[2]{30}$$

$$\sqrt[2]{5} = \sqrt[2]{2^2+1} \qquad \sqrt[2]{6} = \sqrt[2]{2^2+2}$$
$$\sqrt[2]{2^2+1} = 2 + r1 \qquad \sqrt[2]{2^2+2} = 2 + r2$$

Apply property 1

$$=(2+r1) * (2+r2)$$
$$= 2*2 + r\left((r1*2^2) + (r2*2^2) + (r1*r2)\right)$$
$$= 4 + r(4+8+2)$$
$$\mathbf{= 4 + r14}$$

$$Correct\ rest$$
$$0 < r < 5^2 - 4^2 = 9$$
$$Por\ lo\ tanto\ 5 + (r14 - 9)$$

$$\mathbf{= 5 + r\,5}$$
$$then\ 5^2 + rest\ 5 = 30$$

<u>Rest correction formula</u>

Check:	$0 < r < (k+1)^n - k^n$
Correction:	$(\boldsymbol{k}+\mathbf{1}) + \boldsymbol{r} - ((k+1)^n - k^n)$

New root algorithm for prime numbers.

We apply the same concept developed previously, but to decompose a prime number we will start by subtracting 1. We will add this remainder at the end of the exercise.

Example 2: $\sqrt[2]{31}$

$$31 - 1 = 30$$

Prime factor decomposition

30/2

15/3

5/5

1

$$31 = 2^1 * 3^1 * 5^1 + r1$$

$$31 = 6 * 5 + r1$$

$$then\ \sqrt[2]{31}$$

$$\sqrt[2]{30} + r1 = \sqrt[2]{5 * 6} + r1 = \sqrt[2]{5} * \sqrt[2]{6} + r1$$

$$\sqrt[2]{5} = \sqrt[2]{2^2 + 1} \qquad \sqrt[2]{6} = \sqrt[2]{2^2 + 2}$$

$$\sqrt[2]{2^2 + 1} = 2 + r1 \qquad \sqrt[2]{2^2 + 2} = 2 + r2$$

Apply property 1

$$= (2 + r1) * (2 + r2)$$
$$= 2 * 2 + r\left((r1 * 2^2) + (r2 * 2^2) + (r1 * r2)\right)$$
$$= 4 + r(4 + 8 + 2)$$
$$= \mathbf{4 + r14}$$

Correct rest

$$0 < r < 5^2 - 4^2 = 9$$
$$then\ 5 + (r14 - 9)$$

$$= \mathbf{5 + r\,5}$$
$$then\ 5^2 + rest\ 5 = 30$$

Now I add the $\mathbf{r1}$ that we left aside at the beginning of the exercise

$$= \mathbf{5 + r\,5 + r1}$$

$$\mathbf{5 + r6}$$

Example 3: $\sqrt[2]{3.250}$=

$$\sqrt[2]{3.250} = \sqrt[2]{2^1 * 5^3 * 13^1}$$
$$\sqrt[2]{3.250} = \sqrt[2]{2^1 * 5^2} * \sqrt[2]{5 * 13^1}$$
$$\sqrt[2]{3.250} = \sqrt[2]{50} * \sqrt[2]{65}$$

We can easily calculate these roots mentally

$$\sqrt[2]{50} = 7 + r1 \qquad \sqrt[2]{65} = 8 + r1$$

$$= (7 + r1) * (8 + r1)$$
$$= 7 * 8 + r\big((r1 * 8^2) + (r1 * 7^2) + (r1 * r1)\big)$$
$$= 56 + r(64 + 49 + 1)$$
$$\equiv \mathbf{56 + r114}$$

$$Correct\ rest$$
$$0 < r < 57^2 - 56^2 = 113$$

Then:

$$\mathbf{57} + (\boldsymbol{r}\mathbf{114} - \mathbf{113})$$

$$\equiv \mathbf{57 + r\ 1}$$
$$then\ 57^2 + rest\ 1 = 3.250$$

The Roots Algorithm will be the pillar on which this document is developed to solve any type of settlement with any index.

Example 4: $\sqrt[2]{4.346}$

Calculate $\sqrt[2]{4.346}$ Prime decomposition 4346 | 2 2173 | 41 53 | 53 1 |	*I form the 4.346 with the product of two divisors.* $\sqrt[2]{4.346} = \sqrt[2]{82 * 53}$ $82 = 9^2 + r1$ $53 = 7^2 + r4$

Square root product

$$\sqrt[2]{4.346} = \sqrt[2]{82 * 53} = \sqrt[2]{82} * \sqrt[2]{53}$$
$$\sqrt[2]{82} = 9 + r1 \qquad \sqrt[2]{53} = 7 + r4$$

I apply Property 1

$$\equiv (9 + r1) * (7 + r4)$$
$$\equiv 9 * 7 + r\big((r1 * 7^2) + (r4 * 9^2) + (r1 * r4)\big)$$
$$\equiv 63 + r(49 + 324 + 4)$$
$$\boldsymbol{\equiv 63 + r377}$$

Correct rest

$$0 < r < 64^2 - 63^2 = 127$$

$$= 64 + r377 - 127$$
$$\boldsymbol{\equiv 64 + r250}$$

Correct rest

$$0 < r < 65^2 - 64^2 = 129$$

We repeat the correction process

$$65 + (r250 - 129)$$
$$\boldsymbol{\equiv 65 + r121}$$

Correct rest

$$0 < r < 66^2 - 65^2 = 131$$

Then $65^2 + r121 = 4.346$

$$\boldsymbol{\sqrt[2]{4.346} = 65 + rest\ 121}$$

Cube root

Example 5:

Calculate $\sqrt[3]{270}$ Prime decomposition 270	2 135	3 45	3 15	3 5	5 1		$I\ form\ the\ 270\ with\ the\ product\ of\ two\ divisors.$ $\sqrt[3]{270} = \sqrt[3]{30 * 9}$ $30 = 3^3 + r3$ $9 = 2^3 + r1$

$$\sqrt[3]{270} = \sqrt[3]{30 * 9} = \sqrt[3]{30} * \sqrt[3]{9}$$

$$\sqrt[3]{30} = 3 + r3 \qquad \sqrt[3]{9} = 2 + r1$$

Apply property 1

$$\equiv (3 + r3) \ * (2 + r1)$$

$$\equiv \ 3 * 2 + r\left((r3 * 2^3) + (r1 * 3^3) + (r3 * r1)\right)$$

$$\equiv \ 6 + r(24 + 27 + 3)$$

$$\equiv \ \mathbf{6 + r54}$$

$Correct\ rest$

$$0 < r < \ 7^3 - 6^3 \ = 127$$

$$Then\ \ 6^3 + \ r54 = 270$$

$$\sqrt[3]{\mathbf{270}} = \mathbf{6 + rest\ 54}$$

Fourth root

Example 6:

Calculate $\sqrt[4]{1.660}$	$I\ form\ the\ 1.660\ with\ the\ product\ of\ two\ divisors.$
Prime decomposition 1660 | 2 830 | 2 415 | 5 83 | 83 1 |	$\sqrt[4]{1.660} = \sqrt[4]{20 * 83}$ $20 = 2^4 + r4$ $83 = 3^4 + r2$

$$\sqrt[4]{1.660}$$

$$= \sqrt[4]{20 * 83} = \sqrt[4]{20} * \sqrt[4]{83}$$

$\sqrt[4]{20} = 2 + r4$ $\sqrt[4]{83} = 3 + r2$

Apply property 1

$$\equiv (2 + r4) * (3 + r2)$$
$$\equiv 2 * 3 + r\left((r4 * 3^4) + (r2 * 2^4) + (r4 * r2)\right)$$
$$\equiv 6 + r(324 + 32 + 8)$$

$\equiv \mathbf{6 + r364}$

$\boldsymbol{rest\ correct}$

$$0 < r < 7^4 - 6^4 = 1.105$$

$$Then\ 6^4 + r364 = 1.660$$
$$\boldsymbol{\sqrt[4]{1.660} = 6 + rest\ 364}$$

Fifth root

Example 7:

Calculate $\sqrt[5]{1.800}$	$I\ form\ the\ 1.800\ with\ in\ the\ product\ of\ two\ divisors.$
Prime decomposition	$\sqrt[5]{1.800} = \sqrt[5]{40 * 45}$
1800 | 2 900 | 2 450 | 2 225 | 5 45 | 5 9 | 3 3 | 3 1 |	$40 = 2^5 + r8$ $45 = 2^5 + r13$

$$\sqrt[5]{1.800}$$

$$= \sqrt[5]{40 * 45} = \sqrt[5]{40} * \sqrt[5]{45}$$

$\sqrt[5]{40} = 2 + r8$ $\sqrt[5]{45} = 2 + r13$

Apply property 1

$$\equiv (2 + r8) * (2 + r13)$$

$$\equiv 2 * 2 + r\left((r8 * 2^5) + (r13 * 2^5) + (r8 * r13)\right)$$

$$\equiv 4 + r(256 + 416 + 104)$$

$$\boldsymbol{\equiv 4 + r776}$$

rest correct

$$0 < r < 5^5 - 4^5 = 2.101$$

$$then\ 4^5 + r776 = 1.800$$

$$\boldsymbol{\sqrt[5]{1.800} = 4 + rest\ 776}$$

Root with negative rest

Example 8

Calculate $\sqrt[3]{600}$	$I\ form\ the\ 270\ with\ the\ product\ of\ two\ divisors.$
Prime decomposition 600 | 2 300 | 2 150 | 2 75 | 5 15 | 5 3 | 3 1 |	$\sqrt[3]{600} = \sqrt[3]{24 * 25}$ $24 = 3^3 - r3$ $25 = 3^3 - r2$

$$\sqrt[3]{600} = \sqrt[3]{24 * 25} = \sqrt[3]{24} * \sqrt[3]{25}$$
$$\sqrt[3]{24} = 3 - r3 \qquad \sqrt[3]{25} = 3 - r2$$

Apply property 1

$$\equiv (3 - r3) * (3 - r2)$$
$$\equiv 3 * 3 + r\big((-r3 * 3^3) + (-r2 * 3^3) + (-r3 * (-r2))\big)$$
$$\equiv 9 + r(-81 - 54 + 6)$$
$$\equiv \mathbf{9 - r129}$$

$I\ apply\ correction\ of\ the\ rest\ for\ negatives$

$$0 < r < 9^3 - 8^3 = 217$$

$$Then\ 8 - r129 + r217$$
$$= \mathbf{8 + r88}$$

Negative rest correction formula	
check:	$0 < r < k^3 - (k-1)^3$
Correction:	$(\boldsymbol{k-1}) + \boldsymbol{r} + (k^3 - (k-1)^3)$

We can also solve by factoring and simplifying

$(h, A, n, a, ra) \in N$
ra=rest of a

Root Multiplication

$$\boldsymbol{h\sqrt[n]{A}}$$

$$\sqrt[n]{\mathbf{A}} = a + ra$$

then, $\boldsymbol{h}(\mathbf{a} + \boldsymbol{ra})$

Multiplication Property 2

$$h\sqrt[n]{A} = h(\mathrm{a} + r\mathrm{a})$$
$$= \boldsymbol{h * a + h^n * ra}$$

Example 9

$$\sqrt[2]{\mathbf{180}}$$

$$\sqrt[2]{9 * 4 * 5}$$
$$\sqrt[2]{3^2 * 2^2 * 5^1}$$
$$3 * 2\sqrt[2]{5} = 6\sqrt[2]{5}$$

$$\mathbf{6\sqrt[2]{5} = 6(2 + \boldsymbol{r}1)}$$

Apply property 2

$$6 * 2 + 6^2 * r1$$
$$\mathbf{12 + \boldsymbol{r}36}$$

I apply correction of the rest

$$0 < r < 13^2 - 12^2 = 25$$

$$13 + r36 - r25$$
$$\mathbf{13 + \boldsymbol{r}11}$$

$$then\ 13^2 + 11 = 180$$

Example 10

$$\sqrt[3]{\mathbf{3.600}}$$

$$\sqrt[3]{2^4 * 3^2 * 5^2}$$
$$2\sqrt[3]{2 * 9 * 25} = 2\sqrt[3]{450}$$

$\sqrt[3]{450}$ can be calculated using property 1

$$\mathbf{2\sqrt[3]{450} = 2(7 + r107)}$$

Apply property 2

$$2 * 7 + 2^3 * r107$$
$$14 + r856$$

Apply correction of the rest

$$0 < r < 15^3 - 14^3 = 631$$

$$15 + (r856 - 631)$$
$$\mathbf{15 + r225}$$
$$then\ 15^3 + 225 = 3.600$$

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Root Division (Root in the numerator)

$$(h, A, n, a, ra) \in N$$
$$ra = rest\ of\ a$$

$$\frac{\sqrt[n]{\boldsymbol{A}}}{h}$$

$$\sqrt[n]{\boldsymbol{A}} = a + ra$$

then, $\frac{\mathbf{a+ra}}{h}$

Property 3 of the Division

$$\sqrt[n]{\frac{\boldsymbol{A}}{\boldsymbol{h^n}}} = \frac{\sqrt[n]{\boldsymbol{A}}}{h} = \frac{\mathbf{a + ra}}{h}$$

$$= \frac{\boldsymbol{a}}{\boldsymbol{h}} + \frac{\boldsymbol{ra}}{\boldsymbol{h^n}}$$

Example 11

$$\frac{\sqrt[2]{\mathbf{20}}}{2}$$

$$\frac{4 + r4}{2}$$

Apply property 3

$$\frac{\mathbf{4}}{\mathbf{2}} + \frac{\boldsymbol{r4}}{\mathbf{2^2}}$$

$$\mathbf{2 + r1 = \sqrt[2]{5}}$$

$$\mathbf{2}(2 + r1) = \mathbf{2} * 2 + \mathbf{2^2} * r1 = 4 + r4 = \sqrt[2]{\mathbf{20}}$$

The same exercise solved in two different ways.

Example 12: Using Simplification	Example 12: using Properties
$\frac{\sqrt[2]{\mathbf{45}}}{\mathbf{3}} = \frac{\sqrt[2]{3^2 * 5}}{3} =$ $\frac{3\sqrt[2]{5}}{3} = \frac{\not{3} * (2 + r1)}{\not{3}}$ $= \mathbf{2 + r1}$ Correct rest $0 < r < 3^2 - 2^2 = 5$	$\frac{\sqrt[2]{\mathbf{45}}}{\mathbf{3}} = \frac{\sqrt[2]{3^2 * 5}}{3} =$ $\frac{3\sqrt[2]{5}}{3} = \frac{3 * (2 + r1)}{3}$ Apply property 2 (multiplication) $= \frac{3 * 2 + 3^2 * r1}{3}$ $= \frac{6 + r9}{3}$ Apply property 3 (division) $= \frac{6}{3} + \frac{r9}{3^2}$ $= \mathbf{2 + r1}$

Conclution

The roots algorithm works with great accuracy for all roots of any index, it aims to be a new way of interpreting and solving exercises.
This method is more intuitive since we solve roots of small numbers instantly mentally, and then apply their properties and find the final result.

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