

The generalized Bargman-Michel-Telegdi equation for the FERMILAB muon experiment

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Abstract

The influence of the bremsstrahlung on the spin motion of muon is expressed by the equation which is the analogue and generalization of the Bargmann-Michel-Telegdi equation. The new constant is involved in this equation. This constant can be immediately determined by the experimental measurement in FERMILAB of the spin motion of muon, or, it follows from the classical limit of generalized SM electrodynamics with radiative corrections (Pardy, 2008; 2009).

Key words: Bargmann-Michel-Telegdi equation, synchrotron radiation, spin light, muon.

1 Introduction

Fermi National Accelerator Laboratory near Chicago announced that muons elementary particles similar to electrons wobbled more than expected while whipping around a magnetized ring. Both measurements of the muons wobbliness, or magnetic moment, significantly overshoot the theoretical prediction, as calculated by theoretical physicists. The Fermilab researchers estimate that the difference has grown to a level that physicists need to claim a discovery.

The discrepancy is probably caused by unknown particles giving muons an extra push. It is the breakdown of the 50-year-old Standard Model of particle physics describing the known elementary particles and interactions. According to Graziano Venanzoni, one of

the leaders of the Fermilab Muon g-2 experiment and a physicist at the Italian National Institute for Nuclear Physics, the existence of the new particle is plausible.

On the other hand Dominik Stöckinger, a theorist at the Technical University of Dresden and the Fermilab Muon g-2 team, said that physicists can't say whether exotic new particles are pushing on muons until they agree about the effects of the 17 Standard Model particles they already know about (Quanta magazine, 2021).

The similar scepticism is involved in the statement by Andreas Crivellin of CERN and by Hoferichter of the University of Bern: "it could be that the data, or the way it is interpreted, is misleading".

The crucial problem is also the calculation of the magnetic moment of elementary particles in QED, SM, and QCD, which was in the best form performed by Julian Schwinger in QED. His method was applied by author (Pardy, 1979; 2019a; 2019b) in case of the Lee model of elementary particle. The author approach is of deep pedagogical meaning.

We know that the measurement of the g-factor is based on the rotation of spin. However spin rotation is described by the Bargaman-Michel-Telegdi equation which is in this experiment ignored. At the same time there is the influence of the synchrotron radiation on the spin motion in the electromagnetic field. The equation which involves the bremsstrahlung interaction with spin was derived by author (Pardy, 2008; 2009). The theory of the g-factor without the generalized BMT equation, or, so called the Bargaman-Michel-Telegdi-Pardy equation (BMTP) is evidently incomplete and must be revised. So, the crucial problem is the synchrotron radiation interaction of muon in FERMILAB, which evidently influences the motion of the electron in accelerators in CERN and in FERMILAB.

The equation which describes the classical motion involving radiative reaction is so called the Lorentz-Dirac equation, which differs from the so called Lorentz equation only by the additional term which describes the radiative corrections. The equation with the radiative term is as follows (Landau et al., 1988):

$$mc \frac{du_\mu}{ds} = \frac{e}{c} F_{\mu\nu} u^\nu + g_\mu, \quad (1)$$

where u_μ is the four-velocity and the radiative term was derived by Landau et al. in the form (Landau et al., 1988):

$$g_\mu = \frac{2e^3}{3mc^3} \frac{\partial F_{\mu\nu}}{\partial x^\alpha} u^\nu u^\alpha - \frac{2e^4}{3m^2c^5} F_{\mu\alpha} F^{\beta\alpha} u_\beta + \frac{2e^4}{3m^2c^5} (F_{\alpha\beta} u^\beta) (F^{\alpha\gamma} u_\gamma) u_\mu. \quad (2)$$

It is possible to show that the space components of the 4-vector force g_μ is of the form (Landau et al., 1988)

$$\begin{aligned} \mathbf{f} = & \frac{2e^3}{3mc^3} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \left\{ \left(\frac{\partial}{\partial t} + (\mathbf{v}\nabla) \right) \mathbf{E} + \frac{1}{c} \left[\mathbf{v} \left(\frac{\partial}{\partial t} + (\mathbf{v}\nabla) \right) \mathbf{H} \right] \right\} + \\ & \frac{2e^4}{3m^2c^3} \left\{ \mathbf{E} \times \mathbf{H} + \frac{1}{c} \mathbf{H} \times (\mathbf{H} \times \mathbf{v}) + \frac{1}{c} \mathbf{E}(\mathbf{v}\mathbf{E}) \right\} - \\ & \frac{2e^4}{3m^2c^5 \left(1 - \frac{v^2}{c^2}\right)} \mathbf{v} \left\{ \left(\mathbf{E} + \frac{1}{c} (\mathbf{v} \times \mathbf{H}) \right)^2 - \frac{1}{c^2} (\mathbf{E}\mathbf{v})^2 \right\}. \quad (3) \end{aligned}$$

Bargmann, Michel and Telegdi (Berestetskii et al., 1989;) derived so called BMT equation for motion of spin in the electromagnetic field, in the form

$$\frac{da_\mu}{ds} = \alpha F_{\mu\nu} a^\nu - \beta u_\mu F^{\nu\lambda} u_\nu a_\lambda, \quad (4)$$

where a_μ is so called axial vector describing the classical spin and constants α and β were determined after the comparison of the postulated equations with the non-relativistic quantum mechanical limit. The result of such comparison is the final form of so called BMT equations:

$$\frac{da_\mu}{ds} = 2\mu F_{\mu\nu} a^\nu - 2\mu' u_\mu F^{\nu\lambda} u_\nu a_\lambda, \quad (5)$$

where μ is magnetic moment of electron following directly from the Dirac equation and μ' is anomalous magnetic moment of electron which can be calculated as the radiative correction to the interaction of electron with electromagnetic field and follows from quantum electrodynamics. The BMT equation has more earlier origin. The first attempt to describe the spin motion in electromagnetic field using the special theory of relativity was performed by Thomas (1926). However, the basic ideas on the spin motion was established by Frenkel (1926, 1958). After appearing the Frenkel basic article, many authors published the articles concerning the spin motion (Ternov et al., 1980; Tomonaga, 1997). The mechanical model of spin was constructed by Uhlenbeck and Goudsmith (1926), or, in the very sophisticated form by Ohanian (1984) and other authors. However, we know at present time that spin of electron is its physical attribute which follows only from the Dirac equation. Also the Schrödinger Zitterbewegung of the Dirac electron as a point-like particle follows from the Dirac equation.

It was shown by Rafanelli and Schiller (1964), (Parady, 1973) that the BMT equation can be derived from the classical limit, i.e. from the WKB solution of the Dirac equation with the anomalous magnetic moment. Equation (5) is also the basic equation of the non-dissipative spintronics.

2 Equation of motion for the spin-vector

If we introduce the average value of the vector of spin in the rest system by the quantity ζ , then the 4-pseudovector a^μ is of the form $a^\mu = (0, \zeta)$. The momentum four-vector of a particle is $p^\mu = (m, 0)$ in the rest system of a particle. Then the equation

$$a^\mu p_\mu = 0 \quad (6)$$

is valid not only in the rest system of a particle but in the arbitrary system as a consequence of the relativistic invariance. The following general formula is also valid in the arbitrary system

$$a^\mu a_\mu = -\zeta^2. \quad (7)$$

The components of the axial 4-vector a^μ in the reference system where particle is moving with the velocity $\mathbf{v} = \mathbf{p}/\varepsilon$ can be obtained by application of the Lorentz transformation to the rest system and they are as follows (Berestetskii et al., 1989):

$$a^0 = \frac{|\mathbf{p}|}{m} \zeta_{\parallel}, \quad \mathbf{a}_{\perp} = \zeta_{\perp}, \quad a_{\parallel} = \frac{\varepsilon}{m} \zeta_{\parallel}, \quad (8)$$

where suffices \parallel, \perp denote the components of \mathbf{a} , ζ parallel and perpendicular to the direction \mathbf{p} . The formulas for the components can be also rewritten in the more compact form as follows (Berestetzki et al., 1989):

$$\mathbf{a} = \zeta + \frac{\mathbf{p}(\zeta\mathbf{p})}{m(\varepsilon + m)}, \quad a^0 = \frac{\mathbf{a}\mathbf{p}}{\varepsilon} = \frac{\zeta\mathbf{p}}{m}, \quad \mathbf{a}^2 = \zeta^2 + \frac{(\mathbf{p}\zeta)^2}{m^2}. \quad (9)$$

The equation for the change of polarization can be obtained immediately from the BMT equation in the following form (Berestetzki et al., 1989):

$$\begin{aligned} \frac{d\mathbf{a}}{dt} = & \frac{2\mu m}{\varepsilon} \mathbf{a} \times \mathbf{H} + \frac{2\mu m}{\varepsilon} (\mathbf{a}\mathbf{v})\mathbf{E} - \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{a}\mathbf{E}) + \\ & + \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{v}(\mathbf{a} \times \mathbf{H})) + \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{a}\mathbf{v})(\mathbf{v}\mathbf{E}), \end{aligned} \quad (10)$$

where we used the relativistic relations $c = 1$, $ds = dt\sqrt{1-v^2}$, $\varepsilon = m\sqrt{1-v^2}$ and the following components of the electromagnetic field (Landau et al., 1988):

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{pmatrix} \stackrel{d}{=} (\mathbf{E}, \mathbf{H}); \quad F_{\mu\nu} = (-\mathbf{E}, \mathbf{H}). \quad (11)$$

Inserting equation \mathbf{a} from eq. (9) into eq. (10) and using equations

$$\mathbf{p} = \varepsilon\mathbf{v}, \quad \varepsilon^2 = \mathbf{p}^2 + m^2, \quad \frac{d\mathbf{p}}{dt} = e\mathbf{E} + e(\mathbf{v} \times \mathbf{H}), \quad \frac{d\varepsilon}{dt} = e(\mathbf{v}\mathbf{E}), \quad (12)$$

we get after long but simple mathematical operations the following equation for the polarization ζ

$$\begin{aligned} \frac{d\zeta}{dt} = & \frac{2\mu m + 2\mu'(\varepsilon - m)}{\varepsilon} \zeta \times \mathbf{H} + \\ & \frac{2\mu'\varepsilon}{\varepsilon + m} (\mathbf{v}\mathbf{H})(\mathbf{v} \times \zeta) + \frac{2\mu m + 2\mu'\varepsilon}{\varepsilon + m} \zeta \times (\mathbf{E} \times \mathbf{v}). \end{aligned} \quad (13)$$

The special interest is concerned not only in the change of the absolute quantity of the polarization, but in the change with regard to the direction of motion represented by the unit vector $\mathbf{n} = \mathbf{v}/v$. We write the polarization in the form:

$$\zeta = \mathbf{n}\zeta_{\parallel} + \zeta_{\perp}. \quad (14)$$

Then using eqs. (12), (13) and (14), we get the following equation for the parallel component of the polarization (Berestetzki et al., 1989):

$$\frac{d\zeta_{\parallel}}{dt} = 2\mu'(\zeta_{\perp}(\mathbf{H} \times \mathbf{n})) + \frac{2}{v} \left(\frac{\mu m^2}{\varepsilon^2} - \mu' \right) (\zeta_{\perp} \mathbf{E}). \quad (15)$$

3 Spin motion equation with the bremsstrahlung reaction

It is meaningful to consider the BMT equation with the radiative corrections to express the influence of the synchrotron radiation on the motion of spin. To our knowledge such equation, the generalized BMT equation, was not published and we here present the conjecture of the form of such equation. The equation of the spin motion under the influence of the synchrotron radiation is suggested as an analogue to the BMT construction (Parady, 2008; 2009):

$$\frac{da_\mu}{ds} = 2\mu F_{\mu\nu} a^\nu - 2\mu' u_\mu F^{\nu\lambda} u_\nu a_\lambda + \Lambda f_\mu(axial), \quad (16)$$

where the term $f_\mu(axial)$ is generated as the "axialization" of the force elaborated from the radiation term g_μ . The axialization is the operation which was used by Bargmann, Michel and Telegdi and it consists in the construction of the axial vector from the four-vector force. We see from the right side of the BMT equation how to construct such axial equation. Or, the additional axial 4-vector constructed from the bremsstrahlung force is as following:

$$f_\mu(axial) = \Lambda u_\mu (g^\alpha a_\alpha) = \Lambda u_\mu [g_0 a_0 - \mathbf{g} \cdot \mathbf{a}]. \quad (17)$$

So, the generalized BMT equation which involves also the influence of synchrotron radiation on spin motion is as follows:

$$\begin{aligned} \frac{da_\mu}{ds} = & 2\mu F_{\mu\nu} a^\nu - 2\mu' u_\mu F^{\nu\lambda} u_\nu a_\lambda + \\ & \Lambda u_\mu \left\{ \frac{2e^3}{3mc^3} \frac{\partial F_{\lambda\nu}}{\partial x^\alpha} u^\nu u^\alpha - \right. \\ & \left. \frac{2e^4}{3m^2 c^5} F_{\lambda\alpha} F^{\beta\alpha} u_\beta + \frac{2e^4}{3m^2 c^5} (F_{\alpha\beta} u^\beta) (F^{\alpha\gamma} u_\gamma) u_\lambda \right\} a^\lambda. \end{aligned} \quad (18)$$

Using eq. (17), we can write eq. (18) in the form

$$\frac{da_\mu}{ds} = 2\mu F_{\mu\nu} a^\nu - 2\mu' u_\mu F^{\nu\lambda} u_\nu a_\lambda + \Lambda u_\mu [g_0 a_0 - \mathbf{g} \cdot \mathbf{a}]. \quad (19)$$

The constant Λ is new physical constant, which cannot be determined from the classical theory of the spin motion. This constant can be determined immediately from the spin motion observed experimentally. However, this constant follows logically from the classical limit of quantum electrodynamics (QED) involving radiative corrections. The solution of this problem in the framework of the WKB limit of the Dirac equation with radiation term was not still published. On the other hand, Bayer (1971), Bayer et al. (1973) derived, by the different method, the equation of the spin motion in electromagnetic field where the influence of radiative reaction on the spin motion is involved. This equation was used later for the determination of the polarization of electrons in the bent crystals (Baryshevsky et al. 1979).

While the 3-vector components of the radiative force are involved in the equation (3) the zero component must be determined by the extra way. We have:

$$g_0 = P_1 + P_2 + P_3, \quad (20)$$

where the terms of eq. (20) follow from eq. (2) in the form ($c = 1$):

$$P_1 = \left(\frac{2e^3}{3m} \right) \frac{\partial F_{0\nu}}{\partial x^\alpha} u^\nu u^\alpha, \quad (21)$$

$$P_2 = \left(-\frac{2e^4}{3m^2} \right) F_{0\alpha} F^{\beta\alpha} u_\beta, \quad (22)$$

$$P_3 = \left(\frac{2e^4}{3m^2} \right) (F_{\alpha\beta} u^\beta) (F^{\alpha\gamma} u_\gamma) u_0 \quad (23)$$

with

$$u = \left(\frac{1}{\sqrt{1-v^2}}, \frac{\mathbf{v}}{\sqrt{1-v^2}} \right). \quad (24)$$

After some algebraic operations, we write the set of quantities P_1, P_2, P_3 as follows:

$$P_1 = \left(\frac{2e^3}{3m} \right) \frac{1}{1-v^2} \times \{(\partial_t \mathbf{E}) \cdot \mathbf{v} + (\partial_x \mathbf{E}) \cdot \mathbf{v} v_x + (\partial_y \mathbf{E}) \cdot \mathbf{v} v_y + (\partial_z \mathbf{E}) \cdot \mathbf{v} v_z\}, \quad (25)$$

$$P_2 = \left(\frac{2e^4}{3m^2} \right) \frac{1}{\sqrt{1-v^2}} \{E^2 - (\mathbf{H} \times \mathbf{E}) \cdot \mathbf{v}\} \quad (26)$$

$$P_3 = \left(-\frac{2e^4}{3m^2} \right) \frac{1}{(1-v^2)^{3/2}} \{(\mathbf{E} + (\mathbf{v} \times \mathbf{H}))^2 - (\mathbf{E} \cdot \mathbf{v})^2\}. \quad (27)$$

The relation of this equation to the (dissipative) spintronics cannot be a priori excluded. Such equation will have fundamental meaning for the work of LHC where the synchrotron radiation influences the spin motion of protons in LHC.

4 The general solution of the spin precession equation

The equation(13) involving the 3-vector of the radiation term (17) can be in general written in the following form:

$$\frac{d}{dt} \zeta_k = \sum_{l=1}^3 a_{kl} \zeta_l + \Lambda \sum_{l=1}^3 b_{kl} \zeta_l, \quad (28)$$

where the coefficients a_{kl} are the corresponding coefficient in eq. (13) and b_{kl} are the corresponding coefficient in eq. (17).

It follows from the theory of the differential equations that the solution of the system (28) is in general of the following form:

$$\zeta_k(t) = \alpha_k e^{i\Omega t}, \quad (29)$$

where α_k and Ω are some constants. The time derivative of eq. (29) is now

$$\frac{d\zeta_k}{dt} = \alpha_k(i\Omega)e^{i\Omega t}. \quad (30)$$

After insertion of eqs. (29) and (30) into eq. (28), we get the following system after some elementary modification:

$$(a_{11} + \Lambda b_{11} - i\Omega)\alpha_1 + (a_{12} + \Lambda b_{12})\alpha_2 + (a_{13} + \Lambda b_{13})\alpha_3 = 0 \quad (31a)$$

$$(a_{21} + \Lambda b_{21})\alpha_1 + (a_{22} + \Lambda b_{22} - i\Omega)\alpha_2 + (a_{23} + \Lambda b_{23})\alpha_3 = 0 \quad (31b)$$

$$(a_{31} + \Lambda b_{31})\alpha_1 + (a_{32} + \Lambda b_{32})\alpha_2 + (a_{33} + \Lambda b_{33} - i\Omega)\alpha_3 = 0. \quad (31c)$$

The nontrivial solution of the system (31) for the determination of α_i is possible if and only if the determinat of the system is zero, or,

$$\det(A + \Lambda B - i\Omega E) = 0, \quad (32)$$

where A, B, E are matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (33)$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad (34)$$

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (35)$$

Equation (32) is the equation for the determination of the three complex frequencies $\Omega_k = \Omega_1, \Omega_2, \Omega_3$. To the every frequency corresponds the solution

$$\zeta_k(t) = \beta_k e^{i\Omega_k t} \quad (36)$$

and it means that the solution of the system (31) is given as the linear combination of the particular solutions. Or,

$$\zeta_k = \sum_{l=1}^3 \beta_{kl} e^{i\Omega_l(\Lambda)t}, \quad (37)$$

where β_{kl} are some coefficients which can be determined by insertion of (37) in eq. (31).

However, Ω -s are the complex quantities depending on the small parameter Λ . So we can write:

$$\Omega_l = \Re\Omega_l + i\Im\Omega_l \quad (38)$$

Using eq. (38) we can write eq. (37) in the following form:

$$\zeta_k = \sum_{l=1}^3 \left(\beta_{kl} e^{i\Re\Omega(\Lambda)t} \right) e^{-\Im\Omega_l(\Lambda)t}. \quad (39)$$

It may be easily see that for $\Lambda = 0$ we get the solution of the spin motion which is not influenced by the synchrotron radiation. The corresponding $\Omega(0)$ -s follow from eq. (32) with $\Lambda = 0$.

We also observe that that solution (39) involves term with $\exp\{-\Im\Omega_l(\Lambda)t\}$, where Λ is small parameter. The physical meaning of this term is that it expresses the damping of spin precession caused by the bremsstrahlung. The damping is possible only if $\Im\Omega_l(\Lambda) > 0$. Bayer, Katkov and Fadin used the specific method for determination of such factor for the case of the motion of electron in electromagnetic magnetic field (Bayer, 1971; Bayer et al., 1973). Baryshevsky (1979) applied the Bayer-Katkov-Fadin results for the determination of the polarization of electrons caused by the bent crystals.

The result of the Bayer-Katkov-Fadin method is the term $\exp\{-\delta_l(t/T)\}$, where δ_l are some appropriate constants. They calculated T in the form

$$\frac{1}{T} = \frac{5}{8} \sqrt{3} \alpha \frac{\hbar^2}{m^2} \gamma^5 |\dot{\mathbf{v}}|^3, \quad (40)$$

where $\alpha \approx 1/137$ and γ is the Lorentz factor.

We see that we can define T_l by the relations

$$\frac{1}{T_l} = \Im\Omega_l(\Lambda) \quad (41)$$

and for the small parameter Λ it is possible to use approximation

$$\Im\Omega_l(\Lambda) \approx \Im\Omega_l(0) + \Lambda \left. \frac{d\Im\Omega_l(\Lambda)}{d\Lambda} \right|_0 + \dots \quad (42)$$

In other words, we get also three damping factors as Bayer et al. (1973) by the different approach to the bremsstrahlung problem. The method of Schiller and Rafanelli (Rafanelli and Schiller, 1964) based on the WKB solution of the Dirac equation with bremsstrahlung term was not used by Bayer (1971) and Bayer et al. 1973). To our knowledge, the Schiller and Rafanelli method was not still applied to the problem of the influence of the bremsstrahlung on the spin motion.

5 Discussion

We have considered here the influence of the synchrotron radiation on the spin motion of a charged particle moving in the homogeneous magnetic field. It is well known that the synchrotron radiation also influences the trajectory of the charged particle. However we do not consider this influence. It is well known that not only the the synchrotron radiation is produced during the motion of a particle in the magnetic field but also the so called spin light, which is generated by spin motion of a particle. We suppose that the influence of the spin light on the spin motion is so small that it is possible to neglect such influence.

The intensity of the synchrotron radiation is, as it is well known, given by the formula (Ternov, 1994; Bordovitsyn et al., 1995):

$$W_{class. synch. rad.} = \frac{2 e^2 c}{3 R^2} \gamma^4; \quad \gamma = \frac{\varepsilon}{m_0 c^2}, \quad (43)$$

where R is the radius of the circular motion, ε is the energy of the moving particle.

The intensity of the spin light is expressed by the formula:

$$W_{spin light} = \frac{2}{3} \frac{1}{c^3} \left(\frac{d^2 \boldsymbol{\mu}}{dt^2} \right)^2 = \frac{2}{3} \frac{\mu_0^2}{c^3} \omega_R^4 \zeta_{\perp}^2. \quad (44)$$

After comparison of formula (28) and (29), we see that the the intensity of the spin light is smaller than the intensity of the synchrotron radiation. So, the influence of the spin light on the spin motion can be neglected.

There is the second possibility how to generalize the BMT equation. It consists in axialization of the bremsstrahlung force in the following way:

$$g_{\mu}(axial) = \frac{2e^3}{3mc^3} \frac{\partial F_{\mu\nu}}{\partial x^{\alpha}} u^{\nu} a^{\alpha} - \frac{2e^4}{3m^2 c^5} F_{\mu\alpha} F^{\beta\alpha} a_{\beta} + \frac{2e^4}{3m^2 c^5} (F_{\alpha\beta} u^{\beta}) (F^{\alpha\gamma} u_{\gamma}) a_{\mu}. \quad (45)$$

Then, such force multiplied with the appropriate constant can be add to the original BMT equation. We think that the second conjecture which is presented in this article cannot be a priori excluded.

The verification of the bremsstrahlung equation (16) - the Bargman-Michel-Telegdi-Pardy equation - can be evidently verified by all circular accelerators over the world, including LHC and FERMILAB muon accelerator.

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