New Coordinate Vacuum Solution
in Cosmological General Theory of Relativity

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ABSTRACT

In the general relativity theory, we discover new vacuum solution by Einstein's gravity field equation. We investigate the new coordinate in cosmological general theory of relativity (CGTR).

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1. Introduction

We solve new vacuum solution by gravity field equation in cosmological general theory of relativity.

New spherical coordinate is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + V(t,r)\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$$V(t,r) = C_1(act+br)^2, \quad C_1 = \frac{1}{b^2 - a^2}$$

a, b, C_1 is constant, c is light's velocity. (1)

In this time, Einstein's gravity equation is

$$R_{tt} = \frac{\ddot{V}}{V} - \frac{\dot{V}^2}{2V^2}$$

$$= \frac{2a^2}{(act+br)^2} - \frac{1}{2}\frac{4a^2}{(act+br)^2} = 0 \qquad (2)$$

$$R_{rr} = \frac{V''}{V} - \frac{1}{2}\frac{V'^2}{V^2}$$

$$= \frac{2b^2}{(act+br)^2} - \frac{1}{2}\frac{4b^2}{(act+br)^2} = 0$$

(3)

$$R_{\theta\theta} = -\frac{\ddot{V}}{2} + \frac{V''}{2} - 1$$

$$= -C_1a^2 + C_1b^2 - 1 = 0 \qquad (4)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} = 0 \qquad (5)$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2}$$

$$= \frac{2C_1ab}{(act+br)^2} - \frac{1}{2}\frac{4C_1ab}{(act+br)^2} = 0 \qquad (6)$$

In this time,

$$V' = 2C_1b(act+br), \dot{V} = 2C_1a(act+br), V'' = 2C_1b^2, \ddot{V} = 2C_1a^2$$

$$A' = \frac{\partial A}{\partial r}, \dot{A} = \frac{1}{c}\frac{\partial A}{\partial t}$$

2. New vacuum solution in cosmological general theory of relativity

Hence, new vacuum solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + \frac{1}{b^2 - a^2}(act + br)^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

a, b, C_1 are constant, c is light's velocity. (7)

In this time, if r' is

$$r' = \frac{1}{\sqrt{b^2 - a^2}}(act + br)$$

As

$$dr' = \frac{1}{\sqrt{b^2 - a^2}}(acdt + bdr)$$

Or

$$dr = \frac{\sqrt{b^2 - a^2}}{b}dr' - \frac{a}{b}cdt \quad (8)$$

If new solution Eq(7) is inserted by transformation Eq(8),

$$dr^2 = \frac{b^2 - a^2}{b^2}dr'^2 - 2\frac{a}{b^2}\sqrt{b^2 - a^2}dr'cdt + \frac{a^2}{b^2}c^2dt^2 \quad (9)$$

In this time, if α_0 is

$$\alpha_0 = \frac{a}{b} \quad (10)$$

Hence, proper time $d\tau$ of new solution is

$$d\tau^2 = (1 - \alpha_0{}^2)dt^2 + 2\alpha_0\sqrt{1 - \alpha_0{}^2}dr'\frac{dt}{c} - \frac{1}{c^2}[(1 - \alpha_0{}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

(11)

In this time, if dt' is

$$dt' = \sqrt{1 - \alpha_0{}^2}dt \quad (12)$$

Therefore, new solution is

$$d\tau^2 = dt'^2 + 2\alpha_0 dr'\frac{dt'}{c} - \frac{1}{c^2}[(1 - \alpha_0{}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \quad (13)$$

If we rewrite dt, dr instead of dt', dr', the proper time $d\tau$ of new solution is

$$d\tau^2 = dt^2 + 2\alpha_0 dr\frac{dt}{c} - \frac{1}{c^2}[(1 - \alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \quad (14)$$

Therefore, new spherical solution in general relativity theory is

$$d\tau^2 = dt^2 + 2\alpha_0 dr\frac{dt}{c} - \frac{1}{c^2}[(1 - \alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$$\alpha_0 \neq 1 \ , \qquad \alpha_0 \ \text{is constant} \tag{15}$$

In this time, the coordinate transformation in cosmological general theory of relativity [1-3] is

$$r \to r\Omega(t_0), t \to t \quad ,$$

t_0 is cosmological time. $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 . (16)

Hence, this vacuum solution is by the coordinate transformation in cosmological general theory of relativity,

$$d\tau^2 = dt^2 + 2\alpha_0\Omega(t_0)dr\frac{dt}{c} - \frac{\Omega^2(t_0)}{c^2}[(1-\alpha_0^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\varphi^2\}]$$

$$\alpha_0 \neq 1 \ , \qquad \alpha_0 \ \text{is constant} \tag{17}$$

3. Conclusion

In the general relativity theory, we discover new vacuum solution by Einstein's gravity field equation. We investigate the new coordinate in cosmological general theory of relativity.

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