# Tetrad in Curved Space-Time
# in Cosmological General Theory of Relativity

**Sangwha-Yi**

**Department of Math , Taejon University 300-716,South Korea**

**ABSTRACT**

In the cosmological general theory of relativity, we define the tetrad that moves in r-axis in the curved space-time. We study an accelerated motion in curved space-time.

**PACS Number**:**04,04.90.+e**

**Key words:Cosmological General Theory of Relativity;**

**Tetrad in Curved Space-Time;**

**Constant Accelerated Motion**

**e-mail address:**sangwha1**@nate.com**

**Tel:010-2496-3953**

## 1.Introduction

This theory's aim is to define tetrad that moves in r-axis in the curved space-time.

Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (1)$$

In this case, the cosmological time $t_0$ is the present cosmological time for constant accelerated motion in cosmological general theory of relativity [2,3]. The ratio of the universe's expansion is

$$\Omega(t_0) = \quad (1\text{-i})$$

Hence, in this time, the cosmological general theory of relativity and the cosmological special theory of relativity do the general relativity theory and the special relativity theory.

In this time, a moving matter's acceleration is the constant acceleration $a_0$ in the Schwarzschild space-time.

$$a_0 = \frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}) \quad (2)$$

$$a_0 t = \frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}} \quad , \quad u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}} \quad (3)$$

If $\frac{d\theta}{dt} = \frac{d\phi}{dt} = 0$, the solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \quad (4)$$

In this time, if we use $\psi$ ,

$$1 = (1-\frac{2GM}{rc^2})(\frac{dt}{d\tau})^2 - \frac{1}{c^2}\frac{1}{1-\frac{2GM}{rc^2}}(\frac{dr}{d\tau})^2$$

$$\cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau} \quad , \quad \sinh\psi = \frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau} \quad (5)$$

Therefore, r-axis's velocity $V_r$ is

$$V_r = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} = u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}} \tag{6}$$

According to Eq(5),Eq(6),

$$\frac{1}{c}\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}} = \frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}}\sqrt{1-\frac{2GM}{rc^2}}dt\,,\ \cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau}$$

$$= \frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}}\cosh\psi d\tau = \sinh\psi d\tau$$

$$\frac{1}{\cosh^2\psi} = 1-\left(\frac{\sinh\psi}{\cosh\psi}\right)^2 = 1-\left(\frac{a_0 t/c}{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}}\right)^2 = \frac{1}{1+\frac{{a_0}^2 t^2}{c^2}} \tag{7}$$

Hence,

$$\cosh\psi = \sqrt{1+\frac{{a_0}^2 t^2}{c^2}}\quad , \quad \sinh\psi = \frac{a_0 t}{c} \tag{8}$$

$$\cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau} = \sqrt{1+\frac{{a_0}^2 t^2}{c^2}}\,, \sinh\psi = \frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau} = \frac{a_0 t}{c} \tag{9}$$

Therefore,

$$\frac{dt}{d\tau} = \frac{\sqrt{1+\frac{{a_0}^2 t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}}\quad , \quad \frac{1}{c}\frac{dr}{d\tau} = \frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}} \tag{10}$$

## 2. Tetrad in Curved Space-Time

The tetrad ${e_a}^{\mu}$ is the unit vector that is each other orthographic.

$$e_a{}^{\mu} e_b{}^{\upsilon} g_{\mu\upsilon} = \eta_{ab} \tag{11}$$

Therefore, Eq(11) is

$$g_{\mu\upsilon} e_0{}^{\mu}(r,t) e_0{}^{\upsilon}(r,t) = \eta_{00} = -1$$

$$d\tau^2 = -\frac{1}{c^2} g_{\mu\upsilon} dx^{\mu} dx^{\upsilon}$$

$$\rightarrow \ -1 = g_{\mu\upsilon}\left(\frac{1}{c}\frac{dx^{\mu}}{d\tau}\right)\left(\frac{1}{c}\frac{dx^{\upsilon}}{d\tau}\right) = g_{\mu\upsilon} e_0{}^{\mu}(r,t) e_0{}^{\upsilon}(r,t) \tag{12}$$

According to Eq(10),Eq(12)

$$e_0{}^{\alpha}(r,t) = \frac{1}{c}\frac{dx^{\alpha}}{d\tau}$$

$$= \left(\frac{\sqrt{1+\frac{a_0{}^2 t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}}, \frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}}, 0, 0\right) \tag{13}$$

About $\theta$ -axis's and $\phi$ -axis's orientation

$$g_{22} e_2{}^2(r,t) e_2{}^2(r,t) = \eta_{22} = 1, \qquad e_2{}^{\alpha}(r,t) = (0,0,\frac{1}{r},0) \tag{14}$$

$$g_{33} e_3{}^3(r,t) e_3{}^3(r,t) = \eta_{33} = 1, \ \ e_3{}^{\alpha}(r,t) = (0,0,1/r\sin\theta,0) \tag{15}$$

And the other vector $e_1{}^{\alpha}(r,t)$ has to satisfy the tetrad condition, Eq (11)

$$g_{00} e_0{}^0(r,t) e_1{}^0(r,t) + g_{11} e_0{}^1(r,t) e_1{}^1(r,t) = \eta_{01} = 0$$

$$e_1{}^{\alpha}(r,t) = \left(\frac{a_0 t / c}{\sqrt{1-\frac{2GM}{rc^2}}}, \sqrt{1+\frac{a_0{}^2 t^2}{c^2}}\sqrt{1-\frac{2GM}{rc^2}}, 0, 0\right) \tag{16}$$

## 3. Conclusion

In the cosmological general theory of relativity, we define the tetrad that moves in r-axis in the curved space-time.

# References

[1]S.Yi,"Curvature Tensor of the Stationary Accelerated Frame in Gravity Field", African Review of

Physics,**9**,59,(2014)
[2]S.Yi,"Cosmological Special Theory of Relativity", International Journal of Advanced Research in Physical Science,**7**,11,(2020),pp 4-9
[3]S.Yi,"Cosmological Gpecial Theory of Relativity", International Journal of Advanced Research in Physical Science,8,2,(2021),pp 22-26
[4]S.Weinberg, Gravitation and Cosmology (John wiley & Sons,Inc,1972)
[5]P.Bergman, Introduction to the Theory of Relativity (Dover Pub. Co.,Inc., New York,1976),Chapter V
[6]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)
[7]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)
[8]R.Adler, M.Bazin and M.Schiffer, Introduction to General Relativity(McGraw-Hill,Inc.,1965)
[9]M.Schwarzschild, Structure and Evolution of the Stars(Princeton University Press,1958;reprint,Dover,N.Y.1965),chapter II
[10]S.Chandrasekhar, Mon,Not.Roy.Astron.Soc.95.207(1935)
[11]C.Rhoades, "Investigations in the Physics of Neutron Stars", doctoral dissertation, Princeton University
[12]J.Oppenheimer and H.Snyder, phys.Rev.56,455(1939)