A Derivation for the Gravitational Potential from the Heisenberg Uncertainty Principle and the Principle of Least Action

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γνοὺς πρᾶττε - act from knowledge

Abstract: In 2013 Mick McCulloch demonstrated a model for Gravity from the interactions of n, N quantum states acting under Heisenberg's Uncertainty Principle in his paper “Gravity from the uncertainty principle.”[1]. In contrast to McCulloch, this paper demonstrates a broader derivation of Newtonian gravity using the Heisenberg Uncertainty Principle and the Hamilton’s Principle of Least Action, generalizing and systematizes McCulloch’s model; it also seeks to show that gravitational time dilation and General Relativity as a whole might have their basis in Quantum Mechanics; and finally gives a program for the Inflationary model of the Big Bang, a mechanism for the pause of the Matter Dominated universe after the Big Bang, and the continuing Expansion of the Universe from considerations of the energy density of Universe.

§1.0 Theory

§1.1 Quantum Uncertainty as Action

The Lagrangian L with Lagrangian density $\mathcal{L}$

$$L = \iiint \mathcal{L}\, dx\, dy\, dz = \int \mathcal{L}\, d^3 x = \int \mathcal{L}\, dv \tag{1}$$

has the Action S

$$S = \int \mathcal{L}\, d^4 x = \iint L\, dt\, d^3 x \tag{2}$$

this is subject to the Hamiltonian principle of least action

$$\delta S = 0 \tag{3}$$

Now assume an uncertainty σ_s for the average action $\bar{s}$ at any point in spacetime,

$$S \rightarrow \bar{s} \pm \sigma_s \tag{4}$$

here the ± is also required to give the bounds of ΔS above and below the average $\bar{s}$, in other words σ_S is the distribution of the action around the average $\bar{s}$ at any point,

$$\bar{s} \pm \sigma_s = \int\int \bar{L}\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \pm \int\int \sigma_L\,\mathrm{dt}\,d^3\,\mathrm{x} \tag{5}$$

We can introduce Heisenberg's uncertainty relation for energy and time by substituting the uncertainty in energy ΔE for ΔL, and treat the Δt as an infinitesimal dt, we can do this by looking at smaller and smaller regions for time until the time variable becomes infinitesimal (in fact we are letting Δt tend to Planck time t_p, but since $t_p \approx 5.39 \times 10^{-44}$ (s) is trivially small we can ignore this for macroscopic systems)

$$\sigma_s = \int (\sigma_E\,\sigma_t)\,\mathrm{d}^3\,\mathrm{x} \equiv \int\int \sigma_E\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \tag{6}$$

that is to say, take the limit as the uncertainty in time goes to zero (or at the very least near to zero) and integrate

$$\sigma_s = \int\int \sigma_E\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \tag{7}$$

The Heisenberg uncertainty principle places a lower bound of Planck's constant on the uncertainty of the action

$$\sigma_s \geq \hbar/2 \tag{8}$$

In other words, the lowest action is bounded below by $\frac{\hbar}{2}$,

$$\sigma_s = \int\int \sigma_E\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \geq \hbar/2 \tag{9}$$

We can remove the 1/2 by assuming we are integrating for at least the interactions of two quantums of action, and integrating over all of spacetime

$$\sigma_s = \int\int \sigma_E\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \geq \int 2\,\frac{\hbar}{2}\,\mathrm{d}^3\,\mathrm{x} \geq \hbar \tag{10}$$

or

$$\bar{s} \pm \sigma_s \geq \int\int \bar{\mathrm{L}}\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \pm \int\int \sigma_E\,\mathrm{dt}\,\mathrm{d}^3\,\mathrm{x} \geq \hbar \tag{11}$$

Clearly this suggests that Action itself is bound below by Planck's constant, which makes perfect sense since the units of Planck's constant are the units of Action.

§1.2 Quantum Uncertainty under Hamilton's Principle of Least Action

By definition Hamilton's Principle of Least Action requires

$$\delta \mathrm{S} = 0 \tag{12}$$

on adding the uncertainty ΔS to the action S we find a deviation to the minimal action

$$\delta \bar{\mathrm{s}} \pm \sigma_s \geq \hbar \tag{13}$$

where the minimal action is bounded below by the <u>minimal</u> of the equality

$$\delta \bar{\mathrm{s}} + \sigma_s = \hbar \tag{14}$$

Therefore to satisfy both Hamilton's principle of least action and Heisenberg's uncertainty relation (we could call this combination Heisenberg's principle of least action) requires an additional action term such that

$$\delta \bar{\mathrm{s}} + \sigma_s - s' = 0 \tag{15}$$

such that

$$\delta \bar{\mathrm{s}} = 0 \tag{16}$$

or

$$\sigma_s - \mathrm{s}' = 0 \tag{17}$$

Without loss of generality write the action principle in terms of the time integral of the energy uncertainty

$$\int \sigma_E \, \mathrm{dt} = \hbar \tag{18}$$

or for the minimal system of two quantum in the ground state

$$\mathrm{s}' = \int\int \sigma_E \, \mathrm{dt}\, \mathrm{d}^3\,\mathrm{x} = \int \hbar \, \mathrm{d}^3\,\mathrm{x} = \hbar \tag{19}$$

It is now necessary to show that this additional term s' generates the gravitational field.

§1.3 McCulloch's[1] Model

Following McCulloch's[1] model let's substitute

$$\Delta \mathrm{t} = \frac{\Delta \mathrm{x}}{c} \text{ with } \sigma_t = \frac{\sigma_x}{c} \tag{20}$$

and rearrange, σ_x is the uncertainty in spatial distance between the two quantum states

$$\sigma_E \frac{\sigma_x}{c} = \hbar \tag{21}$$

again making the substitution for the infinitesimal of the spatial uncertainty

$$\frac{1}{c}\int^{\Delta x}\sigma_E \, dx = \hbar \tag{22}$$

This is where McCulloch introduces the definition of force F

$$E = \int^{\Delta x} F \, dx \tag{23}$$

$$\frac{1}{c}\int^{\Delta x}\int^{\Delta x} F \, dx \, dx = \hbar \tag{24}$$

integrate over the double integrals

$$\frac{1}{c} F x^2 = \hbar \tag{25}$$

This equation is for the mass of a two quantum states subject to the principle of least action, McCulloch's model assumes there are n and N planck masses producing a force F between them, (the following is from McCulloch's 2013 paper [1]),

"...and x is again the separation between m and M, r, so we get"

$$\frac{F \times x^2}{c} \sim \sum_{i=1}^{n} n \sum_{j=1}^{N} N \hbar \tag{26}$$

"The double summation on the right hand side is equal to the number of Planck masses in mass m (m/m_p) times the number in M (M/m_p), where m_p is the reduced Planck mass, so

$$\frac{F \times x^2}{c} \sim (\hbar \, m \, M) \big/ m_p^2 \; \ldots " \tag{27}$$

on rearranging

$$F = \frac{\hbar c}{m_p^2} \, \frac{mM}{x^2} \tag{28}$$

From here McCulloch was able to derive the Newtonian gravitational field equation

$$F = \frac{GmM}{x^2} \tag{29}$$

McCulloch's model can be generalized as follows...

§1.4 Generalizing the Model

We can arrive at the same result from the considering the action of the two different macroscopic Action systems S_1 and S_2, for instance these two action systems could be two planetary masses floating in space near each other, and without worrying about their distribution or makeup we are only interested in the amount of energy and where they are in time and space. Then noting the minimal action that is physically possible under H.U.P. is the Planck action S_p

$$S_p = \hbar \tag{30}$$

on taking the ratio of the action for N states to the Planck action is

$$\frac{S_1}{S_p} = \frac{S_i}{\hbar} = N \tag{31}$$

or

$$S_1 = N S_p = N \frac{\hbar}{2} \tag{32}$$

similarly S_j the ratio of the action for n states to the Planck action is

$$S_2 = n\, S_p = n \frac{\hbar}{2} \tag{33}$$

we can combine the interaction of these disparate systems with a single action S_{12}

$$S_{12} = n\, N\, \hbar \tag{34}$$

The Planck action is dependent on the Planck mass so we can introduce that here

$$E_p = M_p\, c^2 \tag{35}$$

$$S_p = E_p\, t_p \tag{36}$$

or

$$S_p = \int M_p\, c^2\, dt \tag{37}$$

Now consider the ratio of components of the action S_i to the Planck action

$$S_1 = E\, t \tag{38}$$

$$\frac{S_1}{S_p} = \frac{E\, t}{E_p\, t_p} = \left(Mc^2\, t\right) / \left(M_p\, c^2\, t_p\right) = Mt / \left(M_p\, t_p\right) \tag{39}$$

on choosing the minimal temporal distance in time as the Planck time, this is equivalent to taking the limit as $\delta t \to t_p$ under the assumption that the minimal bound for time t_p and essentially this is saying the time period for the large action is the same as for the Planck action

$$t = t_p \tag{40}$$

this reduces to

$$\frac{S_1}{S_p} = \frac{M\,t_p}{m_p\,t_p} = \frac{M}{m_p} = N \tag{41}$$

similarly for the smaller action

$$\frac{S_2}{S_p} = \frac{m\,t_p}{m_p\,t_p} = \frac{m}{m_p} = n \tag{42}$$

we can now substitute the mass ratios into S_{12}

$$S_{12} = n\,N\,\hbar = \frac{m\,M}{m_p^2}\,\hbar \tag{43}$$

and arrive at the same result as McCulloch, and even though this generalizes McCulloch method there is really little difference in the overall method, the only real difference is that this is an exact result while McCulloch’s is an approximation and McCulloch assumed the use of the Planck mass while here it’s use is a necessary requirement from the action principle.

§1.5 Gravitational Field

If we now apply this result to our previous formulation (19) of the four-vector integral and integrating over the volume of the Lagrangian density

$$S'_{12} = \int \frac{1}{c}\,F\,x^2\,d^3\,x = \int n\,N\,\hbar\,d^3\,x = \int \frac{m\,M}{m_p^2}\,\hbar\,d^3\,x \tag{44}$$

we can determine the force factor F under the integrals

$$\frac{1}{c}\,F\,x^2 = \frac{m\,M}{m_p^2}\,\hbar \tag{45}$$

rearranging

$$F = \frac{\hbar c}{m_p^2}\,\frac{m\,M}{x^2} \tag{46}$$

from the definition of the Planck mass this reduces to the Newtonian gravitational field equation

$$F = (G\,m\,M)\big/x^2 \tag{47}$$

and in so doing it is possible quantize spacetime under the Principle of Least Action and Heisenberg’s Principle of Uncertainty to derive the gravitational field.

The above result can then be rewritten as the gravitational potential g

$$g = -G \int \frac{M}{x} \, dx \tag{48}$$

and having shown the gravitational potential can be derived in this manner it is a matter of equation manipulation to derive the gravitational time dilation T_d,

$$T_d = \exp\left(\frac{1}{c^2} \int_{h_1}^{h_2} g \, dh \right) \tag{49}$$

where h is the height above ground, g is the gravitational field.

This can be trivially expressed as a Lorentz contraction, if T_d is the time dilation near a mass M, T_f is the time far from the mass for an object with escape velocity v_e, and Schwarzschild radius $2\,G\,M/c^2$

$$T_d = T_f \sqrt{\left(1 - (2\,G\,M)/\left(r\,c^2\right)\right)} = T_f \sqrt{\left(1 - \frac{v_e}{r}\right)} = T_f \sqrt{\left(1 - \frac{v^2}{c^2}\right)} < T_f \tag{50}$$

From the Lorentz contraction ultimately the theory of Relativity can be derived, and while Relativity is based on the constancy of the speed of light and the Equivalence principle, the model presented here approaches this solution from the opposite direction, the two methods achieve the same end result - in that time dilation is functionally dependent on gravity potential. It doesn't matter whether the potential is derived from Relativity or the quantum action - the outcome is the same. In a sense this presented model could be seen as a bottom-up approach to General Relativity, while the historical path of Relativity is a top-down approach, this is the key to the bottom-up approach, ignore the big picture just focus on the infinitesimal of the energy distribution at any point in spacetime.

§1.6 From Heisenberg Action to General Relativity

So what does all this mean? At the heart of this is the idea that spacetime itself should be quantized; it must be subject to the Heisenberg uncertainty principle; and above all it must obey Hamilton's principle of least action.

$$\sigma_s = \int\int \sigma_E \, dt \, d^3 x = \hbar \tag{51}$$

In this model Spacetime itself becomes a quantity that possesses energy just like electrons and photons, in other words spacetime is the quantum vacuum, and by this virtue must also have an intrinsic uncertainty of its energy and it is that uncertainty that causes the curvature of spacetime. It's not that a mass exerts a gravitational field a distance R, rather it is the energy distribution of the particles of that mass over all spacetime that determines the motion of energy in the form of light and matter at distance R from the mass, the gravitational field is generated by the variation in energy at that point in spacetime - uncertainty drives gravity.

We can generalize this theory to write the Hilbert action as a special case of the Heisenberg action. First note after Carroll [3 p161]

"Hilbert figured out that this (sic the Ricci scalar) was therefore the simplest possible choice for a Lagrangian $S_{\text{Hilbert}} = \int \sqrt{(-g_{\mu\nu})}\ \mathrm{R}\ \mathrm{d}^4\mathrm{x}$..." (52)

Now if we have the Hilbert action then we have the field equations

$$\mathrm{R}_{\mu\nu} - \frac{1}{2}\mathrm{R}\, g_{\mu\nu} = \frac{8\,\pi\,G}{c^4}\mathrm{T}_{\mu\nu} \tag{53}$$

To achieve this first write the metric tensor in terms of the Minkowski metric for flat spacetime, with the addition and for want of a better term the "Heisenberg metric" for the quantum uncertainty variation of spacetime becomes

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{54}$$

after transforming to appropriate coordinates the Minkowski metric is

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{55}$$

and the "Heisenberg metric" is where the σ_t is proportional to σ_E

$$h_{\mu\nu} = \begin{pmatrix} -\sigma_t & 0 & 0 & 0 \\ 0 & \sigma_x & 0 & 0 \\ 0 & 0 & \sigma_y & 0 \\ 0 & 0 & 0 & \sigma_z \end{pmatrix} \tag{56}$$

and for a volume devoid of energy this reduces to flat spacetime

$$h_{\mu\nu} = 0 \tag{57}$$

Here the uncertainties in time and space are given once more by the Heisenberg uncertainty relations, and the minimal equality imposed on the system by the Principle of least action this "Heisenberg Action", so it can be seen the "Heisenberg metric" imposes a deviation on Euclidean spacetime, while obviously for flat spacetime this reduces the metric tensor to the Minkowski tensor and the Hilbert action to the Euclidean action.

$$\mathrm{S} = \int \mathrm{R}\, \mathrm{d}^4\, \mathrm{x} \tag{58}$$

To illustrate how we can work bottom-up to the field equations, remember the ground state of the Hilbert action should be bound below by the Heisenberg action

$$S_{\text{Hilbert}} \geq S_{\text{Heisenberg}} \tag{59}$$

if we now rewrite the Heisenberg action to include the metric tensor for a macroscopic system we find

$$S_{\text{Hilbert}} \geq \int \sqrt{(-g_{\mu\nu})}\, \mathrm{R}\, \mathrm{d}^4 \mathrm{x} \geq \hbar \tag{60}$$

where again

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{61}$$

From the Hilbert action Einstein was able to derive the field equations, but the field equations are based on the Ricci curvature and the Scalar curvature, so consider how these are constructed:

Ricci curvature

$$\mathrm{R}_{\alpha\beta} = \partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\, \Gamma^\lambda_{\beta\alpha} + \Gamma^\rho_{\beta\lambda}\, \Gamma^\lambda_{\rho\alpha} \tag{62}$$

Scalar curvature

$$\mathrm{R} = \mathrm{g}^{\alpha\beta} \left(\partial_\gamma \Gamma^\gamma_{\alpha\beta} - \partial_\beta \Gamma^\gamma_{\alpha\gamma} + \Gamma^\gamma_{\alpha\beta}\, \Gamma^\gamma_{\gamma\delta} - \Gamma^\delta_{\alpha\gamma}\, \Gamma^\gamma_{\beta\delta}\right) \tag{63}$$

with Christoffel symbol

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\, \mathrm{g}^{\alpha\beta} \left[\partial \mathrm{g}_{\alpha\beta} / \partial \mathrm{x}^\gamma + \partial \mathrm{g}_{\alpha\gamma} / \partial \mathrm{x}^\beta - \partial \mathrm{g}_{\beta\alpha} / \partial \mathrm{x}^\alpha\right] \tag{64}$$

If we put these in the field equations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{65}$$

you can see that ultimately the left hand side of the field equations are driven by variations of the metric tensor, in fact the entire L.H.S. of the field equations are nothing but metric tensors and their derivatives, but the whole premise of this paper is that the uncertainty in energy at any point in Spacetime is what drives the variation of the metric tensor. Therefore we are left with no other option but to suggest the field equations themselves are based upon and driven by quantum uncertainty. In other words, if we can write the field equations from the Hilbert action, then we can we can write the field equations from the "Heisenberg action".

§1.7 "Dark Matter" Huh?

To explain the observed anomalies around galaxies and the general expansion of the universe, let's see if we can modify the gravitational field using this idea of quantum uncertainty, by considering how the vacuum energy density modifies the Einstein equations with the addition of a vacuum term to the energy on the R.H.S. [3 p172]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\,\pi\, G}{c^4} \left(T_{\mu\nu} - \rho_{\text{vac}}\, g_{\mu\nu}\right) \tag{66}$$

The real on-mass shell part of the universe is contained within the stress-energy tensor $T_{\mu\nu}$, and if this is zero or if we have a volume of space with no on-mass shell energy, this reduces to the off-mass shell particles of the vacuum,

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\frac{8\,\pi\, G}{c^4}\, \rho_{\text{vac}}\, g_{\mu\nu} \tag{67}$$

where the vacuum energy density in volume of space is simply

$$E_v = \int \rho_v\, d^3\, x \tag{68}$$

which under Heisenberg's uncertainty principle yields (note how the Lagrangian density formulation conveniently gives up the density)

$$\int E_v\, \text{dt} = \int\int \rho_v\, \text{dt}\, d^3\, x \tag{69}$$

You can see we can still use the model of quantum uncertainty to determine the gravitational field for the vacuum energy, if we can find a way to vary the density of the vacuum over spacetime, then the energy density and its concomitant energy density uncertainty will also vary.

So let's ignore the underlying mechanics of quantum uncertainty and gravity, and make a suggestion - the presence of matter energy density modifies the vacuum energy density because the presence of matter excludes the quantum vacuum by taking up the volume the vacuum could exist within, e.g. as the density of matter goes to a maximum inside a proton or electron then correspondingly the density of the vacuum falls to a minimum, or on a macroscopic scale consider a galaxy with clumps of matter scattered through it, then at each point in the galaxy the observed density of the quantum vacuum energy is determined by the distribution of the density of matter energy, quite simply matter excludes the vacuum (probably why it's called a vacuum).

We can formulate this by noting the vacuum density ρ_v is proportional to the matter energy density

$$\rho_v \propto -\frac{1}{\rho_m} \tag{70}$$

and the vacuum density ρ_v is proportional to the Planck density ρ_p

$$\rho_v \propto \rho_p \tag{71}$$

we can 'guess' an equation by combining these with constants of proportionality α, β

$$\rho_v = \alpha \rho_p - \beta \rho_p \frac{\rho_p}{\rho_m} = \rho_p \left(\alpha - \beta \frac{\rho_p}{\rho_m} \right) \tag{72}$$

you can see ρ_v is dependent on the square of the Planck density,and you'll note the Planck density is a huge number (but more on that later)

$$\rho_p = \frac{m_p}{L_p} = \frac{c^5}{\hbar G^2} = 5.155 \times 10^{96} \left[\mathrm{kg} / \mathrm{m}^3 \right] \tag{73}$$

1: If the energy density of the two terms is equal there should be no curvature and no gravity

$$\rho_v = \rho_p \left(\alpha - \beta \frac{\rho_p}{\rho_m} \right) = 0 \tag{74}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8 \pi G}{c^4} \left(T_{\mu\nu} - \rho_v g_{\mu\nu} \right) = 0 \tag{75}$$

2: If the matter density goes to infinity or is simply large enough to dominate the Planck density,

$$\rho_v = \alpha \rho_p - \beta \rho_p \frac{\rho_v}{\infty} = \alpha \rho_p - 0 = \alpha \rho_p \tag{76}$$

then the curvature due to the vacuum is minimal as the energy density of matter overwhelms the contribution due to the vacuum, and we can equate the vacuum energy density to the Planck density.

3: If the density of matter goes to zero as would be the case far away from center of a galaxy then

$$\rho_v = \alpha \rho_p - \beta \frac{\rho_m}{0} = \alpha \rho_p - \beta \infty < -\infty < 0 \tag{77}$$

which gives rises to a negative curvature or expansion of spacetime, this suggests in regions far away from the galactic center the reduced vacuum density increases to a maximum and correspondingly the curvature of spacetime increases, which does sound a lot like dark matter. So I’m going to suggest the curvature of space between the galaxies is not due to the presence of “dark matter” but rather the absence of real matter energy (sans matter). Strictly speaking because even in the empty space between the galaxies there is still light carrying energy (and in this case we are lumping the light energy with the matter energy) so the vacuum energy density never reaches the Planck density. This should be written as

$$\rho_v = \alpha \rho_p - \beta (\lim / (\rho_m \to 0)) \frac{\rho_p}{\rho_m} < 0 \tag{78}$$

The important part is the vacuum pressure p_{vac} is proportional to the vacuum energy density

$$p_{\text{vac}} = -\rho_v c^2 = -\alpha \rho_p c^2 + \beta (\lim / (\rho_m \to 0)) \frac{\rho_p}{\rho_m} c^2 < 0 \tag{79}$$

which as you can see should be negative iff the vacuum dominates matter in a volume of space, and therefore there should be an expansion of space between the galaxies - as is observed.

It should also be pointed out that in this model the density of the vacuum varies over all of Time with the changes in the density of matter, accordingly the cosmological constant Λ (read the vacuum energy density) must be a function of time and even explode in a universe without matter,

$$\Lambda = \frac{8\pi}{c^4} \rho_v = \frac{8\pi}{c^4} \rho_p \left(\alpha - \beta \frac{\rho_p}{\rho_m} \right) \to \infty \tag{80}$$

and immediately the suggestion must be made that in a universe with only the quantum vacuum (sans matter) as what may have existed before the Big Bang there must be a huge expansion in spacetime due to the infinite negative curvature - and that is so very clearly the hallmark the Inflationary era of the Big Bang.

Further note, that when energy density of matter equals the energy density of the vacuum

$$\rho_v = \rho_p \left(\alpha - \beta \frac{\rho_p}{\rho_m} \right) = 0 \tag{81}$$

the expansion pauses or slows down, and this pause has also been observed during the matter dominated era after the Big Bang.

Finally this may also explain the Vacuum Catastrophe where from quantum field theory the vacuum energy density blows up by 120 orders of magnitude, for what is of importance is not the Planck energy density it's the ratio of the ρ_p to the matter energy density,

$$\rho_v = \rho_p \left(\alpha - \beta \frac{\rho_p}{\rho_m} \right) \tag{82}$$

ρ_p appears as a baseline for the ρ_v and ρ_m to sit upon, and we can always scale the proportionality constants α, β to fudge a solution that matches observations, in other words we can use the Planck energy density to tamp down the Vacuum Catastrophe.

This remarkably simple model does cover a lot of the universe with remarkably few assumptions. It gives a mechanism for the huge Inflation of the Big Bang; it explains why the universe pauses in it's expansion during the matter dominated era after the Big Bang; it offers a way out from the Vacuum Catastrophe; it suggests a reason why the distance between the galaxies is continuing to expand and in so doing explains away the missing Dark matter. That's an awful lot for such a simple equation, however this model isn't perfect, after all I haven't given an explicit reason for why the vacuum energy density is proportional to the square of the Planck energy density,

$$\rho_v = \rho_p\left(\alpha - \beta \frac{\rho_p}{\rho_m}\right) \tag{83}$$

this equation is just a guess, and the constants α, β are just fudges, the constants could also be time dependent α(t), β(t), they could also just be arbitrary constants constructing an arbitrary equation for an arbitrary universe, which is akin to asking the question : "Why is the duck in the bath? Well, it just is."

§2 Discussion

§2.1 Gravitational Bending of Light Paths

To illustrate how this model can affect spacetime, notice that according to the uncertainty principle the energy of any subatomic particle is distributed over all spacetime, in turn this means the energy at any point in spacetime is determined by energy of all particles in the universe. It is the energy at that point that is important - not the distant masses that source that energy, and since the path of any test particle at that point is determined by the action principle at that point any deviation (random or otherwise) must be taken into account. For instance, if a photon is passing a point near the Sun, then quantum mechanically the energy of all the particles in the Sun exists at that point (by virtue of those particles being wave-functions distributed over all spacetime), and because of quantum uncertainty the vectors of each of those particles will be distributed about that point. However, the path the photon follows at that point must be determined by the Hamilton's Principle of Least action, but as discussed above there is a lower bound to Hamilton's Principle which very definitely means of all the paths the photon it must take a path nearest towards the Sun and deviate from an orthogonal path, and this is expressed as a gravitational field or time caused by a gravitational potential - in other words what causes the gravitational deflection of light about the Sun should be quantum uncertainty of the energy of all of the particles in the Sun.

§2.1 Quantum Unification

Apparent Weakness of Gravity: This model should explain the weakness of gravity in that the quantum uncertainty of a quantity like mass and energy is necessarily a fraction of the quantity itself, and on the assumption it is quantum uncertainty that bounds gravity it follows the energy of the gravitational field must be much smaller than the energy of the particle that generates it. Planck's constant is itself mind-boggling small, one of the smallest known quantities, and this gives as clear an indication that gravity should be a trivial force, so it's not surprising the gravitational field is the weakest of all the forces. In a real sense it also suggests the Weak Nuclear force and the gravitational force are two sides of the same duck, as the exchange particle of the Weak Nuclear field are generated by quantum uncertainty, the exchange particles of the weak force are generated by quantum uncertainty as they bounce back and for drawing the particles together, so in a very real sense this model, this quantization of spacetime unifies the four fundamental forces of Physics.

§2.2 Tests

Potentials are instantaneous over spacetime, and to quote David Griffiths - "*There is a very peculiar thing about the scalar potential in the Coulomb gauge: it is determined by the distribution of charge right now. If I move an electron in my laboratory, the potential V on the moon immediately records this change. That sounds particularly odd in the light of special relativity, which allows that no message to travel faster than c. The point is that V by itself is not a physically measurable quantity-all the man in the moon can measure is* $\mathbb{E}$ *...* $\mathbb{E}$ *will change only after sufficient time has elapsed for the news (read electromagnetic waves) to arrive."* [p441 2].

This means (assuming the electric potentials are the same that cause the energy distribution for gravity) gravitational potentials are instantaneous and suggests there might be a test for quantum gravity and the quantization of spacetime from quantum interference patterns conducted up a gravitational well, however, I can give no explicit test - only that this might be a test.

We could also take an evacuated chamber on Earth and vary the amount of matter to vacuum as light passes through the chamber and look for variations in the curvature of space. Admittedly this is a long shot and a better test would probably require a spacecraft moving out into deep space far from Earth and testing for deviations out beyond the Oort Cloud, but given the Oort Cloud extends to distances on the order of light years that is even more of a long shot and not achievable in our lifetimes.

We could also look for changes in the density of the quantum vacuum across the universe, this is an indirect test and it doesn't necessarily prove that the vacuum causes deviations to the metric only (and at best) that it corresponds to presence of spacetime curvature.

However I think the best possible test would be a negative outcome by looking for the absence of matter or “sans matter” between the galaxies, if Dark matter is ever found or any matter at all that could account for the curvature then things aren’t looking too good for this model.

§2.3 Planck

Planck’s mass versus Planck’s Constant: This model is different from McCulloch’s model in that he bases his derivation on the Planck mass as determining the curvature of spacetime, whereas here I'm suggesting it is Planck's constant that determines the action of spacetime and from this the curvature of spacetime is bound by Heisenberg's uncertainty principle, this may be a moot point as effectively they same are the same thing but it is worth mentioning.

§2.4 Equivalence principle

Einstein constructed his field equations partly from the premise of the Equivalence principle or the equivalence of gravitational and inertial mass, on the other hand implicit in the presented model is the idea that it is the uncertainty in the energy of the gravitational mass and that is the same as the uncertainty of energy of the inertial energy. The Universe doesn't care where the Energy comes from, just how it is distributed as a wavefunction at any moment in spacetime, for the energy of the wavefunction is distributed over all of space, it may not be localized at a particular point but it's field is everywhere and from the Universes' point of view that's all that's important. As far as the Universe is concerned gravitational and inertial energy are the same thing, and that is a restatement of the equivalence principle. Also I should point out (yet once more) this is a bottom-up solution to generating the Einstein field equations, as opposed to Einstein's top-down solution from the Equivalence principle, but I think this model is far more elegant than Einstein's, as it makes fewer assumptions and is rigorously based on conventional quantum mechanics.

§2.5 End Note

On a personal note, I've chosen to call the above the "Heisenberg Principle of Least Action", or the "Heisenberg Action" as opposed to the Planck's Lower Bounded Hamiltonian Principle of Least Action, and used "Heisenberg metric" for the spacetime metric variation of the metric tensor. Now, while yes, Heisenberg didn't come up with it, but I think it's a pretty obvious and convenient name to use, and on the premise we can't have enough things named after Heisenberg I think we should call it that. Come to think of it, this could also be called Planck's Principle of Least Action, or even the Hamilton-Hilbert-Planck Principle of Least Action, and on the principle that Heisenberg's uncertainty is at the basis of all physics we might as well call this the Heisenberg Universe and be done with it. Ah! The hell of nomenclature.

§3 References

[1] Mick McCulloch "Gravity from the uncertainty principle" 2013

[2] David Griffiths "Introduction to Quantum Mechanics" 1995

[3] Sean Carroll "Spacetime and Geometry" 2019